sciendo

TATRA MOUNTAINS
Mathematical Publications

DOI: 10.2478/tmmp-2020-0019
Tatra Mt. Math. Publ. **76** (2020), 71–80

ON THE STABILITY OF THE FUNCTIONAL EQUATION $f(2x+y)+f\left(\frac{x+y}{2}\right)=\frac{2f(x)f(y)}{f(x)+f(y)}+\frac{2f(x+y)f(y-x)}{3f(y-x)-f(x+y)}$

Sadani Idir

University of Mouloud Mammeri, Tizi-Ouzou, ALGERIA

ABSTRACT. The aim of this paper is to obtain the general solution to a reciprocal functional equation of the form

$$f(2x+y)+f\left(\frac{x+y}{2}\right)=\frac{2f(x)f(y)}{f(x)+f(y)}+\frac{2f(x+y)f(y-x)}{3f(y-x)-f(x+y)}$$

and to investigate its generalized Hyers-Ulam-Rassias stability.

1. Introduction

In recent years, a lot of attention has been devoted to the study of the stability of functional equations. It at least goes back Ulam's interesting question (1940, [32]): Given a group G_1, a metric group G_2 with a metric $d(.,.)$, and a positive number ϵ. Does there exist a real number $\delta > 0$ such that if the map $f : G_1 \to G_2$ satisfies $d\big(f(x,y), f(x)f(y)\big) < \delta$ for all $x, y \in G_1$, then a homomorphism $T : G_1 \to G_2$ exists with $d\big(f(x), T(x)\big) < \epsilon$ for all $x, y \in G_1$? The first affirmative answer to this question was given by Hyers [13] and the result of Hyers is stated as follows:

Theorem 1 (Hyers). *Let $f : X \to Y$ be a function between Banach spaces such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \delta$$

for all $x, y \in X$. Then,

$$A(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

exists for all $x \in X$ and $A : X \to Y$ is the unique additive mapping satisfying

$$\|f(x) - A(x)\| \leq \delta.$$

© 2020 Mathematical Institute, Slovak Academy of Sciences.
2010 Mathematics Subject Classification: 39B52, 39B72.
Keywords: Functional Equation, Hyers-Ulam-Rassias stability.
Licensed under the Creative Commons Attribution-NC-ND 4.0 International Public License.

In this case, the Cauchy additive functional equation $f(x+y) = f(x) + f(y)$ is said to satisfy the Hyers-Ulam stability. This highly cited result has led to a number of developments in the area. In 1949–1951, this result was extended by Aoki [1] and Bourgin [3, 4]. In 1978, Rassias [18] provided a generalized version of the theorem of Hyers which permitted the Cauchy difference to become unbounded. He proved the following theorem.

Theorem 2 (Rassias). *If a function $f : X \to Y$ between Banach spaces satisfies the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(\|x\|^p + \|y\|^p)$$

for some $\theta \geq 0, 0 \leq p < 1$ and for all $x, y \in X$, then there exists a unique additive function $A : X \to Y$ such that

$$\|f(x) - A(x)\| \leq \frac{2\theta}{2 - 2^p} \|x\|$$

for all $x \in X$. Moreover, if $f(tx)$ is continuous in t for each fixed $x \in X$, then A is linear.

Furthermore, throughout the year 1982 and 1989, Rassias [23], [24] obtained a generalization of the result of Hyers and showed a result employing weaker conditions controlled by a product of different powers of norms

$$(x, y) \to \|x\|^a \|y\|^a, \quad 0 \leq a < \frac{1}{2}.$$

Latterly, the problem of stability for different functional equations in different spaces have been considerably explored by many authors, (see, e.g., [5], [7], [8], [12], [16], [26], [25]).

In 2001, Gilányi [11] showed that if f satisfies the functional inequality

$$\|2f(x) + 2f(y) - f(x-y)\| \leq \|f(x+y)\|,$$

then f satisfies the Jordan-von Neumann functional equation

$$2f(x) + 2f(y) = f(x+y) + f(x-y).$$

In 2008, Ravi, Arunkumar and Rassias [27] investigated the general Euler--Lagrange type functional equation of the form

$$f(mx+y) + f(mx-y) = 2f(x+y) + 2f(x-y) + 2(m^2-2)f(x) - 2f(y)$$

for any arbitrary but fixed real constant m with $m \neq \{0, \pm 1, \pm\sqrt{2}\}$, controlled by the mixed type product-sum function

$$(x, y) \to \epsilon \left[\|x\|_E^p \|y\|_E^p + \left(\|x\|_E^{2p} + \|y\|_E^{2p} \right) \right].$$

Park and Bae [21] gave the general solution and the stability to the functional equation

$$f(x+y+z,u+v+w)+f(x+y-z,u+v+w)+2f(x,u-w)+2f(y,v-w)=$$
$$f(x+y,u+w)+f(x+y,v+w)+f(x+z,u+w)+f(x-z,u+v+w)+$$
$$f(y+z,v+w)+f(y-z,u+v-w).$$

The function $f(x,y)=x^3+ax+b-y^2$ having level curves as elliptic curves is a solution to the above functional equation which is helpful and interesting for studying cryptography and their applications. Then, functional equations and stability theory is an important subject in mathematics due to its many implications and applications. Some of its most important applications are in physics, biology, numerical analysis, economics, computer graphics, neural networks, digital image processing, finance, information theory, wireless sensor networks, and many other fields.

In 2010, Ravi and Kumar [28] obtained generalized Hyers-Ulam stability for the reciprocal functional equation

$$f(x+y)=\frac{f(x)f(y)}{f(x)+f(y)},\ x,\ y\in\mathbb{R}-\{0\}. \tag{1}$$

Not long ago, Ravi, Rassias and Kumar [29] obtained the Generalized Hyers-Ulam stability, the extended Ulam stability of the generalized reciprocal functional equation

$$f\left(\sum_{i=1}^{m}\alpha_i x_i\right)=\frac{\prod_{i=1}^{m}f(x_i)}{\sum_{i=1}^{m}\left[\alpha_i\left(\prod_{j=1,j\neq i}^{m}f(x_i)\right)\right]} \tag{2}$$

such that $(\alpha_1,\ \alpha_2,\ldots,\ \alpha_m)\neq(0,0,\ldots,0)$ are fixed real numbers, with $0<\alpha=\alpha_1+\alpha_2+\cdots+\alpha_m=\sum_{i=1}^{m}\alpha_i\neq 1$ and $f:X\to Y$ with X and Y are the sets of non-zero real numbers. More recently, the same authors [30, 31] obtained the generalized Hyers-Ulam stability of the system of bi-reciprocal functional equations

$$r(x+uy)=\frac{r(x,y)r(u,y)}{r(x,y)+r(u,y)},$$

$$r(x,y+v)=\frac{r(x,y)r(x,v)}{r(x,y)+r(x,v)}$$

in Fréchet spaces, and

$$r(x+u,y+v)=\frac{r(x,y)r(u,v)}{r(x,y)+r(x,v)+r(u,y)+r(u,v)}$$

in quasi-β-normed spaces.

Jung, Popa and Rassias [15] proved the Hyers-Ulam stability of the homogeneous linear functional equation

$$f\big(\phi(x)\big) = g(x)f(x) + h(x),$$

where f is the unknown function and g, h, ϕ are given functions which defined from an arbitrary nonempty set S into a complete metric group $(G, ., d)$.

In 2019, Park and Rassias [22] solved and investigated the Hyers-Ulam stability in complex Banach spaces of the two following additive functional equations

$$f(x+y+z) - f(x+y) - f(z) = s\big(f(x+y-z) + f(x-y+z) - 2f(x)\big),$$

and

$$f(x+y-z) + f(x-y+z) - 2f(x) = s\big(f(x+y+z) - f(x+y) - f(z)\big),$$

where s is a fixed nonzero complex number.

More results concerning stability theory of various forms of functional equations can be found in [14], [17], [2], [6], [9], [10].

In this paper, we focus on studying a particular reciprocal type functional equation and we investigate its generalized Hyers-Ulam-Rassias stability.

2. The general solution

Throughout this paper, X will denote the set $\mathbb{R} - \{-c\}$.

Theorem 3. *Let $f : X \to \mathbb{R}$ be a continuous real-valued function of a real variable satisfying the reciprocal type functional equation*

$$f(2x+y) + f\left(\frac{x+y}{2}\right) = \frac{2f(x)f(y)}{f(x)+f(y)} + \frac{2f(x+y)f(y-x)}{3f(y-x) - f(x+y)} \tag{3}$$

for all $x, y \in \mathbb{R}$. If $f(x) \neq 0$ for all $x \in X$, then f is of the form

$$f(x) = \frac{a}{x+c}$$

for all $x \in X$, where $c \neq 0$ is a constant.

P r o o f. Suppose a function $f : X \to \mathbb{R}$ satisfies (3) with $f(x) \neq 0$ for all $x \in X$. Replacing (x, y) by (x, x) in (3) we obtain

$$f(3x) = \frac{2f(2x)f(0)}{3f(0) - f(2x)} \tag{4}$$

which implies that

$$\frac{3}{2f(2x)} - \frac{1}{2f(0)} = \frac{1}{f(3x)}. \tag{5}$$

Replacing (x, y) by $(0, 2x)$ in (3), we get

$$\frac{1}{f(x)} = \frac{1}{2f(2x)} + \frac{1}{2f(0)} \tag{6}$$

or

$$\frac{1}{f(2x)} = \frac{2}{f(x)} - \frac{1}{f(0)} \tag{7}$$

and using (7) with equation (5), we have

$$\frac{1}{f(3x)} = \frac{3}{f(x)} - \frac{2}{f(0)}. \tag{8}$$

Using induction on a positive integer n, we get

$$\frac{n}{f(x)} - \frac{n-1}{f(0)} = \frac{1}{f(nx)}. \tag{9}$$

Put $s = f(0)$. When $x = 1$, the equation (9) becomes

$$\frac{n}{t} - \frac{n-1}{s} = \frac{1}{f(n)} \tag{10}$$

for some constant $t = f(1) \neq 0$. Then,

$$f(n) = \frac{st}{(s-t)n + t}. \tag{11}$$

We can put $a = \frac{st}{s-t}$ and $c = \frac{t}{s-t}$ such that $s \neq t$ and $a, c \neq 0$, and we get

$$f(n) = \frac{a}{n+c}. \tag{12}$$

Let x be any fixed element in $\mathbb{R} - \{-c\}$, then there is a sequence (x_n) of non-zero rational numbers such that $\lim_{n\to\infty} x_n = x$. Then, by the continuity of f, we have

$$f(x) = \lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} \frac{a}{x_n + c} = \frac{a}{x+c}.$$

Hence, we conclude that $f(x) = \frac{a}{x+c}$ for all $x \in X$, which completes the proof of Theorem. □

3. Stability of equation (3)

Theorem 4. *Let $\phi : X \times X \to \mathbb{R}$ be a mapping satisfying*

$$\Phi(x, y) = \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{i+1} \phi\left(\frac{3^i x}{2^{i+1}}, \frac{3^i x}{2^{i+1}}\right) < \infty \tag{13}$$

for all $x, y \in X$. Let $f : X \to \mathbb{R}$ be a mapping such that

$$\left| \frac{1}{f(2x+y)} - \frac{1}{\frac{2f(x)f(y)}{f(x)+f(y)} + \frac{2f(x+y)f(y-x)}{3f(y-x)-f(x+y)} - f\left(\frac{x+y}{2}\right)} \right| \leq \phi(x, y) \tag{14}$$

for all $x, y \in X$.

Then, there exists a unique mapping $g : X \to \mathbb{R}$ which satisfies the functional equation (3) *and the inequality*

$$\left|\frac{1}{g(x)} - \frac{1}{f(x)} + \frac{1}{f(0)}\right| \leq \Phi(x,x) \tag{15}$$

for all $x \in X$. The mapping g is defined by

$$g^{-1}(x) = \frac{1}{g(x)} = \lim_{n\to\infty} \frac{2^n}{3^n f(\frac{3^n x}{2^n})}. \tag{16}$$

P r o o f. Replacing (x,y) by $\left(\frac{x}{2},\frac{x}{2}\right)$ in (14) and dividing by 3/2, we get

$$\left|\frac{2}{3f(\frac{3x}{2})} - \frac{1}{f(x)} + \frac{1}{3f(0)}\right| \leq \frac{2}{3}\phi\left(\frac{x}{2},\frac{x}{2}\right) \tag{17}$$

for all $x \in X$. Replacing x by $\frac{3x}{2}$ in (17), multiplying by 2/3, and summing the resulting inequality with (17), we have

$$\left|\frac{4}{9f(\frac{9x}{4})} - \frac{1}{f(x)} + \frac{1}{3f(0)} + \frac{2}{9f(0)}\right| \leq \frac{2}{3}\phi\left(\frac{x}{2},\frac{x}{2}\right) + \frac{4}{9}\phi\left(\frac{3x}{4},\frac{3x}{4}\right). \tag{18}$$

Proceeding further and using induction on a positive integer n, we obtain

$$\left|\frac{2^n}{3^n f(\frac{3^n x}{2^n})} - \frac{1}{f(x)} + \frac{1}{3f(0)}\sum_{i=0}^{n-1}\left(\frac{2}{3}\right)^i\right| \leq \sum_{i=0}^{n-1}\left(\frac{2}{3}\right)^{i+1}\phi\left(\frac{3^i x}{2^{i+1}},\frac{3^i x}{2^{i+1}}\right). \tag{19}$$

Replacing x by $\left(\frac{3}{2}\right)^k x, k \in \mathbb{N}$ in (19) and multiplying by $\left(\frac{3}{2}\right)^{-k}$, we have

$$\left|\frac{2^{n+k}}{3^{n+k}f(\frac{3^{n+k}x}{2^{n+k}})} - \frac{1}{f((\frac{3}{2})^k x)} + \frac{1}{3f(0)}\sum_{i=0}^{n-1}\left(\frac{2}{3}\right)^i\right| \leq \sum_{i=0}^{n-1}\left(\frac{2}{3}\right)^{i+1}\phi\left(\frac{3^{i+k}x}{2^{i+k+1}},\frac{3^{i+k}x}{2^{i+k+1}}\right). \tag{20}$$

Since the right-hand side of the inequality (20) tends to 0 as k tends to infinity, the sequence $\left\{\frac{2^n}{3^n f(\frac{3^n x}{2^n})}\right\}$ is a Cauchy sequence. Therefore, we may define

$$g^{-1}(x) = \frac{1}{g(x)} = \lim_{n\to\infty} \frac{2^n}{3^n f(\frac{3^n x}{2^n})} \quad \text{for all } x \in X.$$

Letting $n \to \infty$ in (19), we arrive at (15). Next, we have to show that g satisfies (3). Replacing x, y by $\left(\frac{3}{2}\right)^k x, \left(\frac{3}{2}\right)^k y$ in (14) and dividing by $\left(\frac{3}{2}\right)^{-k}$, then it follows that

$$\begin{aligned}&\left(\frac{2}{3}\right)^k \left| \frac{1}{f((\frac{3}{2})^k(2x+y))} \right. \\ &\left. - \frac{1}{\frac{2f((\frac{3}{2})^k x)f((\frac{3}{2})^k y)}{f((\frac{3}{2})^k x)+f((\frac{3}{2})^k y)} + \frac{2f((\frac{3}{2})^k(x+y))f\left((\frac{3}{2})^k(y-x)\right)}{3f((\frac{3}{2})^k(y-x))-f((\frac{3}{2})^k(x+y))} - f\left(\frac{(\frac{3}{2})^k(x+y)}{2}\right)} \right| \\ &\leq \left(\frac{2}{3}\right)^k \phi\left(\left(\frac{3}{2}\right)^k x, \left(\frac{3}{2}\right)^k y\right).\end{aligned} \tag{21}$$

Taking the limit as $n \to \infty$, using (16) and (13), we see that

$$\left| \frac{1}{g(2x+y)} - \frac{1}{\frac{2g(x)g(y)}{g(x)+g(y)} + \frac{2g(x+y)g(y-x)}{3g(y-x)-g(x+y)} - g\left(\frac{x+y}{2}\right)} \right| \leq 0 \tag{22}$$

which gives

$$\frac{1}{g\left(2x+y\right)} - \frac{1}{\frac{2g(x)g(y)}{g(x)+g(y)} + \frac{2g(x+y)g(y-x)}{3g(y-x)-g(x+y)} - g\left(\frac{x+y}{2}\right)} = 0. \tag{23}$$

Therefore, we have that g satisfies (3) for all $x, y \in X$. To prove the uniqueness of the mapping g, let us assume that there exists a mapping $g' : X \to \mathbb{R}$ which satisfies (3) and the inequality (15). We have

$$\begin{aligned}\left|\frac{1}{g(x)} - \frac{1}{g'(x)}\right| &= \left(\frac{3}{2}\right)^{-n} \left|\frac{1}{g((\frac{3}{2})^n x)} - \frac{1}{g'((\frac{3}{2})^n x)}\right| \\ &\leq \left(\frac{3}{2}\right)^{-n} \left(\left|\frac{1}{g((\frac{3}{2})^n x)} - \frac{1}{f((\frac{3}{2})^n x)}\right| \right. \qquad (24) \\ &\qquad \left. + \left|\frac{1}{f((\frac{3}{2})^n x)} - \frac{1}{g'((\frac{3}{2})^n x)}\right| \right) \\ &\leq 2\sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{i+n+1} \phi\left(\frac{3^i x}{2^{i+1}}, \frac{3^i x}{2^{i+1}}\right) \qquad (25)\end{aligned}$$

for all $x \in X$. Allowing $n \to \infty$ in (24) and using (13), we find that g is unique. This completes the proof of Theorem. □

Corollary 1. *Let $f : X \to \mathbb{R}$ be a mapping such that*

$$\left| \frac{1}{f(2x+y)} - \frac{1}{\frac{2f(x)f(y)}{f(x)+f(y)} + \frac{2f(x+y)f(y-x)}{3f(y-x)-f(x+y)} - f\left(\frac{x+y}{2}\right)} \right| \leq \theta(|x|^p + |y|^p) \quad (26)$$

for all $x, y \in X$, where $\theta \geq 0$ and $p \leq 0$.

Then, there exists a unique mapping $g : X \to \mathbb{R}$ which satisfies the functional equation (3) *and the inequality*

$$\left| \frac{1}{g(x)} - \frac{1}{f(x)} + \frac{1}{f(0)} \right| \leq \frac{2\theta|x|^p}{3(2^{p-1} - 3^{p-1})}.$$

Proof. Considering $\phi(x, y) = \theta(|x|^p + |y|^p)$ for all $x, y \in X$ in Theorem 4, we arrive at the required result. □

Acknowledgement. We would like to acknowledge the editor and the referee for their kind suggestions that helped to improve this paper.

REFERENCES

[1] AOKI. T.: *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan **2** (1950), 64–66.

[2] BOUIKHALENE, B.—ELQUORACHI, E.: *Ulam-Găvruta-Rassias stability of the Pexider functional equation*, Int. J. Appl. Math. Stat. **7** (2007), no. Fe 07, 27–39.

[3] BOURGIN, D. G.: *Class of transformations and bordering transformations*, Bull. Amer. Math. Soc. **57** (1951), 223–237.

[4] BOURGIN, D. G.: *Multiplicative transformations*, Proc. Nat. Academy Sci. U.S.A. **36**(1950), 564–570.

[5] CHANG, I. S.—KIM, H. M.: *On the Hyers-Ulam stability of quadratic functional equations*, J. Ineq. Appl. Math. **33** (2002), 1–12.

[6] CHANG, I. S.—JUNG, Y. S.: *Stability of functional equations deriving from cubic and quadratic functions*, J. Math. Anal. Appl. **283** (2003), 491–500.

[7] CHOLEWA, P. W.: *Remarks on the stability of functional equations*, Aequationes Math. **27** (1984), 76–86.

[8] CZERWIK, S.: *Functional Equations and Inequalities in Several Variables*. World Scientific Publishing Company, New Jersey, London, Singapore and Hong Kong, 2002.

[9] ESHAGHI GORDJI, M.—ZOLFAGHARI, S.—RASSIAS, J. M.—SAVADKOUHI,M. B.: *Solution and stability of amixed type cubic and quartic functional equation in quasi--Banach spaces*, Abst. Appl. Anal. **1** (2009), Art. ID 417473, 1–14.

[10] GHOBADIPOUR, N.—PARK, C.: *Cubic-quartic functional equations in fuzzy normed spaces*, Int. J. Nonlinear Anal. Appl. **1** (2010), 12–21.

[11] GILÁNYI, A.: *Eine zur Parallelogrammgleichung äquivalente Ungleichung*, Aequationes Math. **62** (2001), 303–309.

[12] HYERS, D. H.—ISAC, G.—RASSIAS, TH. M.: *Stability of Functional Equations in Several Variables*. Birkhauser, Basel, 1998.

[13] HYERS, D. H.: *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A. **27**(1941), no. 4, 222–224.

[14] JUNG, S.-M.: *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*. In: *Springer Optimization and Its Applications Vol. 48*. Springer, New York, 2011.

[15] JUNG, S.-M.—POPA, D.—RASSIAS, M. TH.: *On the stability of the linear functional equation in a single variable on complete metric groups*, J. Global Optim. **59** (2014), no. 1 165–171.

[16] KANG, D.: *On the stability of generalized quartic mappings in quasi-β-normed spaces*, J. Inequal. Appl. (2010), Art. ID 198098, 11 pp.

[17] KANNAPPAN, PL.: *Functional Equations and Inequalities with Applications*. In: *Springer Monographs in Mathematics*. Springer, New York, NY, 2009.

[18] RASSIAS, TH. M.: *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.

[19] PARK, C.—KWON, S. M.,—LEE, J. R.: *Hyers-ulam stability of additive function equations in paranormed spaces*, J. Comput. Anal. Appl. **26** (2019), no. 3, 532–538.

[20] PARK, C.—BODAGHI, A.: *Two multi-cubic functional equations and some results on the stability in modular spaces*, J. Inequal. Appl. **2020**, Art. no. 6.
https://doi.org/10.1186/s13660-019-2274-5

[21] PARK, W. G.—BAE, J. H.: *A functional equation originating from elliptic curves*, Abst. Appl. Anal. **2008**, Art. ID. 135237, 10 pp.

[22] PARK, C.—RASSIAS, M. TH.: *Additive functional equations and partial multipliers in C*-algebras*, R. Acad. Cienc. Exactas, Fís., Nat. Ser. A. Mat., Revista de la Real Academia de Ciencias Exactas, Serie A. Matemáticas, RACSAM **113** (2019), no. 3, 2261–2275.

[23] RASSIAS, J. M.: *On approximately of approximately linear mappings by linear mappings*, J. Funct. Anal. USA. **46** (1982), 126–130.

[24] RASSIAS, J. M.: *Solution of problem of Ulam*, J.Approx. Theory. USA. **57**(1989), no. 3, 268–273.

[25] RASSIAS, J. M.—ARUNKUMAR, M.—RAMAMOORTHI, S.—HEMALATHA, S.: *Ulam-Hyers stability of a 2-variable AC-mixed type functional equation in quasi-β--normed spcaes:direct and fixed point methods*. Malaya J. Math. **2** (2014), no. 2, 108–128.

[26] RAVI, K.—RASSIAS, J. M.—SENTHIL KUMAR, B. V.: *Ulam stability of reciprocal difference and adjoint funtional equations*, Aust. J. Math. Anal. Appl. **8** (2011), no. 1, Art. ID 13, 18 pp. (Erratum ibid. 9, (2012), no. 1, Art. ID. 23, 2 pp.)

[27] RAVI, K.—ARUNKUMAR, M.—RASSIAS, J. M.: *Ulam stability for the orthogonally general Euler-Lagrange type functional equation*, Int. J. Math. Stat. **3** (2008), no. A 08, 36–46.

[28] RAVI, K.,—SENTHIL KUMAR, B. V.: *Ulam-Gavruta-Rassias stability of Rassias reciprocal functional equation*, Glob. J. Appl. Math. Math. Sci.,Vol. **3** (2010), no. 1–2, 57–79.

[29] RAVI, K.—RASSIAS, J. M.—SENTHIL KUMAR, B. V.: *Ulam stability of Generalized Reciprocal Funtional Equation in several variables*, Int. J. Appl. Math. Stat. **19** (2010), 1–19.

[30] RAVI, K.—RASSIAS, J. M.—SENTHIL KUMAR, B. V.: *Ulam Stability of a generalized reciprocal type functional equation in non-Archimedean fields*. Arabian J. Math. **4** (2015), 117–126.

[31] SENTHIL KUMAR, B. V.—RASSIAS, J. M.—RAVI, K.: *Ulam stability of a bi-reciprocal functional equation in quasi-β-normed spaces*, Novi Sad J. Math. **46** (2016), no. 2, 1–11.

[32] ULAM, S. M.: *A Collection of Mathematical Problems*. Interscience, New York, 1960.

Received July 11, 2019

Sadani Idir
Department of mathematics
University of Mouloud Mammeri
15000 Tizi-Ouzou
ALGERIA
E-mail: sadani.idir@yahoo.fr