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Fatigue Life Prediction for Reinforced-Concrete Low Domes

Nacer Logzit [1,2*], Rabie Mezouar [1,4], Salah Daoud [1,3]

University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj, Algeria
[1] Materials and Electronic Systems Laboratory, [2] Department of Civil Engineering, Faculty of Science and Technology, [3] Department of Mechanical Engineering, Faculty of Science and Technology
[4] Institue of Optics and Fine Mechanics, University of Ferhat Abbas Setif 1, 19000 Setif, Algeria
*e-mail: n_logzit@yahoo.fr

Abstract

The study of fatigue of thin reinforced-concrete shells presents a complex modelling approach. This work is devoted to the numerical evaluation of the fatigue life of reinforced-concrete domes, with the concrete class of *C25/30*. For the formulation of a fatigue criterion under a symmetrically alternating load, with a very high number of cycles (10^6), the approach begins with the study of mechanical behavior under monotonic loading, related to the fatigue response. The numerical correlation results of the studied cases, based on a conservative fatigue limit of concrete in compression, enables to estimate the service life of these structures, knowing only the response under monotonic loading. The correlation formula proposed in this article can therefore directly estimate the service life of reinforced concrete domes, based on the maximum stress resulting from monotonic loading.

Keywords: reinforced-concrete domes, long span, monotonic loading, mechanical behavior, fatigue

1 Introduction

Three-dimensional thin-walled reinforced-concrete structures are generally used in civil and industrial constructions. However, they have the common characteristic of being among the most delicate structures to study. Through these structures, the performance of materials and shapes is sought [1]. But for the civil engineer, in the field of shells and slender structures, an essential requirement arises: to know how to choose optimal stable and resistant shapes under different types of loading. This requirement is too often neglected, or left only to the architect, whereas, in this type of structure, the analysis is linked with construction and optimization.

The objective of this work is to propose an approach to deal with complex problems of calculation and simulation of fatigue of slender reinforced-concrete domes. We used the mathematical formulation of the fatigue problem of reinforced-concrete applied to domes, to finish with the modeling and the exploitation of the results of numerical simulation. This approach allowed us to introduce the notion of fatigue criteria, and to know the behavior of the structure first under monotonic loading, followed by the analysis under cyclic loads with a very large number of cycles (i.e., 10^6).

Through this approach, a model based on a probabilistic numerical correlation will be proposed in order to predict the service life of these structures under cyclic loading.

In the case of occasional cyclic loading, where the full design load is repeated, for a very large number of cycles (10^6), the concrete undergoes stress concentration, exhibits excessive cracking, and may eventually lead to failure after a sufficient number of load repetitions, even if the maximum stress is less than the static (monotonic) strength of a similar specimen. This is referred to as the fatigue limit, which is, in this article, defined as the highest stress magnitude for which failure is not observed after 10^6 load cycles.

The subject of fatigue of reinforced concrete is generally treated experimentally or analytically through the study of reduced elements (column, beam, node...) [2-6], the analysis of full-scale structures, are the subjects least addressed by the scientific community; the fatigue of large-span domes is rarely addressed under realistic conditions.

The service life of a reinforced concrete structure depends directly on the stress level, stress range, and number of loading cycles [4]. Several models have been proposed for the evaluation of the service life of reinforced concrete under cyclic loading, the commonly used models are those based on the stress level for service life estimation [7].

This paper is aimed to find a numerical correlation of the mechanical behavior of reinforced concrete domes, between the monotonic loading response and the fatigue response.

2 Approaches and Models of Analysis

To know and control the mechanical behavior in monotonic loading mode, and in a cyclic mode of reinforced concrete domes, the following approaches have been adopted:

- Analysis of material linearity of the materials (concrete and steel).
- Analysis of geometrical non-linearity of the structure.

Shrinkage and creep are not considered here, for more details on these phenomena, it is preferable to refer to the research work developed by Ehab et al. [8]. Regarding the fatigue analysis of the structure under cyclic loading, we have adopted the criteria of the critical plane as these criteria are more suitable for brittle materials. We used the following three fatigue criteria: Mataké criterion [9], Findley criterion [10], and Dang Van criterion [11]. For a detailed comparative study of the fatigue criteria, a reliability analysis is proposed by Bianzeube et al. [12]. Mataké's criterion [9], is based on experimental data from bi-axial torsion-bending tests performed on three different materials (including copper and steel). It describes the critical plane in which the amplitude of the shear stress is maximum, and that the normal to the criticized plane is determined by max (τ_a (φ_c, Θ_c)). The criterion is written as follows (Equation 1):

$$\tau_a + \mu . \sigma_{n\,max} < \lambda \qquad (1)$$

of which τ_a is the shear stress, μ and λ are the material parameters, determined by two fatigue tests:

a) Pure torsion:

$$\tau_a = \tau_{-1} \quad \text{and} \quad \sigma_{n\,max} = 0 \qquad \text{then:}$$

$$\lambda = \tau_{-1} \tag{2}$$

b) Symmetrical alternating traction:

$$\tau_a = \frac{\sigma_{-1}}{2} \quad \text{and} \quad \sigma_{n\,max} = \frac{\sigma_{-1}}{2} \quad \text{then:}$$

$$\mu = 2.\frac{\tau_{-1}}{\sigma_{-1}} - 1 \tag{3}$$

with:

$\boldsymbol{\varphi}$ and θ: are the angles that determine the position of the material plane normal.

τ_{-1} and σ_{-1}: are the endurance limit in symmetric alternating torsion, and endurance limit in symmetric alternating tension, respectively.

The area of validity is:

$$\frac{\boldsymbol{\tau}_{-1}}{\boldsymbol{\sigma}_{-1}} > \frac{1}{2} \tag{4}$$

Findley's criterion [3], uses a linear combination of shear stress τ_a and normal stress σ_n, it is given by the following formula (Equation 5):

$$\tau_a(\varphi_c, \theta_c) + k.\sigma_{n\,max}(\varphi_c, \theta_c) < f \tag{5}$$

with *k*: sensitivity coefficient of the normal stress, and *f*: limit factor. The values of these quantities are given as follows (Equation 6a and 6b):

$$k = \frac{2 - \frac{\sigma_{-1}}{\tau_{-1}}}{2.\sqrt{\frac{\sigma_{-1}}{\tau_{-1}} - 1}} \tag{6a}$$

$$f = \sqrt{\frac{\sigma_{-1}^2}{4.\left(\frac{\sigma_{-1}}{\tau_{-1}} - 1\right)}} \tag{6b}$$

Dang Van [11], developed a microscopic-macroscopic approach based on a change of scale, to calculate the stress in the grains of a material subjected to fatigue, in relation to the stress known at the macroscopic scale. Based on this concept, the author proposed his criterion (Equation 7):

$$max_n max_t\{\|\alpha.\rho(t)\|\} < \beta \tag{7}$$

where ρ(t) is the hydrostatic pressure.

$$\text{with } \beta = \tau_{-1} \text{ and}$$
$$\alpha = 3.\left[\frac{\tau_{-1}}{\sigma_{-1}} - \frac{1}{2}\right] \quad (8)$$

where α and β are two constants defined from the endurance limits τ_{-1} (alternate torsion) and σ_{-1} (alternate tension) of the material. The condition for using this criterion is shown in (Equation 9):

$$\boldsymbol{\alpha > 0,} \quad \text{and}$$
$$\frac{\boldsymbol{\tau_{-1}}}{\boldsymbol{\delta_{-1}}} > \frac{1}{2} \quad (9)$$

Fatigue is estimated through a coefficient of use noted *E*, when the endurance limit of reinforced concrete at 10^6 cycles is reached [13], we will have $E = 1$ for any multiaxial cycle under the assumption that the criterion accurately reflects the real fatigue behavior of the material. The interpretation of the fatigue function of a criterion, i.e., its practical use in fact, is as follows:

- if $E < 1$, the criterion is non-conservative for the designer: it predicts stress levels at endurance greater than those obtained experimentally.
- if $E > 1$, the criterion is safe for the designer. It is conservative with respect to the real behavior of the material in fatigue. The multiaxial cycle at endurance is perceived by the criterion as more severe than it is in reality.

A complete multiaxial fatigue study, applied on a different and innovative structural system, is presented in a recent work of Logzit and Kebiche [14]. Regarding the study under monotonic loading, and before subjecting the structure to cyclic loading, an optimization of the structural system is required, based on the verification of the strength and stability of the dome at the service limit state; the same optimization approach, applied on a different structural system is proposed in [15].

3 Parameters and Basic Data

The class of concrete used is *C25/30*, with a compressive strength measured on a *16/32* cm cylindrical specimen of *25 MPa*. The steel reinforcement is class *FeE400*, with a tensile strength of *400* MPa, longitudinal modulus of elasticity of *200* GPa, and an elastic elongation of 0.002%.

The design of the various dome objects for this work is based on the following principles :

- The span of the domes (diameter *D*) is variable between 10 and 100 m.
- Low-profile form, the deflection at the center (*f*) is close to ($f/D = 0.10$).
- The thickness of the domes is kept constant, it is optimized in monotonic loading by checking the stability and the resistance of the system.
- The circular peripheral edge of the dome is considered as an embedding.

- The reinforcement of the meridian bars is obtained by considering a minimum regulatory section according to [16]. For the influence of the reinforcement rate, three variable sections were chosen:
  - S1: HA 10, 30 cm spacing, i.e., a reinforcement rate of 0.00263.
  - S2: HA 12, 30 cm spacing, i.e., a reinforcement rate of 0.00377.
  - S3: HA 14, spacing 30 cm, that is to say, a rate of reinforcement of 0,00523.
- Based on the principle of uniaxial fatigue in the direction of the meridians, the effect of the reinforcement in hoops is neglected.

In addition to the load due to the self-weight of the dome: *G*, the load applied to the dome is of the uniformly distributed type, $P = 100$ kg / m², largely sufficient for non-accessible covers. For the cyclic loading signal, the *P*-load model was used, with symmetrical fluctuations, with constant amplitude (Figure 1).

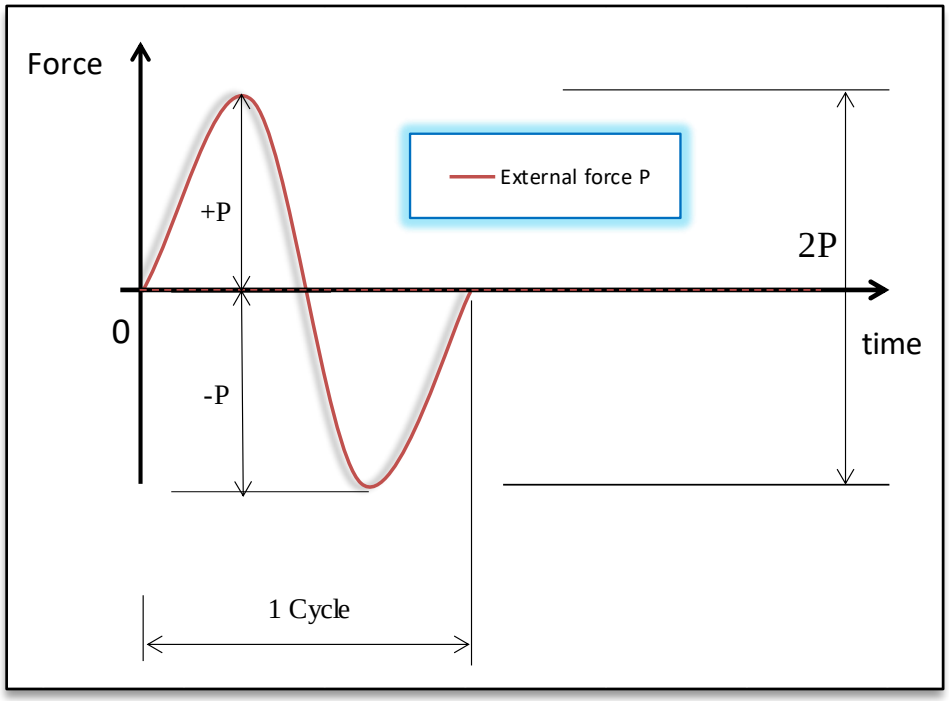


Figure 1: Cyclic loading signal

Concerning the parameters of the fatigue analysis, we considered a fatigue limit of the concrete at a very large number of cycles, close to 0.6 of the static limit of compression of the concrete. This limit is generally admitted by the codes of calculations and regulations, it results from the fatigue limits concerning the traction and the torsion as follows (Table 1).

Table 1: Fatigue limits of concrete

Material	Fatigue tests	Endurance limits	Unit	Value
	Symmetrical alternating traction: $R = -1$	σ_{-1}	MPa	1.50

Concrete class *C25/30*	Symmetrical alternating torsion: $R = -1$	τ_{-1}	MPa	1.00

Through these values, we can deduce the parameters (Coefficient of sensitivity of the normal stress *k*, and the limit factor *f*) of the different criteria, exploiting the formulas of each criterion (Table 2).

Table 2: Parameters of concrete fatigue criteria

Material	**criteria**	**Parameters**	**Unit**	**Value**
Concrete class *C25/30*	Dang Van	*k*	-	0.50
		f	MPa	1.00
	Mataké	*k*	-	0.33
		f	MPa	1.00
	Findley	*k*	-	0.35
		f	MPa	1.06

For the fatigue limits of the steel used, we took the experimental results reported in the literature (Table 3).

Table 3: Steel fatigue criteria parameters

Designation	**Solicitation/Criteria**	**Parameter**	**Unit**	**Value**
Experimental results in fatigue of reinforcing steel	Symmetrical alternating traction: $R = -1$	σ_{-1}	MPa	206.00
	Rotational symmetric bending: $R = -1$	σ_0	MPa	337.00
	Symmetrical alternating torsion: $R = -1$	τ_{-1}	MPa	123.00
Steel fatigue parameters of reinforcing bars	Dang Van	*k*	-	0.291
		f	MPa	123.00
	Mataké	*k*	-	0.194
		f	MPa	123.00
	Findley	*k*	-	0.198
		f	MPa	125.40

4 Numerical Simulation of the Behavior

The simulation of the mechanical behavior under monotonic loading and cyclic loading was done using the Comsol Multiphysics software. In mechanical behavior under monotonic loading, we considered the structure of the dome (Figure 2), the dimensions of the shape are defined as shown in Table 4, the dome is subjected to a permanent load per m^2 noted G (dead weight), and a uniformly distributed operating load, noted P. For the design, the combination of load is fixed at the ultimate limit state: 1.35 G + 1.5 P. For optimization, the verification of the system at the limit state of service under the combination $G + P$, is necessary.

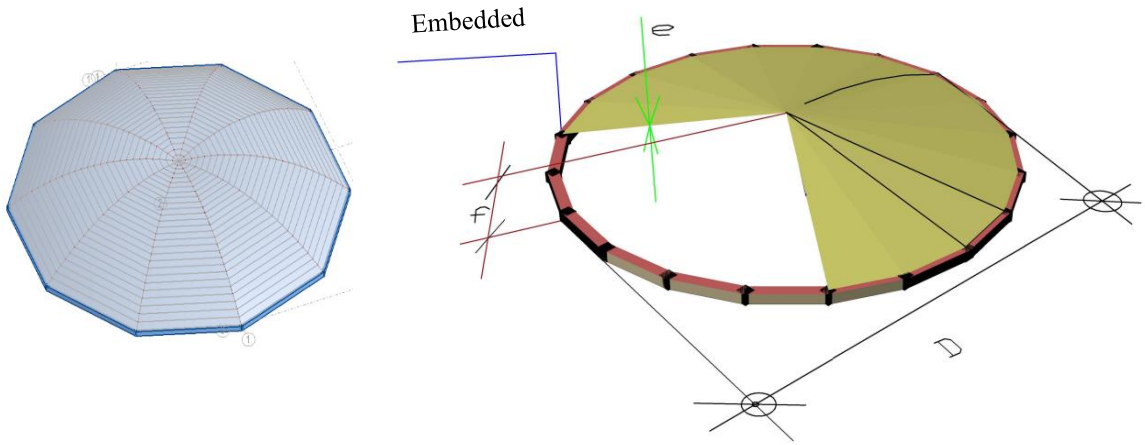


Figure 2: Design and shape dimensions of the dome

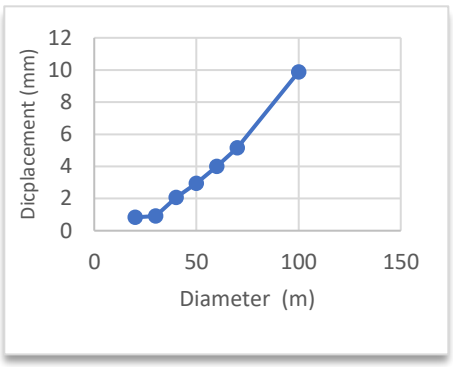
Table 4 shows the different dome shape dimensions, and the verification limits under monotonic loading, at the service limit state.

Figure 3 shows the mechanical behavior of the domes under the combination 1.35 G + 1.5 P, with P = 100 kg/m^2. A non-linear relationship is observed between the normal diaphragm force, calculated at the foot of the dome (at the level of the embedding), as a function of the diameter of the dome. From D = 60 m to D = 100 m, this force has increased by 263%, which implies a particular attention to the balance of this force concerning the slender domes.

Table 4: Dome design and system optimization

Case N°	***D*** **[m]**	***f*** **[mm]**	***e*** **[cm]**	**Maximum compressive stress under the combination *G* + *Q* [MPa]**	**Limit stress 12 [MPa]**	**Max. displacement in the center [mm]**
1	20	2000	8	0.413	0.900	
2	30	3000	10	0.568	0.750	
3	40	4000	13	0.695	0.731	
4	50	5000	19	0.786	0.854	

5	60	6000	25	0.890	0.937
6	70	7000	32	0.956	1.028
7	100	10000	55/70	1.232	1.237



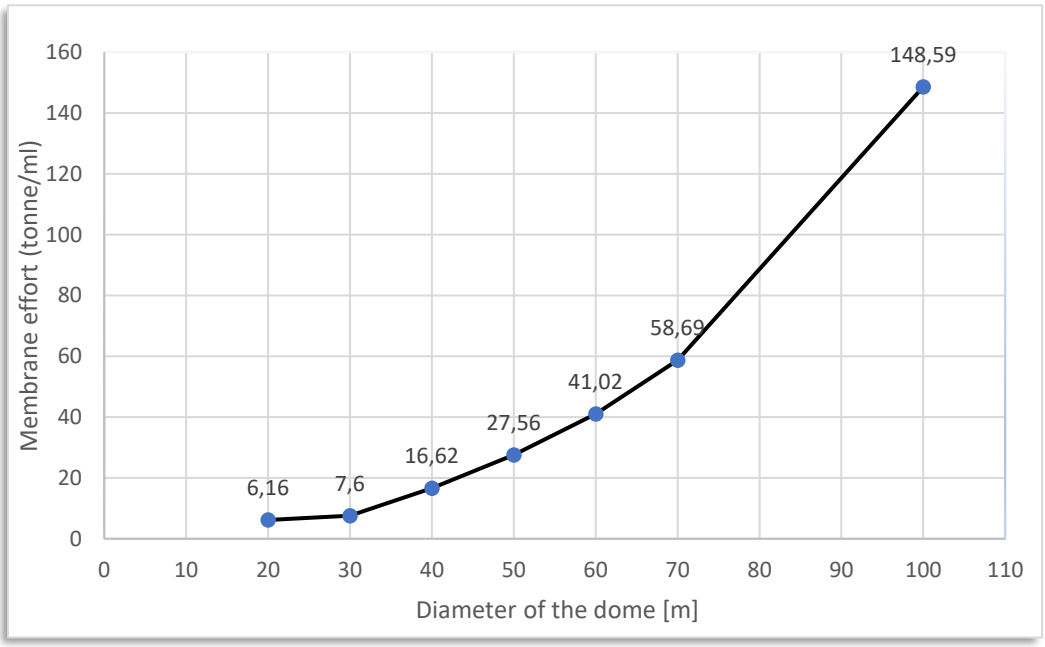
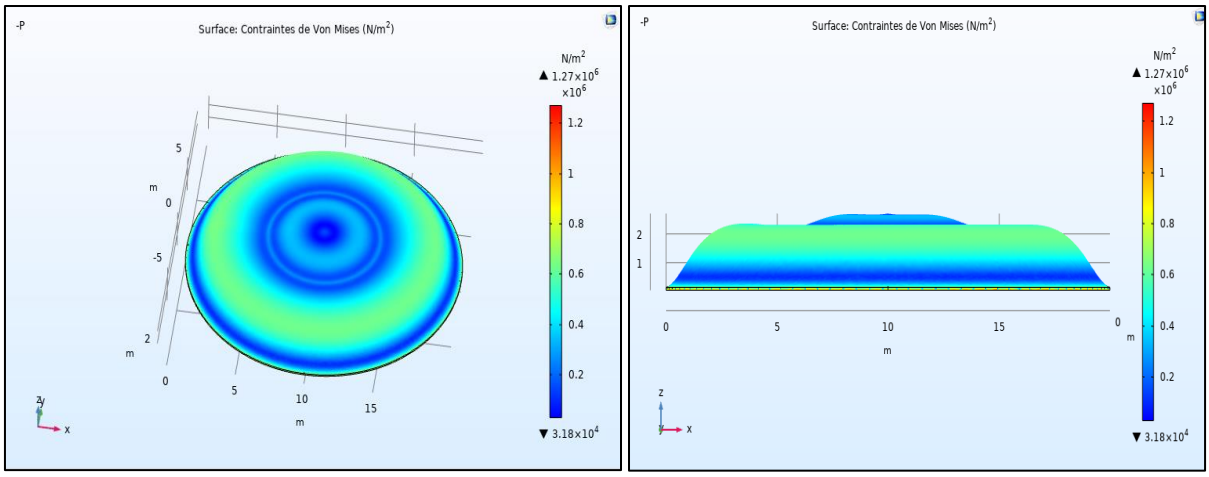


Figure 3: Normal membrane force as a function of dome diameter, a combination of 1.35 G + 1.5 P, with P = 100 kg/m^2

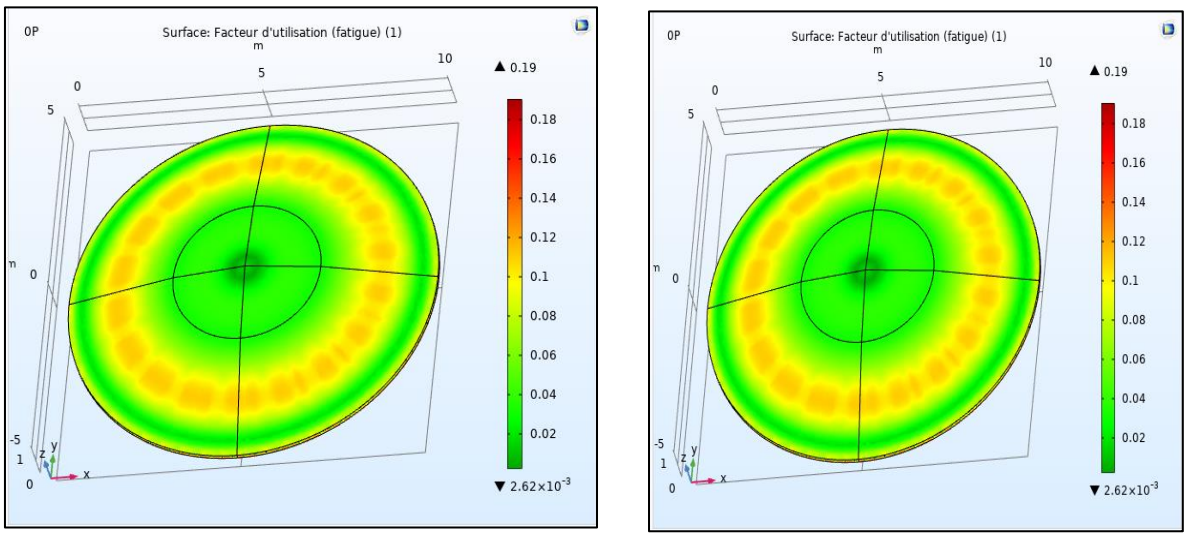
Regarding the behavior under an upwardly directed load (dome uplift), Figure 4 illustrates the stress concentration and displacement.



a) 3D view b) Side view

Figure 4: Behavior under monotonic loading. Dome of D = 20 m, P = 100 kg/m², Combination: $-P$

Through Figure 4, we can see under a monotonic load P (oriented upwards), a concentration of stresses at the junction of the dome with the peripheral edges. This concentration is also noticed in the center of the dome.



a) With reinforcement b) Without reinforcement

Figure 5: Fatigue risk map of a dome of D = 10 m, P = 100 kg/m^2 , Mataké criterion, with and without reinforcement

Figure 5 shows the distribution of the fatigue risk in the dome, the most sensitive areas are located near the transition zones, between the points with a high concentration of stresses and those with low stresses. Fatigue results in damage to the directly connected areas of the dome's peripheral edges. In the absence of the flexural behavior of the dome under fluctuating load P, the effect of the reinforcement is insignificant, which is why the fatigue behavior is similar without and with reinforcement. This last observation is justified by the pure compression behavior of the dome; the flexural behavior requiring a balance by the reinforcement, is not considered here.

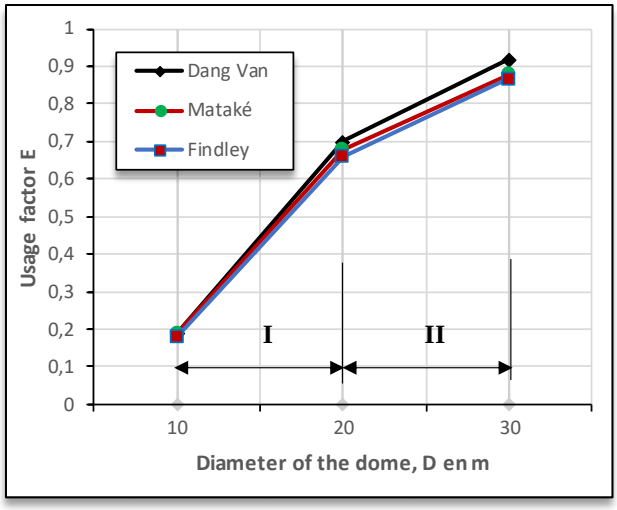


Figure 6: Fatigue risk of domes as a function of diameter D; P = 100 kg/m^2. The three criteria

Figure 6 shows the variation of the fatigue risk of the domes between the diameters D = 10 m and D = 30 m. The three criteria give similar results. It is clear, that the fatigue response becomes faster and faster for larger dome diameters. We can also observe a variation of the fatigue risk of 100% in interval I, while in interval II, a variation of 40%. The decrease of this variation with increasing dome diameter is justified by the flexibility of the dome. The dome, at slender spans, becomes more sensitive to loading, which causes relatively large displacements, the fluctuation of displacements due to repeated cyclic loading at very high numbers, excites fatigue at the most sensitive locations.

The approach of estimating the fatigue life of reinforced concrete domes is based on the notion of the fatigue limit of concrete, at a very high number of cycles (10^6), close to 0.60 of the compressive limit stress in static, this consideration leads us to propose a fatigue criterion for these structures in correlation with the behavior under monotonic loading.

5 Numerical Correlation

The fatigue response of the studied domes is related to the behavior in monotonic loading mode. In this section, we try to find the numerical correlation between these two behaviors, to propose the model of the fatigue response knowing only the response under monotonic loading. For this reason, and through the exploitation of the results of Table 4, the relationship between the maximum stress calculated at the foot of the dome under the combination $G + P$, to the diameter of the dome, is of the linear type, with a very good correlation (coefficient of determination R^2 close to 0.98), which leads us to propose the formula of the mechanical behavior under monotonic loading in the following way (Figure 7).

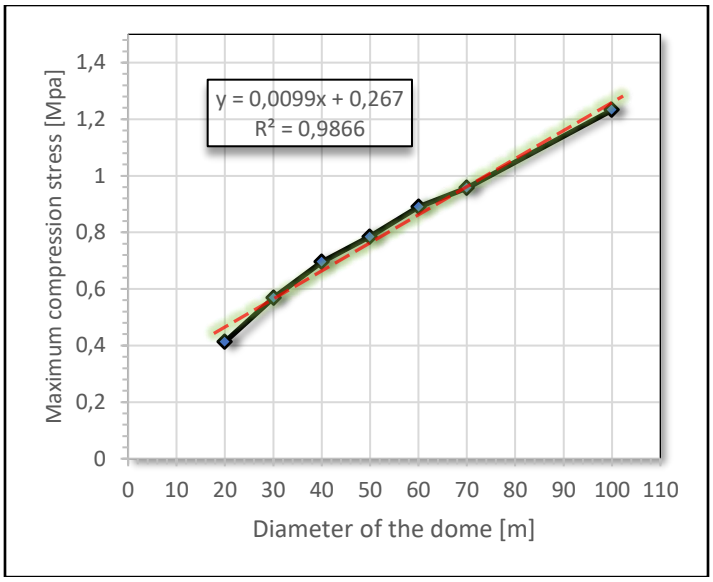


Figure 7: Maximum compressive stress as a function of the diameter of the dome, under the combination: $G + P$

We can therefore write (Equation 10):

$$\sigma_{max} = [0.009D] + (0.267) \quad (10)$$

with σ_{max}: Maximum compressive stress in MPa, at the foot of the dome under the combination $G + P$.

D: Diameter of the dome in m.

Adopting the average value of the usage factor E, deduced from the three fatigue criteria in Figure 6, and replacing the values of D, with the values corresponding to σ_{max}, we obtain the correlation shown in Figure 8.

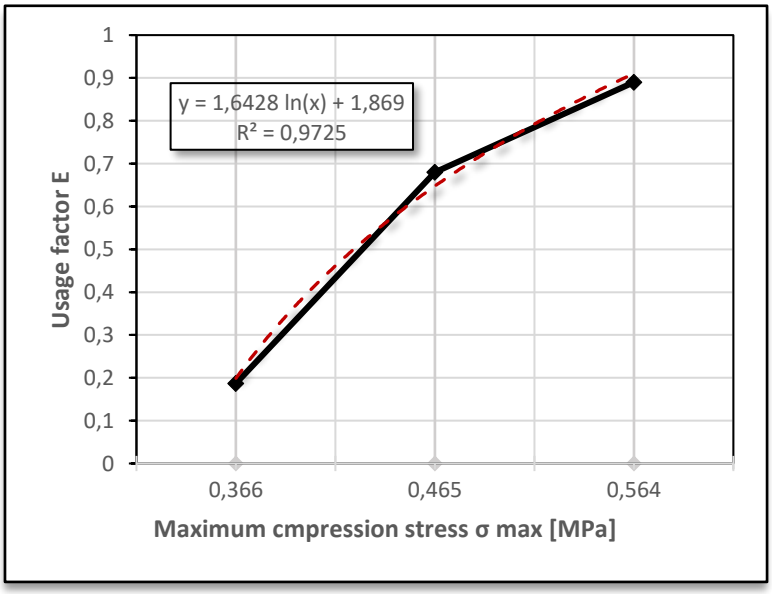


Figure 8: Correlation of fatigue response vs. of the maximum compressive stress σ_{max}

This correlation (Figure 8), with a trend curve in logarithmic form, and with a coefficient of determination R^2 close to 0.97, leads us to propose a fatigue criterion specific to reinforced concrete domes, this criterion can be written in the following form (Equation 11):

$$E = 1.6428\, ln(\sigma_{max}) + 1.869 \quad (11)$$

with :
E: Usage factor indicating the risk of fatigue, $E = 1$ implies a fatigue situation.
σ_{max} : Maximum compressive stress of concrete at the foot of the dome, calculated under the *G* + *P* combination under monotonic loading.
The advantage of this fatigue criterion is to predict the service life of reinforced concrete domes, knowing only the response under monotonic loading. It is valid for the following conditions:

- Reinforced concrete domes, the class of concrete is *C25/30*.
- Reinforcement type *FeE40*, with elastic limit in traction *400* MPa.
- Diameter of the domes *D* from 10 to 100 m.
- Fault in the center *f* is close to 0.10 *D*, corresponding to the lowered domes.
- Dome thickness: Checking the limit of compressive stress of concrete under monotonic loading (*G* +*P*), and the limit of displacement at center.
- Compressive fatigue limit of concrete: close to 0.6 of its limit in static.
- Fatigue under fluctuating load *P* at 10^6 cycles.

6 Conclusion

The work in this article was devoted to the study of the mechanical fatigue behavior of reinforced concrete domes. Through a numerical simulation of this behavior by studying several cases and based on the study of the response under monotonic loading and fatigue, followed by a correlation of the results, a numerical model for predicting the fatigue limit was aimed at. Existing fatigue criteria were exploited to predict this behavior.

In this study, the response under monotonic loading was coupled to the fatigue response. Numerical correlation made it possible to propose a fatigue criterion capable of predicting the service life of these complex structures without the need for a fatigue study. This criterion is of a conservative type, as it is based on a fatigue limit of concrete in compression ensuring sufficient safety in relation to dimensioning aspects and will therefore make a practical and rapid tool for a safety assessment of these structural systems.

The work discussed in this article offers researchers an interesting avenue to push towards a wider field of application in this area.

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