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# Identification of Soil Mechanical Parameters by Inverse Analysis Using Stochastic Methods

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**Abstract**

The mechanical parameters of the soil that must be introduced into geotechnical calculations, in particular those carried out by the Finite Element Method, are often poorly understood. The search for the numerical values of these parameters so that the models best reflect the observed reality constitutes the inverse analysis approach. In this article, we are interested in the identification of the mechanical parameters of the soil based on the principle of inverse analysis using the two methods of stochastic optimization, the genetic algorithm and the hybrid genetic algorithm with Tabu search. Soil behavior is represented by the constitutive soil Mohr-Coulomb model. The identification relates to the following two parameters: The shear modulus ($G$) and the friction angle ($\varphi$). The validation of these two stochastic optimization methods is done on the experimental sheet pile wall of Hochstetten in Germany. The results obtained by applying the genetic algorithm method and the hybrid genetic algorithm method for the identification of the two Mohr-Coulomb parameters ($G$, $\varphi$) show that the hybridization process of the genetic algorithm combined with the Tabu search method accelerated the convergence of the algorithm to the exact solution of the problem whereas the genetic algorithm alone takes a much longer computation time to reach the exact solution of the problem.

**Keywords**: inverse analysis, identification of parameters, stochastic methods, numerical modeling, sheet piling

## 1 Introduction

The mechanical parameters of the soil that must be introduced into the geotechnical calculations are often poorly known. The search for the numerical values of these parameters, so that the models best reflect the observed reality, constitutes the inverse analysis approach [1]. This form of inverse analysis consists of calibrating a numerical soil model on experimental data by iterative modifications of the values of the input parameters of the model until the output values reproduce the observed data as close as possible, i.e., until the error function, which quantifies the difference between the experimental and numerical results, is minimal. Parameter adjustment boils down to a stochastic optimization problem [2, 3, 4].

The objective of this work is to apply the principle of inverse analysis using two methods of stochastic optimization, the genetic algorithm and the hybrid genetic algorithm with the Tabu search method to identify two parameters of the constitutive soil Mohr-Coulomb model, the shear modulus ($G$) and the friction angle ($\varphi$). In order to test the convergence and the efficiency of the two stochastic methods, we have developed two programs under Matlab 07. The first program uses the genetic algorithm method and the second uses the hybrid genetic algorithm method with Tabu search, these two programs are validated from an experimental sheet pile wall from Hochstetten in Germany. In 1993, the University of Karlsruhe in Germany built a full-scale experimental retaining structure on the Hochstetten site. This experiment consisted in digging an excavation in front of a wall of braced sheet piles in a sandy environment and in stressing the wall until the limit equilibrium. The experimental sheet pile wall of Hochstetten is therefore a very well instrumented geotechnical structure. It has been the subject of many studies which constitute a large database [1, 5, 6].

Many studies have been devoted to the determination of the mechanical properties of soils from the experimental sheet pile wall of Hochstetten. This structure has been the subject of numerous studies which constitute an important database such as the research work of Mestat and Arafati [5], Von Wolffersdorff [6] and Levasseur [1]. We propose to complete this research work by studying an optimization problem on a finite dimensional space. The goal of the optimization problem is to identify the best sets of soil parameters ($G$, $\varphi$) that minimize the gap between the numerical model and the experiment. Many participants in the forecasting competition chose the Mohr-Coulomb soil constitutive model. In order to compare the results of the identification method with their predictions. Numerical simulations were carried out using the numerical calculation software Plaxis 08.

## 2 Optimization Principle

A given optimization problem must first be able to be numerically modeled by a Finite Element Calculation. For this study, we chose the Plaxis software. Depending on the stresses imposed on the model and the parameters assumed for a constitutive soil model, a numerical answer is calculated. The numerical answer obtained is then compared to the available experimental data. This comparison results in the calculation of the difference between the observed data and the calculated values, i.e., the calculation of the error function $(F_{err})$. This difference is then minimized by two stochastic optimization algorithms. From this optimization are deduced new values for the parameters of the constitutive soil model [1]. The process repeats until the difference between the calculated values and the measured data is minimal (Figure 1).

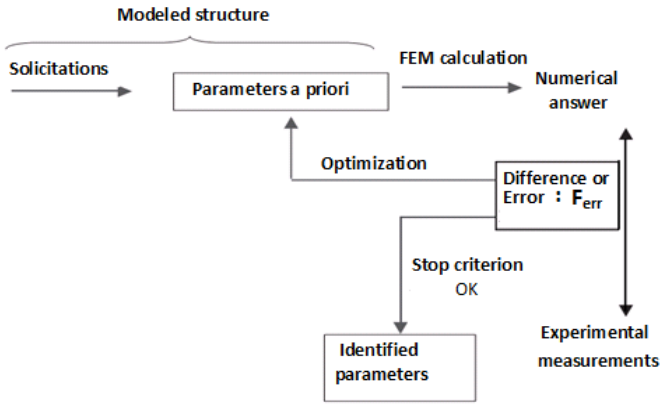



Figure 1: Optimization principle

## 2.1 The Error Function

The choice of the error function ($F_{err}$) is crucial in inverse analysis. Often in geotechnical engineering, the error function evaluates the difference between a numerically calculated curve, described by $U_{ni}$, and an experimental curve measured in situ, described by $U_{ei}$, as shown in the Equation (1):

$$F_{err} = (\frac{1}{N}\sum_{i=1}^{N}\frac{(U_{ei}-U_{ni})}{(0.01+U_{ei})} \quad (1)$$

where:
$N$: is the number of measurement points; $U_{ei}$: is the experimental displacement at a measurement point $i$ and $U_{ni}$: is the numerical displacement at a measurement point $i$.

## 2.2 Optimization Algorithms

### 2.2.1 Optimization by Genetic Algorithm

The genetic algorithm operates on a population of individuals, each individual representing a set of parameters. Selection, crossover and mutation operators are applied to generations of individuals (called parents) in order to create new individuals (called children) [7]. The individuals of the new population will be evaluated from the error function to replace a part of the individuals of the current population. The process of optimization by the genetic algorithm is illustrated in Figure 2.

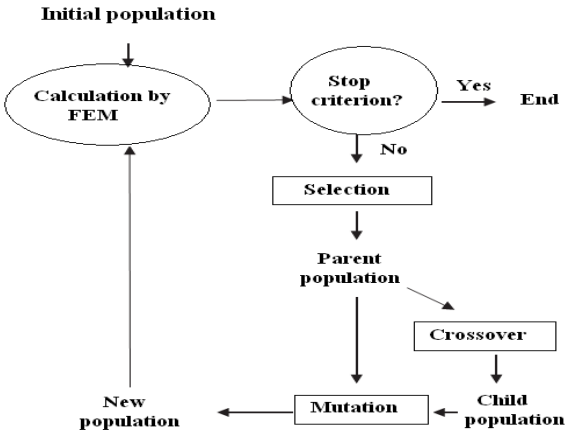



Figure 2: The optimization process by the genetic algorithm

### 2.2.2 Optimization by Hybrid Genetic Algorithm

Genetic algorithms, although robust, are expensive methods because they do not take local information into account much, so it is interesting to hybridize a genetic algorithm with a local search algorithm such as the Tabu search method which allows carrying out a modular analysis of a problem by agreeing to sacrifice generality to gain efficiency [8]. The Tabu search method is introduced after the crossover operation using the intensification operator to find the best neighbor for each solution found. The result will be a new optimized population that will continue to evolve through further genetic operations to produce new generations better suited to the problem [5, 6]. The process of optimization by the genetic algorithm is illustrated in Figure 3.

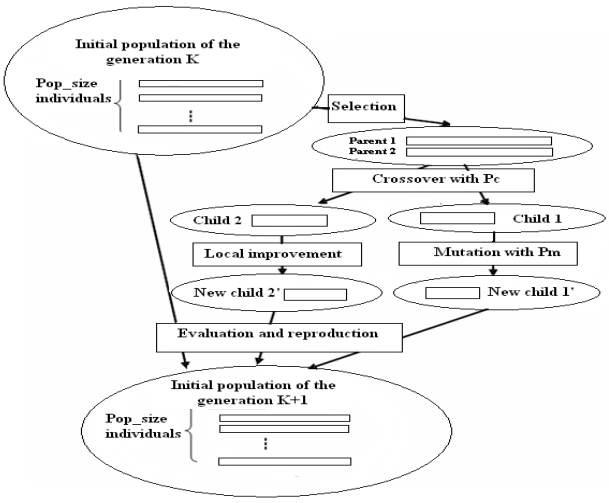



Figure 3: The optimization process by the hybrid genetic algorithm

# 3 Presentation of the Experimental Device An Instrumentation of the Experimental Sheet Pile Wall

The experiment, shown schematically in Figure 4, concerns a KRUPP KD VI type sheet pile wall 7 m long (experimental wall), installed by vibratory driving up to 6 m deep in a mass of unsaturated sandy soil, and held by a level of buttons. This experimental curtain supports one of the longitudinal walls of a rectangular excavation (7 m long and 4 m wide) whose depth reached 5 m at the end of construction. The props, three in number, are spaced 2.4 m apart over a total length of 7 m. The second curtain of ARBED PU 8 type sheet piles, 8 m deep, forms the support for the opposite wall. The sheet and the inertia of this curtain were chosen so that they did not undergo any displacement during the experiment. The water table is located at the depth of 5.5 m [1, 5].

The execution and testing of the work involved the following six stages:
– Step 1: excavation to a depth of 1.75 m;
– Step 2: installation of three struts spaced horizontally by 2.4 m and located at a depth of 1.25 m. The struts are preloaded with a force of 10kN per strut, i.e., a unit force of around 4.5kN per meter of curtain length;
– Step 3: resumption of excavation to a depth of 4 m;
– Step 4: continue the excavation to a depth of 5 m. The excavations are carried out at the same time as the bracing system and screw jacks are operated so as to prevent any displacement of the bracing level;
–Step 5: application of an overload of 10 kPa, on the surface, on an area located between 1 and 5 m behind the head of the experimental curtain. The overload is exerted through a basin filled with water;
– Step 6: reduction of the strut force by shortening the struts to the limit state of the soil block. This state is reached when the forces in the struts remain approximately constant.

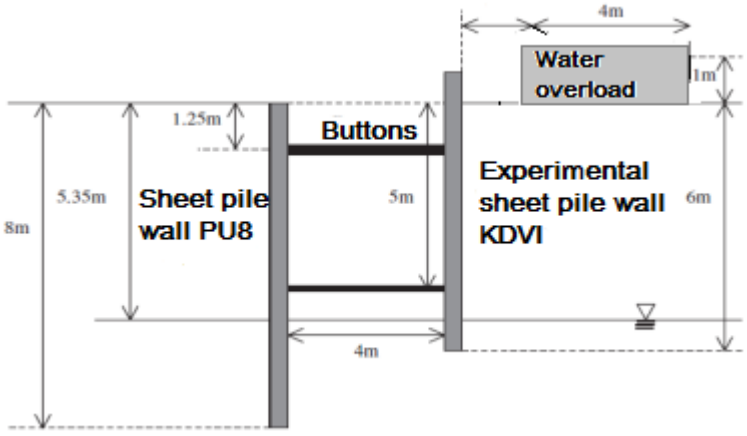



Figure 4: The schematic of the Hochstetten sheet pile wall experiment

## 3.1 Experimentation Modeling

In this experiment, two sheet pile walls with different characteristics are used: the instrumented wall and the retaining wall. It is therefore necessary to model the two structures.

On the other hand, given the length of the excavation and the small spacing between the struts, it is reasonable to produce the model in plane strain. The model then corresponds to a cross section of the structure [1, 5]. The numerical modeling of this structure is shown in Figure 5. Table 1 gives some geometric and numerical characteristics of this model. The characteristics of the curtains and struts are presented in Table 2. The mechanical behavior of Karlsruhe sand is modeled by the Mohr-Coulomb constitutive law, the values of the parameters of which are given in Table 3. These values are used initially to define the experimental data used for the inverse analysis. The experimental curve, shown in Figure 6, corresponds to the horizontal displacements of the KRUPP KD VI type sheet pile wall (experimental sheet piles).

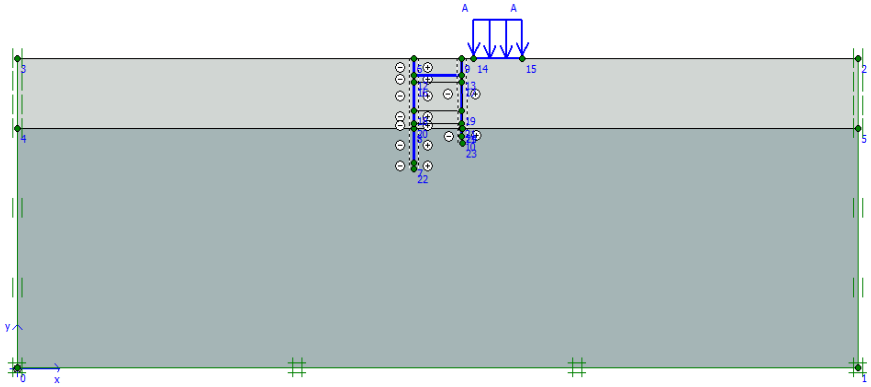


Figure 5: The numerical model

Table 1: Characteristics of the Finite Element Model [5]

| |
|---|
| **Geometric characteristics** |
| Model size: $L$=70 m, $H$=24 m |
| Size of the excavated area: $h$ = 5.35 m, $w$ = 4 m |
| Height of KRUPP KD VI type sheet pile wall (experimental wall): $hw$ = 7 m |
| Height of the ARBED PU 8 type sheet pile wall (rigid wall): $hw$ = 8 m |
| **Finite element characteristics** |
| Type of mesh elements: triangles with 15 nodes |
| Number of elements: 464 |
| Number of knots: 3987 |

Table 2: Values of the parameters of the elastic behavior models used to model the sheet pile wall and the struts [5]

| **Parameter** | **Name** | **Value** | **Unit** |
|---|---|---|---|
| Sheet pile wall | *KRUPP KD VI* | | |
| Normal stiffness | *EA* | 2.2 $10^6$ | kN/m |
| Bending stiffness | *EI* | 2.0 $10^3$ | kN $m^2$/m |
| Weight | *w* | 0 | kN/m/m |
| Equivalent thickness | *d* | 0.105 | m |
| Poisson's ratio | *v* | 0.3 | – |
| Sheet pile wall | *ARBED PU 8* | | |
| Normal stiffness | *EA* | 2.2 $10^6$ | kN/m |
| Bending stiffness | *EI* | 24.4 $10^3$ | kN $m^2$/m |
| Weight | *w* | 0 | kN/m/m |

| | | | |
|---|---|---|---|
| Equivalent thickness | *d* | 0.363 | m |
| Poisson's ratio | *v* | 0.3 | – |
| Buttons | | | |
| Normal stiffness | *EA* | $4.2\ 10^6$ | kN/m |
| Bending stiffness | *EI* | $1.0\ 10^5$ | kN $m^2$/m |
| Equivalent thickness | *d* | 0.535 | m |

Table 3: Values of the parameters of the of the constitutive soil Mohr-Coulomb model for a sand of Karlsruhe [5]

| **Parameter** | **Name** | **Value in the unsaturated layer** | **Value in the saturated layer** |
|---|---|---|---|
| Young's modulus | *E* (kPa) | 30000 | 45000 |
| Shear modulus | *G* (kPa) | 12000 | 18000 |
| Poisson's ratio | *v* | 0.25 | 0.25 |
| Cohesion | *c* (kPa) | 0 | 3 |
| Friction angle | $\varphi$ (◦) | 41.6 | 41.6 |
| Dilatancy angle | $\Psi$ (◦) | 11.6 | 11.6 |
| Earth coefficient at rest | $K_0$ | 0.336 | 0.336 |

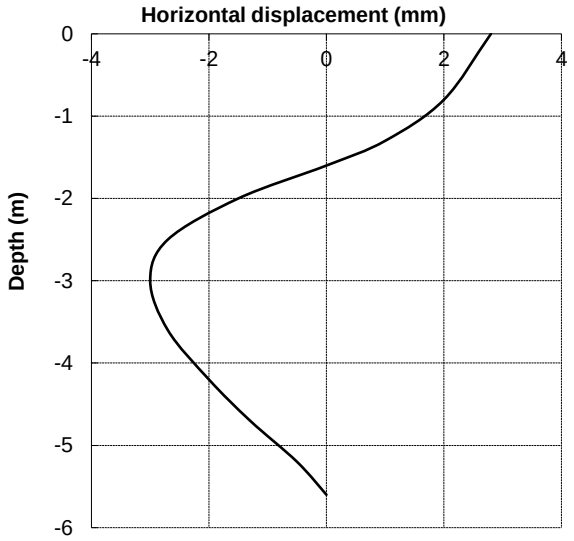



Figure 6: Horizontal displacement of the experimental sheet pile wall according to the depth

(-) Experimental measurements to be optimized

## 4 Validation of the Optimization Methods

In order to test and evaluate the performance of the proposed optimization methods, we have developed two programs under Matlab environment. The first program uses the genetic algorithm method (*GA*) and the second uses the Hybrid genetic algorithm method (*HGA*) with the Tabu search method. These two programs have been validated on the case of Hochstetten sheet piling in Germany, for the purpose of identifying the parameters of the constitutive soil model Mohr-Coulomb, the shear modulus (*G*) and the angle of friction ($\varphi$). Preliminary values

are given to the unknown parameters to simulate the numerical model, using the numerical calculation code Plaxis 08, until the difference between the result of the numerical calculation (numerical curve) and the experimental data (curve experimental) is minimal.

### 4.1 Genetic Algorithm Method

To launch the genetic algorithm program several parameters are necessary to give solutions of very good quality with a reasonable calculation time, these parameters are cited in Table 4.

Table 4: Values Parameters of the genetic algorithm

| **Parameter** | **Value** |
|---|---|
| Number of individuals | $N = 30$ |
| Number of generations | $K = 100$ |
| Length of the bit chain | $l = 18$ bits |
| Probability of crossover | $P_c = 0.6$ |
| Probability of mutation | $P_m = 0.08$ |

#### 4.1.1 Results and Discussion

The optimization method using the genetic algorithm program is tested to identify two parameters of the Mohr-Coulomb model, the shear modulus ($G$) and the friction angle ($\varphi$), from the horizontal displacement of the sheet pile wall depending on the depth. The optimization inside the search space is controlled by the error function ($F_{err}$). Figure 7 shows the evolution of the individuals composing the parent populations during the process of optimization by genetic algorithm. It appears that starting from an initial population uniformly distributed over the entire parameter search space, from the first iteration a solution zone of the space is identified. Then, as the genetic algorithm iterations progress, the identification is refined around the absolute optimum of the problem. The optimization process ends at iteration 11 on a unique combination of solution parameters of the problem for which the error function is equal to zero. This characterizes the convergence of the algorithm. Finally, after 100 Finite Element Calculations, the identified optimum is the exact optimum of the problem, i.e., the following combination of parameters: $G$ = 9800 kPa; $\varphi$ = 44◦. It can be clearly seen that the numerical answer, corresponding to the parameters identified by the genetic algorithm method, are in good agreement with the experimental curve as illustrated in Figure 8.

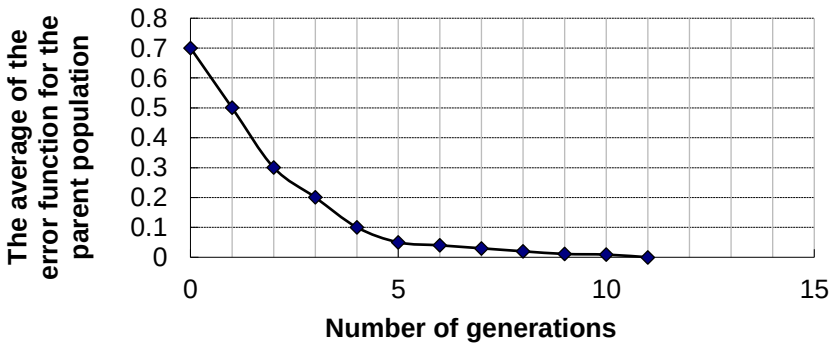



Figure 7: Evolution of the average of the error function on parent population according to the generations of a genetic algorithm

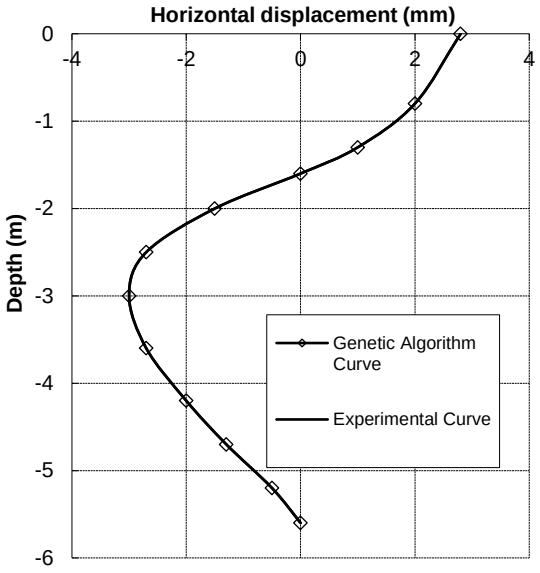



Figure 8: Horizontal displacement of the sheet pile wall according to the depth. (-) Experimental measurements to be optimized; (◊): Modeling of measurements after optimization by genetic algorithm

## 4.2 Hybrid Genetic Method

The tests carried out on the parameters chosen for the hybrid genetic algorithm program have guaranteed the obtaining of a very satisfactory result, these parameters are cited in Table 5.

Table 5: Values Parameters of the hybrid genetic algorithm

| **Parameter** | **Value** |
|---|---|
| Number of individuals | $N$=30 |
| Number of generations | $K$=100 |
| Length of the bit chain | $l$ =18 bits |
| Probability of crossover | $P_c$ = 0.6 |
| Probability of mutation | $P_m$ = 0.04 |
| Size of the tabu list | $T$= 3 |

### 4.2.1 Results and Discussion

The hybrid genetic algorithm program that we have implemented to identify the Mohr Coulomb parameters ($G$ and $\varphi$) relies mainly on the error function which quantifies the difference between the numerical results and the experimental measurements. Figure 9 presents the evolution of the mean error function on the parent populations of the hybrid genetic algorithm according to the generations of the algorithm. For each of these generations, the evolution of the parent population in the search space shows once again that from an initial population uniformly distributed over the search space, the individuals gradually converge towards the areas of the search space where the error function ($F_{err}$) equals zero.

After four iterations and 100 Finite Element Calculations, all the parent individuals are equal to the same combination of parameters, the optimization stops. The identification is refined to the exact optimum of the problem which is the following combination of parameters: $G$ = 9800 kPa; $\varphi$ = 44°. Figure 10 shows that the combinations of parameters obtained allow a very good reproduction of the experimental measurements.

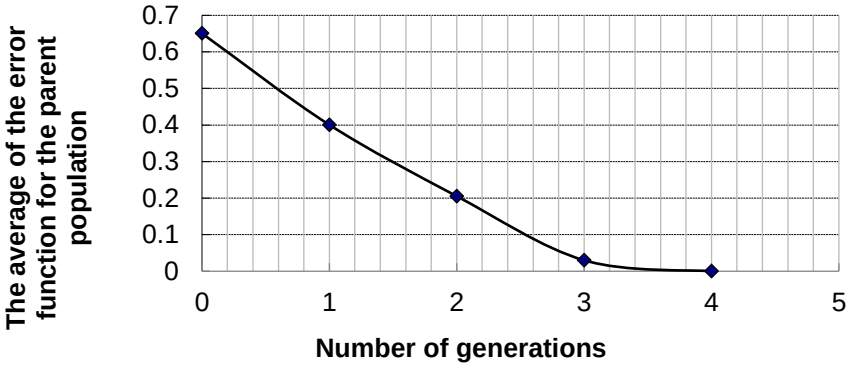



Figure 9: Evolution of the average of the error function on parent population according to the generations of a hybrid genetic algorithm

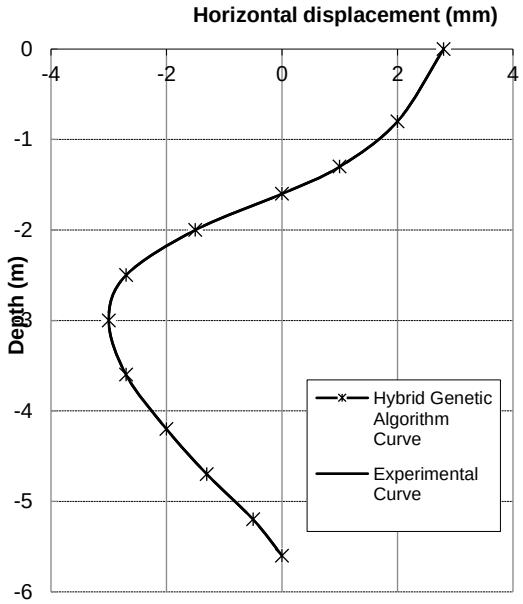



Figure 10: Horizontal displacement of the sheet pile wall according to the depth. (-) Experimental measurements to be optimized; (✳): Modeling of measurements after optimization by hybrid genetic algorithm

## 5 Comparison of Results

The comparison of the horizontal displacements of the sheet pile according to the depth obtained by the two stochastic optimization curves (*GA* and *HGA*) with those obtained experimentally shows good agreement between the measured profiles and those drawn from

the numerical answer corresponding to the parameters identified by the two stochastic optimization methods. This concordance shows that the difference between the experimental curve and the numerical curves which is expressed by the calculation of the error function ($F_{err}$) is equal to zero. So, the identification of the two soil parameters ($G$, $\varphi$) is refined to the exact optimum of the problem (see Figure 11).

If we compare the results obtained by the genetic algorithm method with those estimated by the hybrid genetic algorithm method, it is clear that these two optimization methods provide the same combinations of solution parameters for this problem. However, while it takes 11 iterations for the genetic algorithm to converge, it only takes 4 for the hybrid genetic algorithm method.

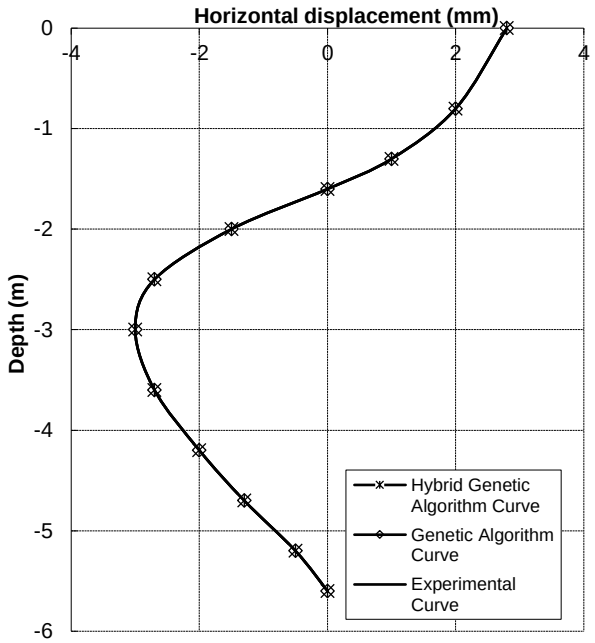



Figure 11: Horizontal displacement of the sheet pile wall according to the depth

(-) Experimental measurements to be optimized; (◊): Modeling of measurements after optimization by genetic algorithm; (✳): Modeling of measurements after optimization by hybrid genetic algorithm

## 6 Conclusion

In this article, we sought to validate two stochastic optimization methods the genetic algorithm and the hybrid genetic algorithm with the Tabu search based on the inverse method from the study of an experimental sheet pile wall of Hochstetten in Germany. For this application, we tested the capacities of two optimization methods to identify two parameters of the constitutive soil model Mohr-Coulomb ($G$, $\varphi$). The results obtained show that each of these two methods identifies the exact solution of the problem. But the hybridization process of the genetic algorithm with the Tabu search method accelerated the convergence of the

algorithm towards the exact solution of the problem, whereas the genetic algorithm alone takes a much longer computation time to reach the exact solution to the problem.

Despite the large number of evaluations of the objective function required by stochastic methods, these methods constitute a class of approximate methods adaptable to a very large number of optimization problems. They have been shown to be highly effective in providing good quality approximate solutions for a large number of optimization problems and real applications. For this reason, the study of these methods is currently in full development.

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