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Explicit Modeling of the Cyclic Behavior under a Large Number of Cycles: Application to the Case of Sand

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Abstract

The estimation of deformations after a large number of cycles is a major concern of geotechnical engineers. This paper presents an explicit formulation of the accumulated cyclic volumetric strain. This study is concerned with cyclic triaxial tests simulated through the calculation by the Finite Element Method. The parameters of identification of the first cycle (loading-unloading) are determined step by step, using, as a soil behavior model, the Soft Soil Hardening Model, then, the behavior of the soil after N cycles ($N > 1000$) is simulated through the formulation proposed as a pseudo-creep. The comparison of the proposed method with several cyclic triaxial tests and a shallow foundation under cyclic loading approves the good compatibility of the proposed formula for this type of problem. This study does not take into account the effect of the grain size and the void ratio, things that can be translated by the coefficients *C1* and *C2*.

Keywords: cyclic, volumetric strain, triaxial test, drained condition, sand

1 Introduction

It is very important to know the prediction of the irreversible cyclic deformations of the soil and their accumulation which may require important maintenance [1-6], to properly measure the dimensioning of structures subject to or located near cyclical excitation such as offshore structures, wind turbine foundations, silos, bridges, industrial installations, railways, tanks.
However, the complexity of mastering the problem of modeling the cyclic behavior of structures, mentioned above, has rendered many Geotechnical researchers interested in finding reliable solutions [7-9]. Among them, a lot of research is concentrated on the behavior of granular material under cyclic loading [10, 11]. Different studies are based on the test from the cyclic triaxial test and focus on the permanent displacements of sand with cyclic loading [3, 13–17, 18-20]. Other researchers have concentrated on the modeling of cyclic behavior under a large number of cycles [21–26]. Several researchers are interested in the settlement of shallow foundations under a high number of cycles [27-31].

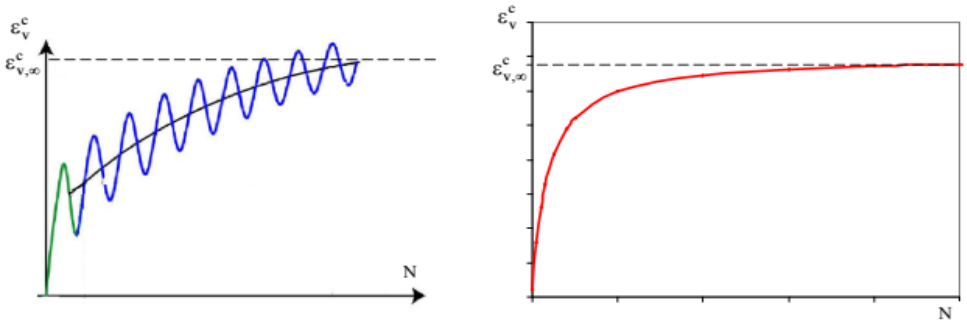
There are two methods to predict the cyclic behavior of soil: the implicit method (incremental) and the explicit method. The implicit method is not recommended [11, 12, 17, 32]. The importance of explicit models appears especially in a large number of cycles (N>1000). In such models, only one or a few initial cycles are calculated using an incremental calculation (implicit method), and the remaining cycles are treated as pseudo-creep.
This paper presents a simpler explicit model for the prediction of volumetric strains after a large number of cycles. The presented model in this work is an improvement from the one presented by [21]. This formulation offers a hyperbolic curve volumetric deformation based on the identification parameters of the average cyclical path and those of the first cycle which will be processed by the finite element calculation using the Plaxis program, in which the behavior of soil is modeled through the hardening soil model (HSM).
Furthermore, this formulation is compared with the measured value applied on a shallow foundation [33].

2 Research Background

In such models, only one or a few first cycles are calculated using an incremental calculation, and the rest of the cycles will be treated as pseudo-creep. Several works have been concentrated in this direction, to better establish models which provide an estimate as faithful as possible to the cyclic behavior of the soil.
Thanopoulos [34] presented a hyperbolic formulation for estimating the delayed axial strains and another for the volumetric strain by introducing the concept of voluminous Poisson's ratio. The model presented by [9], describes the accumulation of axial and volume deformation by means of a logarithmic formulation, although this model was invested with a high number of cycles, it presents some deficiencies according to [31] such as the cyclic flow rule along with the boost in the acceleration of accumulation with the upturn in the mean pressure turns out to be inconsistent with the empirical values of [10], [32] proposed a model for the prediction of the accumulation of volumetric and deviatoric deformations depend on a power formulation, this model doesn't consider the amplitude of the deformation volume and the shape of the cyclic loading loop [7] proposed a model describing the densification of sands, using a logarithmic formulation, his disadvantage is that it is founded on a limited number of cycles and that it doesn't consider the amplitude of the volumetric strain and the shape of the cyclic loading loop [8] proposed a model for the prediction of cumulative deformations based on the results of tests carried out by [32] on sand with a number of cycles of the order of 100.000, the downside of this model is that it calls for implicit models to calculate the parameters of the identification cycle which is the second cycle.
The present study presents a simpler explicit model for the prediction of axial deformations after a very high number of cycles and the constitutive modeling of the cyclic behavior of granular material in a drained condition. A hyperbolic formulation proposed for the axial deformation based on the identification parameters of the average cyclic path and those of the first cycle, which will be treated by a finite element calculation using the Plaxis calculation code and the material modeled by the Hardening Soil Model (HSM).

3 Proposed Model

In this study, the proposed formulation entails determining the evolution of the accumulated volumetric strain, according to the cyclic average path, which is referred to as the pseudo-cyclic (Figure 1). Firstly, the determination of the parameters of identification in the first cycle (load-unload) is achieved through step-by-step calculation by Plaxis 8.2. Secondary, conforming to the cyclic average level, the limit cyclic level η_l, the characteristic cyclic level η_c, and the cumulative strain ($\varepsilon_{v,\infty}^{c}$) when $N \rightarrow \infty$ are determined. With this approach, we consider that cyclic pseudo creep (the evolution of the strain, regarding the number of cycles) is controlled mainly by the effect of the irreversible deformation after the first cycle ($\varepsilon_{v,1}^{c}$), and the calculation of the cumulative strain ($\varepsilon_{v,\infty}^{c}$) when $N \rightarrow \infty$.



a- Complete cyclic path **b-** Average cyclic path

Figure 1: Evolution of cumulative volumetric strain after *N* cycles

The formulation of the cumulative volumetric strain after *N* cycles $\varepsilon_{v,N}^{c}$ can be expressed as follows from Equation 1.

$$\varepsilon_{v,N}^{c} = \frac{\sqrt{N}}{\frac{1}{\varepsilon_{v,1}^{c}} + \frac{\sqrt{N-1}}{\varepsilon_{v,\infty}^{c}}} \tag{1}$$

This last formula is simple and makes it possible to express the cumulative volume strain; it depends on four parameters: The number of cycles *N*, the number of the basic identification cycle and the corresponding strain, as well as the strain for an infinite number of cycles.

$\varepsilon_{v,\infty}^{c}$: The cumulative strain when $N \rightarrow \infty$ from Equation 2.
$\varepsilon_{v,1}^{c}$: The irreversible deformation after the first cycle.

The cumulative volumetric strain at infinity ($\varepsilon_{v,\infty}^{c}$) can be expressed in terms of η_{av}, η_{car} and η_l [21]:

$$\varepsilon_{v,\infty}^{c} = \varepsilon_{v,0}^{\infty} \frac{\left(\frac{\eta_{av}}{\eta_{car}} - 1\right)}{\left(1 - \frac{\eta_{av}}{\eta_l}\right)} \tag{2}$$

$$\eta_{av} = \frac{q_{av}}{p_{av}} \tag{3}$$

where q_{av}: Average of deviator during the cycle

p_{av}: Average of mean pressure during the cycle.

$$\eta_{car} = \frac{6\sin\phi_c}{3 - \sin\phi_c} \tag{4}$$

where ϕ_c: Critical friction angle.

$$\eta_l = \frac{6\sin\phi_l}{3 - \sin\phi_l} \tag{5}$$

where ϕ_l: Limit friction angle.

$\varepsilon_{v,0}^{\infty}$ Function of $\Delta\eta$ from Equation 6.

$$\varepsilon_{v,0}^{\infty} = \frac{C_1 \quad \Delta\eta}{\Delta\eta + C_2} \tag{6}$$

$$\Delta\eta = \eta_{\max} - \eta_{\min} \tag{7}$$

With: $\eta_{max} = \frac{q_{max}}{p_{max}}$, $\eta_{min} = \frac{q_{min}}{p_{min}}$, C_1 and C_2 obviously depend onη_{av} and other parameters such as porosity and particle size. It is proposed for sand C_1= 4 and C_2 = 0.3 [21].

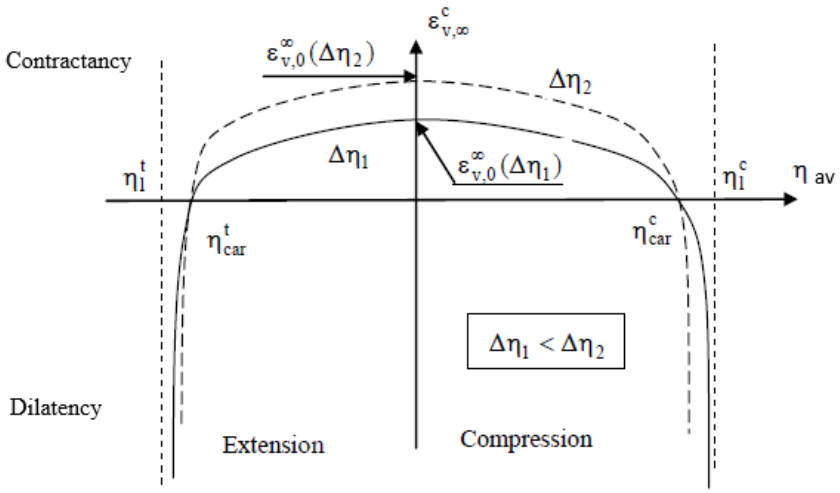


Figure 2: $\varepsilon_{v,0}^{\infty}$ in the function of $\Delta\eta$

Figure 2 illustrates the relationship between the different cyclic parameters where it represents the variation of the cumulative volumetric strain at infinity ($\varepsilon_{v,\infty}^{c}$) with terms of η_{av}, η_{car} and η_l in the contracting domain and the dilatancy domain.

4 Simulation and Results

To test the reliability of the proposed formulation, we proceed to the comparison of results obtained by this method with those presented in the work of [34].
Thanopoulos [34] has conducted cyclic tests on the sand of Plancoët relatively loose characterized by its dry unit weight of 12.5 kN/m^3 and the stress deviation - axial strain and volume strain - axial strain deflection curves for a monotonic triaxial test and three series of cyclic tests. Table 1 summarized the principal cyclical measured and calculated parameters for all tests. The behavior of the material will be reproduced using the HSM (Hardening soil model) model by representing the curves ε_v-N employing the finite element program Plaxis. Based on test 8 (monotonic test), the Plancoët sand will be characterized according to the HSM model: $E_{50} = E_{oed} = 6689$kPa, $E_{ur} = 867.10^2$kPa, $\varphi = 37.6°$, $C = 0$, $\psi = 6°$, $R_f = 0.9$, $m = 0.4$ [21].

Table 1: Parameters of the cyclic tests in compression achieved on the fine sand of Plancoët, according to [34]

Test	$\boldsymbol{\sigma_3}$ kPa	$\boldsymbol{q_{max}}$ kPa	$\boldsymbol{q_{min}}$ kPa	$\boldsymbol{\eta_{moy}}$	$\boldsymbol{\Delta\eta}$	$\boldsymbol{\varepsilon_{v0}^{\infty}}$	$\boldsymbol{\varepsilon_{v}^{\infty}}$Calculated[%]	$\boldsymbol{\varepsilon_{v}^{\infty}}$Test[%]	$\boldsymbol{\varepsilon_{v1}^{\infty}}$Calculated [%]	$\boldsymbol{\varepsilon_{v1}^{\infty}}$Measured[%]
2	40	56	38	0.84	0.23	1.75	1.46	1.60	0.48	0.50
4a	80	166	9	0.80	1.12	3.15	2.70	2.50	1.20	1.50
12	160	148	6	0.41	0.67	2.76	2.63	2.20	0.75	0.80
14a	160	154	37	0.50	0.51	2.53	2.36	1.40	0.79	0.70
14b	160	250	37	0.69	0.81	2.92	2.60	2.60	1.30	1.40
1	40	89	4	0.84	1.18	3.19	2.68	1.50	0.70	0.80
4b	80	237	115	1.27	0.52	2.53	0.91	2.70	1.10	2.50
7	80	166	67	0.98	0.57	2.62	2.01	3.60	1.00	2.00
9	80	172	116	1.13	0.27	1.91	1.21	2.60	1.10	2.30
11	40	125	37	1.21	0.82	2.93	1.47	2.10	0.84	1.60
13	160	323	3	0.76	1.19	3.19	2.78	4.50	1.50	2.60
16	80	182	143	1.21	0.17	1.47	0.73	1.60	1.20	0.95

According to [21], the tests of [34] representing coherence with the monotone test are 4a, 12, 14a, and 14b. For this purpose, this study is restricted to taking into account only those tests.
The simulation results are given in Figures 3 and 4.

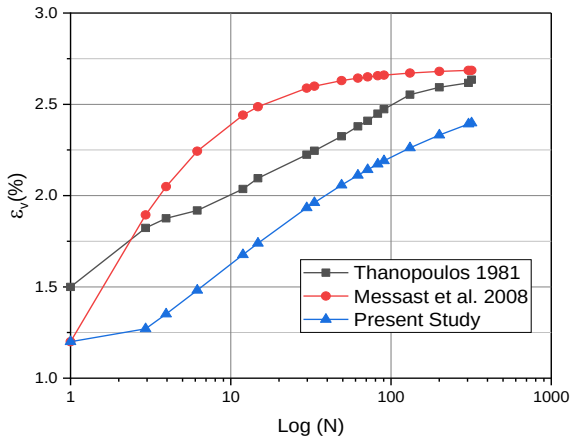


Figure 3: Curve ε_v^c versus log (*N*) for test 4a

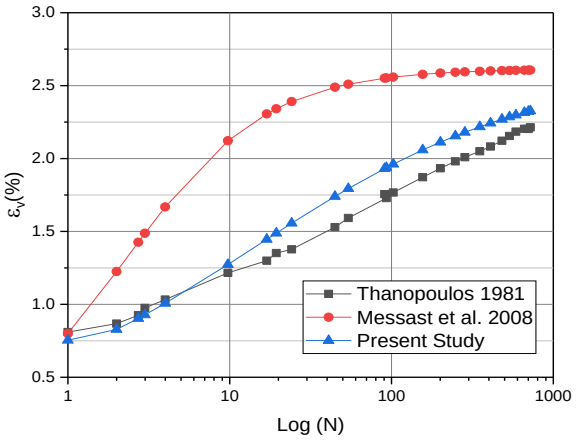


Figure 4: Curve ε_v^c versus log (*N*) for test 12

Figures 3 to 6 show a good improvement in the proposed model. The curves of this proposed formulation are similar to those measured by Thanopoulos [34].

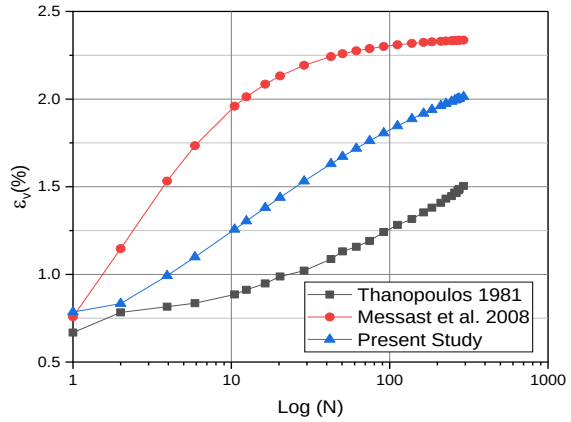


Figure 5: Curve ε_v^c versus log (*N*) for test 14a

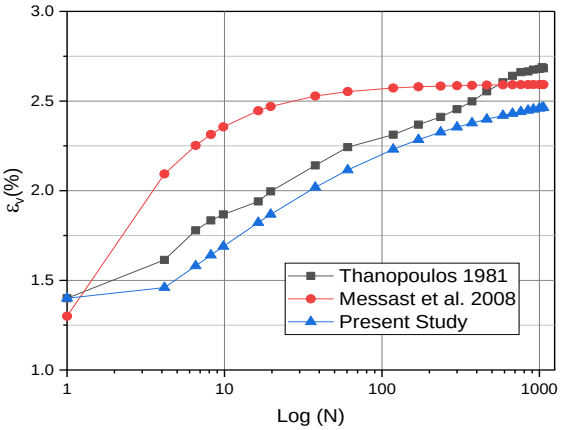


Figure 6: Curve ε_v^c versus log (*N*) for test 14b

5 Application to a Strip Foundation under Cyclic Loading

The numerical calculation of the settlement of shallow foundations using the PLAXIS package 2D version 8.2 is done after the calculation of the parameters q, p,$\Delta\eta$ and η_{av} of the model with the suggested formulations. In order to compare the ability of our numerical approach, we use the same set of parameters obtained from [21] and those found by the experimental test of [33].
The discretization was achieved by the triangular elements of 15 nodes to increase the accuracy of the calculation: the model has 964 triangular elements (average element size $255.64*10^{-3}$ m) and 7910 nodes.
In the experimental centrifuge test of [33], fine sand (poorly graded) was used with these parameters: ρ_s=2.66 g/cm^3, φ=32.8°, Ψ=3°, e_{max}=0.908, e_{min}= 0.575, Cu=2, d_{50}= 0.21. The centrifuge test of [33] was simulated by Plaxis2D with the following boundary conditions:
•Material model of sand: hardening soil model (HSM).
•Dimensions of the test container: Using the symmetry, only half of the sand was discredited (9.05 m, 7.30 m), knowing that the real dimensions of the prototype were: width = 18.1 m, height = 7.3 m (Figure 7).

• Average load σ_{av}= 89 kPa, amplitude σ_{ampl} = 75 kPa.
• Boundary conditions are taken into account by blocking horizontal displacements on vertical faces and blocking horizontal and vertical displacements for the lower boundary.

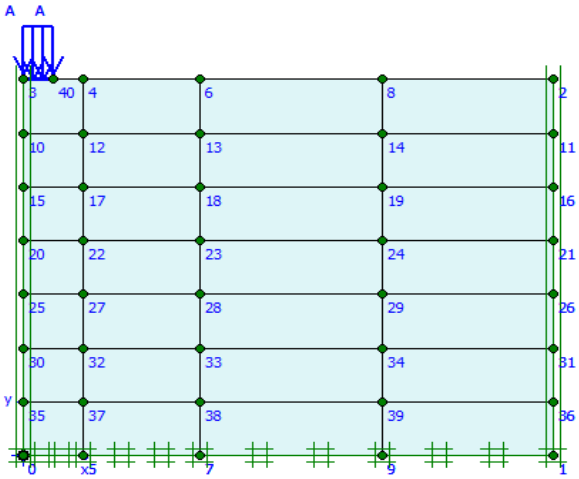


Figure 7: Geometry of the prototype (experimental test, height = 7.30 m, length = 9.05 m)

Table 2: Strip foundation dimension and material

width *b*(m)	**height *h*(m)**	**depth of embedding *t*(m)**	***γ*(g/cm^3)**	***E*(GPa)**	***ν***
1.00	0.60	0 m	2.70	25	0.30

The prototype of the sand studied in Plaxis is split into 28 clusters. After a charging cycle, stresses are created in each bunch whose stresses characterize the parameters of an equivalent triaxial test. The behavior of the global model will be defined by the application of cumulative volumetric strains of each cluster.

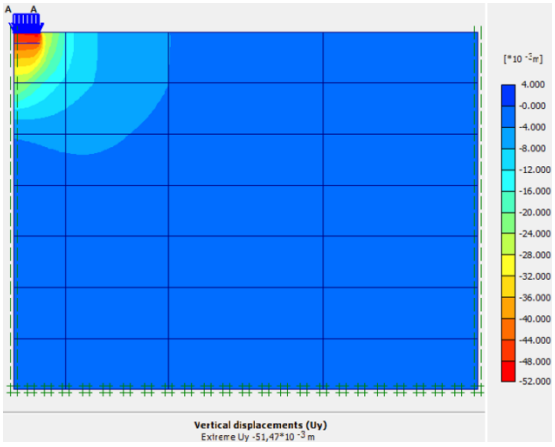


Figure 8: The shape and the field of settlement of the strip foundation (100 cycles)

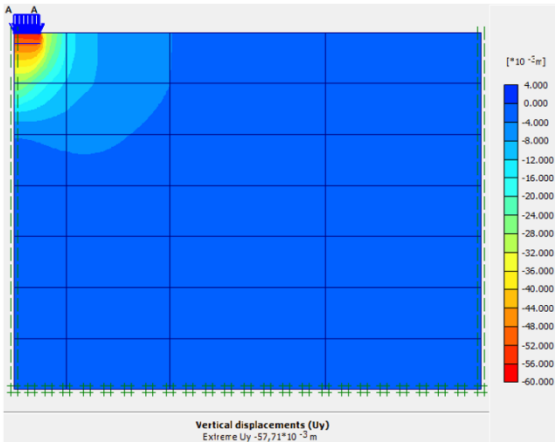


Figure 9: The shape and the field of settlement of the strip foundation (1000 cycles)

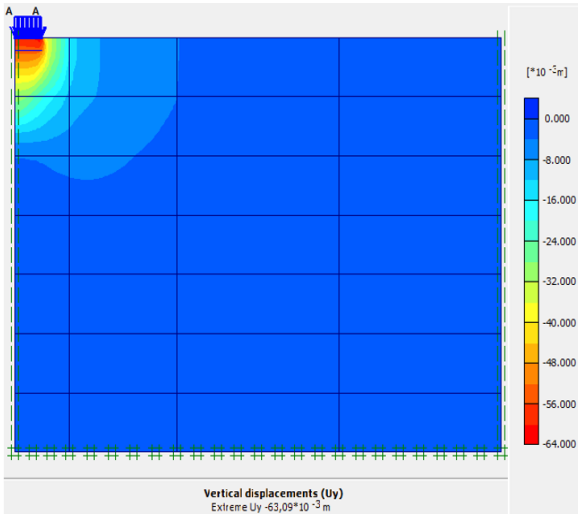


Figure 10: The shape and the field of settlement of the strip foundation (10000 cycles)

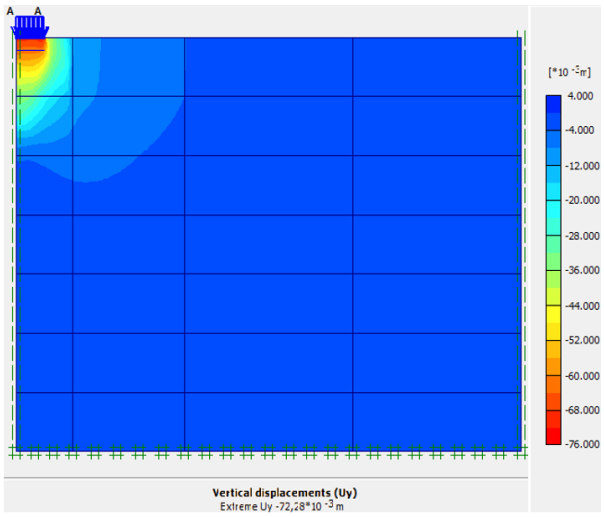


Figure 11: The shape and the field of settlement of the strip foundation (100000 cycles)

Figure 12 shows that the evolution of the settlement of the strip foundation presented in this research is very similar to the experimental curves of Helm et al. [33].

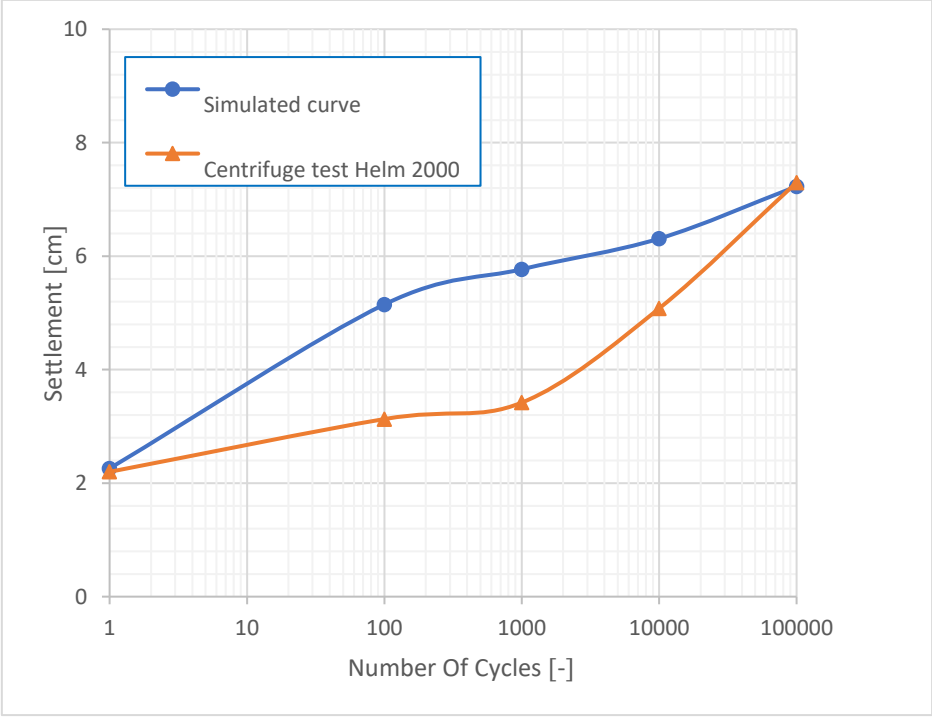


Figure 12: Comparison of the simulated curve with the centrifuge test [33]

6 Conclusion

This paper presents the modeling of the accumulation of volumetric deformations as a function of a large number of cycles. In this approach, the proposed model is based on the parameters of the first cycle (load-unload) and the cyclic parameters characterizing the average cyclic path. The calculation of the first cycle is achieved by a finite element calculation using the software

program Plaxis to simulate a triaxial test (load-unload). For this first cycle, the comportment of the material is described by the HSM model. The change of the volumetric deformations as a function of the number of cycles is formulated by a hyperbolic expression as a function of the number of cycles, the residual deformation after the first cycle, and the volumetric strain estimated at infinity.

The comparison of the results obtained with this formulation to the experimental curves presented by [30] shows a very good agreement, which confirms the reliability of the proposed model as well as its adaptation to this type of problem.

In the second part of this paper, the proposed model has been tested to determine the response of a strip foundation resulting from cyclic loading. The accumulation of deformation is detected at the center of the foundation.

Using the proposed method, a predicted settlement response equal to 7.30 cm of shallow foundation under large cyclic loading (after 10^5 cycles) was observed under the middle of the strip foundation [33]. The proposed approach predicts 7.23 cm. The comparison between the experimental values applied to a strip foundation [33] and those obtained by the proposed model confirms that the proposed model is well adapted to this type of problem.

As continuity of this work, we can cite the following perspectives:

- Examine the extension case.
- Apply this approach to clays.
- Generalize the approach to assess the response of geotechnical structures.
- Study of the cyclic response of interfaces.

References

[1] Gu, C., Wang, J., Cai, Y., Yang, Z. & Gao, Y. (2012). Undrained cyclic triaxial behavior of saturated clays under variable confining pressure. *Soil Dynamics and Earthquake Engineering*, *40*, 118-128.

[2] Wang, J., Cai, Y. & Yang, F. (2013). Effects of initial shear stress on cyclic behavior of saturated soft clay. *Marine Georesources & Geotechnology*, *31*(1), 86-106.

[3] Wang, J., Guo, L., Cai, Y., Xu, C. & Gu, C. (2013). Strain and pore pressure development on soft marine clay in triaxial tests with a large number of cycles. *Ocean Engineering, 74*, 125-132.

[4] Qian, J. G., Wang, Y. G., Yin, Z. Y. & Huang, M. S. (2016). Experimental identification of plastic shakedown behavior of saturated clay subjected to traffic loading with principal stress rotation. *Engineering Geology*, *214*, 29-42.

[5] Qian, J., Zhou, R., Chen, S., Gu, X., & Huang, M. (2018). Influence of pavement roughness on dynamic stresses in saturated subsoil subjected to moving traffic loading. *International Journal of Geomechanics*, *18*(4), 04018012.

[6] Guo, L., Cai, Y., Jardine, R. J., Yang, Z. & Wang, J. (2018). Undrained behaviour of intact soft clay under cyclic paths that match vehicle loading conditions. *Canadian Geotechnical Journal*, *55*(1), 90-106.

[7] Moussa, A., Salah, M. & Rafik, D.(2020). Improvement of a Hypoplastic Model for Granular Materials Under High-Confining Pressures. *Geotechnical and Geological Engineering* 38, 3761–3771. https://doi.org/10.1007/s10706-020-01256-y

[8] Min, G., Fukuda, D., Oh, S., Kim, G., Ko, Y., Liu, H., Chung, M., Cho, S.(2020). Three-Dimensional Combined Finite-Discrete Element Modeling of Shear Fracture Process in Direct Shearing of Rough Concrete–Rock. Joints. *Appl. Sci. 10*, 8033. https://doi.org/10.3390/app10228033

[9] Bellayoune, A., Messast, S. & Boulon, M. (2021). Numerical Modeling of the Degradation of the Normal Stress Under Large Number of Shear cycles. *Civil and Environmental Engineering Reports, vol.31*. (3), pp.118-133. https://doi.org/10.2478/ceer-2021-0037

[10] Sawicki, A., & Swidzinski, W. (1987). Compaction curve as one of basic characteristics of granular soils. In *4th colloque Franco-Polonais de mechanique des sols appliquee* (Vol. 1, pp. 103-115). Grenoble.

[11] Niemunis, A., Wichtmann, T. & Triantafyllidis, T. (2005). A high-cycle accumulation model for sand. *Computers and geotechnics*, *32*(4), 245-263.

[12] Marr, W. A., & Chrisitian, J. T. (1981). Permanent displacements due to cyclic wave loading. *Journal of the Geotechnical Engineering Division*, *107*(8), 1129-1149.

[13] Bouckovalas, G., Whitman, R. V. & Marr, W. A. (1984). Permanent displacement of sand with cyclic loading. *Journal of geotechnical engineering*, *110* (11), 1606-1623.

[14] Ullidtz, P. (1993). Mathematical model of pavement performance under moving wheel load. Transportation Research Record: Journal of the Transportation Research Board, 1384: 94–99.

[15] Gidel, G., Hornych, P., Chauvin, J. J., Breysse, D. & Denis, A. (2001). Nouvelle approche pour l'étude des déformations permanentes des graves non traitées à l'appareil triaxial à chargements répétés. *Bulletin de Liaison des laboratoires des ponts et chaussées*, *233*, 5-21.

[16] Chai, J. C. & Miura, N. (2002). Traffic-load-induced permanent deformation of road on soft subsoil. *Journal of geotechnical and geoenvironmental engineering*, *128*(11), 907-916.

[17] Wichtmann, T., Niemunis, A. & Triantafyllidis, T. (2005). Strain accumulation in sand due to cyclic loading: drained triaxial tests. *Soil Dynamics and Earthquake Engineering*, *25*(12), 967-979.

[18] Cai, Y., Chen, J., Cao, Z., Gu, C. & Wang, J. (2018). Influence of grain gradation on permanent strain of unbound granular materials under low confining pressure and high-cycle loading. *International Journal of Geomechanics*, *18*(3), 04017156.

[19] Ren, X. W., Xu, Q., Teng, J., Zhao, N. & Lv, L. (2018). A novel model for the cumulative plastic strain of soft marine clay under long-term low cyclic loads. *Ocean Engineering*, *149*, 194-204.

[20] Qian, J., Li, S., Gu, X. & Zhang, J. (2019). A unified model for estimating the permanent deformation of sand under a large number of cyclic loads. *Ocean Engineering*, *181*, 293-302.

[21] Messast, S., Boulon, M., Flavigny, E. & Labanieh, S. (2007). Modélisation constitutive du comportement cyclique des sables en condition drainée. In *Congrès français de mécanique*. AFM, Maison de la Mécanique, 39/41 rue Louis Blanc-92400 Courbevoie.

[22] Papon, A. (2010). Modélisation numérique du comportement des sols sous très grands nombres de cycles: homogénéisation temporelle et identification des paramètres (Doctoral dissertation). Ecole centrale de Nantes.

[23] Shariati, M., Hatami, H., Torabi, H. & Epakchi, H. R. (2012). Experimental and numerical investigations on the ratcheting characteristics of cylindrical shell under cyclic axial loading. *Structural Engineering and Mechanics*, *44*(6), 753-762.

[24] Dob, H., Messast, S., Boulon, M. & Flavigny, E. (2016). Treatment of the high number of cycles as a pseudo-cyclic creep by analogy with the soft soil creep model. *Geotechnical and Geological Engineering*, *34*(6), 1985-1993.

[25] Dob, H., Messast, S., Mendjel, A., Boulon, M. & Flavigny, E. (2016). Behavior of sand after a high number of cycles application to shallow foundation. *International Journal of Civil Engineering, 14*(7), *459-465.*

[26] Amrane, M., Messast, S. (2018) Modeling the Behavior of Geotechnical Constructions Under Cyclic Loading with a Numerical Approach Based on J. Lemaitre Model. *Indian Geotech J*, 48**,** 520–528. https://doi.org/10.1007/s40098-017-0275-1

[27] Mohamed, F. M., Vanapalli, S. K. & Saatcioglu, M. (2013). Generalized Schmertmann Equation for settlement estimation of shallow footings in saturated and unsaturated sands. *Geomechanics and Engineering*, *5*(4), 343-362.

[28] Kim, T. H. & You, S. H. (2015). Settlement analysis considering sand mat induced initial settlement in soft ground improved by pbd. *International Journal of Civil Engineering*, *13*(2), 146-152.

[29] Lotfizadeh, M. R. & Kamalian, M. (2016). Estimating bearing capacity of strip footings over two-layered sandy soils using the characteristic lines method. *International Journal of Civil Engineering*, *14*(2), 107-116.

[30] Amrane, M.(2018). *Modélisation du Comportement des Ouvrages Géotechniques avec la Loi de J. Lemaitre, PhD thesis*. University of Skikda, Algeria. http://ftech.univ-skikda.dz/doc2/2018/THESE%20Amrane%20Moussa.pdf

[31] Bellayoune, A. & Messast, S. (2021). Estimation of pile displacements anchored in sand after a large number of cycles. *Selected Scientific Papers-Journal of Civil Engineering*, *16*(1), 35-44.

[32] Wichtmann, T. (2005). *Explicit accumulation model for non-cohesive soils under cyclic loading*. Doctoral dissertation, Inst. Für Grundbau und Bodenmechanik).

[33] Helm, J., Laue, J. & Triantafydillis, T. (2000). Untersuchungen an der RUB zur Verformungsentwicklung von Bödenunterzyklischen Beanspruchungen. *Bödenunter fast zyklischer Belastung: Erfahrungen und Forschungsergebnisse*, 109-133.

[34] Thanopoulos, I. (1981). Contribution à l'étude du comportement cyclique des milieux pulvérulents (Doctoral dissertation). Université Scientifique et médicale & l'institut national polytechnique de Grenoble.