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A STUDY OF THE SEDIMENTATION AND STORAGE CAPACITY DEPLETION OF A RESERVOIR

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Abstract

Theoretical and empirical equations have previously been proposed to compute the sediment trap efficiency of a reservoir. The empirical equations were developed by analyzing a limited amount of field data for reservoirs in selected areas; thus, their suitability is restricted to reservoirs in regions with similar hydrological characteristics. In this study, a sediment trap-efficiency equation, which accounts for the dynamic behavior of sedimentation and the hydraulic conditions of a reservoir, was deduced from the principle of mass conservation. Using this equation, the approximate analytical expression for the storage capacity of the reservoir was obtained. Both equations were applied to investigate real-life problems involving the impacts of reservoir sedimentation, and their predicted solutions were compared with the field data and the results of the theoretical and empirical equations. The results of this investigation reveal that the proposed equations are capable of simulating the sediment trapping and storage capacity depletion characteristics of large reservoirs, which are a measure of the adverse effect of sedimentation on the performance of hydraulic structures. These equations can potentially be incorporated into a large-scale watershed model for estimating a sediment budget.

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Key words

- Trap efficiency,
- Reservoir sedimentation,
- Sediment yield,
- Blue Nile,
- Suspended sediment,
- Sediment-rating curve.

1 INTRODUCTION

Reservoir sedimentation is the gradual process of the accumulation of sediment in a man-made lake. In many parts of the world, sedimentation causes serious problems concerning water management, flood control, and the production of energy. As noted by Auel and Boes (2012) and Schleiss et al. (2010), the worldwide mean annual loss of the storage capacity of reservoirs due to sedimentation is higher than the increase in storage volume from the construction of new reservoirs. It is estimated that between 0.5-1% of the total storage capacity is lost annually (White, 2019). Furthermore, the time rate of the sedimentation in reservoirs varies with their design and the magnitude of their sediment load. Generally, smaller reservoirs lose their gross capacity at higher rates (Heinemann, 1984; Wisser et al., 2013). The results of numerical models showed that 4 to 5 Gt of sediment is trapped by reservoirs annually (Vörösmarty et al., 2003) compared to the Anthropocene-based global sediment load of 12.8 Gt/y (Syvitski and Kettner, 2011). It is obvious that large increases or decreases in such loads can change the dynamic balance of river systems and delicate ecosystems.

A simplified theory of the trap efficiency of a sedimentation basin was first developed by Hazen (1904). He considered the gravitational settling of sediment particles under different hydraulic conditions and obtained an expression for the

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trap efficiency based on residence time, settling velocity, and particle size. Since then, methods to predict reservoir sedimentation have been the theme of several empirical studies. These studies considered trap efficiency as the most informative descriptor of reservoir sedimentation. The trap efficiency of a reservoir depends on many factors such as the capacity and geometric shape of a reservoir, the inflow discharges of sediment and water, the method of operation, and the settling velocity (Brune, 1953; Morris and Fan, 1998; Verstraeten and Poesen, 2000; Xiaoqing, 2003). Many empirical equations have been proposed, as part of the solutions to complex sedimentation problems, to predict the trap efficiency of reservoirs in temperate environments (Espinosa-Villegas and Schnoor, 2009; Heinemann, 1984; Verstraeten and Poesen, 2000). Brown (1943) was the earliest researcher to develop a curve that shows the relationship between trap efficiency and the ratio of the reservoir capacity to a watershed drainage area. A relatively better approach was later presented by Churchill (1948), who employed the ratio of the period of retention to the mean flow velocity through the reservoir as the sedimentation index and related it to the trap efficiency. To some extent, Churchill's approach considered the flow dynamics of a reservoir. For small dry or semi-dry reservoirs with bottom outlets, however, it is difficult to estimate the Churchill sedimentation index as these outlets only operate during infrequent short-lasting events (Dendy, 1974). By using the data of Churchill and those added by Borland (1971), Bube and Trimble (1986) modified the Churchill curves by applying a smoothing spline procedure. As noted by Brune (1953), reservoirs with the same watershed area could have different inflow characteristics and hence different trap efficiencies. Alternatively, he proposed the ratio of the reservoir capacity to the annual inflow and correlated it to the trap efficiency of normal ponded reservoirs. The two different parameters used by Brown and Brune provide only a limited description of the watershed hydrology and reservoir geometry;they are independent of the sediment characteristics. Compared to Brune's approach, the more descriptive Churchill method may provide a better result. The trap-efficiency curves of Brune were further investigated by Dendy (1974), Gill (1979), Heinemann (1981), Lewis et al. (2013), and Siyam et al. (2001). They developed empirical and semi-theoretical equations, which reasonably represented the curves. However, the weighting factor used by Lewis et al. (2013) for estimating the overall trap efficiency was not derived from the daily volumes of sediment transported into the reservoir. The application of the empirical methods described above is restricted to reservoirs in regions with similar hydrological conditions, as noted by Verstraeten and Poesen (2000).

Theoretical models based on the mechanics of sediment deposition were also applied to predict reservoir sediment trap efficiency. Camp (1946) developed a theoretical model based on the simplifying assumptions of a quiescent flow, a constant concentration of suspended particles at a vertical section near the inlet end, and no resuspension of any settled particles. For highly turbulent open-channel flow conditions, Chen (1975) extended Camp's model. He further investigated the applicability of the extended model and compared his results for turbulent flows with the results of the empirical methods of Brune and Churchill. Karaushev (1966) derived an analytical equation for predicting the trap efficiency of small reservoirs. He accounted for the effects of the hydraulic characteristics of a flow and the sediment transported. A comparison of his equation with the Brune trap-efficiency curves clearly reveals the inadequacy of Brune's empirical generalization. For a reservoir having a small annual inflow relative to its storage volume, however, the accuracy of Karaushev's equation is questionable. Following Camp's approach, Curtis and McCuen (1977) developed a theoretical model based on the sedimentation of a small idealized settling tank. They used the model to examine the effects of the distribution of the sediment sizes, initial basin storage, outflow discharge, and basin depth on the trap efficiency of a fictitious pond. An overview of theoretically-based trap-efficiency models for variable discharge conditions was provided by Verstraeten and Poesen (2000).

Numerical approaches have also been used for a detailed analysis of flow-sediment interactions in a reservoir. Dhamotharan et al. (1981) developed a one-dimensional (1D) unsteady convective-diffusive transport model to predict the vertical concentration profile and deposition rate of suspended sediment. Their model is capable of computing the trap efficiency as a function of a reservoir's Peclet number and time. Toniolo et al. (2007) systematically examined the effects of turbidity currents on trapping efficiency using a simplified 1D numerical model that captures the dynamics of an internal muddy pond. By using soft computing data-driven techniques, Garg and Jothiprakash (2010) estimated the trap efficiency of a reservoir. Using a similar approach, Parsaie et al. (2018) also computed the trap efficiency of a retention dam. The results of both studies were verified with the measurements. Compared to empirical methods, numerical models need a large volume of data for their calibration and validation. This raises a practical question about the extent of evaluating the tasks of a model's performance (see Bennett et al., 2013). For a newly constructed reservoir, however, the data necessary for the model's construction are not available. Hence, one could prefer empirical or theoretical methods as a means of studying the effects of reservoir sedimentation, thus providing insight into the dynamics of watersheds.

The current study is aimed at developing a reservoir sediment trap-efficiency equation to overcome the aforementioned drawbacks. The proposed equation is capable of describing the dynamics of reservoir sedimentation and does not require a large volume of suspended sediment concentrations and flow rate data for calibration. This equation is applied to investigate different cases of reservoir sedimentation, and its performance is assessed by comparing its results with the field data and the results of theoretical and empirical methods. It is apparent that accurate quantification of sediment trapping in reservoirs leads to a reasonable prediction of storage capacity losses. Using the trap efficiency equation, an approximate analytical expression for estimating such losses is proposed. Besides, a set of design charts is presented to assist an evaluation of the remaining capacity of the reservoir. The prediction of the sediment trapping and storage capacity depletion characteristics of a reservoir using such equations allows practicing engineers to plan the daily operation and maintenance of a reservoir through dredging or sluicing operations. In the following sections, the derivation of the equations is briefly discussed.

2 MODELING TRAP EFFICIENCY

The sediment mass-balance equation for a reservoir can be expressed as

$$V\frac{dC}{dt} = C_i Q - CQ - w_s A_s C, \tag{1}$$

where V is the volume of the reservoir; C_i is the depth-averaged inflow sediment concentration; Q is the discharge; C is the depth-averaged concentration of sediment in the reservoir; w_S is the settling velocity of sediment; t is the time; and A_S is the surface area of the reservoir. The above equation assumes that the sediment in the reservoir does not interact with the surrounding environment. After dividing both sides of the equation by V and rearranging the terms, the following expression is obtained:

$$\frac{dC}{dt} + \Gamma_0 C = \frac{\Gamma_1}{V}, \tag{2}$$

$$\Gamma_0 = \frac{Q + w_s A_s}{V} = \frac{Q}{V} + \frac{w_s}{H_R}, \tag{3}$$

$$\Gamma_1 = C_i Q, \tag{4}$$

where $H_R(=V/A_S)$ is the mean depth of the reservoir. For a quasi-steady flow situation, the inflow discharge can be described by a step function. Using this approximation and imposing an initial condition of $C = 0$ at $t = 0$, Eq. (2) is integrated by applying an integrating factor method (Polyanin and Manzhirov, 2007), which results in

$$C = \frac{\Gamma_1}{\Gamma_0 V}\left(1 - \exp\left(-\Gamma_0 t\right)\right). \tag{5}$$

The trap efficiency of the reservoir represents the fraction of the incoming sediment that is retained in a reservoir and is expressed as

$$\eta = \frac{q_{si} - q_{so}}{q_{si}}, \tag{6}$$

where q_{si} and q_{so} are the inflow and outflow sediment discharges, respectively, and η is the reservoir sediment trap efficiency. Substituting Eq. (5) into Eq. (6) and simplifying the resulting expression gives

$$\eta = 1 - \left(\frac{1 - \exp\left(\frac{-1 - \Omega\sigma}{\Omega}\right)}{1 + \Omega\sigma}\right), \tag{7}$$

$$\Omega = \frac{V}{Qt} = \frac{V}{I}, \tag{8}$$

$$\sigma = \frac{w_s t}{H_R}, \tag{9}$$

where I is the inflow volume; Ω is the ratio of the reservoir capacity to the inflow volume; and σ is a sedimentation parameter, which is essentially governed by the dynamics of sedimentation in the reservoir. The above equation can be applied to estimate the trap efficiency of a reservoir with a shorter hydraulic residence time due to high intra-annual flow variability. As reported by Dey et al. (2019), several theoretical and empirical equations are available to estimate the settling velocity of natural sediment particles. In this study, a simple but accurate settling velocity equation, which was proposed by Cheng (1997a,b), is employed.

3 RESERVOIR CAPACITY EQUATION

The life of a reservoir primarily depends on its storage capacity loss caused by sediment deposition. The change in a reservoir's capacity over time due to sedimentation is given by

$$\frac{dV}{dt} = -\frac{q_{si}}{\bar{\gamma}}\eta, \tag{10}$$

where $\bar{\gamma}$ is the time-averaged sediment density. The variation of the sediment density over time is approximated by the Lane and Koelzer (1943) equation as follows:

$$\gamma(t) = \gamma_1 + K\log(t) = \gamma_1 + 0.4343K\ln(t), \tag{11}$$

where $\gamma(t)$ is the density of the sediment; γ_1 is the density of the sediment deposited at the end of the first year of filling the reservoir; and K is the compaction factor. The recommended values of γ_1 and K for various reservoir operational conditions can be found in Lane and Koelzer (1943) and USBR (2006). Following the Miller (1953) approach, the time-averaged sediment density at the end of the operating period T becomes

$$\bar{\gamma} = \left(\frac{1}{T-1}\right)\int_1^T \gamma(t)\,dt = \gamma_1 + 0.4343K\left(\frac{T}{T-1}\ln(T) - 1\right). \tag{12}$$

The above equation is used here to simplify the integration of Eq. (10). Inserting Eq. (7) into Eq. (10) and rearranging similar terms together gives

$$\frac{dV}{dt} = \frac{-q_{si}}{\bar{\gamma}}\left(\frac{(\Gamma_3 - 1)\exp\left(\frac{\Gamma_3}{\Omega}\right) + 1}{\Gamma_3 \exp\left(\frac{\Gamma_3}{\Omega}\right)}\right), \tag{13}$$

$$\Gamma_3 = 1 + \Gamma_2 = 1 + \Omega\sigma. \tag{14}$$

The value of Γ_3 depends on the nature of the reservoir sedimentation problem. For the case of low sediment gravitational settling, $\Gamma_2 \to 0$ and hence $\Gamma_3 \cong 1$. The exponential expression on the right-hand side of Eq. (13) is simplified by using the second-order expansion of the Maclaurin series (Polyanin and Manzhirov, 2007) as follows:

$$\frac{dV}{dt} = \frac{-q_{si}}{\bar{\gamma}}\left(\frac{\Gamma_3 V^2 + I\Gamma_3^2 V + 0.5I^2\Gamma_3^3}{(\Gamma_3 - 1)\left(V^2 + I\Gamma_3 V + 0.5I^2\Gamma_3^2\right) + V^2}\right)^{-1}. \tag{15}$$

Integrating the above equation from T_0 to T and substituting the conditions that at $t = T_0$, $V = V_0$ and at $t = T$, $V = V$, the following equation is obtained:

$$V - V_0 + \frac{I}{2}\left(\ln\left(\frac{V^2 + (\Gamma_3 - 1)\left(IV + 0.5I^2\Gamma_3\right)}{V_0^2 + (\Gamma_3 - 1)\left(IV_0 + 0.5I^2\Gamma_3\right)}\right)\right) = -\frac{q_{si}}{\bar{\gamma}}(T - T_0). \tag{16}$$

Dividing both sides of Eq. (16) by $-V_0$ yields non-dimensional expressions as follows:

$$1 - V_R - \frac{1}{2\beta}\left(\ln\left(\frac{2\beta^2 V_R^2 + 2\beta(\Gamma_3 - 1)V_R + \Gamma_3(\Gamma_3 - 1)}{2\beta^2 + 2\beta(\Gamma_3 - 1) + \Gamma_3(\Gamma_3 - 1)}\right)\right) = \Psi, \tag{17}$$

$$\Psi = \frac{q_{si}}{V_0\bar{\gamma}}(T - T_0), \tag{18}$$

$$\Gamma_3 = 1 + \Omega\sigma = 1 + \sigma\beta V_R, \tag{19}$$

where V_R is the ratio of the reservoir capacity at any time T to the original volume, and β is the ratio of the original volume to the inflow volume. Eq. (17) is a generalized expression for estimating the remaining capacity of the reservoir after T years of operation. For a given sediment inflow discharge, initial reservoir volume, and time-averaged sediment density, Eq. (18) is used to predict the dimensionless sediment discharge parameter Ψ. Using this value, Eq. (17) is solved iteratively for the normalized remaining capacity of the reservoir. In the next section, the suitability of this equation for reservoir sedimentation problems will be examined.

4 APPLICATIONS

In this section, the results of the application of Eqs. (7) and (17) to real-life problems are presented and discussed. To establish an appropriate level of confidence in their performance, Eq. (7) will first be calibrated, and then both equations will be validated by comparing their results with the field data for the sediment trap efficiency and storage capacity of a reservoir. Reservoirs with different storage capacities and inflow volumes from various river basins are considered to evaluate the predictions of these equations. These reservoirs are situated in regions having different hydrological characteristics. Additionally, the impact of sedimentation on the reservoir capacity of the Grand Ethiopian Renaissance Dam is investigated by considering two long-term sedimentation scenarios.

4.1 Sediment Trap Efficiency

Equation (7) was calibrated using the sediment trap-efficiency data of Brune (1953) and Kira (1978). Fig. 1 shows the results of the calibration. As discussed before, σ is not a universal physical constant. It depends on the hydraulic and sediment characteristics of the reservoir. Using a non-linear optimization technique which utilizes the generalized reduced gradient method (Lasdon et al., 1974), the errors between the predicted trap efficiencies and the measured data for a normal ponded reservoir were minimized. The coefficient of determination (R^2) and the root mean square error for the best-fit curve (σ = 74.0) were 0.93 and 7.62%, respectively. As the flow in a semi-dry reservoir is very turbulent, the Brune data for this reservoir were well fitted by Eq. (7) for σ = 0.51. In addition, the envelope around Brune's mean curve was described well by this equation with σ values of 19.9 and 136.9 for the lower and upper envelope curves, respectively.

The performance of Eq. (7) was further examined by comparing its results with the results of previous equations proposed by Karaushev (1966) and Gill (1979). The sediment trap-efficiency equation of Karaushev for small reservoirs reads as:

$$\eta = 1 - \left((1-\Omega) \exp\left(\frac{-\Omega\varphi}{1-\Omega} \right) \right), \tag{20}$$

$$\varphi = \frac{w_s T_s}{H_R}, \tag{21}$$

where T_S is the duration of the spill-over period in seconds. For primarily highly flocculated and coarse-grained sediments, Gill (1979) introduced the following trap-efficiency equation:

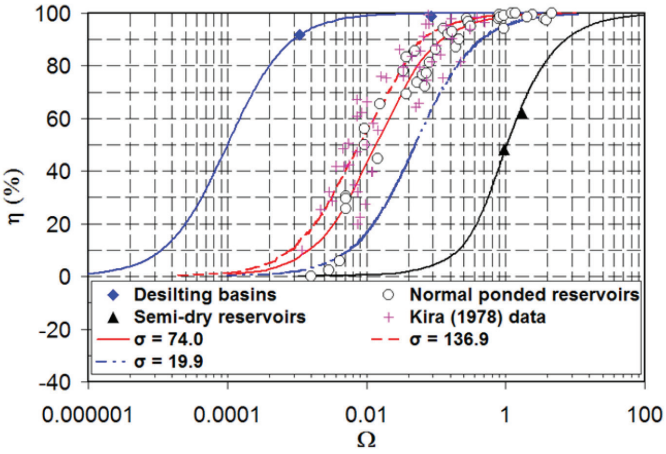


Fig. 1 *Comparison of the calibration results of the proposed equation for reservoir trap efficiency with the field data of Brune (1953) and Kira (1978). The Brune data are depicted by three different geometric symbols, namely a diamond, circle, and triangle*

$$\eta = \frac{\Omega^2}{0.9947\,\Omega^2 + 62.97\times10^{-4}\Omega + 3\times10^{-6}}. \tag{22}$$

Figure 2 depicts a comparison of the results of these equations with the field data (Li et al., 2018; Tan et al., 2019). The data encompass the trap efficiency of small and large reservoirs in the major river basins of China. The active storage capacities of these reservoirs vary from 10 to 165000 Mm^3. As can be seen, the results of Eq. (7) agree reasonably well with the measured data. Compared to the results of the Gill equation, the performance of the Karaushev equation is not satisfactory, especially for the higher values of the reservoir capacity to the inflow ratio ($0.025 < \Omega < 1.0$). The computed mean absolute relative errors for Eq. (7) and those of the Gill and Karaushev equations are 10.5, 11.8 and 15.8%, respectively. This implies that Eq. (7) is significantly more accurate than the Karaushev equation and is slightly more accurate than the Gill equation, in addition to providing a better description of the sediment trapping characteristics.

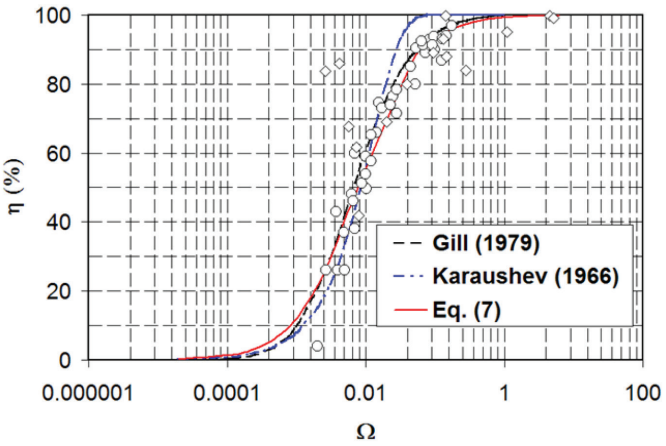


Fig. 2 *Variation of the sediment trap efficiency η with the reservoir capacity to the inflow ratio Ω. The field data of Li et al. (2018) and Tan et al. (2019), which are indicated by the geometric symbols of a diamond and circle, respectively, were used to validate the numerical results*

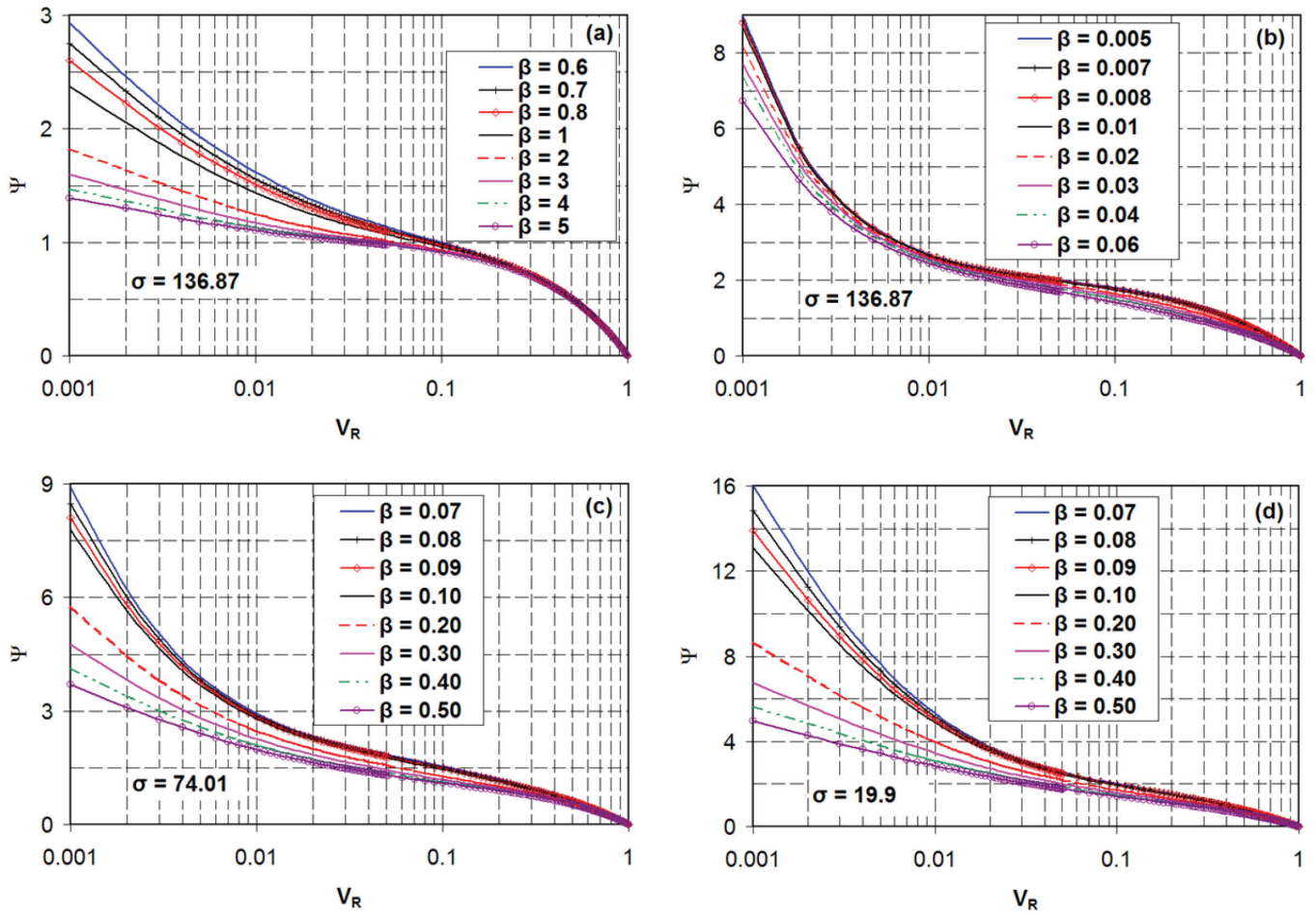


Fig. 3 *Variations of the normalized total sediment inflow* Ψ *with the normalized remaining capacity of the reservoir* V_R *for various values of the initial reservoir capacity to the inflow volume ratio* β*. The calibrated values of* σ *were used to develop the charts*

4.2 Storage Capacity of a Reservoir

4.2.1 Design charts for storage capacity prediction

Equation (17) is a non-linear algebraic equation and requires an iterative numerical procedure for its solution. The iterative procedure can be avoided by producing suitable charts that include the entire range of practical interest. The charts are prepared by plotting the normalized total sediment inflow rate Ψ against the dimensionless remaining capacity of the reservoir V_R for different values of σ and β. Fig. 3 shows the charts developed for typical values of β ranging from 0.005 to 5. For the long-term mean values of sediment discharge and inflow volume, the charts can be used to estimate the capacities of the reservoir after different periods of operation. Using a similar procedure, the useful life of a reservoir is easily predicted by considering various scenarios of reservoir sedimentation. For a given depletion volume of a reservoir, the chart can be used to determine Ψ, from which the required T can be computed.

4.2.2 Verification of the numerical results

The accuracy of Eq. (17) was examined by analyzing the storage capacities of six reservoirs in Kansas, USA, and comparing the predictions with the field data and the results of the Gill (1979, 1988) equation. The following empirical equation was proposed by Gill for fine-grained sediment:

$$1.0266\left(1-V_R\right)+\frac{262.1\times10^{-4}}{\beta}\ln\left(\frac{1}{V_R}\right)+\frac{133\times10^{-6}}{\beta^2}\left(1-\frac{1}{V_R}\right)$$
$$-\frac{5.1\times10^{-7}}{\beta^3}\left(1-\frac{1}{V_R^2}\right)=\frac{q_{si}T^{1-n}}{\gamma_1 V_0\left(1-n\right)}, \tag{23}$$

where n is an exponent for the sediment density equation introduced by Gill (1988) and is given by

$$n=0.5\log\left(1+\frac{2K}{\gamma_1}\right). \tag{24}$$

The selected reservoirs were built between 1948 and 1977, and their original capacities at the conservation-pool elevation vary from 34 to 524 Mm³. For some of these reservoirs, the volume-based sediment accumulation and yield values were estimated by analyzing bathymetry data (e.g., Kansas Biological Survey, 2011, 2014; Mau, 2001). In addition, continuous streamflow and turbidity data were collected at the gaging stations upstream and downstream from the reservoirs for estimating the total suspended sediment loads delivered to and released from the reservoirs as well as quantifying the capacity losses due to sedimentation (Juracek, 2011, 2013; Lee et al., 2008; Lee and Foster, 2013; Stone et al., 2015). In this study, the results of both methods were used to examine the performance of the proposed equation. Tab. 1 summarizes the results of the analysis. It is important to note that the dimensionless storage

Tab. 1 *Comparison of the estimated dimensionless storage capacities with the measured data for selected reservoirs in Kansas, USA*

Reservoir	Year built	V_0 (Mm^3)	Year of capacity survey	Measured V_R (%)	V_R (%) of Eq. (17)	V_R (%) of Eq. (23)
Cheney	1965	206	1998	96	97	97
Clinton	1977	159	2012	91	92	92
John Redmond	1964	126	2014	62	60	66
Kanopolis	1948	90	2010	66	70	70
Toronto	1960	34	2010	60	62	64
Tuttle Creek	1962	524	2010	57	57	57

capacities were estimated as of the years listed in Column 4 of Tab. 1. The dimensionless capacities of the reservoirs predicted by the two equations were slightly different, with mean absolute relative errors of 2.6 and 3.6% for Eqs. (17) and (23), respectively. This suggests that the accuracy of the proposed equation is acceptable for analyzing the impact of sedimentation on the capacity of the reservoirs after years of operation. The estimated annual loss rates of the reservoirs decrease as the size of the reservoir increases. A similar general trend for annual capacity loss has been noted by Crowder (1987).

4.3 The Grand Ethiopian Renaissance Dam

4.3.1 Description of the study area

The Grand Ethiopian Renaissance Dam (GERD) is currently under construction on the Upper Blue Nile (Abbay) River approximately 750 km northwest of Addis Ababa and 20 km upstream of the Ethiopian–Sudanese border. The GERD is a roller-compacted concrete dam with a structural height of 145 m, and is complemented by a concrete-faced rockfill saddle dam approximately 5000 m long and 50 m high. The reservoir has a surface area of 1874 km^2 at a full supply level of 640 m above sea level (a.s.l.) and is classified as a very large reservoir according to Graf (2005). The total storage capacity of this single-purpose reservoir is 74 billion cubic meters (BCM) with an active storage volume of 59.2 BCM. Further details of the technical specifications of the dam can be found in IPoE (2013). The locations of the GERD and other proposed dam sites are shown in Fig. 4.

In the Upper Blue Nile River Basin (UBNRB), the predominant soil types are leptosols, alisols, nitosols, and vertisols. The texture of most of the soils consists of clay, clay loam, silt clay, and loam (BCEOM, 1999). Slopes in the basin range from as steep as 45% in the eastern part of the area

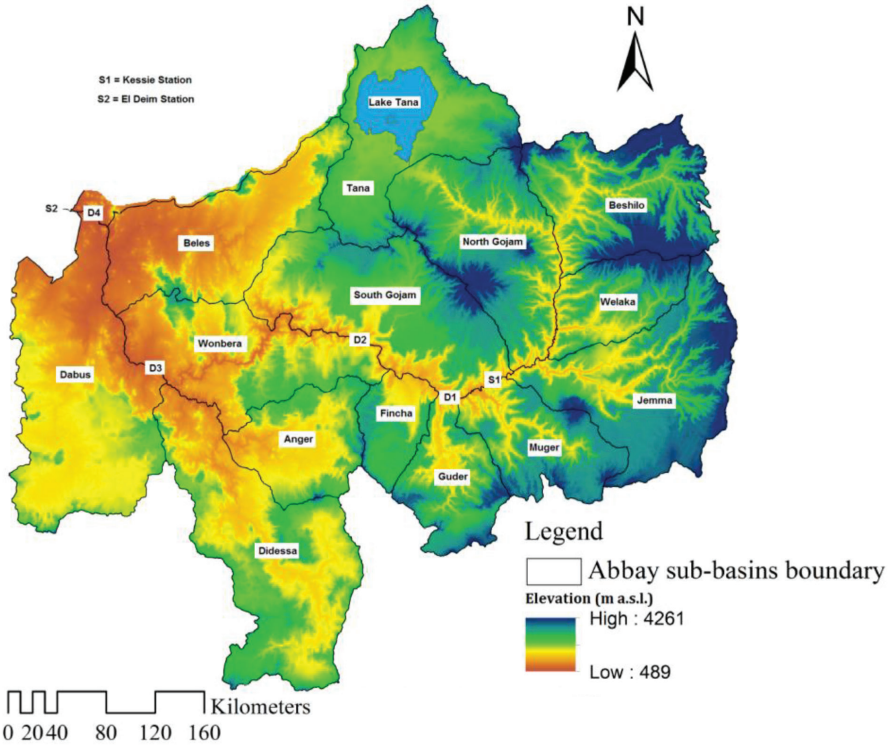


Fig. 4 *The Upper Blue Nile River Basin and its sub-basins. The D1 to D4 symbols on the map indicate the locations of the Karadobi, Beko-Abo, Mandaya, and Grand Ethiopian Renaissance dams, respectively. The first three dams are proposed projects for hydropower generation*

Tab. 2 *Summary of the details of the selected streamflow gaging stations in the UBNRB*

Station name and number	Location		Catchment area (km²)	Percentage of the GERD catchment area
	Longitude (deg)	Latitude (deg)		
Anger (114002)	36.52	9.43	4,674	2.71
Didessa near Arjo (114001)	36.42	8.68	9,981	5.79
Guder (113005)	37.75	8.95	524	0.30
Tato (114010)	36.65	9.03	43	0.02
Teme at Motta (112030)	37.98	10.42	156.3	0.09
Blue Nile at Kessie (112001)	38.18	10.07	65,784	38.19
Blue Nile at Border (116002)	34.96	11.23	172,254	Downstream of GERD
El Deim (900012)	34.98	11.23	178,200	Downstream of GERD

to nearly level in Sudan with an average of about 4%. Land use is mostly agricultural with grasslands and croplands accounting for about 23 and 34% of the basin, respectively (BCEOM, 1998). Woodland, bushland, and shrubland cover 30% of the basin. Urban areas account for less than 1% of the basin.

The mean annual rainfall over the UBNRB varies from 800 to 2000 mm and increases with the altitude. The rainfall distribution is highly variable on both the spatial and temporal scales. Most of the annual precipitation is received during the wet season (June to September). Steep slopes and a lack of vegetative cover result in relatively high rates of sediment delivery to the main rivers. A recent study (Gebremicael et al., 2013) showed an increasing trend of soil erosion in the basin due to land-use/land-cover changes over the past 28 years.

4.3.2 Sediment-rating curves

Streamflow and suspended sediment data for the UBNRB were obtained from the Hydrology Department of the Ministry of Water, Irrigation, and Electricity, Ethiopia. The quality of the data from the selected gaging stations (see Tab. 2) was thoroughly examined, and some of the missing data were filled in with site-specific regression equations between neighboring stations. Because of the scarcity of long-term suspended sediment and bed-load data, the mean annual load was estimated by combining the short-term sediment-rating curve with a long-term flow-duration curve (Miller, 1951). A preliminary analysis of the streamflow and sediment concentration data at the Kessie gaging station indicated that a physically meaningful relationship between the two parameters could be obtained by developing sediment-rating curves for the wet and dry seasons of the water year. Accordingly, two logarithmically transformed data sets were developed and then analyzed using a linear regression approach. A statistical bias correction factor (Ferguson, 1986) was applied to improve the accuracy of the results. Fig. 5 depicts the suspended sediment concentration and sediment discharge rating curves. As expected, the sediment discharge relationships show significantly higher correlation coefficients than does the sediment concentration ($0.72 \leq R^2 \leq 0.98$). Additionally, flow-duration curves for a 30-year record (1973–2005) at this gaging station were developed, and sediment ratings were applied to them. For the intermediate drainage area between Kessie station and GERD, the average specific sediment yields of the sub-basins such as Beles, Dabus, Didessa, etc., were estimated using sediment concentration and daily streamflow (with a record length of 10 to 20 years) data from their gaging stations. Combining the contribution of these sub-basins and the sediment load at the Kessie station yields a mean annual suspended sediment inflow of 157.6 Mt/y at GERD. Due to the lack of measured data, the bed load can be estimated as a percentage of the suspended sediment load. The UBNRB is characterized by different topographic conditions, which range from flat plains to steep areas of the eastern part of the basin. Therefore, an allowance of 20% of the suspended sediment load was made for the bed-load component, giving a long-term mean annual total sediment load of 189.1 Mt. This is equivalent to 1098 t/km²/y when averaged over the total area of the dam catchment and is slightly higher than the instantaneous-sediment load measured at the El Deim station during the wet season of 2009 (Ali et al., 2014). As noted by Vanmaercke et al. (2012), sediment load estimates based on short measuring periods (less than 5 years) are more likely to underestimate the actual long-term mean annual load.

The current results were compared with the results of previous studies (e.g., Ali et al., 2014; Betrie et al., 2011; Easton et al., 2010; Gebremicael et al., 2013), which applied the Soil and Water Assessment Tool (SWAT) model to predict the annual sediment load from the UBNRB. These studies reported different results of the suspended sediment yield, varying from 117 to 147 Mt/y, at the basin outlet. The soil erosion rate is computed in the SWAT model using the Modified Universal Soil Loss Equation [MUSLE] (Williams, 1975). As an empirically-based equation, the MUSLE does not accurately simulate the soil erosion processes in the Ethiopian Highlands, where the contribution of sediment flux by gully erosion is significant. Consequently, the earlier studies underestimated the sediment yield. The trend of the sediment flux at the Kessie station was also examined, revealing that the sediment load was increased at an annual rate of 1.5% between 1989 and 2009. The results obtained are consistent with the trend analy-

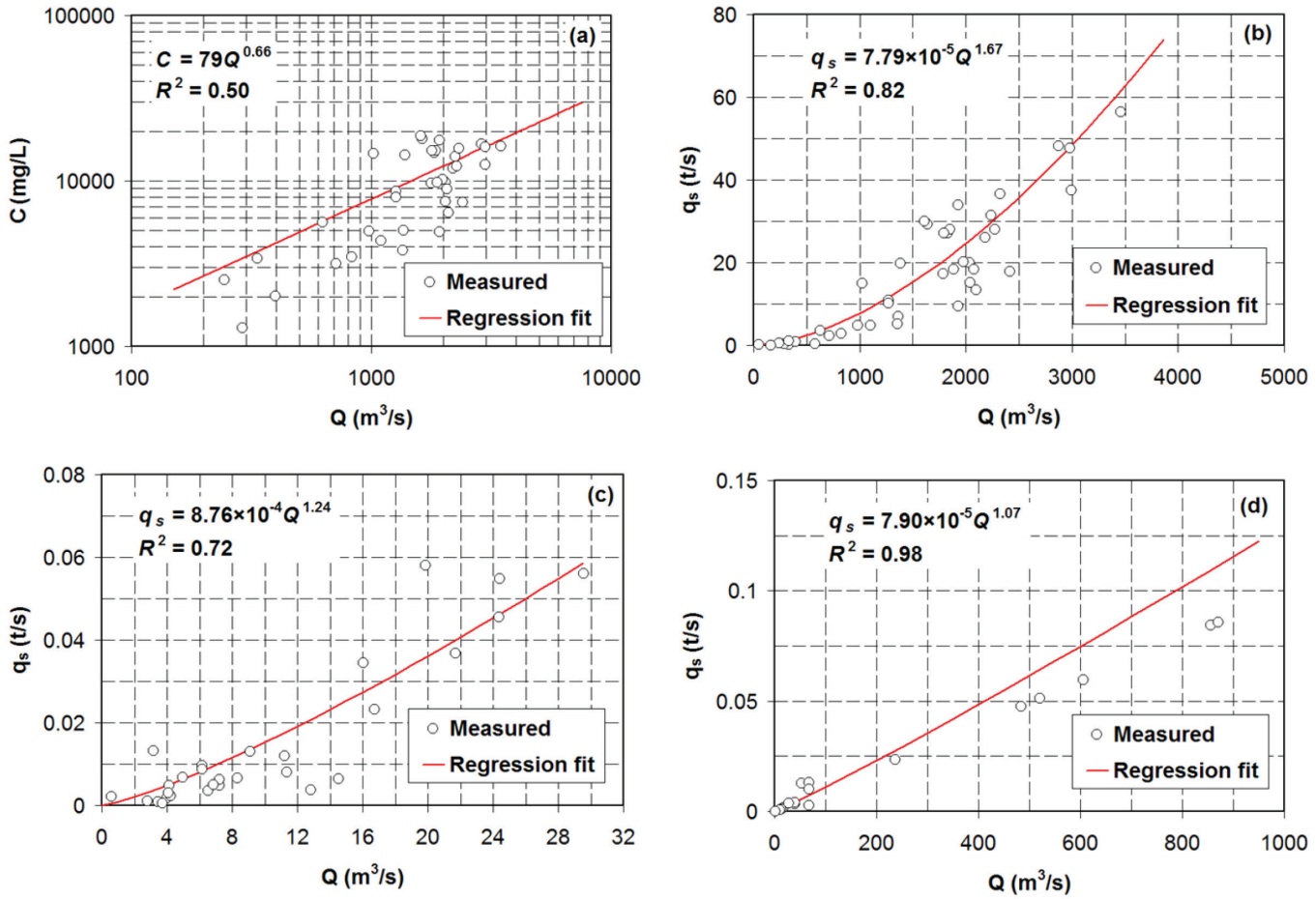


Fig. 5 *Sediment concentration (a) and sediment discharge (b) rating curves for the wet season of the year at the Kessie gaging station on the Blue Nile River. The sediment discharge as a function of streamflow for the Teme and Guder Rivers is also shown in (c) and (d), respectively. The sediment discharge at a particular section of the river is denoted by* q_s

sis results of Gebremicael et al. (2013).

4.3.3 Depletion of the reservoir storage capacity

Because of the longer record period of the available streamflow data and the greater frequency of measurements, the data at the El Deim station rather than those recorded at the Border gaging station were analyzed to compute the long-term mean annual inflow to the GERD reservoir. The computations resulted in a mean annual inflow of 48 BCM on the basis of the 1963–2010 streamflow records, which were obtained from the Ministry of Water Resources, Irrigation, and Electricity, Sudan. Using the computed mean annual sediment discharge and inflow rate, the storage capacities of the reservoir after various years of operation were estimated based on the following two long-term reservoir sedimentation scenarios:

1. Scenario 1: sedimentation of the reservoir is based on a constant mean annual inflow of sediment flux. This scenario assumes that the dam catchment area will not be subjected to significant land-use changes, so that its sediment yield remains unaltered.
2. Scenario 2: sedimentation of the reservoir is based on sediment discharge continuing to rise at 1.5% annually.

Examination of the field data demonstrated that the Blue Nile River transports significant quantities of very fine material composed of silt and clay. Hence, the equations for fine-grained sediment were used to evaluate the remaining capacity of the reservoir.The results of the computation are summarized in Tab. 3. For both scenarios, the predictions of Eqs. (17) and (23) are almost similar. The storage capacity of the reservoir continuously decreases from the sediment load due to soil erosion resulting from land-cover changes. As can be seen, the reservoir loses 21 and 52% of its gross storage through sediment deposition in Scenarios 1 and 2, respectively, within 100 years. The initial trap efficiency of the reservoir was estimated at 96.9% and is expected to decrease with the age of the reservoir. The results of this

Tab. 3 *Normalized storage capacities of the GERD reservoir after various years of operation*

T (y)	Scenario 1		Scenario 2	
	Eq. (17) V_R (%)	Eq. (23) V_R (%)	Eq. (17) V_R (%)	Eq. (23) V_R (%)
25	94	94	92	92
50	89	89	81	80
75	84	84	66	65
100	79	79	48	47
150	69	69	13	2

study reveal that the reservoir loses its storage capacity at an average annual rate of 0.21% in Scenario 1. This rate is doubled in Scenario 2. In light of this sediment infill problem, a sediment management strategy, both upstream and within the reservoir, might be necessary for more sustainable energy production. Such a strategy should consider the sediment trapping effect of the proposed upstream dams on the GERD reservoir.

5 CONCLUSIONS

A sediment trap-efficiency equation, which aims to overcome the drawbacks of the existing methods, was developed based on the principle of mass conservation. The proposed equation considers the dynamic behavior of the sediment transported and the flow dynamics of the reservoir. Compared to a higher resolution numerical model, it does not require extensive field data for calibration and validation. Using this equation, the approximate analytical expression for predicting the storage capacities of the reservoir after various years of operation was obtained. These equations were applied to investigate real-life problems related to the impacts of reservoir sedimentation, and their results were compared with the field data and the results of theoretical and empirical methods.

The results of the analysis for the trap efficiency of the reservoir confirmed that the performance of Eq. (7) was satisfactory compared to the trap-efficiency equations of Gill (1979) and Karaushev (1966). For a hydrologically small reservoir with $\Omega > 0.025$, the Karaushev equation overestimated the sediment trap efficiency. Furthermore, the suitability of Eq. (17) for predicting the loss of reservoir capacity due to sedimentation was examined. Compared to the reservoir capacity equation of Gill (1979,1988), the remaining capacity of the reservoirs was more accurately predicted by Eq. (17) with a mean absolute relative error of 2.6%. Overall, the results revealed that the accuracy of Eqs. (7) and (17) is acceptable for analyzing the sediment trapping and storage capacity depletion characteristics of large reservoirs.

As a test case, the impact of sedimentation on the reservoir capacity of the Grand Ethiopian Renaissance Dam was investigated based on two long-term sedimentation scenarios. Scenario 1 assumed a constant mean annual inflow of sediment flux, whereas Scenario 2 considered sedimentation based on a sediment discharge continuing to rise at 1.5% annually. The results demonstrated that the reservoir loses 21 and 52% of its gross storage through sediment deposition in Scenarios 1 and 2, respectively, within 100 years. In addition, Scenario 2 resulted in a mean annual storage capacity depletion rate of 0.45%, which is twice the resulting rate of Scenario 1. It is concluded that the equations developed can be applied to examine the impact of sediment accumulation on the life of a complex multi-purpose reservoir. Also, they can be coupled with a hydrological model for analyzing the sediment budget on a watershed scale.

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