

STRESS ANALYSIS OF PRESSING JOINT OF TWO CONICAL PIPES – PART 2

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Abstract: The paper presents a strength analysis of a pressed joint of two conical pipes with the aim of finding out whether this method of connection is suitable for telecommunication masts. In this paper, a solution using an engineering approach and a finite element method (FEM) numerical analysis for transversal direction will be compared.

KEYWORDS: conical pipes, finite element analysis, thin-walled shell, contact pressure

1 Introduction

The paper presents a continuation of Article [8] of the strength analysis of the pressed joint of two conical pipes in order to determine whether this method of connection is suitable for telecommunication masts.

In this paper, a solution using an engineering approach and numerical analysis using the finite element method (FEM) for the transverse direction of the mast will be compared.

2 Theoretical background

The model consists of two conical pipes with a taper ratio "k" and contact length a, as can be seen in the fig. 1 (up). The parameters of the pipes are: the wall thickness of the inner pipe is h, the wall thickness of the outer pipe is h_v and the smaller radius outer of pipe is r₁. Because the systems of two conical pipes are loaded by assembling load F_z, the contact pressure is nonzero, in fig. 1 (down). Loading by assembling load will be displacement between outer and inner pipes increase by Δa (in fig. 1 down).

From the described loading of the pipe with contact pressures in article [8], in addition to the longitudinal bending stress, a significant stress in the transverse direction (section) will arise in the pipe (shell). To determine it, we will use the following model: let's choose a ring of unit width from the pipe at location x, loaded by contact pressures with a cosine distribution $p(\varphi) = p_o(x) \cos \varphi$ Fig. 1b) and on the walls of the cross sections by shear flows with a sinusoidal distribution (see Fig. 1c), which give the resulting shear flow $\Delta t(\varphi) = \Delta t_o(x) \sin \varphi = \frac{p_o(x)}{2} \sin \varphi$ (when the dependence $\Delta t_o(x) = \frac{p_o(x)}{2}$ results from the condition of force balance).

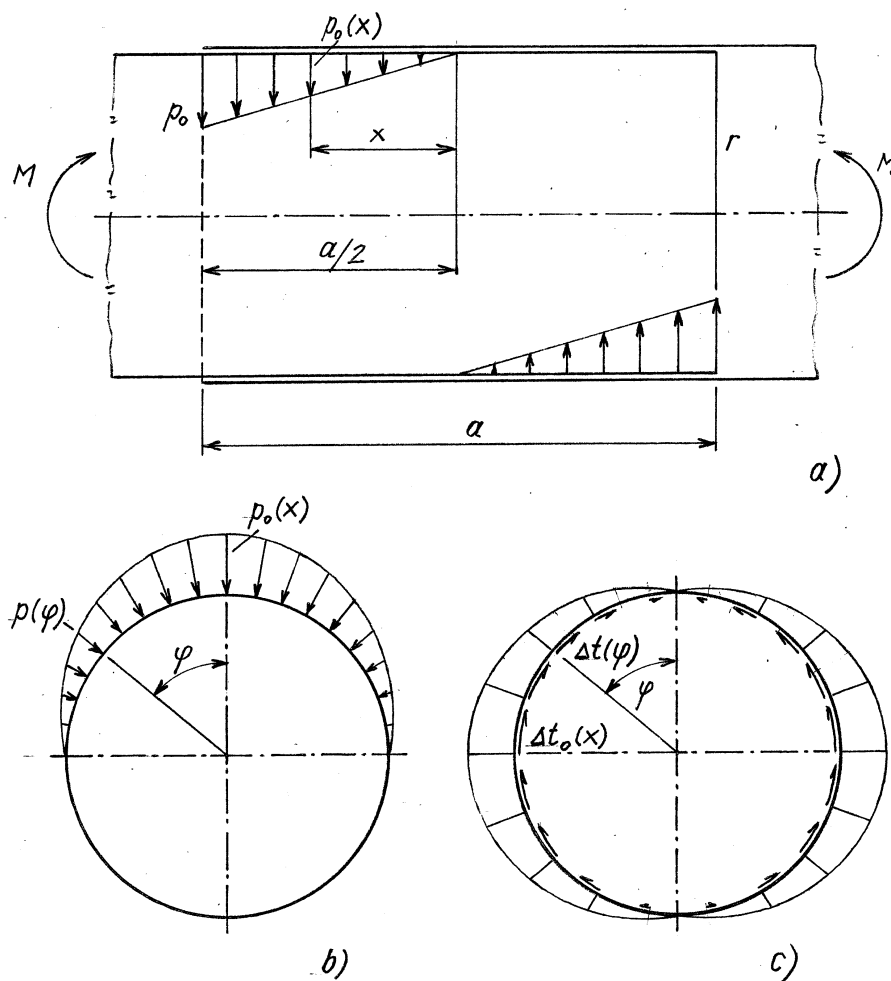


Fig. 1 Contact pressure a) for loading by bending moment b) pressure in inner pipe c) assumed distribution [8]

For the ring at the edge of the inserted pipe ($x = a/2$) the value of the contact pressure is the largest (p_0), and therefore the transverse stress of the ring will be the largest here.

By using the axis of symmetry of the ring, we get a semi-ring, loaded according to Fig. 2; here M_B and N_B are as yet unknown components of the resultant of internal forces in the section on the axis of symmetry. The bending moment at the general point (determined by the angle α) will be:

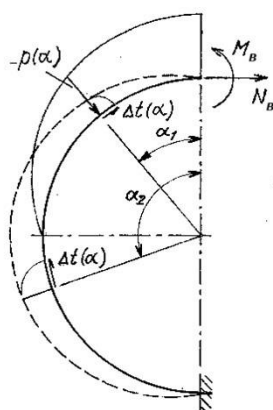


Fig. 2

$$\begin{aligned}
M(\alpha) &= M_B - N_B r (1 - \cos \varphi) - p_o r^2 \int_0^\alpha \cos \varphi \sin(\alpha - \varphi) d\varphi - \frac{p_o r^2}{2} \int_0^\alpha \sin \varphi [1 - \cos(\alpha - \varphi)] d\varphi = \\
&= M_B - N_B r (1 - \cos \varphi) - \frac{p_o r^2}{2} \left(\frac{\alpha}{2} \sin \varphi - \cos \varphi + 1 \right), \quad \text{for } \alpha \in \left(0, \frac{\pi}{2}\right) \\
M(\alpha) &= M_B - N_B r (1 - \cos \varphi) - p_o r^2 \int_0^{\pi/2} \cos \varphi \sin(\alpha - \varphi) d\varphi - \frac{p_o r^2}{2} \int_0^\alpha \sin \varphi [1 - \cos(\alpha - \varphi)] d\varphi = \\
&= M_B - N_B r (1 - \cos \alpha) - \frac{p_o r^2}{2} \left(\frac{\pi - \alpha}{2} \sin \alpha - 2 \cos \alpha + 1 \right), \quad \text{for } \alpha \in (\pi/2, \pi)
\end{aligned} \tag{1}$$

The stress on the ring (frame) and the slightly curved rod (half-ring in Fig. 2) will be the same if

$$N_B = -0,750 p_o r \quad M_B = 6,83 \cdot 10^{-2} p_o r^2 \tag{2}$$

The value of the axial force at the general location α of the semi-ring (but also of the frame) will be:

$$\begin{aligned}
N(\alpha) &= N_B \cos \alpha - p_o r \int_0^\alpha \cos \varphi \sin(\alpha - \varphi) d\varphi + \frac{p_o r}{2} \int_0^\alpha \sin \varphi \cos(\alpha - \varphi) d\varphi = \\
&= N_B \cos \alpha - \frac{p_o r}{4} \alpha \sin \alpha = -p_o r \left(\frac{3}{4} \cos \alpha + \frac{\alpha}{4} \sin \alpha \right), \quad \text{for } \alpha \in (0, \pi/2)
\end{aligned} \tag{3}$$

$$\begin{aligned}
N(\alpha) &= N_B \cos \alpha - p_o r \int_0^{\pi/2} \cos \varphi \sin(\alpha - \varphi) d\varphi + \frac{p_o r}{2} \int_0^\alpha \sin \varphi \cos(\alpha - \varphi) d\varphi = \\
&= N_B \cos \alpha - \frac{p_o r}{2} \left(\frac{\pi - \alpha}{2} \sin \alpha - \cos \alpha \right) = -p_o r \left(\frac{\cos \alpha}{4} + \frac{\pi - \alpha}{4} \sin \alpha \right), \quad \text{for } \alpha \in (\pi/2, \pi)
\end{aligned}$$

Based on relations (1) to (3), we get:

$$\begin{aligned}
\text{for } \alpha = 0 \quad N(\alpha = 0) &= -0,750 p_o r \quad M(\alpha = 0) = 6,83 \cdot 10^{-2} p_o r^2 \\
\alpha = \pi/2 \quad N(\alpha = \pi/2) &= -0,393 p_o r \quad M(\alpha = \pi/2) = 6,83 \cdot 10^{-2} p_o r^2 \\
\alpha = \pi \quad N(\alpha = \pi) &= 0,250 p_o r \quad M(\alpha = \pi) = 6,83 \cdot 10^{-2} p_o r^2
\end{aligned} \tag{4}$$

3 Numerical solution

To check the theoretically derived relations (1) to (4), we will use the numerical solution using the finite element method in the ANSYS program, which additionally provides an illustrative graphic display. In fig. 3 is shown the bending moment in [N.mm] and fig. 4 is shown the axial force in [N] when they fall on a ring of unit width $b = 1$ mm and $p_o = 1$ MPa. Other parameters are according to the article [8].

Fig. 5 shows the displacement (vector) in [mm] of the individual points of the center of the ring under the given load.

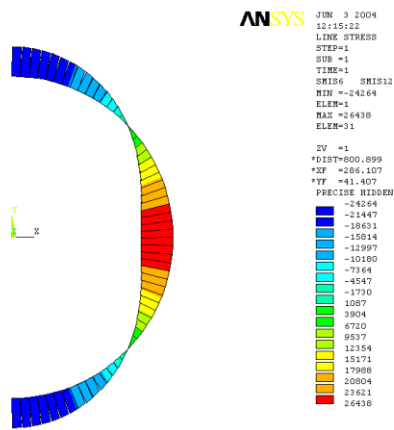


Fig. 3 Bending moment diagram (N.mm)

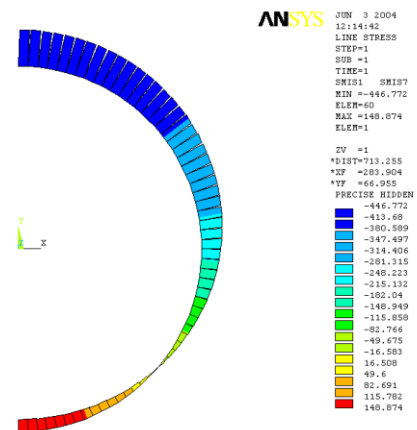


Fig. 4 Normal force diagram (N)

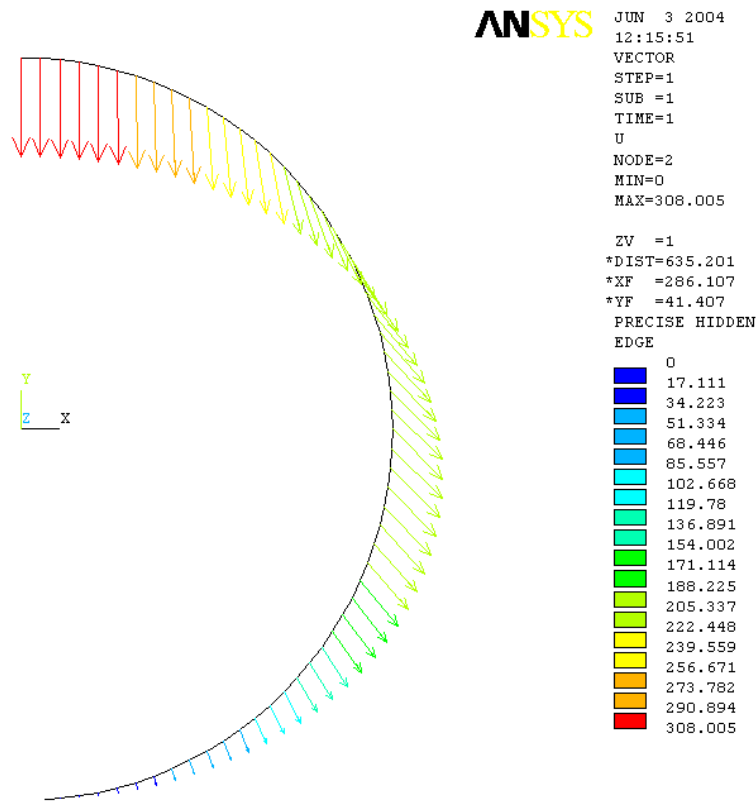


Fig. 5 Displacement vector sum (mm)

CONCLUSION

The resulting bending moment or the axial force for our case in places with a spacing of $\pi/4$ in the interval are:

$$M(\alpha = 0) = 2175 \text{ Nmm}$$

$$M(\alpha = \pi/4) = -1820 \text{ Nmm}$$

$$M(\alpha = \pi/2) = 2210 \text{ Nmm}$$

$$M(\alpha = 3\pi/4) = -1820 \text{ Nmm}$$

$$M(\alpha = \pi) = 2175 \text{ Nmm}$$

$$N(\alpha = 0) = -681 \text{ MPa}$$

$$N(\alpha = \pi/4) = -583 \text{ MPa}$$

$$N(\alpha = \pi/2) = -383 \text{ MPa}$$

$$N(\alpha = 3\pi/4) = -168 \text{ MPa}$$

$$N(\alpha = \pi) = -85 \text{ MPa}$$

The corresponding stresses (oriented in the tangential direction) will be:

- axial stresses
 $\sigma_N(\alpha) = \frac{N(\alpha)}{h}$ with maximal values $(\sigma_N)_{\max} = -\frac{681}{10} = -68 \text{ MPa}$
- bending stresses
 $\sigma_B(\alpha) = \frac{M(\alpha)}{\frac{1}{6}h^2}$ with maximal values $(\sigma_B)_{\max} = \frac{2200}{\frac{1}{6}10^2} = 132 \text{ MPa}$

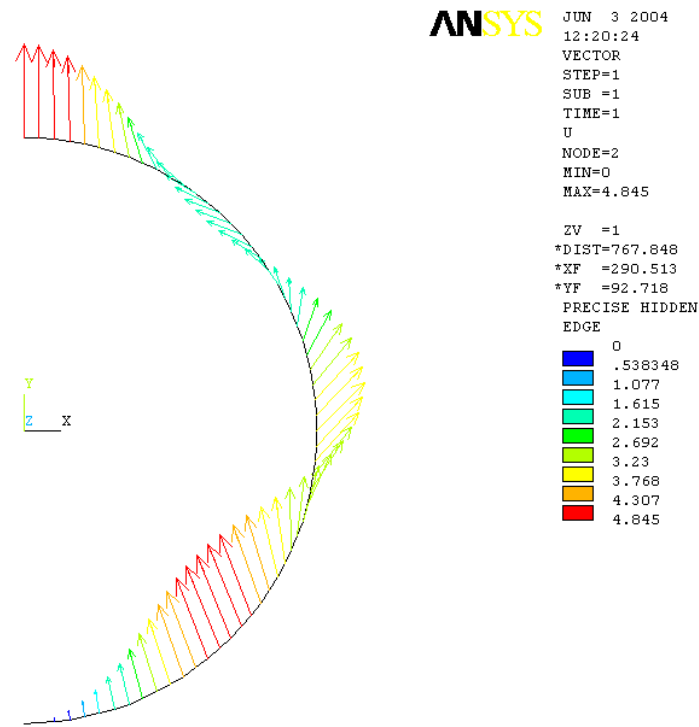


Fig. 6 Displacement vector sum (mm) for real problem

The radial centric deformation of the ring will be:

$$\begin{aligned} u(\alpha = 0) &= 2,34 \text{ mm} & u(\alpha = 3\pi/4) &= -2,60 \text{ mm} \\ u(\alpha = \pi/4) &= -2,50 \text{ mm} & u(\alpha = \pi) &= 2,50 \text{ mm} \\ u(\alpha = \pi/2) &= 2,42 \text{ mm} \end{aligned}$$

For the arrangement with the ring fixed in place, the vector representation of the displacements of the center points is shown in Fig. 6.

In conclusion, we can say that the analytical solution coincides with the numerical solution.

ACKNOWLEDGMENT

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