

COMPUTATIONAL PROCEDURE FOR DETERMINING THE SHEAR YIELD STRESS-SHEAR DEFORMATION RELATIONSHIP IN MAGNETORHEOLOGICAL FLUIDS FROM ANTON-PAAR MEASUREMENT DATA

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Abstract: In the past decade, magnetorheological fluids have been applied in various devices. This article focuses on developing a new computational procedure to determine the shear strength-shear strain relationship of magnetorheological fluids from the measurement data obtained by using an Anton-Paar rotational viscometer.

KEYWORDS: Anton – Paar; disc viscometer; shear strength; shear path; torque.

1 Introduction

The application of magnetorheological fluid (MRF) is currently being investigated across a growing range of fields. In addition to its use in vibration control, damping, and semi-active shock absorption in automobiles, the potential of MRF in magnetorheological (MR) clutches and brakes is also being explored. [1] [2] [3] In an MR fluid, a particle placed in a magnetic field becomes magnetized and transforms into a dipole. Models consider two factors: the absolute value of shear strength, determined by various parameters (magnetic field strength, particle and fluid permeability, particle radius, etc.), and a function related to the shear angle. For instance, the Bossis, Song Cheng, Yi, and Peng models incorporate angle-dependent factors normalized in Figure 1. [4]

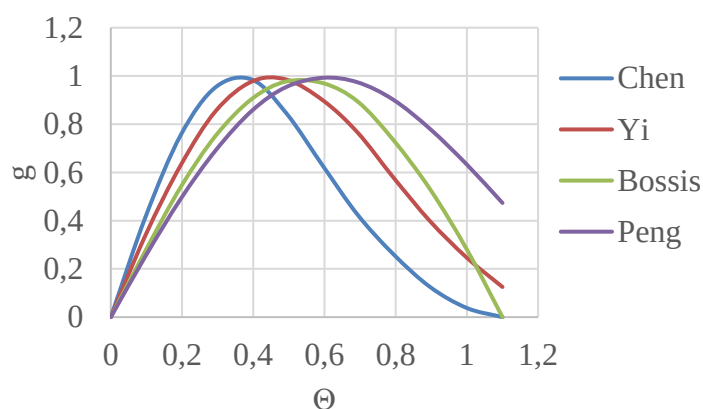


Fig. 1 Angle-dependent factors for dipole models

After reaching its peak during the rising phase, shear stress subsequently decreases. Variations are observed in the shear strain angle corresponding to the function's maxima (yield

strain). In particular, for disc brakes or couplings, an increase in the shear angle during startup causes the maximum shear stress to initially occur at the largest radius, followed by a decrease. Therefore, when considering the entire disc, the applied torque must slightly decrease after reaching its maximum, as the stress curves shift to the descending phase at both small and large radii. If the applied torque exceeds this maximum torque, the system transitions into a motion phase. At this point, chains may break or slip at surfaces experiencing relative motion, new chains form, and dynamic equilibrium is established. This assumption leads to the investigation of the transitional process as the device approaches the motion threshold from a resting state. This phenomenon is analogous to Coulomb friction, which distinguishes between static and kinetic friction coefficients. Using the Anton-Paar disc viscometer, we aim to determine the τ - x relationship under static conditions, effectively $\dot{\gamma} \rightarrow 0$ shear rate (Figure 2). [5]

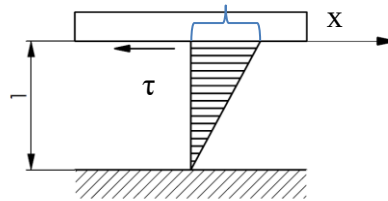


Fig. 2 Shear path

During continuous motion, the shear strength of the magnetorheological fluid becomes independent of the shear path, allowing it to be considered constant and easily determined from the measured torque. However, in the case of a disc rotor, shear paths vary across different radii. The outer radius experiences significant shear displacement, while the inner radii experience less. If shear strength depends on the shear path under static conditions, the fluid's shear strength varies radially.

Under these static conditions, it is impossible to determine the radial variation of shear strength from the measured torque at a specific disc rotation angle. However, theoretically, the shear strength parameters for “ n ” interconnected torque discs can be determined from “ n ” pairs of rotation angles. This raises the question: What should the “ n ” parameters of the τ - x function be when there are “ n ” measurement points?

A straightforward approach would be to seek the continuous τ - x function in the form of an “ n ”-degree polynomial, where the parameters would be the polynomial coefficients:

$$\tau(x) = a_1x + a_2x^2 + \dots + a_nx^n \quad (1)$$

Unfortunately, this approach was ineffective due to the high-order exponents of the polynomial when dealing with a large number of measurement points. The resulting linear system of equations had significantly different magnitudes for the matrix elements, rendering the matrix ill-conditioned. Consequently, the computation either did not start or produced entirely unrealistic outcomes.

Using trigonometric polynomials was also unfeasible, as they are suitable only for periodic functions. Thus, instead of an analytical solution, numerical methods had to be employed.

2 Generating torque and angular displacement measurement data with a Disc Anton-Paar viscometer

The Anton-Paar viscometer used for data extraction is shown in Figure 3. The measurement setup also includes a Gauss and Tesla meter, a heat pump, and a computer. One of the key components of the A-P viscometer is the cylindrical sensor, which terminates in a disc-shaped design with a 6mm inner and a 20mm outer diameter. By adjusting the height of the

measurement sensor, a 1mm measurement range can be established between the circular ring of the measuring body and the base designed for storing the MR fluid.

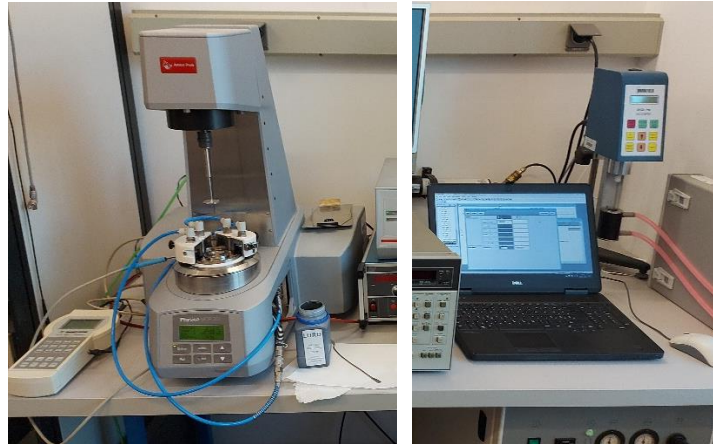


Fig. 3 Measurement setup

A suitable magnetic coil generates the magnetic field, positioned beneath the gray-colored MR fluid, as shown in Figure 4. The measurement can be performed by inserting the appropriate fluid (0.32 ml), lowering the measuring probe, and closing the upper part of the white magnetic circuit. [6]



Fig. 4 The placement of the MR fluid in the viscometer

2.1 Measurement using an Anton-Paar type viscometer

The MR fluid exhibits non-Newtonian behavior. To ensure measurement repeatability, test measurements were conducted to determine the optimal measurement settings. Initial measurements revealed that once the fluid has been exposed to a magnetic field, it does not revert to its previous state, characterized by a homogeneous distribution of metallic particles. Restoring the homogeneous state can be achieved by either manually mixing the fluid or replacing it by removing the used fluid from the viscometer and introducing fresh fluid.

Measurement Settings:

- Number of measurement points: 100.
- Excitation current: 2A.
- Temperature: 25°C.
- Rotational speed of the measuring probe: 0.1 revolutions per minute.

The results are shown in Figure 5.

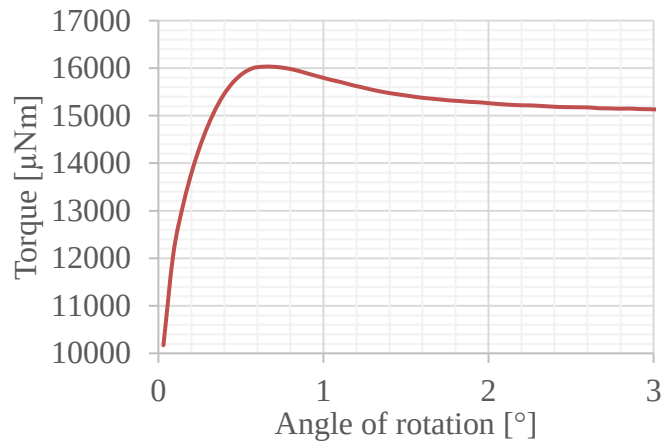


Fig. 5 Torque–Angular Displacement Diagram

3 Computational procedure for determining the Tau-x relationship from Anton-Paar measurement data

The initial step involves selecting n reference points x_i for shear strain, where we aim to determine the shear strength function τ_i based on n measured torque values. The function can be approximated most simply by using a linear relationship between the reference points, as illustrated in Figure 6.

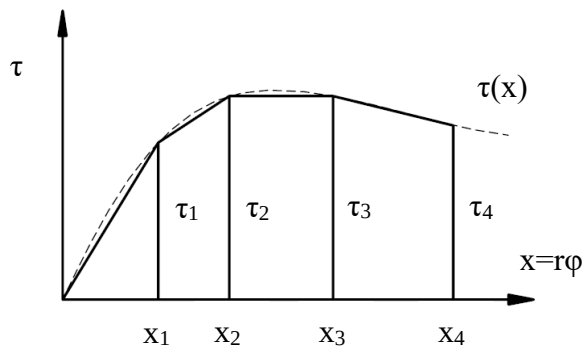


Fig. 6 Approximation of the Shear Strength Function

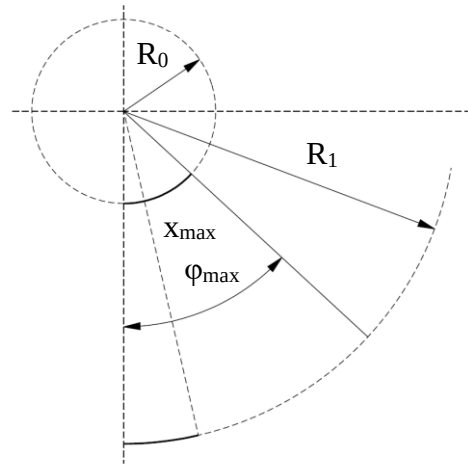


Fig. 7 Shear Path at the Inner Radius of the Disc

When recording data points, it is crucial to ensure that the disc reaches the shear path at even the smallest radius, up to the x_{max} value where the $\tau(x)$ function is to be determined (see Figure 7). For the rotational viscometer, the required angular displacement is $\varphi \geq x_{max}/R_0$.

Starting from the equilibrium position, at an arbitrary angle of rotation φ_j , the shear base points correspond to the rays

$$r_i = \frac{x_i}{\phi_j} \quad (2)$$

on the viscometer disc. The circular regions bounded by the rays r_{i-1} and r_i are denoted as $i=1, 2, 3, 4$ in Fig. 8. For a more accurate solution, the number of regions can be increased arbitrarily, although presenting the method with a small number of regions is more effective¹.

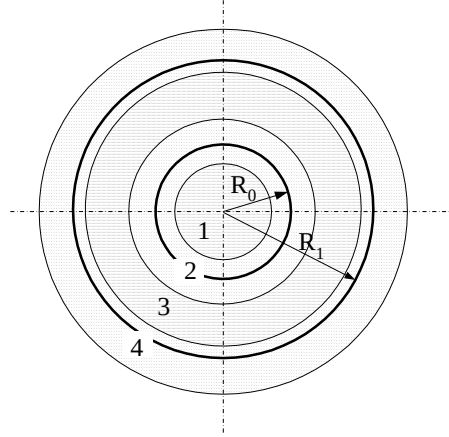


Fig. 8 The division of the disc into regions

In the case of domain 1, r_0 is understood to be 0. The diagram shows that there are regions that fall outside the surface characterized by radii R_0 and R_1 . Some regions are entirely within the disc surface, while others may lie outside. There are also regions where a certain part is inside the disc, while another part is outside. At different rotation angles, the boundaries of these regions shift, and the width of the regions decreases. In Fig. 9, at rotation angles φ_1 and $\varphi_2 > \varphi_1$, we can observe the A, B, C, D, and modified A*, B*, C*, D* domain boundaries. Consequently, the stress distribution along the radius of the disc also changes. The shaded portions of the diagram represent the distribution of τ along the radius.

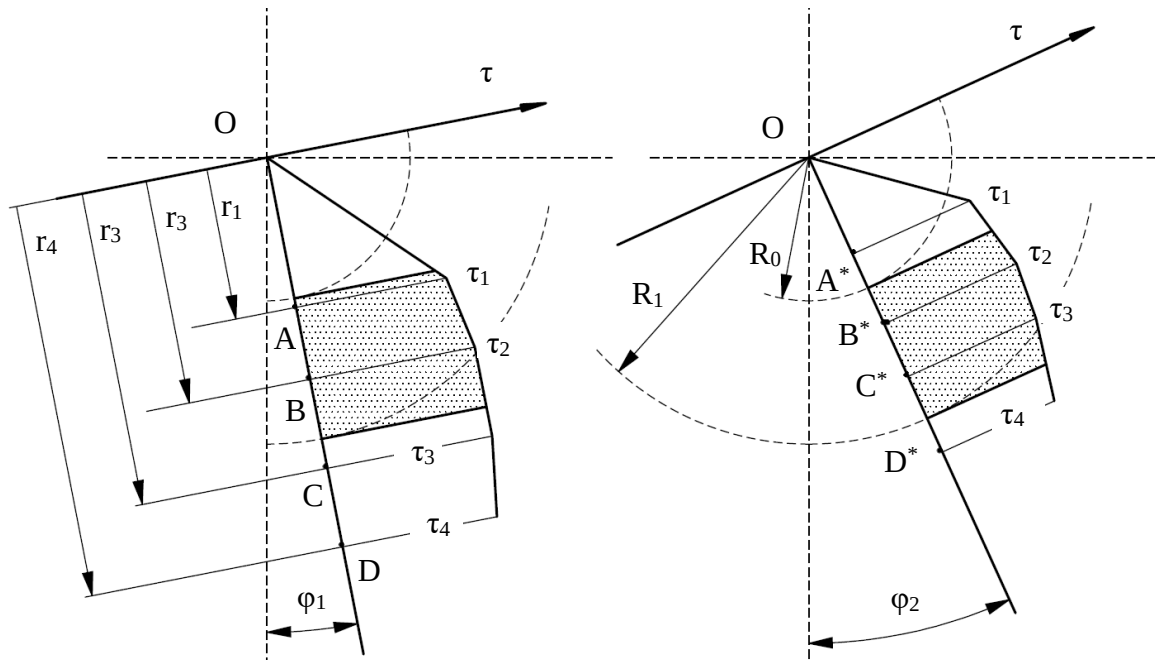


Fig. 9 Radial Variation of Shear Strength at Different Angular Displacements

We aim to determine the shear stresses τ_i at the boundaries of the domains, assuming a linear stress distribution between these boundaries. By densely sampling the division points, a piecewise linear approximation of the function can be effectively obtained. In the i -th domain, bounded by the radii r_{i-1} and r_i , We illustrate the calculation of stress distribution and the

resultant torque for the scenario where both the lower radius r_{i-1} and the upper radius r_i are within the disc's radii R_0 and R_1 (see Figure 10).

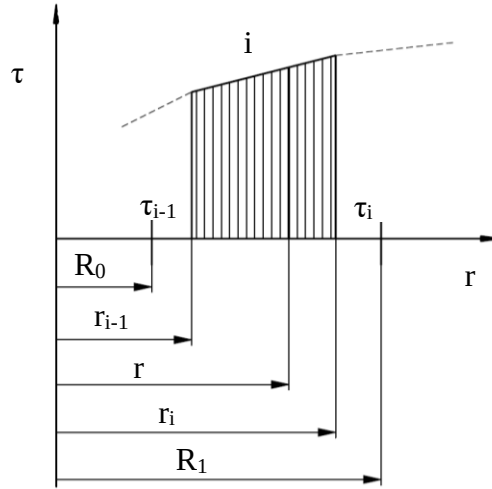


Fig. 10 Shear Stress Within the Domain

The relationships of cases differing from this will be discussed later. If we know the stress values τ_{i-1} and τ_i at the boundaries of the r_{i-1} and r_j domains, then within the domain, the shear stress value can be determined for any arbitrary radius r .

$$\tau(r) = \tau_{i-1} + (\tau_i - \tau_{i-1}) \frac{r - r_{i-1}}{r_i - r_{i-1}} = \frac{\tau_{i-1}r_i - \tau_i r_{i-1}}{r_i - r_{i-1}} + \frac{\tau_i - \tau_{i-1}}{r_i - r_{i-1}} r \quad (3)$$

The torque exerted by the i -th section.

$$M^{(i)} = 2\pi \int_{r_{i-1}}^{r_i} \left(\frac{\tau_{i-1}r_i - \tau_i r_{i-1}}{r_i - r_{i-1}} + \frac{\tau_i - \tau_{i-1}}{r_i - r_{i-1}} r \right) r^2 dr \quad (4)$$

The integral is given by:

$$M^{(i)} = 2\pi \left\{ \frac{\tau_{i-1}r_i - \tau_i r_{i-1}}{r_i - r_{i-1}} \cdot \frac{r_i^3 - r_{i-1}^3}{3} + \frac{\tau_i - \tau_{i-1}}{r_i - r_{i-1}} \cdot \frac{r_i^4 - r_{i-1}^4}{4} \right\} \quad (5)$$

The torque exerted by the i -th domain, according to equation (68), depends on the values of τ_{i-1} and τ_i beyond the radii defining the domain. To enhance clarity in subsequent calculations, we transform equation (5) as follows:

$$M^{(i)} = \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left(\frac{r_i(r_i^3 - r_{i-1}^3)}{3} - \frac{r_i^4 - r_{i-1}^4}{4} \right)}_{e_i} \tau_{i-1} + \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left(\frac{-r_i(r_i^3 - r_{i-1}^3)}{3} + \frac{r_i^4 - r_{i-1}^4}{4} \right)}_{f_i} \tau_i \quad (6)$$

In the i -th domain, the coefficients of stress are denoted by e_i and f_i :

$$M^{(i)} = e_i \tau_{i-1} + f_i \tau_i \quad (7)$$

Depending on whether the boundaries of the relevant domain lie inside or outside the inner and outer radii of the disc, there are several ways for torque calculation. As it depends only on one stress (τ_i), the first domain requires a separate treatment, whereas in other domains, the

developed torque relies on the two stresses at the domain boundaries. The possible scenarios, considering radial direction from inside to outside, are as follows:

Case 1: $r_1 < R_0$

The boundary of the first region is smaller than the inner radius of the disc. The developed torque is zero, $e_1=0$, $f_1=0$.

Case 2: $R_0 < r_1 < R_1$

$$M_2^{(1)} = \underbrace{2\pi \frac{r_1^4 - R_0^4}{4r_1}}_{f_1} \cdot \tau_1 \quad (8)$$

Case 3: $r_1 > R_1$

$$M_3^{(1)} = \underbrace{2\pi \frac{R_1^4 - R_0^4}{4r_1}}_{f_1} \cdot \tau_1 \quad (9)$$

Case 4: $r_i < R_0$

The i-th region does not fall within the useful range of the disc. The developed torque is zero ($e_i=0$, $f_i=0$)

Case 5: $r_{i-1} < R_0$ and $R_0 < r_1 < R_1$

$$M_5^{(i)} = \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[r_i \frac{r_i^3 - R_0^3}{3} - \frac{r_i^4 - R_0^4}{4} \right]}_{e_i} \cdot \tau_{i-1} + \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[-r_{i-1} \frac{r_i^3 - R_0^3}{3} + \frac{r_i^4 - R_0^4}{4} \right]}_{f_i} \cdot \tau_i \quad (10)$$

Case 6: $R_0 < r_{i-1} < R_1$ and $R_0 < r_i < R_1$

$$M_6^{(i)} = \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[r_i \frac{r_i^3 - r_{i-1}^3}{3} - \frac{r_i^4 - r_{i-1}^4}{4} \right]}_{e_i} \cdot \tau_{i-1} + \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[-r_{i-1} \frac{r_i^3 - r_{i-1}^3}{3} + \frac{r_i^4 - r_{i-1}^4}{4} \right]}_{f_i} \cdot \tau_i \quad (11)$$

Case 7: $R_0 < r_{i-1} < R_1$ and $r_1 > R_1$

$$M_7^{(i)} = \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[r_i \frac{R_1^3 - r_{i-1}^3}{3} - \frac{R_1^4 - r_{i-1}^4}{4} \right]}_{e_i} \cdot \tau_{i-1} + \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[-r_{i-1} \frac{R_1^3 - r_{i-1}^3}{3} + \frac{R_1^4 - r_{i-1}^4}{4} \right]}_{f_i} \cdot \tau_i \quad (12)$$

Case 8: $r_{i-1} < R_0$ and $r_i > R_1$

$$M_8^{(i)} = \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[r_i \frac{R_1^3 - R_0^3}{3} - \frac{R_1^4 - R_0^4}{4} \right]}_{e_i} \cdot \tau_{i-1} + \underbrace{\frac{2\pi}{r_i - r_{i-1}} \left[-r_{i-1} \frac{R_1^3 - R_0^3}{3} + \frac{R_1^4 - R_0^4}{4} \right]}_{f_i} \cdot \tau_i \quad (13)$$

Case 9: $r_{i-1} > R_1$ and $r_i > R_1$

The i -th region does not fall within the useful range of the disc. The developed torque is zero ($e_i=0, f_i=0$).

When we sum up the torques $M^{(i)}(\phi_j)$ exerted in different regions for a given ϕ_j rotation angle, we obtain the torque measured by the viscometer at this rotation angle:

$$M(\phi_j) = [e_1 \cdot 0 + f_1(\phi_j)\tau_1] + [e_2(\phi_j)\tau_1 + f_2(\phi_j)\tau_2] + [e_3(\phi_j)\tau_2 + f_3(\phi_j)\tau_3] + [e_n(\phi_j)\tau_{n-1} + f_n(\phi_j)\tau_n] \quad (14)$$

Grouping the terms in another way:

$$M(\phi_j) = [f_1(\phi_j) + e_2(\phi_j)]\tau_1 + [f_2(\phi_j) + e_3(\phi_j)]\tau_2 + \dots + [f_{n-1}(\phi_j) + e_n(\phi_j)]\tau_{n-1} + f_n(\phi_j)\tau_n \quad (15)$$

So, based on the observed regularity, we can express the relationship more concisely:

$$M(\phi_j) = \sum_{i=1}^{n-1} [f_i(\phi_j) + e_{i+1}(\phi_j)]\tau_i + f_n(\phi_j)\tau_n \quad (16)$$

Introducing the notation of:

$$c_{ji} = f_i(\phi_j) + e_{i+1}(\phi_j), c_{jn} = f_n(\phi_j) \quad (17)$$

we obtain the following linear system of equations relating the torques and the shear stresses τ_i at the pivot points:

$$\begin{bmatrix} M(\phi_1) \\ M(\phi_2) \\ \vdots \\ M(\phi_n) \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdot & \cdot & c_{1n} \\ c_{21} & c_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{n1} & c_{n2} & \cdot & \cdot & c_{nn} \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} \quad (18)$$

The vectors of shear stresses within the shear planes can be straightforwardly determined using the equation:

$$\underline{\tau} = \underline{C}^{-1} \underline{M} \quad (19)$$

The measurement data obtained during the 2A excitation on the Anton-Paar viscometer and the reference points of the selected shear path were as follows:

Table 1 Measurement data on A-P viscometer

j	φ [rad]	M [Nm]
1	0.000535	0.01017
2	0.00158	0.01209
3	0.003680	0.01391
4	0.00473	0.01453
5	0.00684	0.01542
6	0.01320	0.01601
7	0.02670	0.01540
8	0.04345	0.01518

Table 2 Shear path reference points

i	x_i [m]
1	0.0000035
2	0.0000130
3	0.0000260
4	0.000040
5	0.000060
6	0.00013
7	0.00026
8	0.00043

The elements of the coefficient matrix calculated using the Matab program were as follows:

$$c = \begin{bmatrix} 1,761 \cdot 10^{-6} & 1,67 \cdot 10^{-7} & 0 & 0 & 0 & 0 & 0 & 0 \\ 3,43 \cdot 10^{-7} & 1,589 \cdot 10^{-6} & 1,06 \cdot 10^{-7} & 0 & 0 & 0 & 0 & 0 \\ 3 \cdot 10^{-9} & 2,86 \cdot 10^{-7} & 1,166 \cdot 10^{-6} & 5,82 \cdot 10^{-7} & 0 & 0 & 0 & 0 \\ 0 & 1,07 \cdot 10^{-7} & 5,8 \cdot 10^{-7} & 1,92 \cdot 10^{-6} & 1,59 \cdot 10^{-7} & 0 & 0 & 0 \\ 0 & 1,1 \cdot 10^{-8} & 1,78 \cdot 10^{-7} & 6,05 \cdot 10^{-7} & 1,201 \cdot 10^{-6} & 4,3 \cdot 10^{-8} & 0 & 0 \\ 0 & 0 & 0 & 6,2 \cdot 10^{-8} & 7,68 \cdot 10^{-7} & 1,207 \cdot 10^{-6} & 1 \cdot 10^{-9} & 0 \\ 0 & 0 & 0 & 0 & 5,6 \cdot 10^{-8} & 7,92 \cdot 10^{-7} & 1,185 \cdot 10^{-6} & 3 \cdot 10^{-9} \\ 0 & 0 & 0 & 0 & 0 & 1,538 \cdot 10^{-7} & 9,017 \cdot 10^{-7} & 9,179 \cdot 10^{-7} \end{bmatrix} \quad (20)$$

As it can be seen, there is no significant difference in the magnitudes of the elements that would poorly condition the matrix. This is the computational method's main virtue, especially considering that the approximations using n th-degree polynomials exhibited several orders of magnitude discrepancies. Plotting the computed stresses at nodal points revealed the shear stress distribution as a function of shear displacement, as shown in Figure 11. The calculated points are intentionally connected with straight lines in the diagram, indicating that the computational model assumed piecewise linear stress distribution.

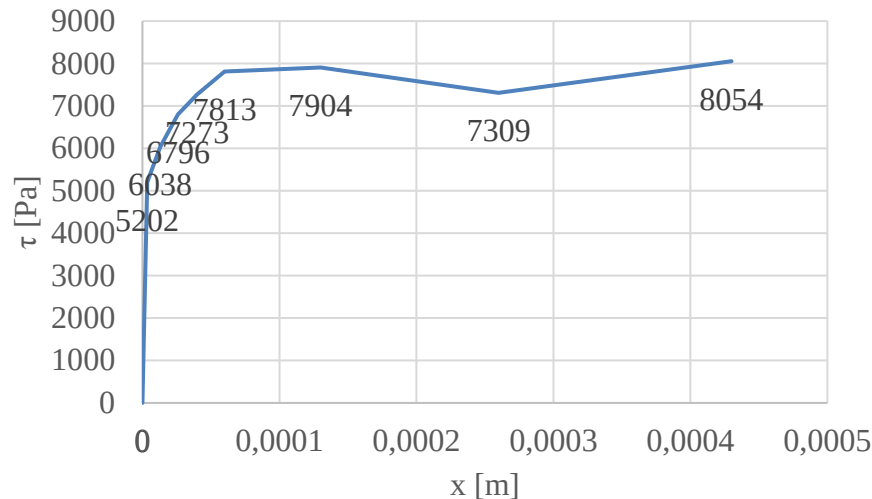


Fig. 11 Shear strength-shear path diagram extracted from the M- ϕ measurement points measured on the Anton-Paar viscometer for a gap size $h=1$ mm.

CONCLUSION

From the investigation of microscopic models of MR fluids, it is evident that reaching its maximum value after the rising phase, the shear stress decreases. In the case of a disc-type configuration, upon initiation, by increasing the shear angle, the shear stress initially reaches its maximum at the largest radius and then begins to decrease. This assumption led to the idea of examining the transitional process as the device transitions from a resting state to the limit of motion. To address this, we developed a computational procedure to determine the relationship between MRF shear strength and shear path based on the A-P disc viscometer measurement data.

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