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GLOBAL SENSITIVITY STUDY OF A DUFFING-TYPE NONLINEAR VIBRATION SYSTEM

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Abstract: An interesting field of studying nonlinear systems is their sensitivity study. With sensitivity study the most influential parameters on a system can be obtained and then the simplification and improvement of the model will be possible. In this paper the global sensitivity study of a Duffing-type vibration system is carried out with Sobol's variance-based method taking the root mean square of acceleration and the maximum acceleration as output variables. With the sensitivity study it was observed that the parameters of the excitation signal like the amplitude and the angular velocity are the most influential. It was also found that a single parameter has less influence on the system than the parameter combinations. The aim of the research is to carry out the global sensitivity study of a relatively simple nonlinear system. The study is the basis for further research tasks in order to perform the sensitivity study of more complex systems.

KEYWORDS: sensitivity study, nonlinear system, numerical simulation, vibration

1 Introduction

An interesting field of studying nonlinear systems is their sensitivity study. There are countless methods for determining the sensitivity of a system, from partial differential techniques to statistical methods [1]. The three main methods of sensitivity analysis are screening, local, and global sensitivity testing. Within these, there are still countless methods, like sampling-based methods, variation-based methods, the Bayesian method, and graphical methods [2].

The local sensitivity study methods are usually based on the derivatives. The main drawback of the local methods is that the derivatives are only informative at the base point. For nonlinear systems, the global methods allow the exploration of a more detailed parameter space [3]. Global sensitivity study can be widely used in the case of nonlinear systems with a lot of parameters. In this session, some methods and applications of it are presented. In Paper [4] a novel sampling strategy is described, which is a combination of one-factor-at-a-time and latin-hypercube. The method allows a global sensitivity analysis in the case of a lot of different parameters with a limited number of model runs. The method was tested with the water flow and water quality parameters. In Paper [5] we can read about the numerical modelling and sensitivity study of cardiac electrophysiology. Instead of Monte Carlo simulations nonintrusive polynomial chaos-based approximation was used for obtaining the atrial contribution to a realistic electrocardiogram. The parameter influence was determined using Sobol's indices. In Paper [6] 5 global sensitivity methods, which are squared standardized regression, coefficient, squared Spearman correlation coefficient, key issue analysis, Sobol's indices, and random balance design were compared in the case of a life cycle assessment. Two case studies were examined: a theoretical electricity production and a production system of a fishery with 115 parameters. In Paper [7] Sobol's method was used on

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a mathematical model of covid-19. The aim of the research was to investigate the individual and combined effects of the parameters on the output variables. In Paper [8] 3 different sensitivity analysis methods are compared: the local sensitivity analysis and 2 global, which are Morris' method and Sobol's method. The test example was pressure propagation induced by fluid injection and leakage in a CO_2 storage system with 7 parameters. It was found that the local sensitivity method is sufficient in this case. The Morris method provided a lot of useful information at a relatively small computational cost. The Sobol's method was computationally expensive, but it gave a more quantitative sensitivity measure. In Paper [9] we can read about the global sensitivity study of the water productivity model AquaCrop. The sensitivity study was carried out using Morris' method and EFAST (Extended Fourier Amplitude Sensitivity Test). Different environmental conditions were examined, which means 22 different climate-crop-soil-meteorology combinations. In Paper [10] the global sensitivity study of the Urban Weather Generator is carried out using a regression-based sensitivity analysis and the Monte Carlo technique. There were 30 inputs including urban characteristics, meteorological factors, building systems, and vegetation variables. In Paper [11] we can read about the sensitivity study of a floating wind turbine design. 2 sensitivity analysis methods were used: a variance-based method and a density- based method. 10 design parameters were used as an input and the output variables were the manufacturing cost, the stiffness, the natural frequencies, and the hydrodynamic coefficients. In Paper [12] surrogate modelling is used in the case of a nuclear reactor assembly. A global sensitivity analysis was carried out using a variance-based method, which is the FAST (Fourier Amplitude Sensitivity Test). For sample generation besides FAST, LHS (Latin Hypercube Sampling) and LPSS (Latinized Partially Stratified Sampling) methods were used. The model included 6 independent input parameters and one output variable. Sensitivity study can also be used to validate the stability of implemented Multi-Attribute Decision-Making methods to select a proper material for a piston [13].

In this paper Sobol's method was chosen for sensitivity study as it is widely used in the literature and gives a quantitative sensitivity measure [8]. The selected system is a Duffing-type nonlinear vibration system with 5 parameters. 3 different output variables were selected for sensitivity analysis, which are the root mean square (RMS) of acceleration, the maximum value of acceleration, and the peak frequency [14]. The One-at-a-Time sensitivity study of a similar system is presented in [15]-[16]. The paper is organised as follows: first Sobol's sensitivity method is described, which is followed by the description of the examined nonlinear system. Then the results are discussed. The paper concludes with further research tasks.

2 SOBOL'S SENSITIVITY STUDY

For sensitivity study Sobol's variance-based global sensitivity analysis method was chosen, as it is able to capture the nonlinear input-output relation of the system and the structure of the system can be examined via indices [17]. To perform sensitivity study the parameter boundaries and the output variable are necessary. The parameter values are randomly generated using Saltelli's sampling method [17]. The number of samples generated by the method is the following [18]:

$$N_s = N(2D + 2) \tag{1}$$

where N_s is the number of samples, D is the number of parameters and N is a number power of 2. There were 5 parameters and $N = 512$ the total number of samples was 6144.

Then the Sobol' sensitivity indices can be calculated:

$$S_T = \frac{var(\mu_T(x_T))}{var(Y)} \tag{2}$$

$$S_i = \frac{var(\mu_i(x_i))}{var(Y)} \tag{3}$$

$$S_{ij} = \frac{var(\mu_{ij}(x_{ij}))}{var(Y)} \tag{4}$$

where $var(\mu_T(x_T))$, $var(\mu_i(x_i))$ and $var(\mu_{ij}(x_{ij}))$ are the conditional variance of a selected parameter and $var(Y)$ is the variance of the output variable [19].

For the sample generation and the sensitivity analysis Python SALib's built-in commands were used [18]. Numerical simulations were carried out with Python using rkf45 numerical solver. The time step was 0.01 s and the simulation time was 50 s.

3 DUFFING-TYPE NONLINEAR SYSTEM

The examined nonlinear system is shown in Figure 1.

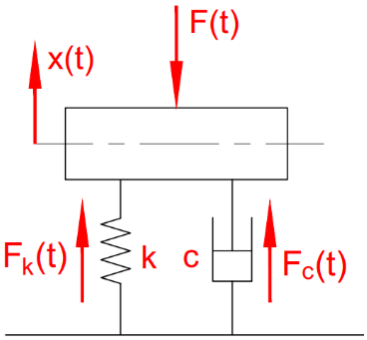

Figure 1. Duffing-type nonlinear system

It was assumed that the spring has a Duffing-type restoring force, which can be modelled with the following equation:

$$F_k = -\beta x - \alpha x^3. \tag{5}$$

It was assumed that the damping coefficient is constant. The excitation signal was sinusoidal. The system equation can be given by the following formula.

$$\frac{d^2}{dt^2}x(t) + \frac{c}{m}\frac{d}{dt}x(t) + \frac{k}{m}\left(-x(t) + 1000x(t)^3\right)x(t) = Asin(\omega t) \tag{6}$$

where m is the mass, k is the spring stiffness, c the damping coefficient, A the amplitude of the excitation signal and ω is the angular velocity of the excitation signal. The boundaries of the variables are shown in the Table 1.

The RMS of acceleration was selected as the output variable as it is widely used in the case of vibrational systems [20]. The RMS of acceleration can be calculated with the following formula [21]:

Table 1. Boundaries of the parameters

Variable	Minimum	Maximum
m	100	800
k	10000	60000
c	300	3000
A	0	0.5
ω	1	30

$$RMS = \sqrt{\frac{\sum_{i=1}^{n} a_i^2}{n}} \tag{7}$$

where a_i is the i_{th} value of acceleration and n is the number of acceleration values.

The other 2 output values for the sensitivity analysis were the maximum value of acceleration and the peak frequency based on [16].

4 RESULTS

The sensitivity indices in the case of the RMS of acceleration are shown in Figure 2.

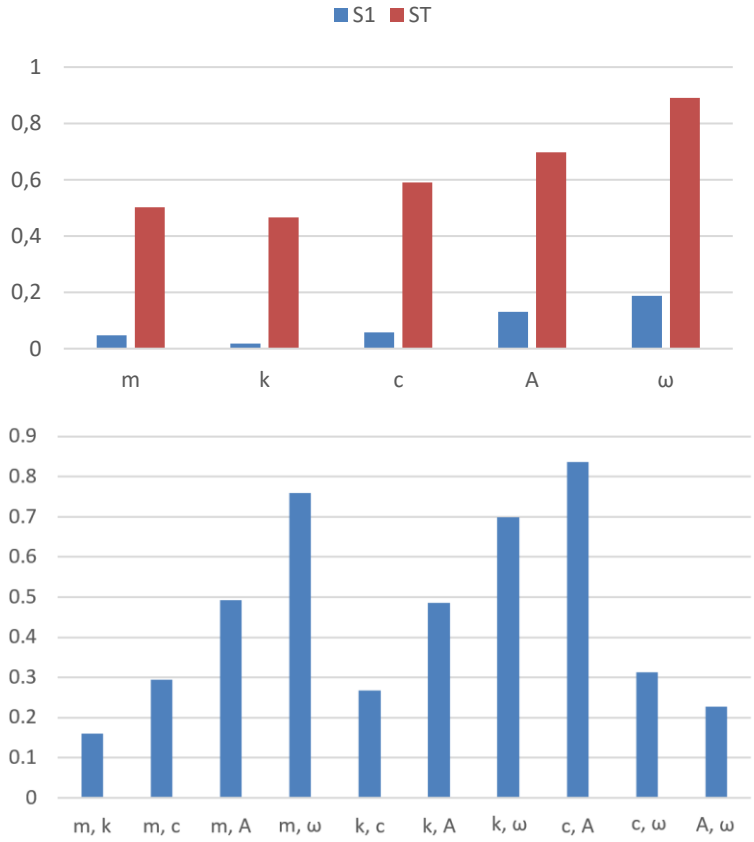

Figure 2. First-order and total sensitivity indices (above) and second-order sensitivity indices (below) in the case of the RMS of acceleration

It can be seen that the most influential parameter is the angular velocity of the excitation signal as S1, and ST are both the largest. It is followed by the amplitude of the excitation signal and the damping coefficient. The next most influential parameter is the mass, and the

least influential parameter is the spring stiffness. It is interesting to observe that S1 is small compared to ST, which means that the parameters have less influence on the RMS of acceleration alone than their interactions. The most influential parameter interaction is the mass and the damping coefficient, which is followed by the spring stiffness and the damping coefficient. The next most influential parameter interaction is the mass and the angular velocity followed by the mass and the damping coefficient. The next most influential parameter interactions are the mass and the spring stiffness, the spring stiffness and the angular velocity, and the spring stiffness and the amplitude. The least influential parameter interactions are the damping coefficient and the amplitude, the damping coefficient and the angular velocity, and the amplitude and the angular velocity.

The sensitivity indices in the case of the maximum value of acceleration are shown in Figure 3.

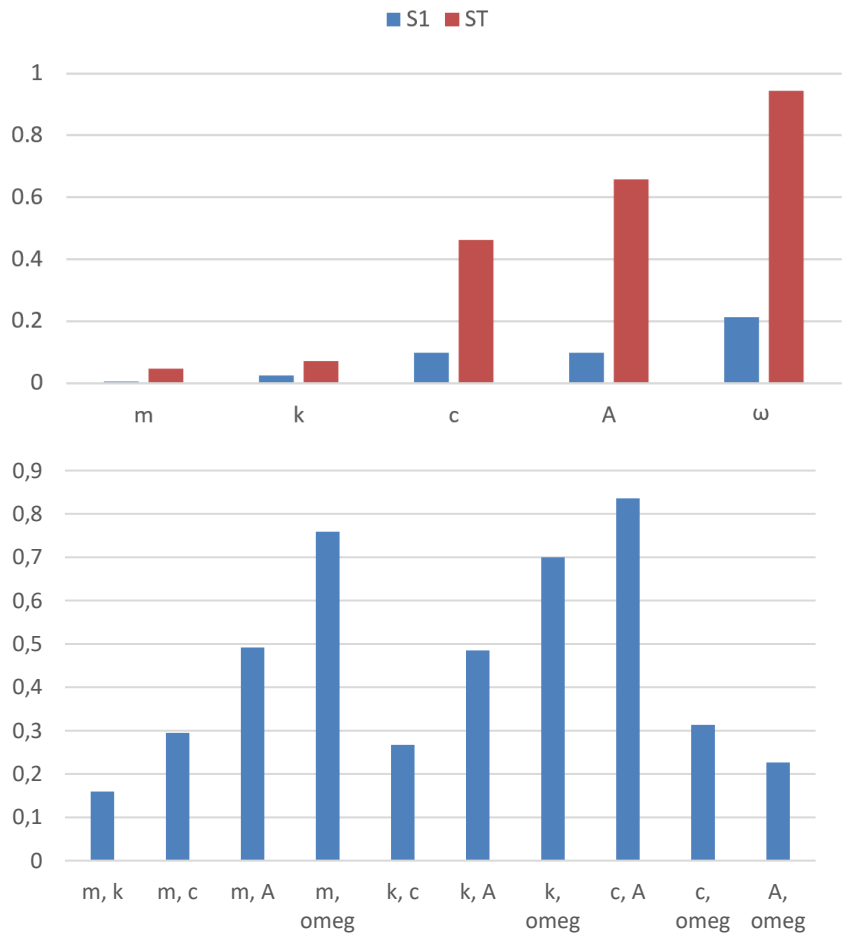

Figure 3. First-order and total sensitivity indices (above) and second-order sensitivity indices (below) in the case of the maximum value of acceleration

It can be seen that the most influential parameter is the angular velocity of the excitation signal in this case too as S1, and ST are both the largest. It is followed by the amplitude of the excitation signal and the damping coefficient. The next most influential parameter is the spring stiffness, and the least influential parameter is the mass. It is interesting to observe that S1 is small compared to ST, which means that the parameters have less influence on the maximum acceleration alone than their interactions. The most influential parameter interaction is the damping coefficient and the amplitude of the excitation signal, which is followed by the mass and the angular velocity. The next most influential parameter interaction

is the spring stiffness and the angular velocity followed by the mass and the amplitude of the excitation signal. The least influential parameter interactions are the mass and the spring stiffness, the amplitude and the angular velocity of the excitation signal, and the spring stiffness and the damping coefficient.

In the case of the peak frequency, the total sensitivity indices were larger than one or were negative. This means that more samples are necessary to carry out the sensitivity analysis in this case. Also, the peak frequency is not the best choice for sensitivity analysis, as the values change between 2 intervals (Figure 4)

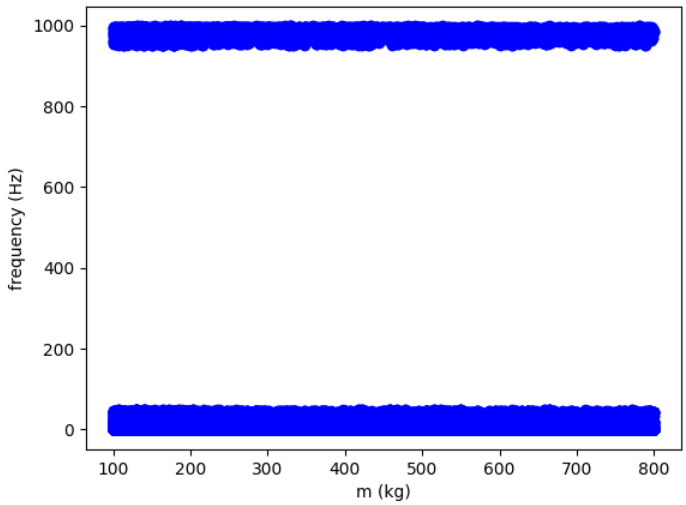

Figure 4. Scatterplot of the sensitivity of the mass to the peak frequency of the acceleration

An attempt was made to carry out the global sensitivity analysis of a fire truck suspension system with 23 parameters using 49152 samples and a suspension system with hysteresis and with 8 parameters using 73728 samples, but some indices were larger than one or a negative number. This means that more samples will be necessary. It is ongoing research for which parallelization and supercomputers will be used. Also, the numerical simulation will be carried out using C++, and Python will be used only for the sample generation and the global sensitivity study.

CONCLUSION

In this paper the global sensitivity analysis of a Duffing-type nonlinear vibration system was carried out using Python SALib. The output variable was the RMS of acceleration and the maximum value of acceleration, and the input parameters were the mass, the spring stiffness, the damping coefficient, the amplitude of the excitation signal, and the angular velocity of the excitation signal. Based on Sobol' indices the angular velocity and the amplitude of the excitation signal were the most influential parameters in both cases. However, the first-order sensitivity indices were small compared to the second-order sensitivity indices in all cases. This means that the parameters alone have less influence on the model output, than the parameters combined. The most influential parameter combinations are the mass and the damping coefficient in the case of the RMS of acceleration, and the damping coefficient and the amplitude of the excitation signal, and the mass and the angular velocity in the case of the maximum value of acceleration. In the case of the peak frequency, the sensitivity indices were negative or larger than 1, which means more parameter combination is necessary. Further research task is to carry out the global sensitivity study of more complex

nonlinear systems as well and to speed up the calculation using other methods (e.g. latin-hypercube) and supercomputers.

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