

THE EFFECT OF RATIO OF FIBER BUNDLES IN THE TEXTILE COMPOSITE LAYER

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Abstract: The article shows how the orthotropic material properties of the textile composite layer change if the ratio of longitudinal and transversal fiber bundles in the textile is modified in the case of a composite reinforced with plain weave textile. I determine the material properties using model-cell and finite element simulation. The density of the longitudinal fiber bundles is the same in each of the five examined layer models. I change the ratio of the fiber bundles of the textile by varying the distance of the transversal roving's in each model.

KEYWORDS: composite; textile; roving; ratio of fiber bundles; orthotropic material properties; finite element method

1 Introduction

Laminated textile composite plates contain one flat woven fabric per layer. The planar 2D fabric is one whose thickness size is small compared to the other two sizes. In the fabric, the elementary reinforcing fibers are arranged in fiber bundles, in roving's, which can be untwisted or twisted. In untwisted roving, the elementary fibers are arranged parallel to each other, in twisted roving like a helix. The majority of textiles have two mutually perpendicular fiber directions. The fabric is obtained by regular tying of warp and weft roving's that are perpendicular to each other. There are many types of weaving. This article analyses composites reinforced with plain weave textiles, in which the warp roving runs once under and once above a weft fiber bundle. Figure 1 shows a plain weave textile. In the examined textile composites, the micron-thin elementary carbon fibers are arranged in untwisted, flat roving. The matrix of the composite is polyester resin.

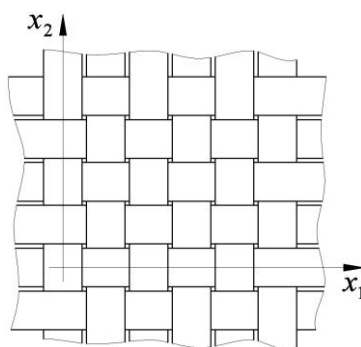


Fig. 1 Plain weave textile

If the density of the roving's in a textile is increased in one direction, the volume ratio of the fibers will also be higher. From this it is expected that the stiffness of the composite increases not only in the direction of densification, but also in the direction perpendicular to densification, since the roving also has a higher stiffness in the transverse direction than the

matrix material. However roving's perpendicular to densification will have more waves within a given length, and the waviness will reduce the stiffness of the composite. I analyzed the combined result of these two effects using the numerical method described in article [4].

2 Linear Elastic, Orthotropic Material Law

Figure 1 shows the x_1x_2 material coordinate system of the textile composite plate, where x_1 is the warp direction of the textile, which is called the longitudinal direction, and x_2 is the weft direction, which is called the transverse direction.

The material structure of the composite is heterogeneous. Each layer consists of matrix-impregnated textile and matrix. The behavior of such a textile composite layer can be treated with a macroscopic, orthotropic material law. The linear elastic, orthotropic material law in the generalized plane stress state, in the material coordinate system of the layer, has the following form [2], [8]:

$$\begin{bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ \bar{\gamma}_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \cdot \begin{bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\tau}_{12} \end{bmatrix}. \quad (1)$$

The average strain and stress of a textile composite layer with volume V can be defined as follows [6], [9], [11]:

$$\bar{\varepsilon}_{ij} = \frac{1}{V} \int_{(V)} \varepsilon_{ij} dV, \quad (2)$$

$$\bar{\sigma}_{ij} = \frac{1}{V} \int_{(V)} \sigma_{ij} dV, \quad (i, j = 1, 2, 3). \quad (3)$$

Five macroscopic material characteristics give the relationship between the homogenized stresses and deformations: the E_1 , E_2 moduli of elasticity, the ν_{12} , ν_{21} Poisson's ratios and the G_{12} shear modulus of elasticity. Four of these are independent, as the following equation can be written for energetic reasons [2], [5]:

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}. \quad (4)$$

3 Material Properties of the Fiber, Matrix and the Roving

In the textile composite there are linear elastic, orthotropic, transversely isotropic carbon fiber bundles embedded in a polyester matrix. The type of the roving is Torayca T300-3K. 3K means that there are 3000 elementary carbon fibers with a diameter of $d_f = 7 \mu\text{m}$ in the roving. The material properties of the carbon fiber from the literature source [7], [10] are given in Table 1, in the $x_{f1}x_{f2}x_{f3}$ material coordinate system shown in Figure 2. It is transversely isotropic.

Table 1 Material properties of the carbon fiber

$E_{f1} = 230\,000 \text{ MPa}$	$\nu_{f12} = 0.166$	$G_{f12} = 6\,432 \text{ MPa}$
$E_{f2} = 15\,000 \text{ MPa}$	$\nu_{f23} = 0.400$	$G_{f23} = 5\,357 \text{ MPa}$
$E_{f3} = 15\,000 \text{ MPa}$	$\nu_{f13} = 0.166$	$G_{f13} = 6\,432 \text{ MPa}$

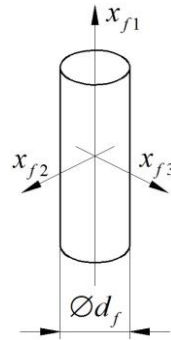


Fig. 2 Material coordinate system of a carbon fiber

The matrix is a polyester resin of type AROPOL M105TB, which is a linear elastic, isotropic material. I determined the E_m modulus of elasticity and the ν_m Poisson's ratio of the matrix material by measurement according to the standard [1], and I calculated the G_m shear modulus of elasticity based on formula (5) valid for isotropic materials:

$$E_m = 3677 \text{ MPa}, \nu_m = 0.346,$$

$$G_m = \frac{E_m}{2(1+\nu_m)} = 1365.9 \text{ MPa}. \quad (5)$$

The cross section and $x_{r1}x_{r2}x_{r3}$ material coordinate system of the matrix-impregnated roving are shown in Figure 3. The x_{r1} axis is parallel to the fibers (Figure 2). I produced the macroscopic material constants of the roving with the finite element model described in article [3]. The roving model is a periodic unit cell (Figure 4). The results of the calculations are in Table 2. The matrix-impregnated roving is linear elastic, orthotropic, transversely isotropic.

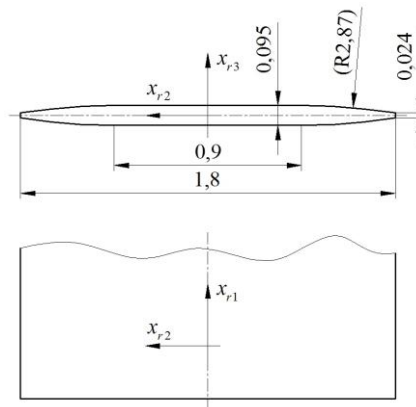
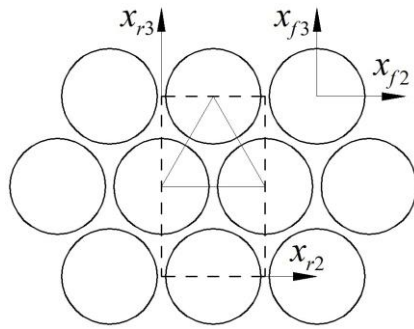
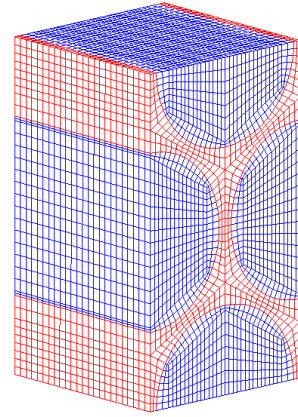


Fig. 3 The cross section of the roving and its material coordinate system



(a) One periodic unit in the roving



(b) Finite element mesh of the roving model-cell

Fig. 4 The model-cell of the fiber bundle

Table 2

Macroscopic material properties of the matrix-impregnated roving

$E_{r1} = 177\,236 \text{ MPa}$	$\nu_{r12} = 0.202$	$G_{r12} = 4115 \text{ MPa}$
$E_{r2} = 10\,352 \text{ MPa}$	$\nu_{r23} = 0.430$	$G_{r23} = 3620 \text{ MPa}$
$E_{r3} = 10\,352 \text{ MPa}$	$\nu_{r13} = 0.202$	$G_{r13} = 4115 \text{ MPa}$

4 The Ratio of the Longitudinal and Transversal Roving's

Figure 5 shows a sketch of a plain weave textile. The x_1x_2 is the material coordinate system of the plain weave textile composite layer. The structure of the textile can be described with the following geometric characteristics:

Table 3 Geometric characteristics of the textile

	x_1 direction	x_2 direction
Density of roving's	n_1 [count/cm]	n_2 [count/cm]
Distance, spacing of roving's	t_1 [mm]	t_2 [mm]

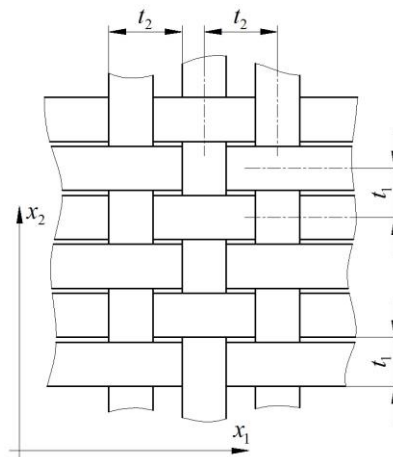


Fig. 5 Distance of roving's in the textile

The following relationship exists between the density and distance of the roving's:

$$t = \frac{1}{n}, \quad t[\text{mm}] = \frac{1}{n[\text{count/cm}]} \left[\frac{10 \text{ mm}}{1 \text{ cm}} \right] = \frac{10}{n[\text{count/cm}]}, \quad (6)$$

$$n = \frac{1}{t}, \quad n[\text{count/cm}] = \frac{10}{t[\text{mm}]}. \quad (7)$$

The ratio of longitudinal and transversal roving's in the textile:

$$\lambda = \frac{n_2}{n_1} = \frac{t_1}{t_2}. \quad (8)$$

The volume ratio of the fibers in the roving's in x_1 and x_2 directions in the composite layer with volume V :

$$\varphi_{f1} = \frac{V_{f1}}{V}, \quad \varphi_{f2} = \frac{V_{f2}}{V}. \quad (9)$$

The volume ratio of the textile in the composite layer:

$$\varphi_f = \varphi_{f1} + \varphi_{f2} = \frac{V_{f1} + V_{f2}}{V_f + V_m} = \frac{V_f}{V}. \quad (10)$$

The roving is impregnated by matrix material, so also between the elementary fibers there is bedding material. The volume ratio of the matrix in the composite layer:

$$\varphi_m = \frac{V_m}{V_f + V_m} = \frac{V_m}{V}. \quad (11)$$

I changed the ratio of longitudinal and transversal fiber bundles during the tests that I took the distance of the roving's to be constant in the x_1 direction, and I modified the density of the roving's in x_2 direction. I examined five cases. The spacing of the longitudinal roving's in each textile is $t_1 = 2 \text{ mm}$. The structure of the textile used in the five cases and the dimensions of the model-cell used in the finite element analysis are summarized in Table 4 and shown in Figure 6. The model-cell contains one periodic unit of textile in cases (b), (c), (d) and (e).

Table 4 Geometric characteristics

	λ [—]	n_1 [$\frac{\text{count}}{\text{cm}}$]	n_2 [$\frac{\text{count}}{\text{cm}}$]	φ_{f1} [—]	φ_{f2} [—]	φ_f [—]	φ_m [—]
(a)	0	5	0	0.231	0	0.231	0.769
(b)	0.5	5	2.5	0.231	0.116	0.347	0.653
(c)	0.67	5	3.33	0.231	0.155	0.386	0.614
(d)	0.8	5	4	0.231	0.186	0.417	0.583
(e)	1	5	5	0.232	0.232	0.464	0.536

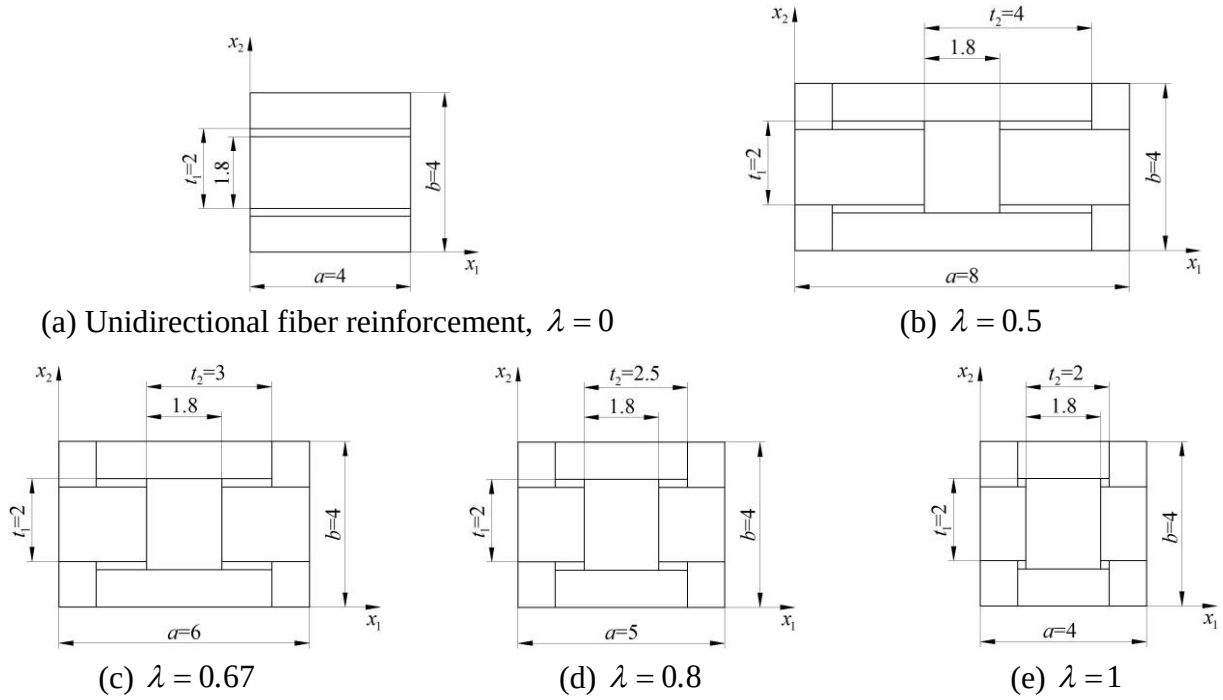


Fig. 6 Dimensions of the textile in the finite element model-cell

The smallest roving ratio is $\lambda = 0$, this is the case with unidirectional fiber reinforcement (Table 4, Figure 6 (a)). Then there is no transversal roving, so it is essentially not a textile. At the highest fiber ratio, at $\lambda = 1$, the density of the roving's is the same in the longitudinal and transverse directions (Table 4, Figure 6 (e)). In this case, the fiber bundle is most wavy in the x_1 direction. Because of the waviness, $\varphi_{f1} = 0.232$ and not $\varphi_{f1} = 0.231$ as in the others. The longitudinal sections of two of the examined textiles are shown in Figure 7. In Figure 7 (a), the spacing is $t_2 = 2$ mm (Table 4, Figure 6 (e)), here the textile contains 3 repeating units on a length of 12 mm. In Figure 7 (b), the spacing is $t_2 = 3$ mm (Table 4, Figure 6 (c)), in this case, the textile contains 2 repeating units on a length of 12 mm. The transversal roving's are the same in all textiles, because the distance of the longitudinal roving's is the same in the models. In the Figure 7, the $x_1x_2x_3$ is the material coordinate system for the textile composite layer, the $x_{r1}x_{r2}x_{r3}$ is for the roving. The layer thickness for all models is $h = 0.25$ mm. Figure 8 shows a textile composite layer.

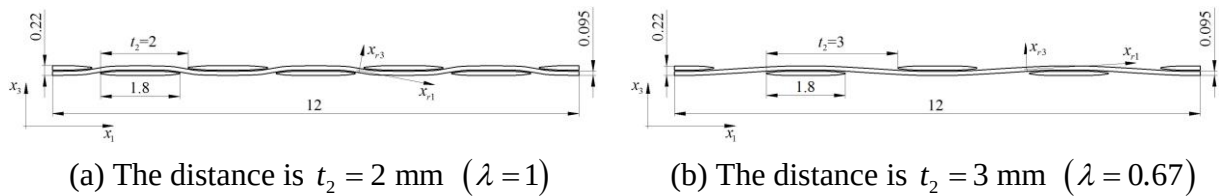


Fig. 7 Longitudinal section of plain weave textile

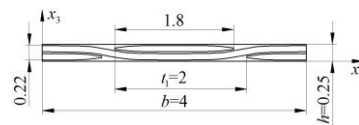


Fig. 8 Textile composite layer

5 The Results of the Finite Element Simulation

I performed the finite element calculations as described in article [4]. I generated a finite element mesh consisting of quadratic tetrahedron elements for the textile composite layer. Figure 9 shows the finite element mesh of the textile, in the case of $\lambda = 0.8$. The longitudinal roving's are illustrated by blue elements, the transversal roving's by green elements. Figure 10 shows the meshed layer model-cell. The red elements mark the matrix.

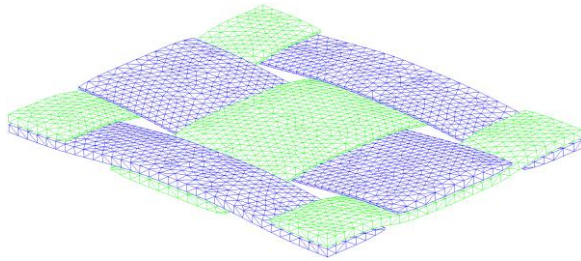


Fig. 9
Finite element mesh of the textile
($\lambda = 0.8$)

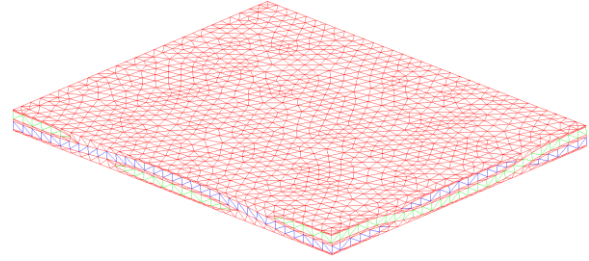


Fig. 10
Finite element mesh of the layer model-cell
($\lambda = 0.8$)

To determine the material properties of the composite layer, three load cases must be examined on each model-cell: An x_1 directional tension for computation of E_1 , ν_{12} , an x_2 directional tension for computation of E_2 , ν_{21} and a shear on the surface x_1x_2 for the calculation of G_{12} shear modulus of elasticity. In the case of $\lambda = 1$, $E_1 = E_2$ and $\nu_{12} = \nu_{21}$, so there is no need to model separate x_1 and x_2 directional tensions, for example x_1 directional tension is sufficient.

Table 5 summarizes the material properties calculated using the five finite element model-cells. The moduli of elasticity and Poisson's ratios satisfy equation (4) in all cases.

Table 5 Material properties calculated with the finite element model-cells

λ [–]		Material properties from the modell-cell		
(a)	$\lambda = 0$	$E_1 = 55700$ MPa	$\nu_{12} = 0.276$	$G_{12} = 2100$ MPa
		$E_2 = 5850$ MPa	$\nu_{21} = 0.029$	
(b)	$\lambda = 0.5$	$E_1 = 56300$ MPa	$\nu_{12} = 0.088$	$G_{12} = 2450$ MPa
		$E_2 = 27575$ MPa	$\nu_{21} = 0.043$	
(c)	$\lambda = 0.67$	$E_1 = 55850$ MPa	$\nu_{12} = 0.094$	$G_{12} = 2600$ MPa
		$E_2 = 35065$ MPa	$\nu_{21} = 0.059$	
(d)	$\lambda = 0.8$	$E_1 = 54700$ MPa	$\nu_{12} = 0.100$	$G_{12} = 2705$ MPa
		$E_2 = 41040$ MPa	$\nu_{21} = 0.075$	
(e)	$\lambda = 1$	$E_1 = 50700$ MPa	$\nu_{12} = 0.103$	$G_{12} = 2900$ MPa
		$E_2 = 50700$ MPa	$\nu_{21} = 0.103$	

6 The Effect of Changing the Roving Ratio on the Material Properties

For each of the material properties E_1 , E_2 , ν_{12} , ν_{21} , G_{12} I examined the effect of the change in the ratio of longitudinal and transversal roving's λ . For the points obtained with the model-cells, I took the following diagrams with third degree spline interpolation:

$$E_1 = f(\lambda), \quad E_2 = f(\lambda), \quad \nu_{12} = f(\lambda), \quad \nu_{21} = f(\lambda), \quad G_{12} = f(\lambda).$$

In the range $0 < \lambda < 0.5$, the roving's are very far from each other in the x_2 direction. This does not even occur in practice. Therefore, I did not calculate material properties with the finite element model-cell for this interval. However, I also indicated the values for $\lambda = 0$ (for unidirectional fiber reinforcement) in the diagrams. Table 5 shows that the material properties E_2 , ν_{21} , G_{12} are strictly monotonically increasing as a function of λ . Between $\lambda = 0$ and $\lambda = 0.67$, the modulus of elasticity E_1 has a maximum and the Poisson's ratio ν_{12} has a minimum. The ν_{12} takes its maximum value at $\lambda = 0$ in the examined interval $\lambda \in [0;1]$. I took the diagram $E_1 = f(\lambda)$, $E_2 = f(\lambda)$, $\nu_{21} = f(\lambda)$, $G_{12} = f(\lambda)$ at $\lambda = 0, 0.5, 0.67, 0.8, 1$ with the third degree spline interpolation fitted to five points. This is shown in Figures 11–13. The points marked in black are values calculated with finite element model-cells. I determined the Poisson's ratio ν_{12} between $\lambda = 0$ and $\lambda = 0.5$ in two more places using equation (4) to make the diagram $\nu_{12} = f(\lambda)$ more accurate. One is $\lambda = 0.2$, the other where the modulus of elasticity E_1 has a maximum ($\lambda = 0.411$). I read the function values of the diagram $E_1 = f(\lambda)$, $E_2 = f(\lambda)$, $\nu_{21} = f(\lambda)$ at $\lambda = 0.2$ and $\lambda = 0.411$. I marked these points with red crosses in Figures 11 and 12:

$$\begin{aligned} E_1(\lambda = 0.2) &= 56125 \text{ MPa}, & E_2(\lambda = 0.2) &= 14428 \text{ MPa}, & \nu_{21}(\lambda = 0.2) &= 0.031, \\ E_1(\lambda = 0.411) &= 56340 \text{ MPa}, & E_2(\lambda = 0.411) &= 23626 \text{ MPa}, & \nu_{21}(\lambda = 0.411) &= 0.037. \end{aligned}$$

The Poisson's ratio ν_{12} is obtained as follows at $\lambda = 0.2$ and $\lambda = 0.411$ according to equation (4):

$$\nu_{12}(\lambda = 0.2) = \frac{E_1(\lambda = 0.2)}{E_2(\lambda = 0.2)} \nu_{21}(\lambda = 0.2) = 0.121, \quad (12)$$

$$\nu_{12}(\lambda = 0.411) = \frac{E_1(\lambda = 0.411)}{E_2(\lambda = 0.411)} \nu_{21}(\lambda = 0.411) = 0.088. \quad (13)$$

I marked the two points obtained from equations (12) and (13) with green squares in Figure 12. I fitted the spline of the diagram ν_{12} to seven points.

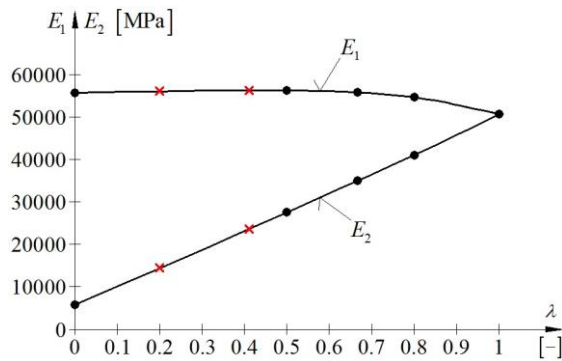


Fig. 11 The change of E_1 , E_2 moduli of elasticity as a function of λ

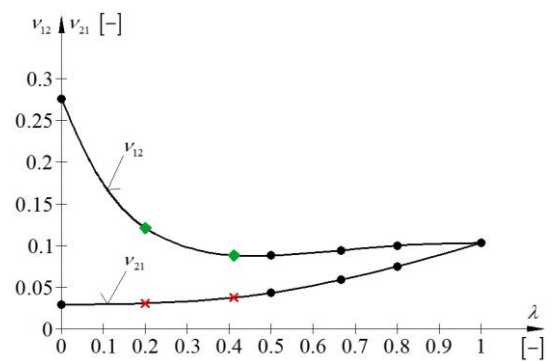


Fig. 12 The change of ν_{12} , ν_{21} Poisson's ratios as a function of λ

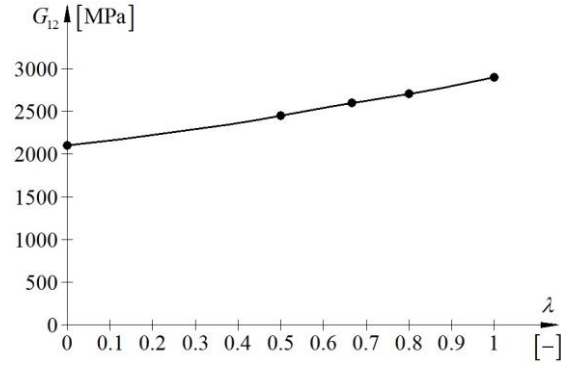


Fig. 13 The change of G_{12} shear modulus of elasticity as a function of λ

Increasing the density of the transversal roving's, and thus increasing the volume ratio of the reinforcing fibers, increase the waviness of the longitudinal roving's, which reduces the longitudinal stiffness. The geometry and waviness of the transversal roving's are the same in all cases except for $\lambda = 0$, therefore the E_2 transversal modulus of elasticity increases almost linearly as a function of λ (Figure 11).

In contrast, the E_1 longitudinal modulus of elasticity has a local maximum. Figure 14 shows the material coordinate system x_1x_2 of the textile composite layer and $x_{r1}x_{r2}$ of the roving's. By densification of the roving's in the x_2 direction, the stiffness of the composite plate also increases in the x_1 direction, since the roving also has a higher stiffness in the x_{r2} direction than the matrix material. The maximum of E_1 can be explained by the fact that the densification of the transversal roving's has a stiffening effect in the x_1 direction only up to a certain λ value. If the longitudinal roving's reach a certain waviness, the textile composite layer loses its stiffness (Figure 11). The composite reinforced with textile in Figure 7 (b) has higher stiffness in the x_1 direction than the composite with textile in Figure 7 (a).

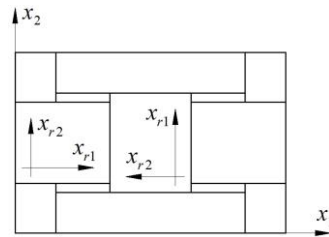


Fig. 14 The material coordinate system x_1x_2 of the textile composite layer and $x_{r1}x_{r2}$ of the roving's

The Poisson's ratio ν_{21} increases monotonically, i.e. the cross contraction increases monotonically in the case of tension in x_2 direction, if there are more waves in the x_1 directional roving's (Figure 12).

The Poisson's ratio ν_{12} is the largest when there is no transversal reinforcement. If the number of roving's is increased in the transverse direction, the cross contraction decreases only up to a certain roving ratio. After the local minimum, the diagram $\nu_{12} = f(\lambda)$ increases with a small slope (Figure 12).

The shear modulus of elasticity G_{12} increases monotonically, approximately linearly, if the proportion of transversal roving's is increased (Figure 13).

CONCLUSION

The article determines the change of the macroscopic material properties of the textile composite layer as a function of the longitudinal and transversal roving's ratio. The material properties calculated using the finite element model-cell and the diagrams fitted to the points are valid for roving, polyester resin and layer thickness of given geometry and material. As the roving ratio increases, the material properties E_2 , G_{12} , ν_{21} increase monotonically, E_1 has a maximum and ν_{12} has a minimum in the examined range. These extreme values can be explained by two opposing effects, the stiffening effect of increasing the roving density and the stiffness weakening effect of the increase in waviness.

REFERENCES

- [1] *ASTM D638-10*, Standard Test Method for Tensile Properties of Plastics, **2010**.
- [2] Barbero, E. J., "Finite Element Analysis of Composite Materials Using Ansys", CRC Press, **2014**.
- [3] Bojtár, G., Csizmadia, B. M., Égert, J. "Numerical Estimation Method of Orthotropic Material Properties of a Roving for Reinforcement of Composite Materials", *Acta Polytechnica Hungarica* 13 (6), pp. 163 – 182, ISSN 1785-8860, **2016**.
- [4] Bojtár, G., Csizmadia, B. M., Égert, J. "Numerical Determination of Orthotropic Material Properties of Textile Composite Layers and their Validation by Measurement", *Acta Polytechnica Hungarica* 14 (2), pp. 47 – 67, ISSN 1785-8860, **2017**.
- [5] Bunsell, A. R., Renard, J. "Fundamentals of Fibre Reinforced Composite Materials", Series in Material Science and Engineering, Institute of Physics Publishing, **2005**.
- [6] Siva Bhaskara Rao Devireddy; Sandhyarani Biswas: "Effect of Fiber Geometry and Representative Volume Element on Elastic and Thermal Properties of Unidirectional Fiber-Reinforced Composites", Hindawi Publishing Corporation Journal of Composites, Article ID 629175, 12 p., **2014**.
- [7] Goda T. "Wear Mechanism of Composite-Steel Sliding Pairs", PhD Thesis, Budapest University of Technology and Economics, Faculty of Mechanical Engineering, Supervisor: K. Váradi (In Hungarian), **2002**.
- [8] Kollár, L. P., Springer, G. "Mechanics of Composite Structures", Cambridge University Press, **2003**.
- [9] Leijan, H. "Determining Micro- and Macro- Geometry of Fabric and Fabric Reinforced Composites", Dissertation, Department of Mechanical and Nuclear Engineering College of Engineering, Kansas State University, Manhattan, Kansas, Advisor: Youqi W, **2013**.
- [10] Torayca T300 Factory catalogue (No. CFA-001), Toray Carbon Fibers America, Inc. (CFA), **2012**.
- [11] Xia, Z., Zhang, Y., Ellyin, F. "A Unified Periodical Boundary Conditions for Representative Volume Elements of Composites and Applications", *International Journal of Solids and Structures* 40 (8), pp. 1907 – 1921, **2003**.