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DYNAMIC STABILITY OF AN INCLINED FLUID-CONVEYING PIPE WITH A ONE-SIDED ELASTIC SUPPORT

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Abstract: Pipeline transport is proven to be an efficient method for conveying fluids over long distances, offering benefits such as continuous transport, significant volume capacity, lower energy costs, and substantial environmental advantages. Damage to pipelines could have detrimental effects on the economy, the environment, and pose health and safety risks for the population. The safety issues associated with pipes garner considerable attention in both scientific research and industry. The focus of the current study is on the dynamic stability of a straight fluid-conveying pipe with an elastic support. The investigation is based on the Euler-beam model and involves the use of Laplace and Fourier transformations. The paper draws attention to the stability of inclined pipes, as they haven't received much attention.

KEYWORDS: dynamic stability, critical velocity, inclined pipe, Laplace transformation, Fourier transformation

1 Introduction

Fluid flowing pipes are used in various fields of engineering. The study of the dynamic behaviour of the fluid-pipe system has been addressed by a number of scientists, [1] - [3].

The article [4] delves into a probabilistic evaluation of the reliability of gas pipelines, employing the direct Monte Carlo method. In [5] the stress state of fluid-conducting pipes lying on an elastic base has been investigated.

Transverse vibrations of polypropylene copolymer (PP-R) pipes conducting water were investigated in [6]. They are pinned at both ends. The task parameters include the inclination angle, temperature variation and the ratio of the pipe length to the outer diameter. An analytical model, utilizing Fourier - Laplace transforms, is introduced. Applying this approach, transverse displacements of the tube axis were obtained and the results were compared with those of the Galerkin method. For the identical pipe-fluid system examined in [6], the circular frequency of oscillations and the critical velocity of the flowing fluid are determined across various eigenforms of vibrations, as detailed in [7]. In [8] the Laplace - Fourier transforms method is employed to solve the differential equation describing the transverse vibrations of a fixed inclined pipe with a flowing fluid under temperature influence. The study determined transverse displacements at various angles of pipe inclination and temperature changes.

2 Problem formulation

The pipe under consideration is shown in Fig.1. It is cantilevered at its left end and on a linear elastic support with rigidity c on its right end. The transverse vibrations of the pipe are described with the following differential equation [9]:

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$$EI\frac{\partial^4 w}{\partial x^4}-\left[\left(m_f+m_p\right)(l-x)g\sin\theta-m_fV^2\right]\frac{\partial^2 w}{\partial x^2}+\left(m_f+m_p\right)g\sin\theta\frac{\partial w}{\partial x}+\left(m_f+m_p\right)g\cos\theta+$$
$$+2m_fV\frac{\partial^2 w}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

In equation (1) t is the time, x is the axial coordinate and l is the length of the pipe (Fig.1). $w(x,t)$ represents the function of transverse displacements of the pipe's axis. The other characteristics of the system are the angle of inclination of the pipe θ, the bending stiffness EI, the mass per unit length of the pipe m_p, the mass of the transported fluid per unit length of the pipe and the fluid velocity V. g is the gravity acceleration.

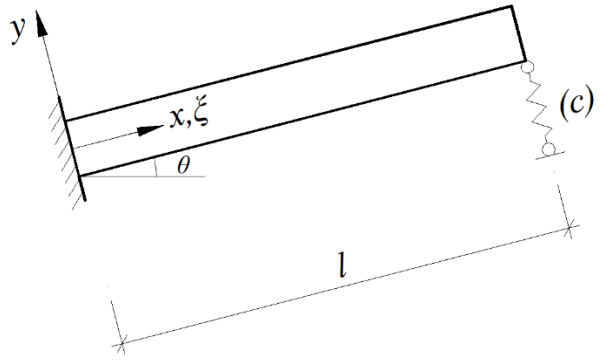


Fig. 1 Static scheme and of the inclined pipe

The dimensional equation (1) is being transformed into non-dimensional by introducing non-dimensional parameters:

$$\eta=\frac{w}{l};\quad \xi=\frac{x}{l};\quad \beta=\frac{m_f}{m_f+m_p};\quad G=\frac{\left(m_f+m_p\right)g\,l^3}{EI};\quad u=Vl\sqrt{\frac{m_f}{EI}};\quad \tau=\frac{t}{l^2}\sqrt{\frac{EI}{m_f+m_p}};$$
$$\bar{c}=\frac{cl^3}{EI};\quad \Omega=\omega\,l^2\sqrt{\frac{m_f+m_p}{EI}} \tag{2}$$

where β is the mass ratio, u is the non-dimensional parameter of the fluid's velocity, τ is the non-dimensional parameter of the time, Ω- dimensionless parameter of the system's circular frequency, $\bar{c}$ is the non-dimensional parameter of the rigidity of the elastic support at the right end of the pipe and G is the dimensionless parameter of the gravity acceleration.

After the above mentioned transformation the equation (1) obtains the following form:

$$\frac{\partial^4\eta}{\partial\xi^4}-\left[(1-\xi)G\sin\theta-u^2\right]\frac{\partial^2\eta}{\partial\xi^2}+G\sin\theta\frac{\partial\eta}{\partial\xi}+2\sqrt{\beta}u\frac{\partial^2\eta}{\partial\xi\,\partial\tau}+\frac{\partial^2\eta}{\partial\tau^2}+G\cos\theta=0 \tag{3}$$

The solution of the differential equation is sought in the form:

$$\eta(\xi,\tau)=\phi(\xi)\,q(\tau),\quad \xi\in[0;1] \tag{4}$$

In equation (4) $q(\tau)$ is unknown function and the function $\phi(\xi)$ has to satisfy the boundary conditions of the pipe. These boundary conditions for the pipe shown in Fig.1 are:

$$\begin{aligned} \xi = 0 \quad &\rightarrow \quad \eta = 0 \\ &\quad\;\; \frac{\partial \eta}{\partial \xi} = 0 \\ \xi = 1 \quad &\rightarrow \quad \frac{\partial^2 \eta}{\partial \xi^2} = 0 \\ &\quad\;\; \bar{c}\,\eta - \frac{\partial^3 \eta}{\partial \xi^3} = 0 \end{aligned} \tag{5}$$

Substituting (4) in (3) one obtains the following equation:

$$\phi^{IV} q - \left[(1-\xi) G \sin\theta - u^2\right] \phi^{II} q + G \sin\theta\, \phi^{I} q + 2\sqrt{\beta} u\ \phi^{I} \dot{q} + \phi\, \ddot{q} + G \cos\theta = 0 \tag{6}$$

In (6), the dots represent the derivatives with respect to the non-dimensional parameter of the time τ and primes with respect to the non-dimensional coordinate ξ.

In order to suppress the dependence on the non-dimensional parameter of the time, the Laplace transformation in the case of zero initial conditions was employed [9].

$$\phi^{IV} q(s) - \left[(1-\xi) G \sin\theta - u^2\right] \phi^{II} q(s) + G \sin\theta\, \phi^{I} q(s) + 2\sqrt{\beta} u\ \phi^{I} s\, \dot{q}(s) + \phi\, s^2\, \ddot{q}(s) + \frac{G}{s} \cos\theta = 0 \tag{7}$$

Where s is a complex variable. After that the finite Fourier sine transformation is applied

$$\begin{aligned} & q(s) \int_0^1 \phi^{IV} \sin n\pi\xi\, d\xi - \left(G \sin\theta - u^2\right) q(s) \int_0^1 \phi^{II} \sin n\pi\xi\, d\xi + G \sin\theta\, q(s) \int_0^1 \xi\, \phi^{II} \sin n\pi\xi\, d\xi + \\ & + G \sin\theta\, q(s) \int_0^1 \phi^{I} \sin n\pi\xi\, d\xi + 2\sqrt{\beta}\, u\ s\, q(s) \int_0^1 \phi^{I} \sin n\pi\xi\, d\xi + s^2\, q(s) \int_0^1 \phi \sin n\pi\xi\, d\xi + \\ & + \frac{G}{s} \cos\theta \int_0^1 \sin n\pi\xi\, d\xi = 0 \end{aligned} \tag{8}$$

In the present paper for the functions $\phi(\xi)$ are used the eigenfunctions of the pipe, expressed in polynomial form

$$\phi(\xi) = a_0 + a_1 \xi + a_2 \xi^2 + a_3 \xi^3 + a_4 \xi^4 \tag{9}$$

$$\phi^{I}(\xi) = a_1 + 2a_2 \xi + 3a_3 \xi^2 + 4a_4 \xi^3 \tag{10}$$

$$\phi^{II}(\xi) = 2a_2 + 6a_3 \xi + 12 a_4 \xi^2 \tag{11}$$

$$\phi^{III}(\xi) = 6a_3 + 24 a_4 \xi \tag{12}$$

Taking into account the boundary conditions (5) for the polynomial coefficients one obtains:

$$a_0 = 0 \tag{13}$$

$$a_1 = 0 \tag{14}$$

$$2a_2 + 6a_3 + 12 a_4 = 0 \tag{15}$$

$$\bar{c}\left(a_2 + a_3 + a_4\right) - \left(6a_3 + 24 a_4\right) = 0 \tag{16}$$

From equations (13) to (16) is obtained

$$a_2 = \frac{3}{2\left(\bar{c} + 3\right)} a_4 \tag{17}$$

$$a_3 = -\frac{1}{6}\left(\frac{3}{\bar{c} + 3} + 12\right) a_4 \tag{18}$$

$$\phi(\xi) = a_2\xi^2 + a_3\xi^3 + a_4\xi^4 \tag{19}$$

The coefficient a_4 could be obtained by using orthogonality of the eigenfunctions of the pipe. It depends on the rigidity of the elastic support.

In the case of $c = 200\,kN/m$, cross section of the pipe with inner and outer radii - $R_{in} = 0.012\ m$ and $R_{out} = 0.014\ m$, length of the pipe $l = 6\,m$ and modulus of elasticity of the material of the pipe $E = 210\,GPa$, the coefficient $a_4 = 5.237$. For the function $\phi(\xi)$ is obtained:

$$\phi(\xi) = 0.000101\,\xi^2 - 2\,\xi^3 + 5.237\,\xi^4 \tag{20}$$

The numerical solutions of the integrals in equation (8) are as follows:

$$A_1 = \int_0^1 \phi \sin n\pi\xi\, d\xi = \frac{(-1)^n\left(-1.0304n^4 + 1.6398n^2 - 0.4107\right) + 0.4107}{n^5} \tag{21}$$

$$A_2 = \int_0^1 \phi^I \sin n\pi\xi\, d\xi = \frac{(-1)^n\left(3.6667n - 4.8224n^3\right) + 0.3870n}{n^4} \tag{22}$$

$$A_3 = \int_0^1 \phi^{II} \sin n\pi\xi\, d\xi = \frac{(-1)^n\left(4.05363 - 16.2484n^2\right) + 0.0643n^2 - 4.05363}{n^3} \tag{23}$$

$$A_4 = \int_0^1 \xi\,\phi^{II} \sin n\pi\xi\, d\xi = \frac{(-1)^n\left(11.404 - 16.192n^2\right) + 0.7752n^2}{n^3} \tag{24}$$

$$A_5 = \int_0^1 \phi^{III} \sin n\pi\xi\, d\xi = \frac{-(-1)^n 36.188n - 3.8197n}{n^2} \tag{25}$$

$$A_6 = \int_0^1 \sin n\pi\xi\, d\xi = \frac{1-(-1)^n}{\pi n} \tag{26}$$

Substituting equations (21) to (26) in (8) one obtains

$$\left[s^2 A_1 + 2\sqrt{\beta}\,u\,A_2\,s + G\sin\theta\left(A_2 + A_4\right) - \left(G\sin\theta - u^2\right)A_3 + 125.688\,A_6\right]q(s) + \\ + \frac{G}{s}\cos\theta\,A_6 = 0 \tag{27}$$

$$\left[s^2 + \frac{2\sqrt{\beta}\,u\,A_2}{A_1}s + \frac{G\sin\theta\left(A_2 + A_4\right) - \left(G\sin\theta - u^2\right)A_3 + 125.688\,A_6}{A_1}\right]q(s) + \\ + \frac{G}{s}\cos\theta\frac{A_6}{A_1} = 0 \tag{28}$$

Let

$$B_1 = \frac{2\sqrt{\beta}\,u\,A_2}{A_1} \tag{29}$$

$$B_2^2 = \frac{G\sin\theta\left(A_2 + A_4\right) - \left(G\sin\theta - u^2\right)A_3 + 125.688\,A_6}{A_1} \tag{30}$$

Finally, the equation (8) obtains the following form:

$$\left[s^2 + B_1\, s + B_2^2\right] q(s) = -\frac{G}{s}\cos\theta\,\frac{A_6}{A_1} \tag{31}$$

$$q(s) = \frac{-\frac{G}{s}\cos\theta\,\frac{A_6}{A_1}}{\left[s^2 + B_1\, s + B_2^2\right]} \tag{32}$$

After application of the Fourier - Laplace inversion the function $\eta\,(\xi,\tau)$ is expressed in the following way [6]

$$\eta\,(\xi,\tau) = \phi(\xi)\left(-\frac{G}{s}\cos\theta\,\frac{A_6}{A_1}\,F(\tau)\right) \tag{33}$$

where

$$F(\tau) = \frac{1}{b_1\, b_2} + \frac{b_1\, e^{-b_2\,\tau} - b_2\, e^{-b_1\,\tau}}{b_1\, b_2\,(b_2 - b_1)} \tag{34}$$

and

$$b_1 = \frac{B_1}{2} + i\sqrt{B_2^2 - \frac{B_1^2}{4}} \tag{35}$$

$$b_2 = \frac{B_1}{2} - i\sqrt{B_2^2 - \frac{B_1^2}{4}} \tag{36}$$

The dimensionless natural frequency of the system is obtained from the roots of the characteristic equation of the system (31) by substituting $s = \pm i\,\Omega$ [7]

$$\Omega_{n,1} = \sqrt{B_2^2 - \frac{B_1^2}{2}} \tag{38}$$

And its complementary value is

$$\Omega_{n,2} = \sqrt{-B_2^2} \tag{39}$$

When $\Omega_{n,1} = 0$ the system is at the edge of losing stability. The natural frequency of the system depends on all its parameters. On that basis, if all system's parameters are fixed (except the fluid velocity), the corresponding critical velocity could be obtained.

RESULTS AND DISCUSSION

Numerical investigations have been conducted to explore the dynamic stability of the system shown in Fig.1.

The characteristics of the system are as follows: length of the pipe $l = 6\,m$, rigidity of the spring support $c = 200\ kN/m$, modulus of elasticity of the material of the pipe $E = 210\,GPa$, inner and outer radii of the cross-section - $R_{in} = 0.012\ m$ and $R_{out} = 0.014\ m$, angle of inclination of the pipe $\theta = 15^\circ$.

Fig.2 presents the relationship between the circular frequency ω of the system and the velocity of the conveyed fluid V for the first three eigenforms. The system is on the verge of losing stability when the circular frequency $\omega = 0$.

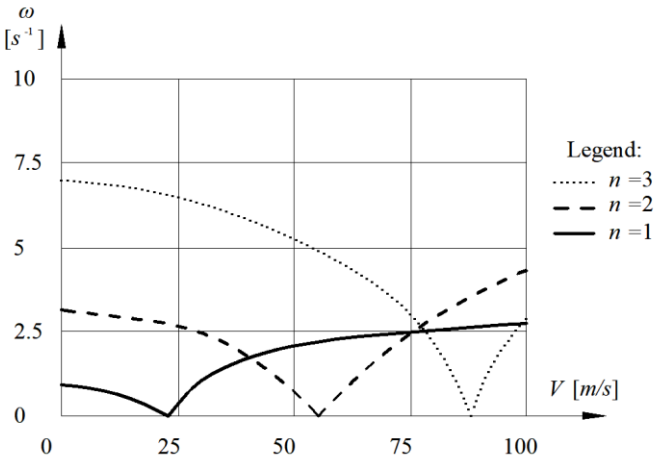


Fig. 2 The relation between the natural frequency ω and the fluid velocity V, at condition $\theta = 15^{\circ}$

CONCLUSIONS

The Laplace and Fourier transformation applied in this paper present another possibility to investigate the dynamic response of pipes conveying fluid. The proposed method could be considered competitive with other well-established approaches in the dynamic investigation of fluid conveying pipes, such as the Matrix method.

The results reveal that the fluid velocity significantly influences the dynamic response of the system and could impact its safety. To prevent damage, the pipe operator should not permit transportation velocities higher than the critical velocity of the system.

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