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THE MODEL OF INTERCONNECTED PIPES IN THE INVESTIGATION OF DYNAMIC STABILITY OF CARBON NANOTUBES EMBEDDED IN AN ELASTIC MEDIUM

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Abstract: The dynamic stability of an interconnected single-walled carbon nanotube is investigated on the basis of the Euler-beam model and with the employment of the spectral Galerkin method. The tubes under investigation are assumed hinged at its both ends and are embedded in an elastic medium. In order to investigate the effect of the surrounding elastic medium on the dynamic stability of the pipe the Pasternak elastic medium is introduced. The critical velocity is calculated for different stiffness of the elastic medium and different density of the conveyed fluid.

KEYWORDS: dynamic stability, critical velocity, nanotubes, interconnected tubes, Pasternak elastic medium

1 Introduction

Carbon nanotubes are widely used in various industrial sectors. The mechanical properties of these tubes are exceptional. Research on carbon nanotubes has been developing since their discovery in 1991. Nowadays the scientific investigation in the field is extensive [1], [2].

A series of articles is dedicated to the dynamic stability of nanotubes with flowing fluid. C.D. Reddy et al. [3] investigated the influence of the mass of the conveyed fluid on the free vibrations and stability of a single-walled carbon nanotube. These vibrations are studied by A.Belhadi et al. [4] when the nanotube rests on Winkler's elastic foundation.

Investigation of the fluid-structure interaction at the nanoscale is associated with significant difficulties and is also expensive. These are the main reasons why the dynamic stability of fluid conveying nanotubes often is investigated through the Euler or Timoshenko beam theories.

In the investigations of the vibrations and stability of tubes conveying fluid, the differential quadrature method is often applied. C. Bert and M. Malik [5] solved various problems in the field through the method, demonstrating its effectiveness. C.H.W. Ng et al. [6] study the stability and accuracy of a number of differential quadrature formulations when studying the vibrations of beams with different support conditions.

In the paper of F. Tornabene et al. [7], a cantilever tube with flowing fluid was investigated. The generalized differential quadrature method was applied, and dynamic stability analyses of the tube were conducted.

Other commonly applied method in the problems of dynamic stability of the tubes is the spectral Galerkin method. The method proves to be particularly suitable. Unlike the traditional Galerkin approach, orthogonal functions are used as basic functions in the approximate

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solution. This significantly improves the accuracy of the solution and avoids poorly conditioned matrices. The possibilities for choosing orthogonal functions include – Eigenfunctions of the form, which are solutions to a similar problem, Fourier series, Lagrange polynomials and Chebyshev polynomials.

In the present study, the spectral Galerkin method is employed to determine the critical velocity of the flowing fluid in a system of two interconnected carbon nanotubes, embedded in elastic medium. The medium is modelled as Pasternak elastic medium. Various parameters of this elastic foundation are considered.

2 Problem formulation

The dynamic stability of two interconnected carbon nanotubes, connected by rotational and linear springs that model the Pasternak elastic medium, is being investigated using the continuous Euler beam model. This model (Sv. Lilkova-Markova and D. Lolov [8]) is extremely simple and reliable for such studies. Stability experiments on carbon nanotubes demonstrate the effectiveness of the Euler model (J. Yoon et al. [9]). The differential equation describing the transverse vibrations of the nanotubes with length l, bending stiffness EI, and fluid flow velocity V is as follows (Talib EH. Elaikh et al. [10]):

$$EI\frac{\partial^4 w_1}{\partial x^4}+\left(m_f V^2-k_r\right)\frac{\partial^2 w_1}{\partial x^2}+k_r\frac{\partial^2 w_2}{\partial x^2}+2m_f V\frac{\partial^2 w_1}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 w_1}{\partial t^2}+k_w\left(w_1-w_2\right)=0 \tag{1}$$

$$EI\frac{\partial^4 w_1}{\partial x^4}+\left(m_f V^2-k_r\right)\frac{\partial^2 w_2}{\partial x^2}+k_r\frac{\partial^2 w_1}{\partial x^2}+2m_f V\frac{\partial^2 w_1}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 w_1}{\partial t^2}+k_w\left(w_2-w_1\right)=0 \tag{2}$$

In equations (1) and (2) x represents the axial coordinate, w_1 and w_2 are the transverse displacement of the both interconnected pipes, t is the time, EI is the rigidity of each tube, m_f is the mass of the conveyed fluid per unit length of the tube, m_p is the mass per unit length of the tube. V is the velocity of the conveyed fluid. k_w and k_r are the parameters of the Pasternak elastic medium, that models the elastic medium in which the nanotubes are embedded.

The static scheme of the two interconnected tubes is shown in Fig.1

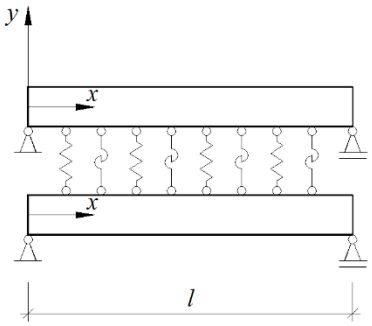


Fig. 1 Static scheme and of the two interconnected pipes

For the tubes shown in Fig.1, the following boundary conditions are valid

$$w\left(0,t\right)=w\left(l,t\right)=0;\left[\frac{\partial^2 w\left(x,t\right)}{\partial x^2}\right]_{x=0}=\left[\frac{\partial^2 w\left(x,t\right)}{\partial x^2}\right]_{x=l}=0. \tag{3}$$

For easier solution of the differential equations (1) and (2), a transition to dimensionless coordinates (ξ, η_1, η_2) is made. As a result, the following dimensionless parameters are introduced:

$$\xi = \frac{x}{l}; \eta_1 = \frac{w_1}{l}; \eta_2 = \frac{w_2}{l}; u = Vl\sqrt{\frac{m_f}{EI}}; \Omega = \omega l^2 \sqrt{\frac{m_f + m_p}{EI}};$$
$$\beta = \frac{m_f}{m_f + m_p}; \tau = \frac{t}{l^2}\sqrt{\frac{EI}{m_f + m_p}}; \bar{k}_w = \frac{k_w l^4}{EI}; \bar{k}_r = \frac{k_r l^2}{EI} \tag{4}$$

u - is dimensionless velocity of the fluid, β - the mass ratio, τ - dimensionless parameter of the time, Ω - dimensionless parameter of the circular frequency, $\bar{k}_w$ and $\bar{k}_r$ are dimensionless parameter of the elastic medium.

The differential equations (1) and (2) written in dimensionless form have the following form:

$$\frac{\partial^4 \eta_1}{\partial \xi^4} + \left(u^2 - \bar{k}_r\right)\frac{\partial^2 \eta_1}{\partial \xi^2} + \bar{k}_r \frac{\partial^2 \eta_2}{\partial \xi^2} + 2\sqrt{\beta} u \Omega \frac{\partial^2 \eta_1}{\partial \xi \partial \tau} + \Omega^2 \eta_1 + \bar{k}_w (\eta_1 - \eta_2) = 0 \tag{5}$$

$$\frac{\partial^4 \eta_2}{\partial \xi^4} + \left(u^2 - \bar{k}_r\right)\frac{\partial^2 \eta_2}{\partial \xi^2} + \bar{k}_r \frac{\partial^2 \eta_1}{\partial \xi^2} + 2\sqrt{\beta} u \Omega \frac{\partial^2 \eta_2}{\partial \xi \partial \tau} + \Omega^2 \eta_2 + \bar{k}_w (\eta_2 - \eta_1) = 0 \tag{6}$$

The spectral Galerkin method is employed to solve the differential equations (5) and (6). The functions η_1 and η_2 are sought in the following form:

$$\eta_1 = \sum_{i=1}^{n} q_i(\tau)\phi_i(\xi); \eta_2 = \sum_{i=1}^{n} q_{i+n}(\tau)\phi_i(\xi) \tag{7}$$

In (7) q_i and q_{i+n} are functions that are unknown, $\phi_i(\xi)$ are functions that satisfy the boundary conditions (basic functions). As basic functions in the present paper are used the eigenfunctions of the tubes filled with stationary fluid ($V = 0$). n is the number of basic functions used in the solution.

If (7) is substituted in equations (5) and (6), the residual functions are obtained, since the functions η_1 and η_2 are not exact solutions of the above mentioned differential equations. From here on, the primes denote the derivatives with respect to the dimensionless coordinate ξ and dots with respect to the dimensionless time τ.

$$R_1(\xi,\tau) = \sum_{i=1}^{n}\left[q_i \phi_i^{IV} + \left(u^2 - \bar{k}_r\right)q_i \phi_i^{II} + \bar{k}_r q_{i+n} \phi_i^{II} + 2\sqrt{\beta} u \Omega \dot{q}_i \phi_i^{I} + \Omega^2 q_i \phi_i + \bar{k}_w (q_i - q_{i+n})\phi_i\right] \tag{8}$$

$$R_2(\xi,\tau) = \sum_{i=1}^{n}\left[q_{i+n} \phi_i^{IV} + \left(u^2 - \bar{k}_r\right)q_{i+n} \phi_i^{II} + \bar{k}_r q_i \phi_i^{II} + 2\sqrt{\beta} u \Omega \dot{q}_{i+n} \phi_i^{I} + \Omega^2 q_{i+n} \phi_i + \bar{k}_w (q_{i+n} - q_i)\phi_i\right] \tag{9}$$

The spectral Galerkin method requires the residual functions $R_1(\xi,\tau)$ and $R_2(\xi,\tau)$ to be orthogonal to the basic functions within the interval $\xi \in [0;1]$.

$$\int_0^1 R_1(\xi,\tau)\phi_s(\xi)d\xi = 0, \quad s = 1,\cdots,n \tag{10}$$

$$\int_0^1 R_2(\xi,\tau)\phi_s(\xi)d\xi = 0, \quad s = 1,\cdots,n \tag{11}$$

As a result, a system of $2n$ differential equations is obtained with respect to the unknown functions q_i and q_{i+n}.

$$\int_0^1 \sum_{i=1}^{n} \phi_s \left[q_i \phi_i^{IV} + \left(u^2 - \bar{k}_r\right) q_i \phi_i^{II} + \bar{k}_r q_{i+n} \phi_i^{II} + 2\sqrt{\beta} u \Omega \ddot{q}_i \phi_i^{I} + \Omega^2 q_i \phi_i + \bar{k}_w (q_i - q_{i+n}) \phi_i \right] d\xi = 0 \quad (12)$$

$$\int_0^1 \sum_{i=1}^{n} \phi_s \left[q_{i+n} \phi_i^{IV} + \left(u^2 - \bar{k}_r\right) q_{i+n} \phi_i^{II} + \bar{k}_r q_i \phi_i^{II} + 2\sqrt{\beta} u \Omega \ddot{q}_{i+n} \phi_i^{I} + \Omega^2 q_{i+n} \phi_i + \bar{k}_w (q_{i+n} - q_i) \phi_i \right] d\xi = 0 \quad (13)$$

The eigenfunctions of a simply supported beam are:

$$\phi_i(\xi) = \sin(\lambda_i \xi) \quad (14)$$

The following expressions are in force:

$$\delta_{si} = \int_0^1 \phi_s \phi_i \, d\xi = \begin{cases} 0.5, s = i \\ 0, s \neq i \end{cases} \quad (15)$$

$$b_{si} = \int_0^1 \phi_s \phi_i^{I} \, d\xi = \begin{cases} 0, s = i \\ \dfrac{\lambda_s \lambda_i \left[1 - (-1)^{s+i}\right]}{\lambda_s^2 - \lambda_i^2}, s \neq i \end{cases} \quad (16)$$

$$d_{si} = \int_0^1 \phi_s \phi_i^{II} \, d\xi = -0.5 \lambda_i^2 \delta_{si} \quad (17)$$

Taking into account expressions (15), (16) and (17), the system (12) and (13) takes the form:

$$\sum_{i=1}^{n} \left[\delta_{si} \lambda_i^4 q_i + \left(u^2 - \bar{k}_r\right) q_i d_{si} + \bar{k}_r q_{i+n} d_{si} + 2\sqrt{\beta} u \Omega \ddot{q}_i b_{si} + \Omega^2 q_i \delta_{si} + \bar{k}_w (q_i - q_{i+n}) \delta_{si} \right] = 0 \quad (18)$$

$$\sum_{i=1}^{n} \left[\delta_{si} \lambda_i^4 q_{i+n} + \left(u^2 - \bar{k}_r\right) q_{i+n} d_{si} + \bar{k}_r q_i d_{si} + 2\sqrt{\beta} u \Omega \ddot{q}_{i+n} b_{si} + \Omega^2 q_{i+n} \delta_{si} + \bar{k}_w (q_{i+n} - q_i) \delta_{si} \right] = 0 \quad (19)$$

The system differential equations (18) and (19) could be represented in matrix form

$$|M| \ddot{q} + |C| \dot{q} + |K| q = 0 \quad (20)$$

The system (20) has the following characteristic equation

$$\det(X) = 0 \quad (21)$$

The matrix members X_{km} are obtained as follows:

$$X_{km} = \lambda^2 M_{km} + \lambda C_{km} + K_{km} \quad (22)$$

The roots λ of the characteristic equation (22) gives information about the stability of the system. If the real parts of all the roots are negative the system is stable. The velocity of the conveyed fluid and the characteristics of the investigated dynamic system influence on the roots of the characteristic equation. On that basis the critical velocity of the transported fluid V_{cr} could be obtained.

RESULTS AND DISCUSSION

To illustrate the proposed method, the stability of two interconnected single-walled carbon nanotubes with an external radius $R = 55\,nm$ of the cross-section and thickness $h = 10\,nm$ is investigated. The tubes are considered simply supported with length $l = 2000\,nm$. The problem is solved for different densities of the conveyed fluid in the range of $\rho = [700; 1600]\,kg/m^3$. The critical fluid velocity V_{cr} is determined for the elastic parameters of the Pasternak foundation $k_w = 100\,kPa; k_r = 150\,Nm/m$ and $k_w = 100\,kPa; k_r = 250\,Nm/m$. The obtained results are presented in Fig.2

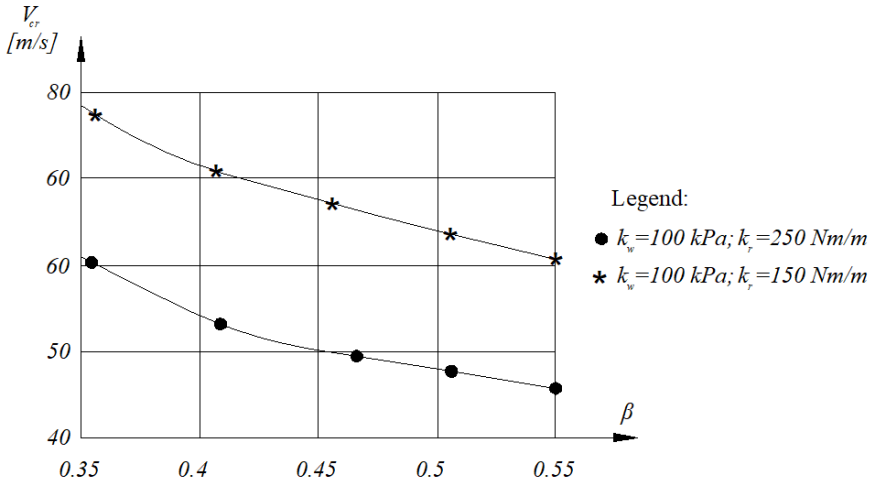


Fig. 2 The relation between the critical velocity V_{cr} and the mass ratio β

CONCLUSIONS

The results from the numerical investigation, presented in Fig.2, show that increase of the parameter k_r of the Pasternak elastic medium has a stabilizing effect on the system. Within the investigated range of the mass ration β, the higher critical velocities are obtained for the elastic foundation with a higher k_r. For the both examined elastic mediums, a reduction in the critical velocity is observed with an increase in the mass ratio β.

The model with interconnected tubes provides a more realistic representation of the dynamic stability of nanotubes embedded in an elastic medium in comparison to the dynamic model of a single pipe resting on an elastic foundation.

The Euler's and Timoshenko's beam theories have proven themselves over the years as effective methods in the investigations of dynamic stability of pipes conveying fluid. The theoretical results have been confirmed by a series of experimental studies.

On the field of nanoscale, the both methods are broadly used as an alternative to the expensive experimental investigations (X.Q. He et al. [11])

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