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DYNAMIC STIFFNESS DETERMINATION OF THE RUBBER BUSHING FEM MODELWITH VISCOELASTIC MATERIAL BEHAVIOUR

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Abstract: Presented paper deals with determination of dynamic stiffness of rubber bushing. Viscoelasticity of rubber is introduced. Next rheological models are shown and determined. Theory of the viscoelastic constitutive relation are done with storage modulus and loss modulus. Rubber bushing FEM model was prepared for calculation of dynamic stiffness through dynamic harmonic analysis in MSC Marc. Results from simulations are compared to experimental measurement data.

KEYWORDS: Rubber bushing, Viscoelasticity, Dynamic harmonic analysis, Storage modulus, Loss modulus, Dynamic stiffness

1 Introduction

Elastomers are typically amorphous polymers with molecular random motion. Cross-linking of molecules over relatively long distances produces a material with a fixed shape and size, i.e. an elastic solid. These materials exhibit hyperelastic and viscoelastic properties under load.

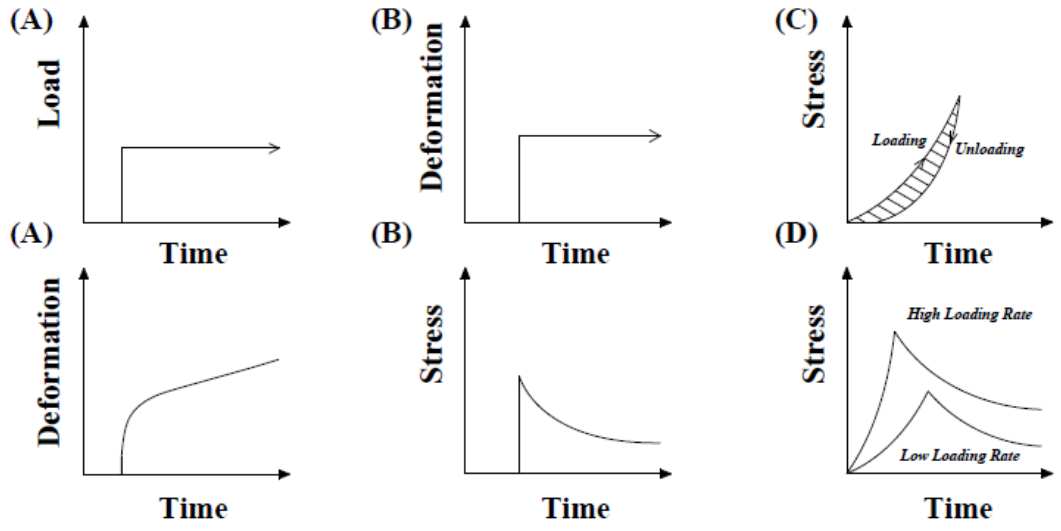


Fig. 1 Typical behavior of viscoelastic materials, A) creep, B) relaxation, C) hysteresis loop, D) relaxation

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The hyperelastic model captures the nonlinear elasticity of the material without time dependence. In contrast, the viscoelastic model is already dependent on time, frequency and heat [1].

Viscoelasticity has three main characteristics: creep, stress relaxation, and hysteresis. Creep is manifested by a gradual increase in deformation over time and a constant application of stress (Figure 1). Stress relaxation has the opposite characteristic of creep, which means that the deformation does not change over time with increasing stress. In the case of cyclic loading, the given material will demonstrate hysteresis. The energy absorbed in one cycle is given by the area in the loop, which is dependent on the rate of loading and unloading stress. Based on these characteristics, the material will reach a higher stress at a higher speed of the same loaded material than at lower speeds, which is due to the short excitation time interval during loading for energy release (Figure1 D).

The behavior of a viscoelastic material is best described through its shear deformation with a time-dependent shear relaxation modulus [1].

2 Rheological models

Differential equations of rheological models are most often used to describe the behavior of viscoelastic substances. The elastic properties of rubber are similar to metal springs, while energy absorption is in the form of a viscous liquid. These properties allow the rubber to maintain a constant shape after deformation and at the same time absorb mechanical energy.

An ideal elastic linear solid works on the principle of Hooke's law, i.e. stress is proportional to strain. On the other hand, an ideal viscous liquid works on the principle of Newton's law, i.e. stress is proportional to the rate of change of stress with time. By combining these two properties, we obtain the basic viscoelastic model. By series connection, we gain the Maxwell model and by connecting parallel, the Voight model, where η is the viscosity coefficient, E is represented stiffness in the spring element.

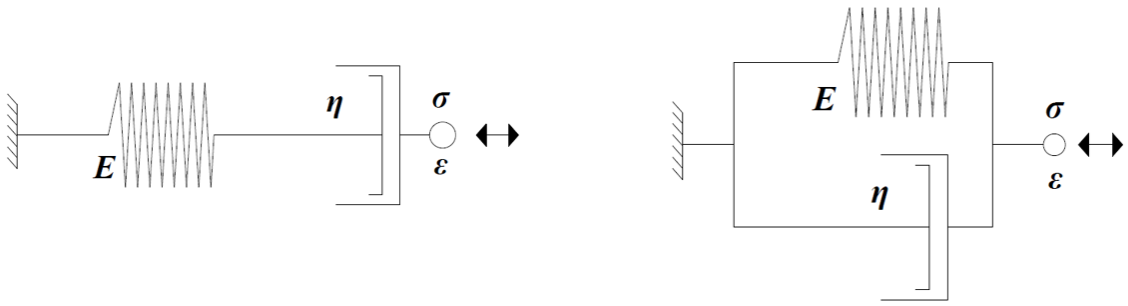


Fig. 2 Maxwell model (left), Voigt model (right)

While Maxwell's model is able to describe stress relaxation, Voigt's model is suitable for the description of creep behavior. However, a model is needed for materials used in practice, which can describe both mentioned phenomena. Therefore, a three-parameter model were proposed. They represent a combination of the mentioned elements and one of the models can be the Zener model as example (Figure 3).

However well they describe viscoelastic behavior, they can only do so relatively in a short range of time, temperature, and frequency. In order to more accurately define states with a large range of time, temperature and frequency values, the Generalized Maxwell model is used (Figure 4).

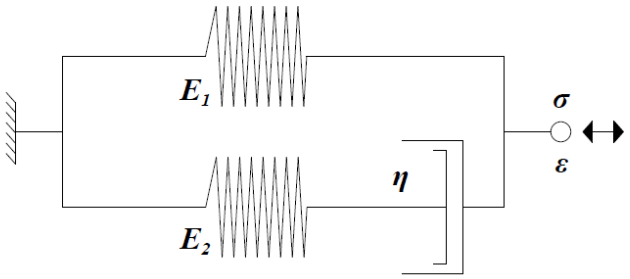


Fig. 3 Zener model

Element properties used in FEM model were Reduced Integration, Hourglass Control (Assumed Strain) and additional integration point.

As the number of Maxwell elements increases, the achieved dynamic stiffness is better described, but the complexity of the model also increases directly. This model represents the properties of more advanced models and it is also possible to introduce it into a model of nonlinear elastic behavior by replacing linear springs with nonlinear counterparts.

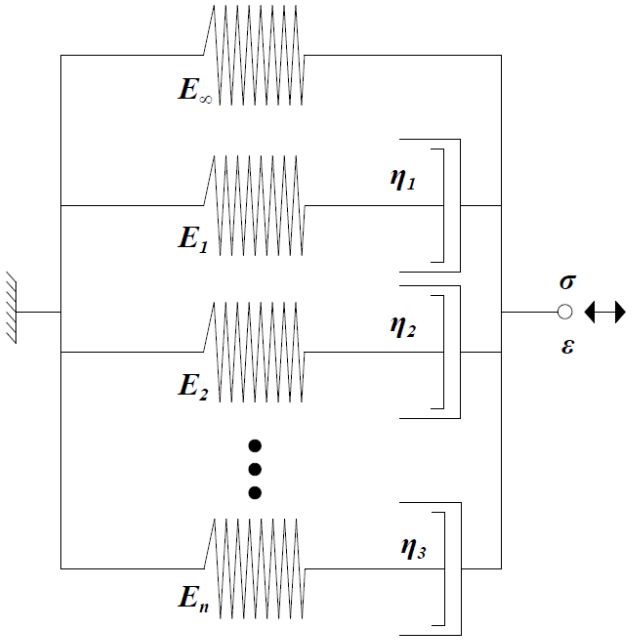


Fig. 4 Generalized model

3 Determination of the complex module

Frequent dynamic loads are influenced by the mechanical response of elastomeric materials, caused by viscous effects. Due to the viscous behavior of the rubber element, there is a phase gap between stress and strain (called loss angle δ). The ratio of stress and strain amplitudes represents the magnitude of the complex modulus $\boldsymbol{G}^*$ (shear).

$$|G^*| = \frac{\sigma_0}{\varepsilon_0} \tag{1}$$

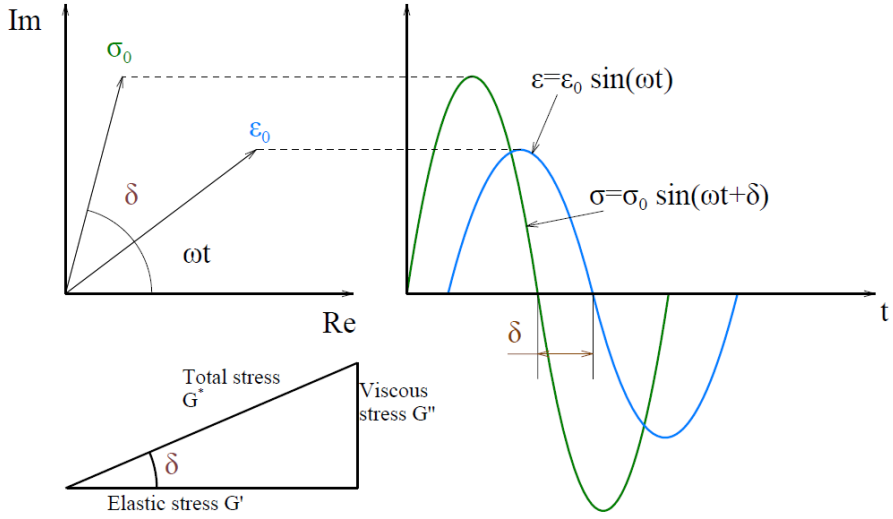


Fig. 5 Stress/strain curve and distribution of total complex module

Subsequently, it is clear from [9] that the complex module is composed of a real component called the storage module G' and an imaginary component called the loss modulus G''.

$$G^* = G' + iG'' \tag{2}$$

Defining the storage module and loss module has the form:

$$G' = \frac{\sigma_0}{\varepsilon_0}\cos(\delta) \tag{3}$$

$$G'' = \frac{\sigma_0}{\varepsilon_0}\tan(\delta) \tag{4}$$

The storage modulus represents the elastic response of the material and the loss modulus represents the viscous behavior of the material.

The loss factor is used to determine energy losses in viscoelastic materials. It represents mechanical damping or internal friction viscoelastic system.

$$\tan\delta = \frac{G''}{G'} \tag{5}$$

4 Theory of the viscoelastic constitutive relation

The viscoelastic constitutive model is defined based on the measured characteristics from dynamic mechanical analysis (DMA). Defining the material model based on DMA is the use of the relaxation function. A shear strain is considered, whose change of angular deformation has the form:

$$\gamma(t) = \gamma_0 \exp(i\omega t) \tag{6}$$

γ_0 – amplitude of angular deformation

ω – angular velocity

t – time

The steady-state shear stress response has a load given in the form:

$$\tau(t) = [G'(\omega) + iG''(\omega)]\gamma(t)\gamma_0 \exp(i\omega t) \quad (7)$$

By introducing a dimensionless quantity of the shear relaxation function whose form is:

$$g(t) = \frac{G_R(t)}{G_\infty} - 1 \quad (8)$$

$G_R(t)$ – time-dependent shear relaxation modulus

G_∞ - long term shear modulus

storage and loss modulus can be defined. Their expression can be written in the complex form of the Fourier transformation $g^*(\omega)$:

$$G'(\omega) = G_\infty[1 - Im(g^*)] \quad (9)$$

$$G''(\omega) = G_\infty[Re(g^*)] \quad (10)$$

While $Im(g^*)$ represents the imaginary element of the relaxation function g(t) after Fourier transformation and $Re(g^*)$ is the real element of the relaxation function g(t) as well.

5 Rubber bushing FEM model for dynamic harmonic analysis in MSC Marc

Preprocessing was done using MSC Marc (Hexagon). For meshing three-dimensional ten-node tetrahedron was used. Figure 6 shows resulted 3D mesh with size of element 2 mm and element type 130 created in MSC Marc.

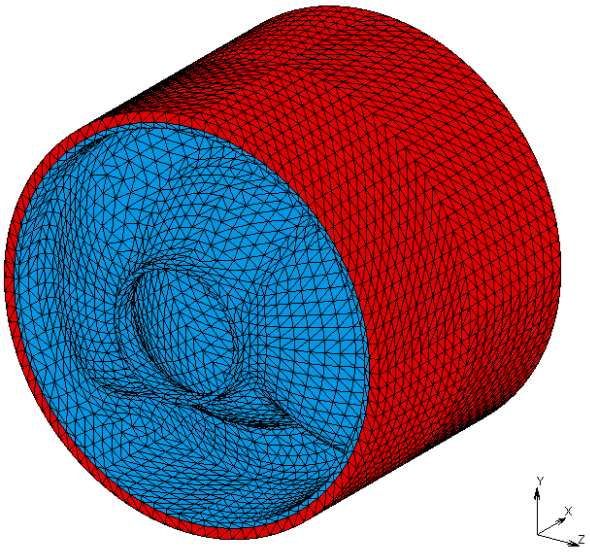


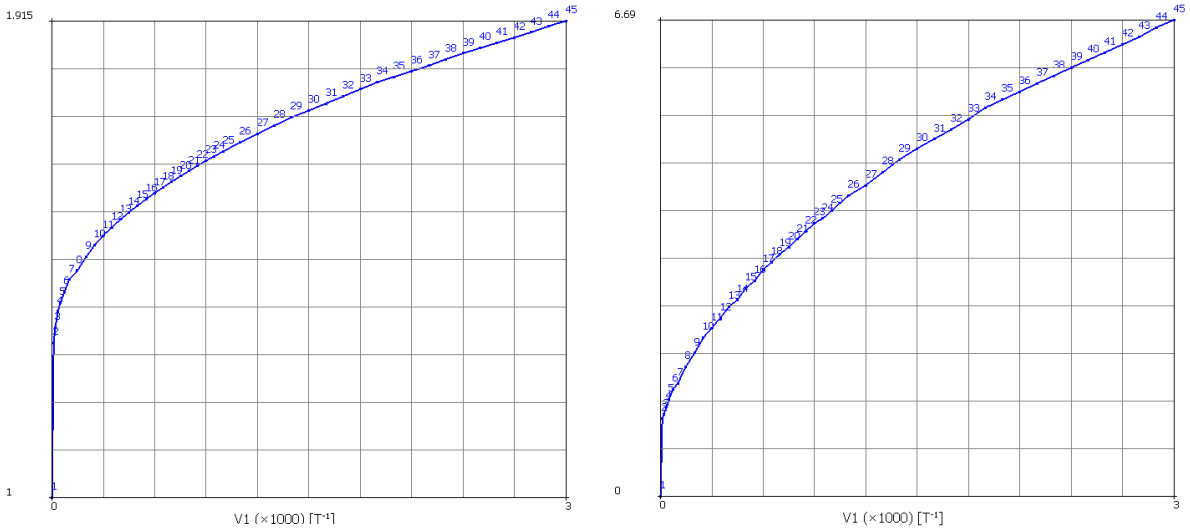
Fig. 6 FEM mesh model of rubber bushing

Rubber bushing consists of two parts - outer steel ring and inner rubber part. Material properties used in FEM model are shown in Tab. 1. Hyperelastic formulation used in the FEM model is based on the Yeoh model.

Viscoelastic properties were obtained from measurements on specimen at an excitation amplitude of 0.1 mm. Subsequently, for use in MSC Marc, they had to be normalized using the G_∞ long term shear modulus. Normalized G' storage modulus and G'' loss modulus was entered into the model using frequency-dependent tables (Figure 7).

Tab. 1 Material properties used in FEM model

Material	**Constitutive Model**	**Material Parameters**		
Steel	Linear Elastic	Young Modulus [MPa]	Poisson's Ratio [-]	Density [Tonne/mm^3]
		210000	0.3	7.8E-09
Rubber 60 ShA	Hyperelastic	Thermal Expansion coefficient [°C^{-1}]		Density [Tonne/mm^3]
		0.0001538		1.2E-09
		C_{10} [MPa]	C_{20} [MPa]	D_1 [MPa]
		0.603179	0.0301589	3015.89



a) b)

Fig. 7 Normalized curves, a) shear storage modulus, b) shear loss modulus

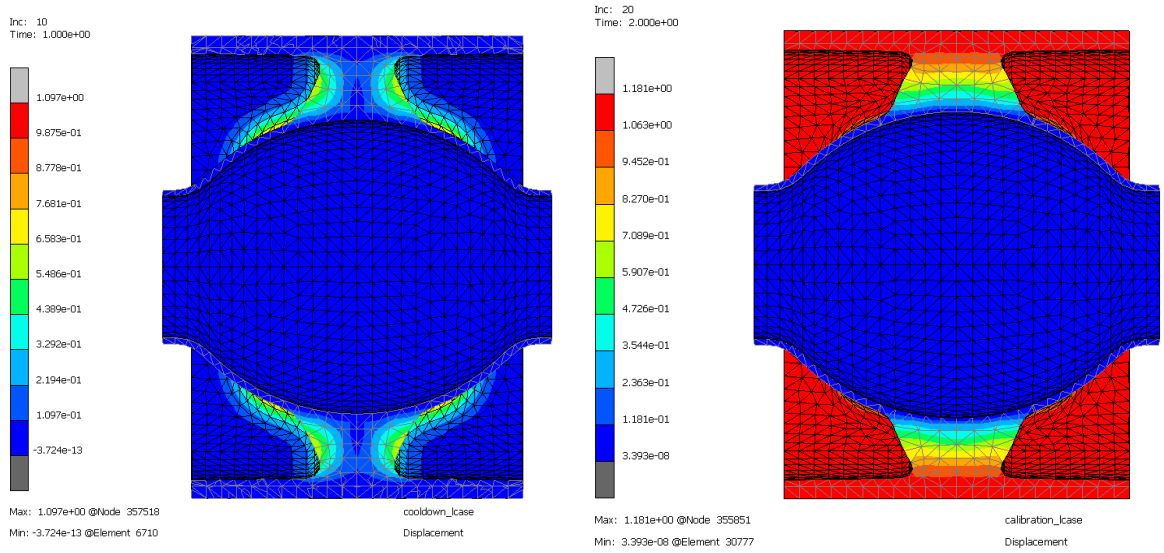
FEM rubber bushing job contains three consecutive load cases:

1. Cooldown load case: In this load case temperature is lowered from initial temperature 150 °C to 20 °C.
2. Calibration load case: In this load case temperature is maintained at 20 °C, independent node is fixed in all 6 DOF, outer ring surface nodes are displaced in radial direction (1.1 mm) using cylindrical coordinate system. Self-contact is turned on.
3. Dynamic harmonic load case: In this load case temperature is maintained at 20 °C, outer ring surface nodes are fixed, on independent node applies the harmonic excitation (0.1 mm in y direction) over a frequency range from 5 to 200 Hz in increments of 5Hz. Self-contact is turned on.

6 Comparison of FEM reults of rubber bushing to experimental measurement data

Figure 7 shows the deformation (resulting displacement) of rubber bushing after cooldown load case and calibration load case. The third load case was performed after these two load

cases. Third load case was dynamic harmonic analysis from which the dynamic stiffness as a function of the frequency was determined (Figure 9).



a) b)

Fig. 8 Deformation of rubber bushing after a) cooldown load case, b) calibration load case

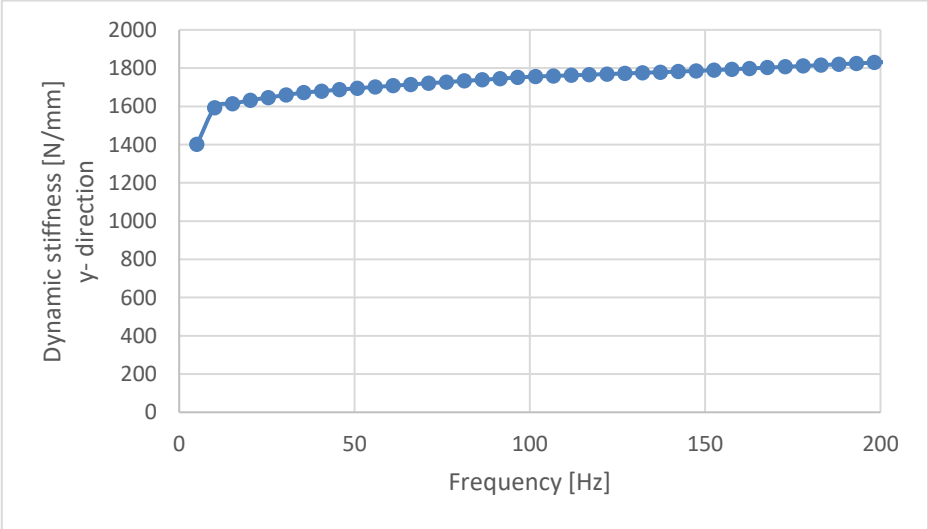


Fig. 9 Dynamic stiffness in y-direction of rubber bushing

Tab. 2 Comparison of simulation and measurement data at selected frequencies

Frequency [Hz]	Dynamic stiffness [N/mm] FEM results	Dynamic stiffness [N/mm] Experimental data	Difference [%]
20	1632	2047	20,3
150	1790	2902	38.3

Table 2 show simulations results compared to experimental measurement data, but measurement data obtain only values of two frequencies, 20 Hz and 150 Hz.

CONCLUSION

Results from FEM simulation using Marc Solver show dynamic stiffness of rubber bushing. FEM model consist of viscoelastic material behaviour defined by normalized tabular data of storage modulus and loss modulus. The first two load cases serve to get closer to the real bushing before starting the dynamic test. The comparison of simulation and measurement data show average coincidence. This will be investigated in further work.

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