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MESHING CHARACTERISTICS OF PROFILE SHIFTED CYLINDRICAL QUASI-INVOLUTE ARC-TOOTH-TRACE GEARS. PART 1. THEORETICAL BASE

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Abstract: A mathematical model of profile shifted quasi-involute arc-tooth-trace gears is obtained. The model includes functional relationships between the geometric-kinematic meshing characteristics and parameters of the generating surface as well as conditions which allow to prevent teeth undercutting and tip pointing. A short analysis of meshing characteristics value along the tooth trace is done for pressure angle value of 20º. This value of the angle was chosen as it is the most common one for general-duty gears, although the basic rack profile angle in the model is a variable quantity. The model can be used in the design and analysis of geometric-kinematic meshing characteristics of profile shifted quasi-involute arc-tooth-trace gears, cut by Gleason-type cutters with different profile angle value.

KEYWORDS: arc-tooth-trace gears, profile shift coefficient, generating surface, meshing characteristics, undercutting, tip pointing

1 Introduction

Gear train is one of the most important units of modern machines. This is why gears make a significant part of any machine-building product. Moreover, annual growth of the global volume of their production is predicted by [1] "from USD 26780 Mn in 2021 to USD 35810 Mn by 2028 at a CAGR of 4.2% during the 2021-2028 period". According to [2] high gears quality provided by advanced manufacturing process is a key step which allows to achieve sustainability in gear manufacturing. Therefore, the objective of gears improvement is relevant in the present time.

2 Literature review

It is well known that gears' operational characteristics are affected by the properties of their material. This is why, one method of gears improvement consists in choosing a proper material [3, 4, 5]. The second method involves choosing of the rational geometrical parameters of gears [3, 4, 6]. Both methods are applied at the designing stage, and they can be used almost

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independently. Geometry improvement can be implemented in different ways among which teeth geometry modification is the most common one. K. Marković and M. Franulović [7] investigated the influence of tip relief profile modification on decreasing of the contact stresses value. The authors of [8] and [9] also established positive influence of the profile modification on serviceability of gears. In addition to modification, the real importance in the present has development of non-standard gearing. In [10] B.E. Berlinger and J.R. Coulbourne proposed convoloid gearing as an alternative to involute gear pair. The convoloid pair showed 20-35% higher loading capacity that the involute one. The rules for design of novel S-gears was developed by J. Hlebanja et al. in [11]. These gears got their name due to their S-shape path of contact. Durability of S-gears is 20% higher compared to the involute ones. Replacement of gears' involute profile by cosine one leads to 20% and 30% decline of contact stresses as well as 35% and 50% decline of bending stresses according to results of [12] and [13] respectively. J. Wang et al. [14] developed method for the design of tooth profile based on parabolic curve as a form of the path of contact. D. Kopiláková and M. Bošanský [15] proposed the similar one to [14] C-C gearing. The path of contact in C-C gearing is outlined by arcs of circle. Both gearings [14] and [15] have advantages by contact and bending stresses compared to the involute analogues. It should be noted that standard cutting tool cannot be used for manufacturing of both modified [7-9] and non-involute [10-15] gears. Kyrychenko I., Stupnytskyy V. et al. [16] considers the method of cutting gears with a hyperboloid tool, which can significantly improve the accuracy and roughness of gear teeth. Unlike the mentioned ways of geometry improvement, profile shifting has one significant advantage because it does not require new tool geometry. At the same time profile shifting provides improving of meshing characteristics [3, 4]. The entire research [7-15] is related to spur gears which does not create axial force while in operation. Arc-tooth-trace (ATT) gears have the same feature, although their production process is more difficult. Nevertheless, heavy duty gear trains in some cases include arc-tooth-trace gears due to their advantages such as 25-50% higher loading ability at smooth operation and less sensitivity to misalignments [17]. Quasi-involute [18] and Novikov's [19] ATT gears are well known. The first ones have a limited loading capacity by contact stresses because of small values of active flank's related curvature radii. Novikov's ATT gears are noisier because their transverse contact ratio is equal to zero. The synthesis of gear profile geometry by meshing characteristics using procedure developed in [20] allows to reach the loading capacity level of synthesised gearing up to 50% higher compared to the quasi-involute one. Such approach requires designing of new non-standard tools, and that does not always make sense. The compromise solution consists in profile shifting which was presented in [18]. But the author of [18] described the technological aspects of profile shifting without its influence on meshing characteristics of profile shifted quasi-involute ATT gears. In [21] the authors described ATT gears' geometry at profile shifting but functional relationships between the geometric-kinematic meshing characteristics and parameters of the generating surface was not obtained. Thus the objective of this research is to develop the mathematical model of profile shifted quasi-involute ATT gears which includes functional relationships between the geometric-kinematic meshing characteristics and parameters of the generating surface taking into account conditions which allow to prevent teeth undercutting and tip pointing.

3 Generating surface

According to [18] teeth of ATT gears is generated by Gleason-type cutters. The segments of cutting bits for teeth active flank generation are delineated by a straight line. When Gleason-type cutter rotates, these segments delineate active flanks of rack's teeth that are basically a generating surface. In the normal cross-section of the generating surface the basic rack of involute gearing can be seen (Fig. 1). Let us present the basic rack using coordinate systems $X_{p1}O_{p1}Y_{p1}$ and $X_{p2}O_{p2}Y_{p2}$ in the form

$$y_{p1} = x_{p1}\tan\alpha; \quad y_{p2} = 0{,}5\pi - x_{p2}\tan\alpha, \tag{1}$$

where α is the profile angle, i.e. the angle between profiling straight line and appropriate $O_{p1}X_{p1}$ or $O_{p2}X_{p2}$ axis.

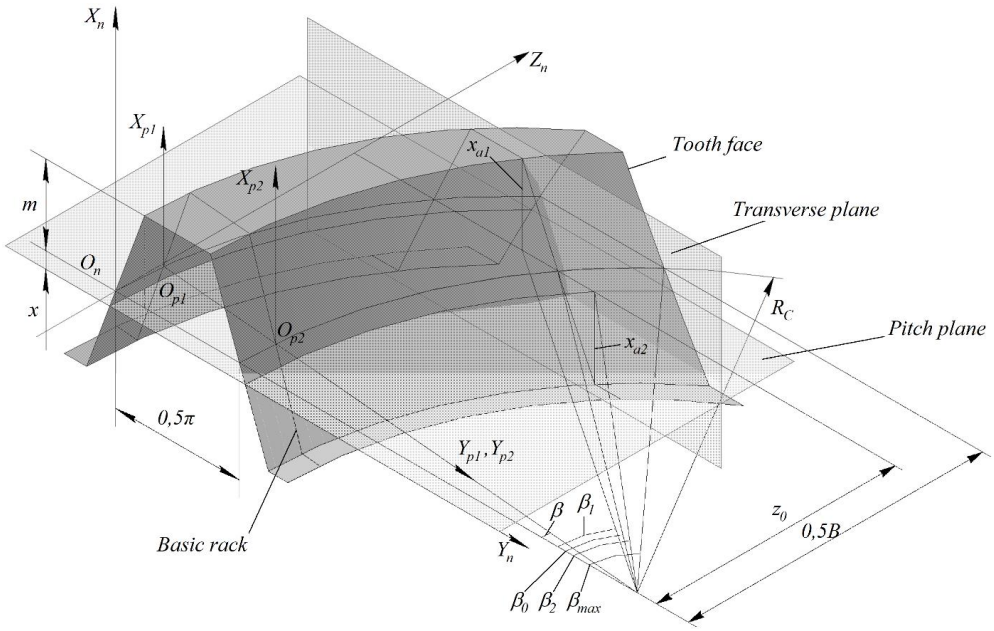


Fig. 1. Generating surface of profile shifted quasi-involute ATT gears

The equation of the generating surface in $X_nY_nZ_n$ system of coordinate, using (1) and results of [20], can be obtained for convex and concave sides respectively

$$\begin{aligned} &x_{n1} = x_{p1} + x; && x_{n2} = x_{p2} + x; \\ &y_{n1} = R_C - \left[R_C - \left(x_{p1} + x\right)\tan\alpha\right]\cos\beta; && y_{n2} = R_C - \left[R_C + \left(x_{p2} + x\right)\tan\alpha\right]\cos\beta + 0{,}5\pi; \\ &z_{n1} = \left(R_C - \left(x_{p1} + x\right)\tan\alpha\right)\sin\beta, && z_{n2} = \left(R_C + \left(x_{p2} + x\right)\tan\alpha\right)\sin\beta, \end{aligned} \tag{2}$$

where R_C is the radius of Gleason-type cutter; β is the angle of tooth trace, i.e. an angle between $\overline{R}_C$ vector and O_nY_n axis; x is the profile shift coefficient. It should be mentioned that Eqs. (2) and the following geometric parameters are normalised for the module m equal to 1 mm in a similar to [18] and [20] way.

To study further the ATT gears' geometry let us obtain the equation of meshing taking similar equation from [21] as a base. When the convex side of pinion tooth (at $i = 1$ and upper signs) or concave side of gear tooth (at $i = 2$ and lower signs) are meshed with generating surface the equation of meshing is as follows:

$$F_i^{CX} = \mp\left[R_C\left(1 - \cos\beta\right) - r_{wi}\varphi_i\right]\sin\alpha \mp \left(x_{pi} + x\right)\cos\beta/\cos\alpha = 0. \tag{3}$$

For the meshing of generating surface with concave side of pinion tooth (at $i = 1$ and upper signs) or convex side of gear tooth (at $i = 2$ and lower signs) the equation is as follows:

$$F_i^{CV} = \mp\left[-R_C\left(1 - \cos\beta\right) + r_{wi}\varphi_i - 0{,}5\pi\right]\sin\alpha \mp \left(x_{pi} + x\right)\cos\beta/\cos\alpha = 0. \tag{4}$$

where φ_i is the angles of rotation of pinion at $i = 1$ and gear at $i = 2$; r_{wi} is the radii of the reference cylinders of pinion at $i = 1$ and gear at $i = 2$.

It is well known that in the meshing of the generating surface and a tooth being generated, the vector of the relative velocity must be perpendicular to the line normal to the generating

surface at the contact point. Eqs. (3) and (4) provide fulfillment of this condition and connect generating surface's geometry with φ_i angles. This allows to obtain the equations of active flanks of profile shifted quasi-involute ATT gears. The equations of pinion tooth convex side (at $i=1$ and upper signs) or gear tooth concave side (at $i=2$ and lower signs) are as follows:

$$\begin{aligned} x_{i1} &= (x_{pi} + x \pm r_{wi})\cos\varphi_i \pm (x_{pi} + x)\cos\beta\sin\varphi_i / \tan\alpha; \\ y_{i1} &= \pm(x_{pi} + x \pm r_{wi})\sin\varphi_i - (x_{pi} + x)\cos\beta\cos\varphi_i / \tan\alpha; \\ z_{i1} &= [R_C - (x_{pi} + x)]\sin\beta. \end{aligned} \tag{5}$$

For the concave side of pinion tooth (at $i=1$ and upper signs) or convex side of gear tooth (at $i=2$ and lower signs) the equations of teeth flanks are as follows:

$$\begin{aligned} x_{i2} &= (x_{pi} + x \pm r_{wi})\cos\varphi_i \mp (x_{pi} + x)\cos\beta\sin\varphi_i / \tan\alpha; \\ y_{i2} &= \pm(x_{pi} + x \pm r_{wi})\sin\varphi_i + (x_{pi} + x)\cos\beta\cos\varphi_i / \tan\alpha; \\ z_{i2} &= [R_C + (x_{pi} + x)]\sin\beta. \end{aligned} \tag{6}$$

In a similar to (3)-(6) way in the following equations, $i=1$ and $i=2$ will be taken for pinion and gear respectively.

4 Meshing characteristics

Let us obtain geometric-kinematic meshing characteristics using Eqs. (2) – (6) and taking into account results of [20, 21]. It should be noted that generalized vertical coordinate x_p will be used in the following equations. It will be taken $x_p = x_{p1}$ for convex side of generating surface and $x_p = x_{p2}$ for concave one. Any other feature of meshing characteristics obtained will be explained separately.

Sliding velocity. This velocity affects teeth wear process and for profile shifted quasi-involute ATT gears it can be obtained as follows:

$$V_S = \omega_1[(u+1)/u](x_p + x)\sqrt{\cos^2\beta/\tan^2\alpha + 1}, \tag{7}$$

where ω_1 is the angular velocity of the pinion; $u = z_2/z_1$ is the gear ratio.

Rolling velocities. Vector of rolling velocity is normal to the contact line. Using this property and taking [20] and [21] results as a base the following formulas can be obtained for pinion and gear respectively:

$$V_1 = \frac{\omega_1[r_{w1}\sin^2\alpha + (x_p + x)(\cos^2\beta + \tau_3)]}{\sin\alpha\sqrt{\cos^2\beta + \tau_3}}; \; V_2 = \frac{\omega_2[r_{w2}\sin^2\alpha - (x_p + x)(\cos^2\beta + \tau_3)]}{\sin\alpha\sqrt{\cos^2\beta + \tau_3}}, \tag{8}$$

where ω_2 is the angular velocity of the gear; τ_3 is defined by the expression

$$\tau_3 = (1 \mp (x_p + x)) / [R_C\tan\alpha \mp (x_p + x)\tan^2\alpha]\sin^2\alpha\sin^2\beta. \tag{9}$$

In Eqs. (8) and (9) "–" sign corresponds to pinion tooth convex side and gear tooth concave side while "+" sign should be taken for pinion tooth concave side and gear tooth convex side.

The real values of R_C is when $R_C \geq 10m$ at $0° \leq \beta \leq 50°$ [18] as a rule. To obtain the maximum value of τ_3 the minimal $R_C = 10$ and maximal $\beta = 50°$ values should be taken. Let us also take $x_p + x \approx 1$ and $\alpha = 20°$, in that case the maximum value of τ_3 parameter is $\tau_3 = 0{,}050 \div 0{,}088$. It

follows that the effect of this parameter's value on the rolling velocity is insignificant, therefore the value of $\tau_3 = 0$ can be taken. At $\tau_3 = 0$ pinion and gear rolling velocities will be as follows:

$$V_1 = \frac{\omega_1\left[r_{w1}\sin^2\alpha + \left(x_p + x\right)\cos^2\beta\right]}{\cos\beta\sin\alpha},\ V_2 = \frac{\omega_2\left[r_{w2}\sin^2\alpha - \left(x_p + x\right)\cos^2\beta\right]}{\cos\beta\sin\alpha}. \tag{10}$$

The rolling velocities equal zero when the condition below is fulfilled

$$r_{wi}\sin^2\alpha \pm \left(x_p + x\right)\left(\cos^2\beta + \tau_3\right) = 0. \tag{11}$$

In Eq. (11) "+" sign corresponds to pinion teeth and "–" sign should be taken for gear teeth.

Total rolling velocity. Using [20] and [21] results the following formula can be obtained:

$$V_\Sigma = \omega_1\left[2r_{w1}\sin^2\alpha + \left(u-1\right)\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)/u\right]\left(\sin\alpha\sqrt{\cos^2\beta + \tau_3}\right)^{-1}. \tag{12}$$

It was demonstrated before that $\tau_3 = 0$ can be taken. Then the approximate value of total rolling velocity can be calculated using the following formula:

$$V_\Sigma = \omega_1\left[2r_{w1}\sin^2\alpha + \left(u-1\right)\left(x_p + x\right)\cos^2\beta/u\right]\left(\sin\alpha\cos\beta\right)^{-1}. \tag{13}$$

It follows from (12) that the total rolling velocity equals zero at:

$$2r_{w1}\sin^2\alpha + \left(u-1\right)\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)/u = 0. \tag{14}$$

The total rolling velocity is infinity at the section where

$$\cot\beta = \sin\alpha\sqrt{-1 \pm \left(x_p + x\right)/\left[R_C\tan\alpha \mp \left(x_p + x\right)\tan^2\alpha\right]}. \tag{15}$$

Relative curvature. This characteristic strongly defines contact stresses value on the teeth active surfaces, and it also affects teeth wear. Based on [20] and [21] results let us define the relative curvature as follows:

$$\chi = \frac{\left(r_{w1} + r_{w2}\right)\sin^3\alpha}{\left[r_{w1}\sin^2\alpha + \left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)\right]\left[r_{w2}\sin^2\alpha - \left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)\right]}. \tag{16}$$

Taking $\tau_3 = 0$ the formula for calculation of relative curvature approximate value is obtained from Eq. (16) as follows:

$$\chi = \left(r_{w1} + r_{w2}\right)\sin^3\alpha\left[r_{w1}\sin^2\alpha + \left(x_p + x\right)\cos^2\beta\right]^{-1}\left[r_{w2}\sin^2\alpha - \left(x_p + x\right)\cos^2\beta\right]^{-1}. \tag{17}$$

Using Eq. (16) in the middle cross-section of the arc on the pitch plane at $\beta = 0$ and $x_p + x = 0$ relative curvature is obtained

$$\chi = \left(r_{w1} + r_{w2}\right)/\left(r_{w1}r_{w2}\sin\alpha\right). \tag{18}$$

It should be noted that Eq. (18) is a well-known expression for calculation of relative curvature at the pitch point of cylindrical spur gear pair [4].

Specific sliding coefficients. This meshing characteristic affects teeth wear. It is defined based on [20] and [21] results for pinion and gear teeth respectively:

$$\eta_1 = \frac{\left(u+1\right)\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)}{u\left[r_{w1}\sin^2\alpha + \left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)\right]};\ \eta_2 = \frac{-\left(u+1\right)\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)}{u\left[r_{w2}\sin^2\alpha - \left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)\right]}. \tag{19}$$

Approximate value of pinion and gear teeth specific sliding at $\tau_3 = 0$ can be defined:

$$\eta_1 = \frac{(u+1)(x_p + x)\cos^2\beta}{u\left[r_{w1}\sin^2\alpha + (x_p + x)\cos^2\beta\right]};\ \eta_2 = \frac{-(u+1)(x_p + x)\cos^2\beta}{u\left[r_{w2}\sin^2\alpha - (x_p + x)\cos^2\beta\right]}.$$

Angle between sliding velocity vector and contact line. This angle along with the rolling velocity defines teeth contact conditions in terms of oil films generation, its most favourable value is $\nu = 0{,}5\pi$. The tangent of this angle can be obtained based on [20] and [21] results as:

$$\tan\nu = \left(\cos^2\beta + \tau_3\right)\left[R_C - (x_p + x)\tan\alpha\right]/\left[(x_p + x)\cos\alpha\sin\beta\cos\beta\right]. \quad (20)$$

The formula for calculation of $\tan\nu$ approximate value can be obtained at $\tau_3 = 0$ from Eq. (20):

$$\tan\nu = \left[R_C - (x_p + x)\tan\alpha\right]/\left[(x_p + x)\cos\alpha\tan\beta\right].$$

Contact ratio. This ratio defines smoothness of teeth meshing. It characterises tooth remating, i.e. it shows the number of teeth pairs meshed at the same time, and it can be defined as the sum of transverse contact ratio and overlap ratio. Based on [20] and [21] results *transverse contact ratio* can be defined as follows:

$$\varepsilon_\alpha = \left(z_1/2\pi r_{w1}\right)\left[\left(2x_{a2}/\sin 2\alpha - R_C\right)\cos\beta_2 - \left(2x_{a1}/\sin 2\alpha - R_C\right)\cos\beta_1\right], \quad (21)$$

where x_{a1} and x_{a2} are the values of $(x_p + x)$ which correspond to the tip radii r_{a1} and r_{a2} of pinion and gear, they can be defined for any transverse cross-section at the appropriate value of β by the expressions

$$x_{a1} = \left[\sqrt{r_{w1}^2 - \gamma_1\left(r_{w1}^2 - r_{a1}^2\right)} - r_{w1}\right]/\gamma_1;\ x_{a2} = \left[r_{w2} - \sqrt{r_{w2}^2 - \gamma_2\left(r_{w2}^2 - r_{a2}^2\right)}\right]/\gamma_2, \quad (22)$$

where $\gamma_1 = 1 + \cot^2\alpha\cos^2\beta_1$, $\gamma_2 = 1 + \cot^2\alpha\cos^2\beta_2$. Since geometric parameters in Eqs. (1) – (22) are normalised for the module m equal to 1 mm, tip radii r_{a1} and r_{a2} can be defined as $r_{ai} = r_{wi} + 1 + x$. It follows from Fig. 1 when $x_{a1} > 0$ then $x_{a2} < 0$.

The values of x_{a1} and x_{a2} should be obtained using Eq. (22) at $\beta = \beta_1$ and $\beta = \beta_2$ respectively. The mentioned values of β angle can be defined from Eqs. (5) and (6) as follows:

$$\sin\beta_1 = z_0/\left(R_C \mp x_{a1}\tan\alpha\right),\ \sin\beta_2 = z_0/\left(R_C \mp x_{a2}\tan\alpha\right). \quad (23)$$

In Eqs. (23), the upper signs correspond to the contact of pinion tooth convex side with mating gear tooth concave side, while the lower signs should be taken for contact of pinion tooth concave side with gear tooth convex side. In order to calculate β_1 and β_2 by Eq. (23) the value of z_0-coordinate should be set from the following range: $0 \le z_0 \le 0{,}5B$, where B is a facewidth, i.e. z_0-coordinate defines the transverse cross-section (Fig. 1). In this case at $z_0 = const$:

$$\sin\beta_1/\sin\beta_2 = \left(R_C \mp x_{a2}\tan\alpha\right)/\left(R_C \mp x_{a1}\tan\alpha\right).$$

Let us calculate the range of $\sin\beta_1/\sin\beta_2$ ratio at $R_C = 10$, $x_{a1} = 1$, $x_{a2} = -1$, $\alpha = 20°$. It equals 1.071 at upper signs and 0.93 at lower signs, i.e. $\beta_1 \approx \beta_2$. Thus the equality $\beta_1 = \beta_2 = \beta_0$ can be taken. The angle β_0 can be defined from:

$$z_0 = R_C\sin\beta_0. \quad (24)$$

At $\beta_1 = \beta_2 = \beta_0$ transverse contact ratio can be approximately defined as follows:

$$\varepsilon_\alpha = \left[z_1\left(|x_{a1}| + |x_{a2}|\right)\cos\beta_0\right] / \left(\pi r_{w1}\sin 2\alpha\right). \tag{25}$$

Taking into account Eq. (24) expression (25) becomes:

$$\varepsilon_\alpha = \left[z_1\left(|x_{a1}| + |x_{a2}|\right)\sqrt{1-\left(z_0/R_C\right)^2}\right] \Big/ \left(\pi r_{w1}\sin 2\alpha\right). \tag{26}$$

Based on [19] results let us define the *overlap ratio* at the range of $0 \le \beta \le \beta_{\max}$ as follows:

$$\varepsilon_\beta = z_1\left[2\left(x_p + x\right)/\sin 2\alpha - R_C\right]\left(\cos\beta_{\max} - 1\right)/\left(2\pi r_{w1}\right). \tag{27}$$

Eq. (27) can be also used for definition of facewidth B which provides the given value of the overlap ratio at the known value of R_C and value of $\beta_{\max}$ which can be defined from:

$$\cos\beta_{\max} = 1 - 2\pi r_{w1}\varepsilon_\beta / \left(z_1 R_C\right). \tag{28}$$

Applying $\beta_0 = \beta_{\max}$ from Eq. (28) and $z_0 = z_{0\max} = 0.5B$ to Eq. (24) we obtain the desired value of facewidth mentioned before:

$$B = 2R_C\sqrt{1-\left[1 - 2\pi r_{w1}\varepsilon_\beta / \left(z_1 R_C\right)\right]^2}\ .$$

If the facewidth is given and the radius of Gleason-type cutter is to be chosen, the following formula can be used to provide the given value of the overlap ratio:

$$R_C = 2\pi r_{w1}\varepsilon_\beta / \left[\left(1 - \cos\beta_{\max}\right)z_1\right].$$

Undercutting. This phenomenon appears under the fulfilling condition (11). Let us change it to the form:

$$\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)/\sin^2\alpha + r_{w1} = 0\,. \tag{29}$$

Applying $r_{w1} = r_{\min} = 0{,}5z_{\min}$ to Eq. (29) we obtain the minimal teeth number that allow to prevent undercutting of quasi-involute ATT gears:

$$z_{\min} = -2\left(x_p + x\right)\left(\cos^2\beta + \tau_3\right)/\sin^2\alpha\,. \tag{30}$$

It follows from Eq. (30) that the weakest section in terms of undercutting is at $\beta = 0°$. At $\tau_3 = 0$ we obtain the formula for approximate calculation of minimal teeth number

$$z_{\min} = -2\left(x_p + x\right)\cos^2\beta/\sin^2\alpha\,. \tag{31}$$

Eq. (31) can be also used for standard 20-degree cylindrical X-zero spur gears at $\beta = 0°$, $x_p + x = -1$ and $\alpha = 20°$. Applying the mentioned parameters, we obtain the known value of $z_{\min} = 17$.

Tip pointing. Tooth thickness on the top land is usually limited by the type of heat or chemicothermal treatment of gears. The procedures for tooth thickness calculation is given in [18] and [20] in different forms. Using parameters mentioned above tooth thickness on the top land can be obtained in the following form:

$$S_{aT} = \sqrt{\left(x_{i1} - x_{i2}\right)^2 + \left(y_{i1} - y_{i2}\right)^2}\,. \tag{32}$$

where $x_{i1}, x_{i2}, y_{i1}, y_{i2}$ -coordinates are to be defined by Eqs. (5) and (6) at $x_{p1} + x = x_{a1}$ and $x_{p2} + x = x_{a2}$ while x_{a1}, x_{a2} values are calculated by Eq. (22).

It follows from [18] results that pinion and gear transverse planes are the weakest in terms of tip pointing. Consequently, tooth thickness S_{aT} should be defined at $z_{i1} = z_{i2} = 0{,}5B$. The value of β that corresponds to the transverse plane should be calculated by the last Eqs. (5) and (6).

5 Results and discussions

When choosing the profile shift coefficient for quasi-involute ATT gears formation before defining the meshing characteristics the following conditions should be checked: $z_1 \geq z_{\min}$, where $z_{\min}$ is calculated by Eq. (30); $S_{aT} \geq 0.3...0.7m$ by [18], where S_{aT} is calculated by Eq. (32); $\varepsilon_\alpha \geq 1$ by [18], where ε_α is calculated by Eq. (26); the value of ε_β calculated by Eq. (27) should provide the given total value of contact ratio.

It follows from Eq. (7) that the value of sliding velocity is in direct proportion to $x_p + x$ distance. The value of sliding velocity declines when the value of β angle increases at $\alpha = const$ and $x_p + x = const$. For the most usual value of pressure angle $\alpha = 20°$ the sliding velocity values are in the range $2{,}03(x_p + x)(u+1)/u \leq V_S \leq 2{,}92(x_p + x)(u+1)/u$, at the corresponding β range $0° \leq \beta \leq 50°$. Consequently, the value of sliding velocity on the transverse profile of the teeth is 44% lower than in the middle cross-section of the arc. It is easily seen from Eqs. (8) and (10) that V_1 and V_2 take their maximum values in the middle cross-section of the arc at $\beta = 0°$ and minimum values on the transverse profile at $\beta = \beta_{\max}$. For example, at $x_p + x = 0$, $\alpha = 20°$ and $0° \leq \beta \leq 50°$ the range of rolling velocities is $0{,}342 r_{wi}\omega_i \leq V_i \leq 0{,}532 r_{wi}\omega_i$, i.e. the value of rolling velocity on the transverse profile of the teeth is 56% higher than in the middle cross-section of the arc. At $x_p + x = 0$, $\alpha = 20°$ and $0° \leq \beta \leq 50°$ the range of total rolling velocity from Eqs. (12) and (13) is $0{,}684 r_{wi}\omega_i \leq V_\Sigma \leq 1{,}064 r_{wi}\omega_i$, i.e. the value of total rolling velocity on the transverse profile of the teeth is also 56% higher than in the middle cross-section of the arc. It shows that the peripheral part of quasi-involute ATT gears provides better contact conditions in terms of oil film thickness, gearing efficiency and wear and scuffing resistance. The geometrical parameters combination that provides fulfillment conditions of Eqs. (11) and (14). Fulfillment condition (14) is impossible for gear pairs with gear ratio $u = 1$. When designing quasi-involute ATT gears with gear ratio $u \neq 1$ fulfillment condition (14) in any contact point should be avoided. Analysis of Eq. (15) shows that this condition cannot be fulfilled at realistic values of teeth geometrical parameters $0 \leq |x_p + x| \leq 2$, $R_C \geq 10m$, $14.5° \leq \alpha \leq 30°$. Analysis of Eqs. (16) and (17) shows that relative curvature in contact of quasi-involute ATT gears' teeth reaches predictably higher values at $\beta = 0$ and lower values when $\beta = \beta_{\max}$. Indefinitely great values of relative curvature are possible when condition (11) is fulfilled. The combination of ATT gears geometrical parameters that provides indefinitely great values of relative curvature should be avoided. It follows from Eq. (19) that specific sliding coefficients equals zero at $x_p + x = 0$, i.e. on the pitch line of the tooth flank, while it takes indefinitely great value at geometric parameters combinations which fulfill condition (11). These combinations should be avoided. Analysis of Eq. (19) also shows that the value of specific sliding coefficients in the middle cross-section of arc at $\beta = 0$ is higher in comparison with peripheral cross-sections at $\beta = \beta_{\max}$. It shows again that peripheral part of quasi-involute ATT gears provides

better contact conditions in terms of wear resistance. It follows from Eq. (20) that the angle ν takes its most favorable value only on the pitch line of the tooth at $x_p + x = 0$ or in any profile point in the middle cross-section of arc at $\beta = 0$ or at $R_C = \infty$, i.e. for cylindrical spur gears. The analysis of Eq. (20) also shows that in cross-sections with higher value of β the angle ν takes lower values. The most undesired value of the angle is $\nu = 0$. This value can be reached just at $R_C = (x_p + x)\tan\alpha$. There is no ATT gears geometrical parameters combination in real gearing that can fulfill the mentioned condition, therefore the angle between sliding velocity vector and contact line is always $\nu \neq 0$ for quasi-involute ATT gears.

CONCLUSION

A mathematical model of profile shifted quasi-involute arc-tooth-trace gears is obtained. Besides the meshing characteristics, the conditions of teeth undercutting and tip pointing for profile shifted quasi-involute arc-tooth-trace gears are obtained. The analysis of the obtained formulas showed that the most undesirable values of geometric-kinematic meshing characteristics take place in the central section of the tooth as expected. When designing quasi-involute arc-tooth-trace gears the shift coefficients should be chosen to improve the meshing characteristics and prevent undercutting in this cross section. However, the most dangerous in terms of the teeth tip pointing are the peripheral transverse section. Therefore, when choosing the shift coefficients, it is recommended to calculate the pinion and the wheel teeth tip thickness by the formulas presented in this paper. Recommendations for determining the rational geometric parameters of the teeth, such as width, radius of the circle delineating the tooth trace, and the angle of inclination of the teeth in the peripheral transverse section are given. These parameters are extremely important because they determine the overlap ratio and, therefore, affect the smoothness of operation.

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