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STABILITY AND ANALYSIS OF VIBRATIONS BIFURCATION BASED ON DYNAMIC MODELING OF A SOLAR TITAN 130 GAS TURBINE

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Abstract: This work is interested in studying the vibrations' stability in gas turbines, compared to a dynamic model of the Jeffcott rotor, the parameters of the gas turbine will be estimated for the purpose of developing a mathematical model which points out the behavior of the turbine rotor in virtue of the data of actual vibration measurements. The addition of the geometric nonlinearities of the vibrations unveils a supercritical Hopf bifurcation, the unstable modes are repulsed towards a stable limit cycle of small radius. The vibration measurements and analyzes were performed on a gas turbine Solar TITAN 130 type mounted in a gas compression station in Djelfa, Algeria. The turbine was in the operating mode when the vibration parameters were read. Bifurcation monitoring bounds the specific operating ranges, extracts efficiently the dynamic behavior core of the studied turbine and consequently affords a new approach to efficiently dimension the turbine rotor.

KEYWORDS: Gas turbine, vibrations, Hopf bifurcation, Jeffcott rotor, spectral vibration analyzes, dynamic modeling.

1 Introduction

In industrial practice, most of the vibration issues of rotating machines are non-linear, these non-linearities unveil bifurcations, resonances, non-periodic periodic movements. However, the strictest security regulations, lower costs and improving performances forcing the managers to use the most advanced machinery and mastering their behavior in both normal or abnormal operating conditions. Furthermore, monitoring non-linear behaviors employing a classical temporal integration approach implies an enormous computation and modeling costs for the parametric study. Hence, a numerical method, both efficient and universal, is developed for the purpose of analyzing in parametric way the dynamic behavior of a nonlinear rotor. Therefore, the implementation of non-linear analysis methods permitting as well as to reduce of the initial non-linear dynamics as to rapidly identify the vibrational responses becomes one of the central points and headline targets of the performers of rotating machinery.

The aim of this study is to conduct tests on a TITAN 130 turbine, to determine the parameters affecting its nonlinear dynamic behavior and the parameter variables influencing

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the first critical speed passage. In fact, the development of a Jeffcott rotor dynamics tool for a rotor with high rotational speed in bending aims to show the different vibration phenomena that are found on rotating gas turbines. The rotating part of the turbine is made up of shafts on which bladed wheels are mounted. The different bodies of the shaft line are supported by bearings, the turbine rotor is sufficiently slender to represent it by the Jeffcott rotor. The gas turbine has unstable modes resulting from damping rotating in the supercritical domain. If the turbine is held in three bearings without sliding, the shaft stiffens when its amplitude increases. This geometric non-linearity (of the oscillator class) is added to the classical Jeffcott rotor equations. We compare the experimental results carried out on a gas turbine with the results obtained by a simulation of the equations of the Jeffcott rotor to estimate the parameters of the turbine. Spectral analyzes on the vibration signal of the turbine made it possible to make a comparison with the Jeffcott rotor dynamic behavior applied to the full model.

However, a parametric study of the bifurcation is proposed with of view to have a rapid comprehension of the dynamic behavior of the modeled turbine system, and to predict the response evolution depending on the turbine system parameters. These parametric studies are often carried out during the design of monitoring and diagnostic strategies, to obtain the value of the parameter for which a system performance index is optimized to control the entire nonlinear dynamic behavior of the rotating machine.

With the intention to complete the vibrational phenomena, we expose several vibro-acoustic aspects which had arisen on the TITAN 130 turbine of the gas compression station SC3 in Djelfa, Algeria. However, the vibrations instabilities that could cause significant damage to the turbine, hence a monitoring system for faults detection and decision-making must be configured to cope with these phenomena. We propose in this work to analyze the stability of the periodic vibrations of the dynamic system, the addition of the geometric nonlinearities of the vibrations displays a supercritical Hopf bifurcation, the unstable modes are pushed towards a stable limit cycle of small radius. This result allows to limit the hazardousness of these modes by blocking the translation of the bearings and will allow the observation and experimental measurement of these modes, and to define the minimum damping to bring to the rotor which will allow to cross critical speeds with a certain level of amplitude and to decide on the influence of the non-linearity compared to the dynamics of rotating machine in the proximity of critical speeds.

2 Dynamic Jeffcott non-linear rotor

The rotor of a gas turbine consists of the shaft, the disc, the unbalance, the bearings and the blade, the model of the disc consists of a mass m concentrated in G and located at a distance e from the center of rotation. C, as shown in Figure 1. With no effect of gravity and at rest, the center of rotation is positioned at O. The rotating reference to the disc $(O, \vec{y}_r, \vec{z}_r)$ rotates at speed ω relative to the fixed reference $(O, \vec{y}, \vec{z})$. The shaft of length l at rest, assumed to be axisymmetric and massless, is modeled by a bending stiffness K_f, a tension stiffness over the half-length K_t and an internal viscous damping c_i in the directions $\vec{y}_r$ and $\vec{z}_r$. The damping of the shaft in the direction of the tension is neglected because of the third order before the damping in bending [1, 9, 16]. The bearings are assumed to be infinitely rigid and blocked laterally but dissipate energy radially by means of an external viscous damping c_e in the directions $\vec{y}$ and $\vec{z}$. From where, the position of the center of rotation in the fixed coordinate system is given by (y, z).

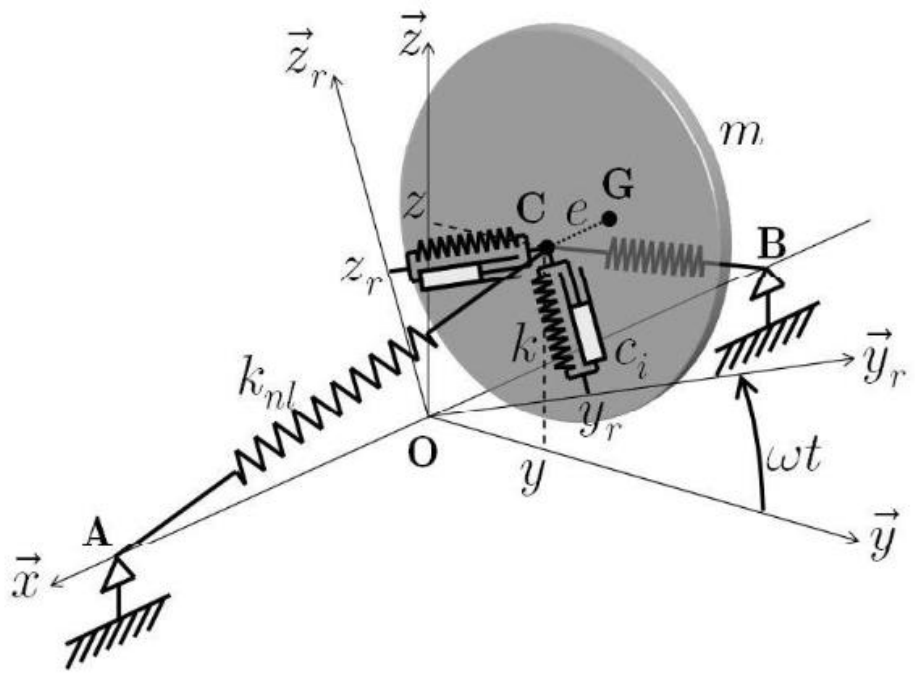


Fig. 1 Model of a gas turbine represented by Jeffcott rotor in large displacement with fixed bearings (external damping is not represented)

The kinetic energy of the system is written as follows:

$$T = \frac{1}{2} m \left(\dot{y}^2 + \dot{z}^2 + e^2 \omega^2 - 2e\dot{y}\omega \sin(\omega t) + 2e\dot{z}\omega \cos(\omega t) \right) + \frac{1}{2} I \omega^2 \tag{1}$$

The potential energy of the system is written:

$$U = \frac{1}{2} k_f \left(y^2 + z^2 \right) + \frac{1}{4} k_t \left(\sqrt{l^2 + 4y^2 + 4z^2} - l \right)^2 + mg\left(y + e\sin(\omega t) \right) \tag{2}$$

The geometric nonlinearity in potential energy is simply derived from Pythagoras' theorem [4, 20, 22]. By definition, the dissipation function is equal to half the power dissipated by the external and internal dampers. The internal viscous dissipation being proportional to the speed of C in the rotating frame, the dissipation function is given by [5, 10]:

$$D = \frac{1}{2} c_e \left(\dot{y}^2 + \dot{z}^2 \right) + \frac{1}{2} c_i \left((\dot{y} + \omega z)^2 + (\dot{z} - \omega y)^2 \right) \tag{3}$$

The equations of motion are determined from the Lagrange equations [6, 17], as follows:

$$\frac{d}{dt}\left[\frac{\partial T}{\partial q} \right] - \frac{\partial T}{\partial q} + \frac{\partial D}{\partial \dot{q}} + \frac{\partial U}{\partial q} = Q, q \in \{y, z\} \tag{4}$$

We get:

$$\begin{aligned} &\ddot{y} + (d_e + d_i)\dot{y} + {\omega_f}^2 y + d_i \omega z + {\omega_t}^2 \left(1 - \frac{l}{\sqrt{l^2 + 4y^2 + 4z^2}} \right) y = e\omega^2 \cos(\omega t) \\ &\ddot{z} + (d_e + d_i)\dot{z} + {\omega_f}^2 z - d_i \omega y + {\omega_t}^2 \left(1 - \frac{l}{\sqrt{l^2 + 4y^2 + 4z^2}} \right) z = e\omega^2 \sin(\omega t) + g \end{aligned} \tag{5}$$

where $d_e = c_e / m, d_i = c_i / m, \omega_f = \sqrt{k_f / m}, \omega_t = \sqrt{2k_t / m}$ are respectively the external viscous mass damping coefficient, the internal viscous mass damping coefficient, the natural pulsation in bending of the non-damped system and the natural pulsation in tension of the non-damping system.

The problem is written in the form:

$$\begin{aligned} &Y'' + (D_e + D_i)Y' + Y + D_i\Omega Z + 2\Omega_t^2\left(1 - \frac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Y = E\Omega^2\cos(\theta) \\ &Z'' + (D_e + D_i)Z' + Z - D_i\Omega Y + 2\Omega_t^2\left(1 - \frac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Z = E\Omega^2\sin(\theta) + G \end{aligned} \tag{6}$$

By tacking $(.)' = (\dot{.})/\omega_f = d(.) = d(\omega_f t)$, $Y = y/l$, $Z = z/l$, $D_e = d_e/\omega_f$, $D_i = d_i/\omega_f$, $\Omega = \omega/\omega_f$, $\Omega_t = \omega_t/\omega_f$, $E = e/l, \theta = \omega t, G = g/l\omega_f^2$.

Assuming Y and Z small in front l, a Taylor expansion to order 3 of the previous equations is written:

$$\begin{aligned} &Y'' + (D_e + D_i)Y' + Y + D_i\Omega Z + 2\Omega_t{}^2\left(Y^2 + Z^2\right)Y = E\Omega^2\cos(\theta) \\ &Z'' + (D_e + D_i)Z' + Z - D_i\Omega Y + 2\Omega_t{}^2(Y^2 + Z^2)Z = E\Omega\sin(\theta) + G \end{aligned} \tag{7}$$

In order to determine the dynamic characteristics of the rotor, it will be a question of solving equation (7). From these equations, the natural pulsations and the eigenvalues of the rotor can be determined. The dynamic study of the system therefore amounts to solving these equations by varying the rotation speed W of the rotor. The solutions will have to check the equilibrium equation may not represent the real behavior of the system if they are unstable [2-3, 7-8, 11, 13, 19]. Therefore, it is important to study stability after solving equilibrium equations, in order to determine the set of stable solutions. For this, the next section aims to propose the Hopf Bifurcation for an analysis related to the nonlinear dynamic stability of the rotor of a gas turbine [15, 18, 23].

3 Hopf bifurcation

The objective of the Hopf bifurcation is to establish the continuous passage of stability from a certain equilibrium position of the system to a periodic bifurcated position. It is known that the Hopf bifurcation can only occur on one side of the critical value of the bifurcation parameter, which gives rise to either a sub-critical bifurcation, a critical bifurcation or a bifurcation supercritical [12, 23].

In this work, we are interested in the case of free oscillations and the effect of gravity is neglected, the equation system can be written in the form of a dynamic system, as follows [12, 23]:

$$x' = f(x) \tag{8}$$

with:

$$x=\begin{pmatrix} Y \\ V \\ Z \\ W \end{pmatrix},\ f(x)=\begin{pmatrix} V \\ -(D_e+D_i)V-Y-D_i\Omega Z-2\Omega_t{}^2\left(Y^2+Z^2\right)Y \\ W \\ -(D_e+D_i)W-Z+D_i\Omega Y-2\Omega_t{}^2\left(Y^2+Z^2\right)Z \end{pmatrix},\ x,\ f\in\Re^4 \tag{9}$$

where V and W are respectively the dimensionless speeds of the rotor according to $\vec{y}$ and $\vec{z}$.

The fixed points of the system are solution of $0=f(\bar{x})$ either [21, 23]:

$$0=\begin{pmatrix} 0 \\ -\bar{Y}-D_i\Omega\bar{Z}-2\Omega_t{}^2\left(\bar{Y}^2+\bar{Z}^2\right)\bar{Y} \\ 0 \\ -\bar{Z}+D_i\Omega\bar{Y}-2\Omega_t{}^2\left(\bar{Y}^2+\bar{Z}^2\right)\bar{Z} \end{pmatrix},\ \bar{x}\in\Re^4 \tag{10}$$

By writing $-\bar{Z}\times f_2(\bar{x})+\bar{Y}\times f_4(\bar{x})=0$, we get from $D_i\Omega(\bar{Z}^2+\bar{Y}^2)=0$ where $\bar{Y}=\bar{Z}=0$. The only fixed point is $\bar{x}=^T(0\quad 0\quad 0\quad 0)$.

The linear stability of the system around the equilibrium position is obtained by disturbing $\varepsilon(t)\in\Re^4$ the dynamic system. By neglecting the quadratic terms in, the system is reduced to the differential linear system [12, 23]:

$$\varepsilon'=Df(\bar{x})\varepsilon \tag{11}$$

where $Df(\bar{x})$ is the Jacobian matrix f, that is equal to $Df_{ij}=\partial f_i/\partial x_j$.

The solution of equation (11) is given by:

$$\varepsilon(t)=e^{Df(\bar{x})t}\varepsilon(0) \tag{12}$$

with:

$$Df(\bar{x})=\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & -D_e-D_i & -D_i\Omega & 0 \\ 0 & 0 & 0 & 1 \\ D_i\Omega & 0 & -1 & -D_e-D_i \end{pmatrix} \tag{13}$$

The complex values of $Df(\bar{x})$ are [16, 28]:

$$\begin{cases} \lambda_1=\frac{1}{2}\left(-(D_e+D_i)-2\sqrt{-\Omega_d{}^2-iD_i\Omega}\right) \\ \lambda_2=\frac{1}{2}\left(-(D_e+D_i)+2\sqrt{-\Omega_d{}^2-iD_i\Omega}\right) \\ \lambda_3=\frac{1}{2}\left(-(D_e+D_i)-2\sqrt{-\Omega_d{}^2-iD_i\Omega}\right) \\ \lambda_4=\frac{1}{2}\left(-(D_e+D_i)+2\sqrt{-\Omega_d{}^2-iD_i\Omega}\right) \end{cases} \tag{14}$$

By posing $\omega_d^2=\omega_f^2-\frac{1}{4}(d_e+d_i)^2$ either $1-\frac{1}{4}(D_e+D_i)^2$ with $\Omega=\omega_d/\omega_f$, the calculation of complex roots gives the eigenvalues in the form real part (damping) and imaginary part (pulsation) will be given in the following form:

$$\begin{cases} \lambda_1 = \frac{1}{2}\left(-(D_e + D_i) + \sqrt{2}\sqrt{-\Omega_d{}^2 + \sqrt{\Omega_d{}^2 + D_i{}^4\Omega^2}} - i\sqrt{2}\sqrt{\Omega_d{}^2 + \sqrt{\Omega_d{}^4 + D_t{}^2\Omega^2}} \right) \\ \lambda_2 = \frac{1}{2}\left(-(D_e + D_i) + \sqrt{2}\sqrt{-\Omega_d{}^2 + \sqrt{\Omega_d{}^2 + D_i{}^4\Omega^2}} + i\sqrt{2}\sqrt{\Omega_d{}^2 + \sqrt{\Omega_d{}^4 + D_t{}^2\Omega^2}} \right) \\ \lambda_3 = \frac{1}{2}\left(-(D_e + D_i) - \sqrt{2}\sqrt{-\Omega_d{}^2 + \sqrt{\Omega_d{}^2 + D_i{}^4\Omega^2}} - i\sqrt{2}\sqrt{\Omega_d{}^2 + \sqrt{\Omega_d{}^4 + D_t{}^2\Omega^2}} \right) \\ \lambda_4 = \frac{1}{2}\left(-(D_e + D_i) - \sqrt{2}\sqrt{-\Omega_d{}^2 + \sqrt{\Omega_d{}^2 + D_i{}^4\Omega^2}} + i\sqrt{2}\sqrt{\Omega_d{}^2 + \sqrt{\Omega_d{}^4 + D_t{}^2\Omega^2}} \right) \end{cases} \quad (15)$$

These eigenvalues are generally expressed in approximate form to two pairs of conjugate complexes (λ_1, λ_2) and (λ_3, λ_4). Modes 3 and 4 are linearly stable because $\Re(\lambda_3) = \Re(\lambda_4) < 0$.

The other two may be unstable when Ω is large enough $\Re(\lambda_1) = \Re(\lambda_2) > 0$ to get in exact form:

$$\Omega > 1 + \frac{D_e}{D_i} \quad (16)$$

A system having two conjugate complex eigenvalues whose real part changes sign presents a Hopf bifurcation, when $\Omega = 1 + \frac{D_e}{D_i}$, the system is at the Hopf bifurcation point.

The solution branch $\bar{x} = {}^T(0 \quad 0 \quad 0 \quad 0)$ goes from stable to unstable and a stable cycle appears if the bifurcation is supercritical. The equation (7) without second member have an exact limit cycle type solution which demonstrates its existence. Indeed, by posing $X = Y + iZ$, this system is written in the following form:

$$X'' + (D_e + D_i)X' + (1 - iD_i\Omega)X + 2\Omega_t^2 \left| X^2 \right| X = 0 \quad (17)$$

A limit cycle type solution can be written in the form $\bar{X} = \bar{R}e^{i\bar{\Omega}t}$ with $\bar{R}, \bar{\Omega} \in \Re^2$, where $\bar{R}$ is the radius of the cycle and $\bar{\Omega}$ is the rotation speed of the cycle. By injecting this solution into the previous equation, it comes:

$$\left(-\bar{\Omega}^2 + (D_e + D_i)i\bar{\Omega} + (1 - iD_i\Omega) + 2\Omega_t^2 \bar{R}^2 \right) \bar{R}e^{i\bar{\Omega}t} = 0 \quad (18)$$

Either by separating the real part from the imaginary part:

$$\begin{aligned} -\bar{\Omega}^2 + 1 + 2\Omega_t^2 \bar{R}^2 &= 0 \\ (D_e + D_i)\bar{\Omega} - D_i\Omega &= 0 \end{aligned} \quad (19)$$

If $\Omega \geq 1$, i.e. from the Hopf bifurcation, there is a solution to these equations:

$$\begin{aligned} \bar{\Omega} &= \frac{D_i}{D_e + D_i}\Omega \\ \bar{R} &= \frac{1}{\sqrt{2}\Omega_t}\sqrt{\bar{\Omega}^2 - 1} \end{aligned} \quad (20)$$

Hence the solution in the complex plane:

$$\bar{X} = \frac{1}{\sqrt{2}\Omega_t}\sqrt{\left(\frac{D_i}{D_e + D_i}\right)^2 \Omega^2 - 1}\, e^{i\frac{D_i}{D_e+D_i}\Omega t} \tag{21}$$

The evolution of the cycle speed is linear in Ω, so the evolution of the radius is linear in Ω when $\Omega >> 1$, because $\bar{R} \to \bar{\Omega}/\sqrt{2}\Omega_t$. We also note that at the Hopf bifurcation point the radius of the cycle is zero but its speed is not zero. The following section deals with a model of rotor dynamics, to study the stability of the equilibrium points as a function of this dynamic, with an estimate of the distance between the solution of the initial equation and that of the reduced equation to obtaining bifurcated periodic solutions.

4 Gas turbine

A gas turbine is a continuous flow rotating motor, equipped with an axial compressor and combustion chambers; it is able to produce itself a fluid under pressure and at very high temperature which, by undergoing its phase of expansion in the various stages of the turbine, provides mechanical energy to the outside, it is a true autonomous unit which suffices in itself [13, 15]. The studied gas turbine in this work is of the Solar TITAN 130 type, as shown in Figure 2, is a turbine with two separate shafts, with axial flow, Table 1 shows the specifications of the Titan 130 gas turbine. However, the main components of the turbine remain precisely aligned via counter flanges having pilot surfaces and are bolted together to form a rigid assembly.



Fig. 2 TITAN 130 Solar turbine

This work is based on the estimation of the turbine parameters values, we analyze the behavior of the turbine to that of the Jeffcott rotor dynamic system model. The numerical study of the behavior of the rotor is carried out from nonlinear equations without approximation, i.e. equation (5), put in the form of the following dynamic system:

$$x' = h(x) \tag{22}$$

Table 1. Characteristics of the Solar TITAN 130 gas turbine

Quantity	Value
Output Power	14 500 kW (19800 hp)
Heat Rate	9940 kJ/kW-hr (7025Btu /hp-hr)
Exhaust Flow	180050 kg/hr (396,940 lb/hr)
Exhaust Temperature	505°C (940°F)
Max Speed NGP	11222 rpm
Max Speed NGP	8856 rpm

With:

$$x=\begin{pmatrix} Y \\ V \\ Z \\ W \end{pmatrix},\ f(x)=\begin{pmatrix} V \\ -(D_e+D_i)V-Y-D_i\Omega Z-2\Omega_t^2\left(1-\dfrac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Y \\ W \\ -(D_e+D_i)W-Z+D_i\Omega Y-2\Omega_t^2\left(1-\dfrac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Z \end{pmatrix},\ x,\ f\in\Re^4 \tag{23}$$

The dynamic equations system of the rotor is written in the form:

$$\begin{aligned} &Y''+(D_e+D_i)Y'+Y+D_i\Omega Z+2\Omega_t^2\left(1-\frac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Y=E\Omega^2\cos(\theta) \\ &Z''+(D_e+D_i)Z'+Z-D_i\Omega Y+2\Omega_t^2\left(1-\frac{1}{\sqrt{1+4Y^2+4Z^2}}\right)Z=E\Omega^2\sin(\theta) \end{aligned} \tag{24}$$

5 Experimental analyzes

The gas turbine vibration model proposed in this work is non-linear, this model deals with the complex dynamics of the examined turbine. In this section, the obtained results from the stability analysis by the Hopf bifurcation approach are presented. This is a contribution to the study of the Hopf bifurcation issue in the framework of a global study of the dynamic behavior of a turbine vibrations.

In this section, the periodic solutions of nonlinear problems in the turbine model are presented. For this, vibration measurements are carried out with autonomous equipment including an FFT analyzer "Machinery Health Analyzer" and a piezoelectric accelerometer with magnetic positioning, on a gas turbine in operation of the gas compression station SC3, Djelfa, Algeria. However, the use of this discretization of FFT series leads to a complex but manageable algebra which shows all the power of the asymptotic numerical method. To analyze these series carefully, a sufficiently large number of terms of vibratory measurements were made, with the aim of seeking periodic solutions in the form of whole series as a function of a certain turbine operating parameter.

Figure 3 shows the vibration signal along the z axis noted by the device, this vibration signal contains significant information on the operating status of the various components of the turbine, from this signal, it is possible to " to develop a mathematical model of the behavior of the turbine rotor by applying the frequency method which is based on the Fourier transform.

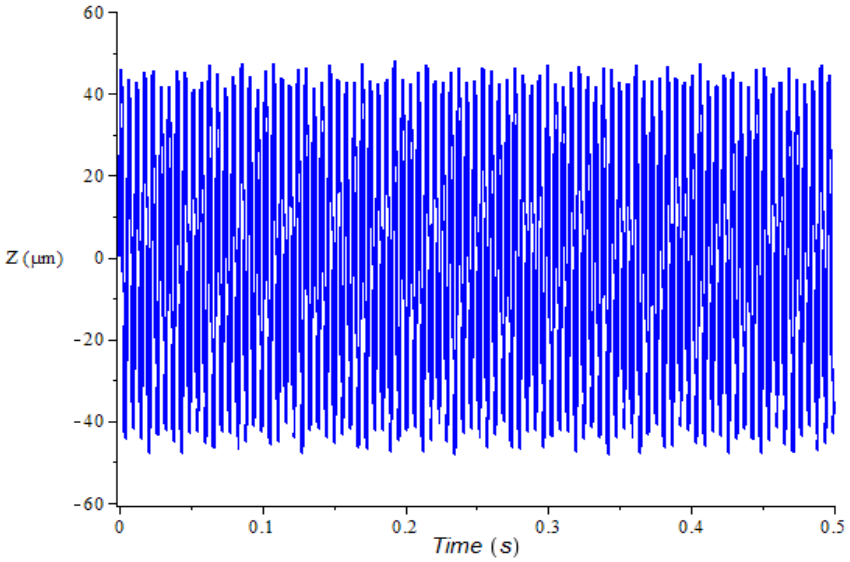


Fig. 3 Vibration signal on the axis Z of the Solar TITAN 130 gas turbine

Knowledge of the characteristic frequencies makes it possible to mathematically identify the vibration signal of the mechanical components by analyzing their spectrum, shown in Figure 4, using the command and control system and the turbine vibration monitoring system we have noted. shaft displacement measurements and turbine speed.

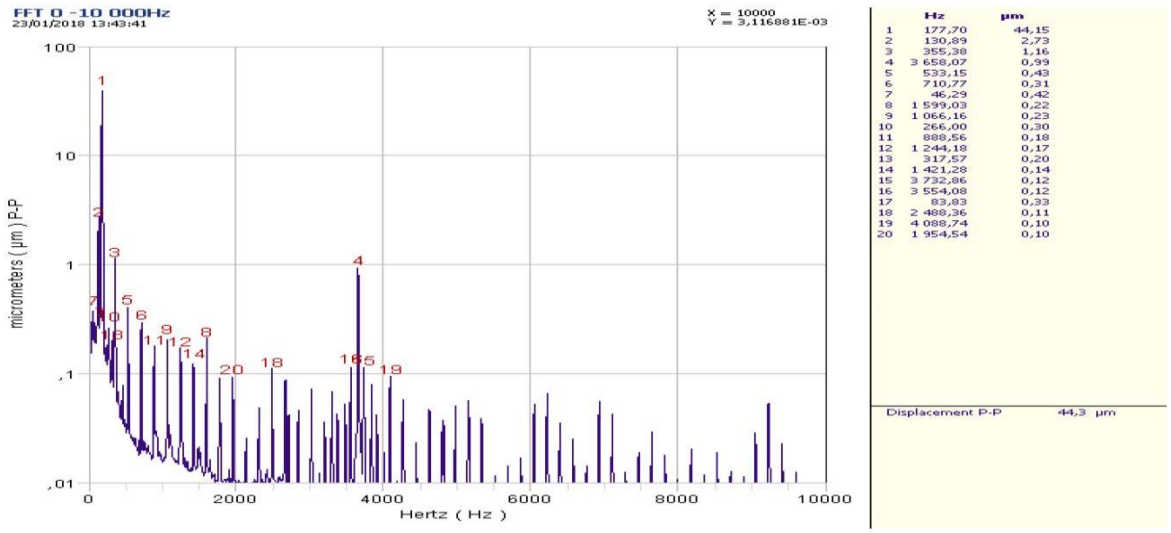


Fig. 4 Spectral analysis of vibrations on the axis Z of the Solar TITAN 130 gas turbine

The temporal results for $\omega = 1116.5 rd/s$, as shown in Figure 5, shows the expected behavior of the turbine, its rotor tends to converge whatever its initial position, the bifurcation indicator is cancelled at the bifurcation points or towards a cycle radius limit R = 45 µm obtained by graphic reading.

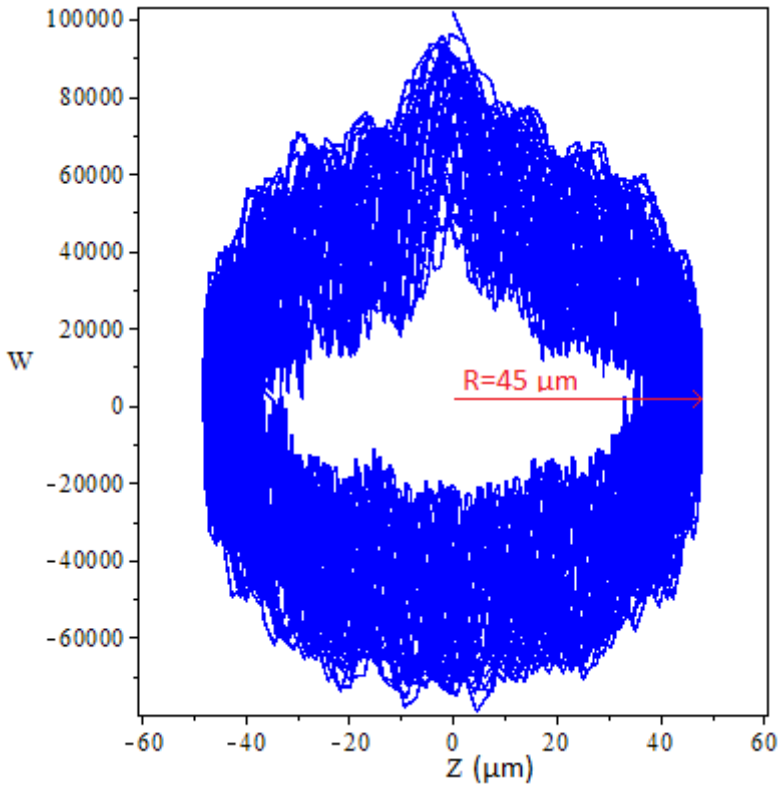


Fig. 5 Temporal behavior of the turbine rotor after Hopf bifurcation with the radius limit cycle $R = 45 \times 10^{-6}$ m

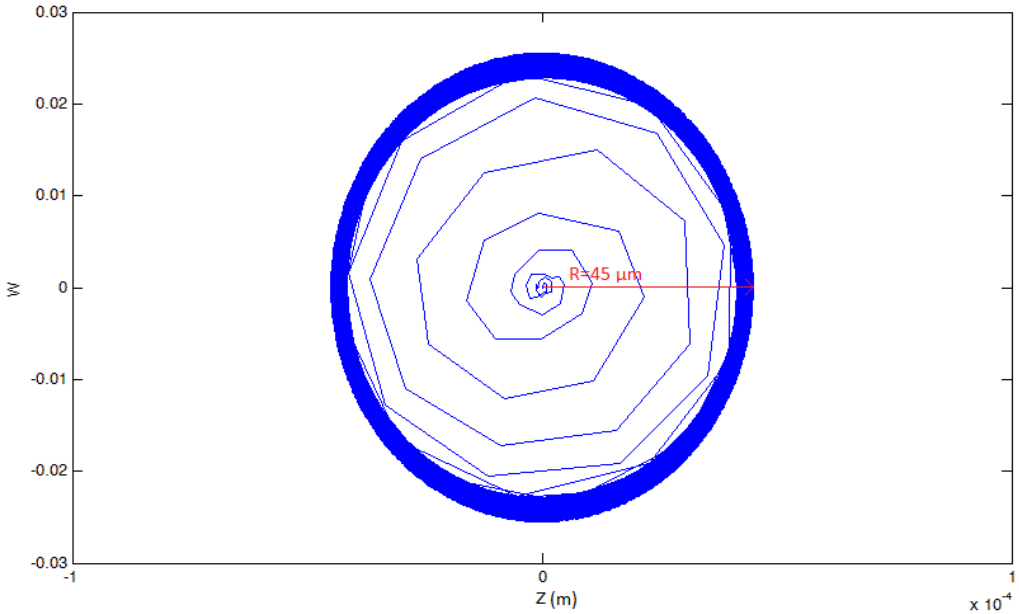


Fig. 6 Time results of the estimated behaviour of the gas turbine after Hopf bifurcation with radius limit cycle $R = 45\mu m$

For a turbine speed at 95% of its nominal speed at 10662 tr/mn which corresponds to $\omega = 1116.5 rd/s$ and $f = 177.7 Hz$, the representative values of the turbine parameters are estimated and obtained by the solutions given by equation (5), the results time of the rotor is represented by Figure 6, we then obtain:

$$(e = 4 \times 10^{7}, \omega = 1116.5, \omega_t = 6.14 \times 10^{6}, \omega_f = 1.4, d_e = 28.92, d_i = 15.56)$$

We can deduce: $(\Omega = 797.5, \Omega_t = 4.39\times10^{6}, D_e = 20.66, D_i = 11.11)$, with radius limit cycle $R = 45\mu m$. The Radius R of the limit cycle is also obtained by equation (20) by entering the values of the parameters of the turbine it results $R = 45\times10^{-6} m$. Figures 7 and 8 show a Hopf bifurcation of the turbine behavior from $\Omega = 2$ and limit cycle of radius R in space (Ω, Z, W), obtained by the analytical solution of equation (20), the Hopf bifurcation is represented by:

$$Z = \frac{1}{\sqrt{2}\Omega_t}\sqrt{\left(\frac{D_i}{D_e + D_i}\right)^2 \Omega^2 - 1} \qquad (25)$$

We obtain the bifurcation point at $\Omega = 2$ either $\omega = 2.8 rd/s$, so, the turbine is stable at $\Omega < 2$, and the turbine is unstable at $\Omega > 2$ with a limit cycle of radius R depends on Ω is limited by equation (24). On the basis of the obtained results, the asymptotic development as a function of parameters (Ω, Y, Z) always diverges at a critical value corresponds to the convergence radius R of the series and which is independent of the order of the model of gas turbine system.

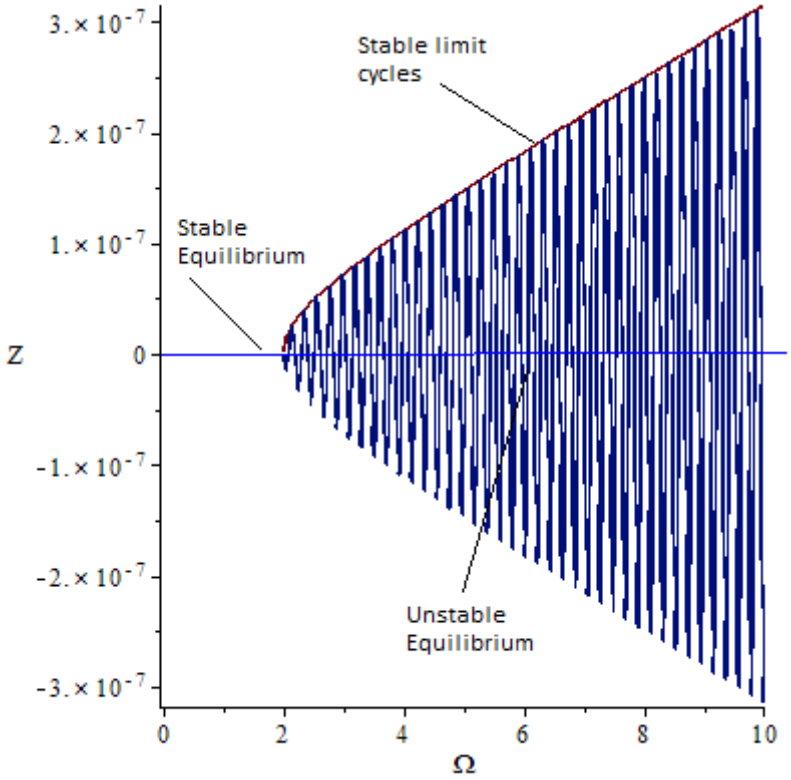


Fig. 7 Hopf bifurcation $\Omega = 2$ and limit cycle of the gas turbine rotor

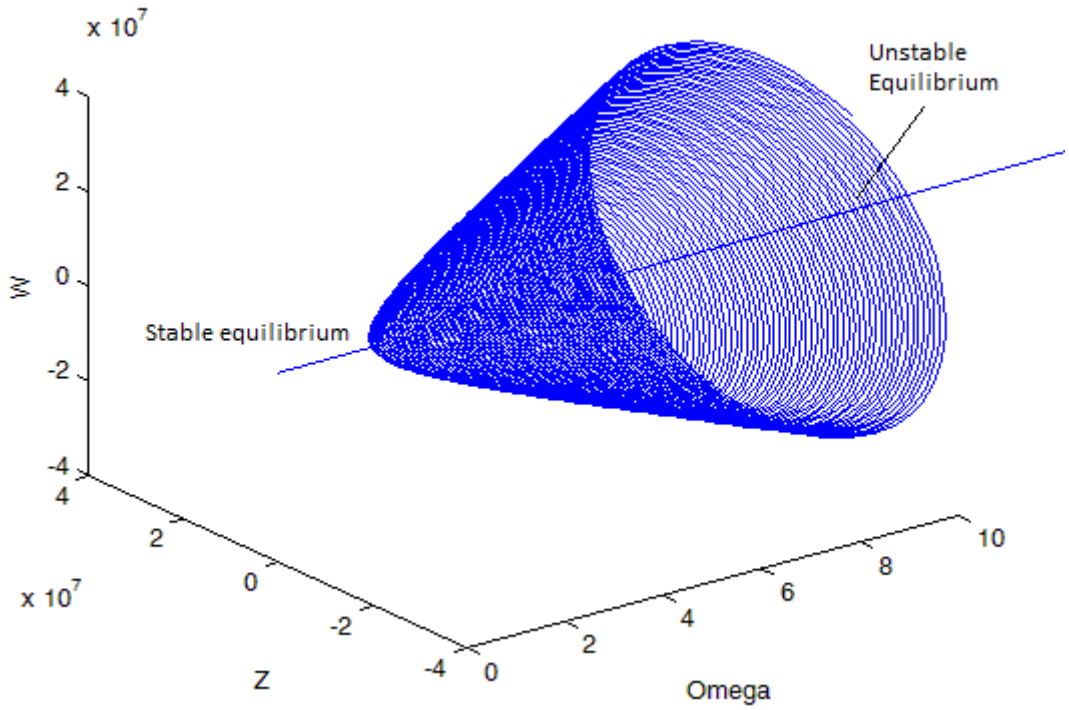


Fig. 8 3D Hopf bifurcation of the gas turbine in physical space (Ω, Y, Z) instability $\Omega > 2$ and limit cycle of radius R depends on Ω

CONCLUSION

In this article, the Jeffcott rotor mathematical equations have been developed and applied for modeling the dynamics of the gas turbine. Furthermore, the values of the turbine parameters compared to the equations of the Jeffcott rotor model are estimated. Nevertheless, the gas turbine is a very complex system being by its physical structure as well as by its dynamic behavior occasioned by the phenomena of vibrations generated by each organ of the turbine, the rotor speed and the disturbances of the rotating auxiliaries. First, measurements of the vibration signal on an operating gas turbine in the compression station SC3 at Djelfa (Algeria) were performed, a frequency analysis was carried out to obtain solutions in the frequency domain where the numerical calculation of the system response is more efficient than in the time domain by direct integration. Given the complexity of the turbine system and the influence of the other auxiliary organs on the vibrations, the acceleration of the vibrations is amplified in the high frequencies, conversely the displacements are important in the low frequencies from where the determination of the values and the turbine parameters reflects its real behavior with acceptable accuracy.

It is clear that the turbine in the presence of internal damping (lubricated bearings) has a Hopf bifurcation at $\Omega = 2$ supercritical and a stable limit cycle of radius R proportional to Ω. It is also noted that at the Hopf bifurcation point the radius of the cycle is zero but its speed is not zero in the supercritical domain, this cycle has a simple analytical solution. Finally, these results seem to be an observation of the experimental measurement of the dynamic modes of a gas turbine and the study of the limit cycle has shown that its radius is very small.

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