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THE CORRECTION FACTOR TAKING INTO ACCOUNT THE EFFECT OF MASS TRANSFER ON THE HEAT TRANSFER COEFFICIENT

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Abstract: Various factors affect the processes that take place in the devices. Knowledge of these factors is important, especially for technical practice. If these were omitted in the design of the device, the expected changes in the properties of the substances would probably not occur in the device. Different processes occur in the devices, often involving simultaneous heat and mass transfer through the interfacial area. In this case, it is true that the description of the heat transfer cannot be based on ordinary laws. It is necessary to proceed from their corrected forms because, generally, the mass transfer influences the heat transfer. This impact can be taken into account using the correction factor adjusting the commonly reported relationships, as mentioned in this article.

KEYWORDS: Fourier's law, Newton's law of cooling, corrected form of the Newton's cooling law, Ackermann correction factor, correction factor, interfacial area, heat transfer coefficient.

1 Introduction

There are a group of equipment that ensure the transfer of heat from the hot medium to the cold medium [(1), (2), (3), (4), (5)]. The special group of these equipment is those in which the hot medium is separated from the cold medium by a wall made of the solid material. For a correct description of these equipment, it is necessary to know the values of the heat transfer coefficients corresponding to the hot and cold media. The values of these coefficients are usually determined on the basis of the criterion equations valid for the specific cases [(3), (5), (6), (7), (8)]. The calculation of such equipment can in principle be considered seamless if the media, hot and cold media, are single component. In the opposite case, if only one of the media is multicomponent, the calculation is already more complicated. The complexity consists on the one hand in determining the properties of mixtures, the knowledge of which is necessary for the determination of the heat transfer coefficients, and on the other hand, also from the point of view of the calculation methodology of such equipment. The special case occurs when at least one of the media undergoes the phase change in the equipment, which can generally also be accompanied by the presence of an inert [(3), (4), (8), (9), (10), (11), (12), (13)]. In such a case, it is true that there is simultaneous heat and mass transfer through the interfacial area in the equipment. This is due to the fact that there are at least two phases in a given space of the equipment. In this case, the amount of heat exchanged between the hot and cold media depends on the amount of energy transferred through the interfacial area from the first phase to the second phase [(14), (15), (16)]. The aim of this article is to state the equation that determines the total heat flux through the interfacial area. Knowledge of the heat flux is crucial in the design of the equipment in which simultaneous heat and mass transfer occur. In these cases, it is true that the heat transfer

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coefficient determined on the basis of the common equations is corrected by a correction factor taking into account the simultaneous heat and mass transfer.

2 Original equation for the description of the simultaneous heat and mass transfer

If the temperature changes from place to place in the observed system, there must be a heat flux in the system. In the simplest case, when the system represents a pure substance (solid, liquid, gas) that does not flow, the heat flux can be described by Fourier's law:

$$\dot{Q} = \vec{q}.\vec{A} = -\lambda(\nabla T).\vec{A} \tag{1}$$

The heat, as already shown above, can also be transferred from one system (from one phase) to another system (to another phase) in such a way that the individual phases are separated from each other by the interfacial area Fig. 1. In this case, also the flow of one phase or both phases is usual. In such cases, it is also true that the heat flux can be expressed near the interfacial area by Eq. 1, if the process description is based on the film theory of heat transfer (film model). The reason is the fact that in the flowing fluid a thermal boundary layer of thickness $\delta_T = z_2 - z_1$ is formed near the interfacial area [(5), (7), (14), (17)]. The thickness of this layer depends on the turbulence intensity of the fluid in a given phase. Behind this layer, the temperature in the phase volume is considered constant (i.e., $T_2 = T_f$ = constant, Fig. 1). In other words, the film model assumes that the temperature change occurs only in the thermal boundary layer and not in the phase volume. It is also assumed (reasonably assumed) within the concept of the film model that there is a laminar flow of the fluid along the interfacial area in the thermal boundary layer. This means that a laminar film exists in a given phase along the interfacial area.

Let λ = constant and let the heat transfer occur only in a direction perpendicular to the interfacial area, whereas the direction of the heat transfer is identical to the direction of the *z*-axis. Then based on Eq. (1), the heat flux near the interfacial area may be expressed by the equation written in the form:

$$q_z\big|_{z=w} = -\lambda\frac{\mathrm{d}T}{\mathrm{d}z} = -\lambda_w\frac{\mathrm{d}T}{\mathrm{d}z} = q_w \tag{2}$$

It should be kept in mind that this equation allows to determine the amount of heat that passes through the interfacial area (i.e., from one phase to another).

Similar considerations apply also in the case of mass transfer through the interfacial area [(5), (15), (18)]. The total molar flux, e.g., of a substance *A* near the interfacial area, can be expressed by a general equation written in the form:

$$N_{Aw} = \frac{-E_{tAmw}}{(z_2 - z_1)\,\varphi}\left(c_{Af} - c_{Aw}\right) = k_c\left(c_{Aw} - c_{Af}\right) \tag{3}$$

in which $k_c = \dfrac{F}{\varphi}$, $F = \dfrac{E_{tAmw}}{z_2 - z_1} = \dfrac{E_{tAmw}}{\delta_D}$, $\varphi = \left(\dfrac{c_{Aw}}{c\gamma} - \dfrac{c_{Af}}{c\gamma}\right) \Bigg/ \ln\left(\dfrac{1 - \frac{c_{Af}}{c\gamma}}{1 - \frac{c_{Aw}}{c\gamma}}\right)$, and $\dfrac{1}{\gamma} = \dfrac{\sum_{i=A}^{n} N_{iw}}{N_{Aw}}$.

It should be kept in mind that Eq. (3) also allows to determine the quantity of substance *A* that passes from one phase to another. Fig. 2 represents the conditions that apply to the transfer of substance *A* through the interfacial area, assuming that there is a unidirectional mass transfer of substance *A* in the device. Here, it is also assumed that the concentration change occurs only in the concentration boundary layer, which thickness is

represented by the variable δ_D. In general, the thicknesses of the concentration and the thermal boundary layers are different (i.e., $\delta_D \neq \delta_T$).

If several substances pass through the interfacial area, similar equations can be written on the basis of Eq. (3) for each additional substance (e.g., for substance *B*, it would apply

$$N_{Bw} = \frac{-E_{tBmw}}{(z_2 - z_1)\,\varphi}\left(c_{Bf} - c_{Bw}\right), \quad \text{where} \quad \varphi = \left(\frac{c_{Bw}}{c\gamma} - \frac{c_{Bf}}{c\gamma}\right) \Bigg/ \ln\left(\frac{1-\frac{c_{Bf}}{c\gamma}}{1-\frac{c_{Bw}}{c\gamma}}\right), \quad \text{and} \quad \frac{1}{\gamma} = \frac{\sum_{i=A}^{n} N_{iw}}{N_{Bw}}).$$

Such equations allow to determine the molar flux of a given substance. The sum of these fluxes gives the total molar flux of the substances passing through the interfacial area, i.e.:

$$N_w = N_{Aw} + N_{Bw} + \ldots + N_{iw} + \ldots + N_{nw} = \sum_{i=A}^{n} N_{iw}\,, \tag{4}$$

alternatively, the next equation can be written based on the Eq. (3):

$$\sum_{i=A}^{n} N_{iw} = \frac{E_{tAmw}}{(z_2 - z_1)}\, c \ln\left(\frac{1-\frac{c_{Af}}{c\gamma}}{1-\frac{c_{Aw}}{c\gamma}}\right) = Fc\ln\left(\frac{\gamma - \frac{c_{Af}}{c}}{\gamma - \frac{c_{Aw}}{c}}\right) \tag{5}$$

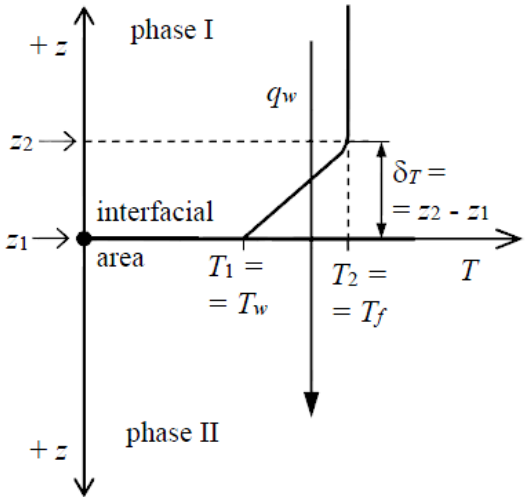


Fig. 1 Representation of the conditions near the interfacial area in the case of heat transfer.

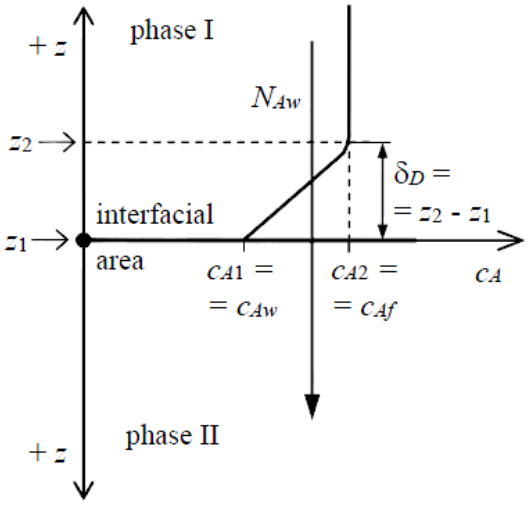


Fig. 2 Representation of the conditions near the interfacial area in the case of the unidirectional mass transfer of substance *A*.

If substances pass through the interfacial area, then from the point of view of balancing, these represent the material stream, which is also the carrier of the energy. The energy flux contained in this stream can be expressed as follows:

$$\begin{aligned} N_w \bar{h}_w &= N_{Aw}\bar{h}_{Aw} + N_{Bw}\bar{h}_{Bw} + \ldots + N_{iw}\bar{h}_{iw} + \ldots + N_{nw}\bar{h}_{nw} = \\ &= \sum_{i=A}^{n} N_{iw}\bar{h}_{iw} = \sum_{i=A}^{n} N_{iw} h_{iw}^{\circ} + \sum_{i=A}^{n} N_{iw}\Delta h_{iw}^{mix}\,, \end{aligned} \tag{6}$$

where: $h_{iw}^{\circ} = h_{iw}^{\circ} - h_{iwref}^{\circ} = h_{iw}^{\circ} - 0 = \int_{T_{ref}}^{T} c_{piw}^{\circ} \mathrm{d}T + \int_{p_{ref}}^{p} \left[v_{iw}^{\circ} - T\left(\frac{\partial v_{iw}^{\circ}}{\partial T} \right)_p \right] \mathrm{d}p + \Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{\alpha\to\beta}\right)$.

Fig. 3 shows the situation corresponding to the simultaneous transfer of heat and mass through the interfacial area. In this case, the total energy flux passing through the interfacial area is the sum of the two components represented by the Eqs. (2) and (6). This can be expressed mathematically by the following equation:

$$e_w = q_w + N_w \bar{h}_w = -\lambda_w \frac{\mathrm{d}T}{\mathrm{d}z} + \sum_{i=A}^{n} N_{iw} \bar{h}_{iw} \qquad (7)$$

or in vector form: $\vec{e} = \vec{q} + \vec{N}\bar{h} = -\lambda(\nabla T) + \sum_{i=A}^{n} \vec{N}_i \bar{h}_i$. In this equation, the first term $\vec{q} = -\lambda(\nabla T)$ expresses the energy flux component comprising the heat transfer by conduction and the second term $\vec{N}\bar{h} = \sum_{i=A}^{n} \vec{N}_i \bar{h}_i$ represents the energy flux transferred by the substances passing through the interfacial area.

Eq. (7) is the original equation for the description of the simultaneous transfer of heat and mass. On the basis of this equation, the corrected form of Newton's law of cooling is derived. Knowledge of this corrected form is important because it is used in the calculation of heat transfer and especially in the design of equipment where simultaneous heat and mass transfer [(14), (15), (16)].

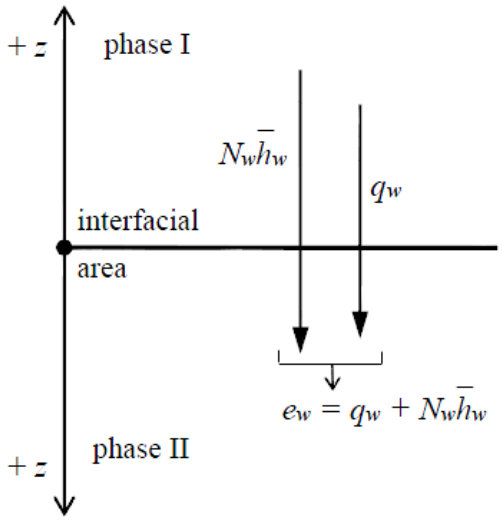


Fig. 3 Simultaneous heat and mass transfer (assuming the unidirectional mass transfer of substance *i*) through the interfacial area.

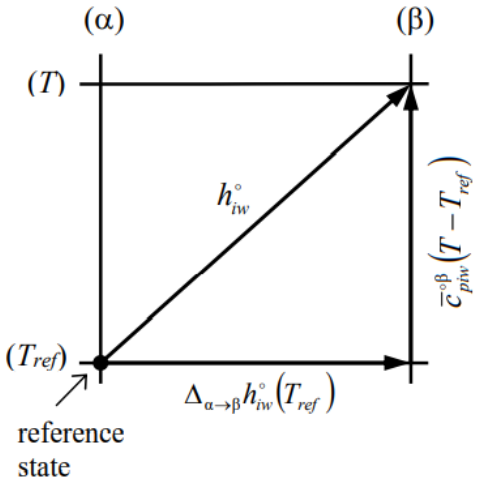


Fig. 4 Change of enthalpy at constant pressure, including the partial changes of enthalpy.

3 The energy flux through the interfacial area

Eq. (7) is the fundamental equation for solving the technical problems in which simultaneous heat and mass transfer through the interfacial area take place [(15)]. When applying a given equation to a specific problem, the conditions that prevail in the given device must be taken into account. Assume further that the pressure in the system is constant (i.e., p = constant),

the mixture of substances can be considered as ideal (i.e., $\sum_{i=A}^{n} N_{iw}\Delta h_{iw}^{mix} = 0$), the state of the substances in the system changes from α to β at a temperature corresponding to the reference state (i.e., $\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{\alpha\to\beta}\right) = \Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right)$), and that the temperature of the substances in the state β changes from T_{ref} to T (i.e., $\int_{T_{ref}}^{T} c_{piw}^{\circ}\mathrm{d}T = \int_{T_{ref}}^{T} c_{piw}^{\circ\beta}\mathrm{d}T = \bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right)$). Taking this into account, the enthalpy of the pure substance can be expressed by the equation: $h_{iw}^{\circ} = h_{iw}^{\circ}\left(T; p; \beta\right) - h_{iwref}^{\circ}\left(T_{ref}; p; \alpha\right) = h_{iw}^{\circ}\left(T; p; \beta\right) - 0 = \Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) + \bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right)$ (Fig. 4). The advantage of such an adjustment is that within a given range of temperatures, the enthalpy of the pure substance can be considered constant (i.e., h_{iw}° = constant). On the basis of the above facts, Eq. (7) takes the form:

$$e_w = -\lambda_w \frac{\mathrm{d}T}{\mathrm{d}z} + \sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right) + \sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) \tag{8}$$

If the system operates under steady conditions (i.e., e_w = constant, N_{iw} = constant), and when also λ_w = constant, Eq. (8) can be solved. By separating the variables, the following equation can be obtained:

$$\int \frac{\mathrm{d}z}{\lambda_w} = \int \frac{\mathrm{d}T}{\left[-e_w + \sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) + \sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right)\right]} \tag{9}$$

which, after integration, turns into a form:

$$\frac{z}{\lambda_w} = \frac{\ln\left[-e_w + \sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) + \sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right)\right]}{\sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}} + \text{constant} \tag{10}$$

In this equation, a constant has to be expressed. From a mathematical point of view, it can be formally written that at a distance $z = z_{ref}$ is $T = T_{ref}$. Taking into account the just mentioned, Eq. (10) takes the form:

$$\frac{z - z_{ref}}{\lambda_w} = \frac{\ln\left[\dfrac{-e_w + \sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) + \sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}\left(T - T_{ref}\right)}{-e_w + \sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right)}\right]}{\sum_{i=A}^{n} N_{iw}\bar{c}_{piw}^{\circ\beta}} \tag{11}$$

When working with this equation, it applies that all variables appearing in it relate to only one phase. This should always be remembered. It also follows from the derivation of this equation that in the practical application of this equation, a reference state (i.e., the value of the quantity T_{ref} and the corresponding value of z_{ref}) must be chosen. This follows from the fact that the derivation of this equation is based on Eq. (7) in which occurs the enthalpy, which always depends on the choice of the reference state. If it is necessary to know the value of the quantity e_w, the following relation can be obtained according to this equation (i.e., Eq. (11)):

$$e_w = \frac{\sum_{i=A}^{n} N_{iw} \bar{c}_{piw}^{\circ\beta} \left(T - T_{ref}\right)}{\left[1 - \exp\left[\sum_{i=A}^{n} N_{iw} \bar{c}_{piw}^{\circ\beta} \left(\frac{z - z_{ref}}{\lambda_w}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw} \Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right)\right] = e_{sen} + e_{lat} \qquad (12)$$

from which the equations commonly mentioned in the literature follow.

Eq. (12) is used in the technical calculations to express the total energy flux through the interfacial area if there is simultaneous heat and mass transfer through the interfacial area in the system. The first term on the right side of this equation (i.e., e_{sen}) represents the quantity of energy needed to change the temperature of the substances, and the second term (i.e., e_{lat}) represents the quantity of energy needed to change the state of the substances. In other words, Eq. (12) allows determining the amount of energy that needs to be transferred from a mixture of substances of a given state (in this case, the state indicated by the symbol β) in order to change their state into another (into the state indicated by the symbol α).

4 Ackermann correction factor

Eq. (12) is a key equation for deriving further equations. If this equation is to be solved, it is necessary to specify the conditions that represent the reference state. These can generally be chosen arbitrarily. Let the reference state be represented by the conditions prevailing at the interfacial area (i.e., $T_{ref} = T_w = T_1$ and $z_{ref} = z_w = z_1$), and let the state be represented by phase II (i.e., α = II and β = I) (see Figs. 3 and 4). In the sense just given, it will apply in**Chyba! Nenašiel sa žiaden zdroj odkazov.** Eq. (12): $\bar{c}_{piw}^{\circ\beta} = \bar{c}_{piw}^{\circ\mathrm{I}}$ and EMBED Equation.3 $\Delta_{\alpha\to\beta} h_{iw}^{\circ}\left(T_{ref}\right) = \Delta_{\mathrm{II}\to\mathrm{I}} h_{iw}^{\circ}\left(T_w\right)$. It can be written similarly for other quantities, i.e., $N_{iw} = N_{iw}^{\mathrm{I}}$ and $\lambda_w = \lambda_w^{\mathrm{I}}$. Then, Eq. (12) takes the form:

$$e_w = \frac{\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \bar{c}_{piw}^{\circ\mathrm{I}} \left(T - T_w\right)}{\left[1 - \exp\left[\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \bar{c}_{piw}^{\circ\mathrm{I}} \left(\frac{z - z_1}{\lambda_w^{\mathrm{I}}}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \Delta_{\mathrm{II}\to\mathrm{I}} h_{iw}^{\circ}\left(T_w\right)\right] = e_{sen} + e_{lat} \qquad (13)$$

The equation written in this way basically shows how much energy needs to be transferred from a system containing substances represented by state I (or substances representing phase I) to change their state to state II (or phase II). This equation (i.e., Eq. (13)) applies, of course, provided that there is a simultaneous transfer of heat and mass through the interfacial area in the system.

It is also often the case that the substance, the mixture of substances, flows through a device (Fig. 5). In such a case, it is supposed in terms of film theory that the change of temperature (i.e., $T_f \rightarrow T_w$) occurs only in a thermal boundary layer. In the sense just given, the temperature T_f can be substituted for the temperature T (i.e., $T = T_f = T_2$) in Eq. (13), which also means that the equality $z = z_2$ has to apply (Fig. 5). That means that Eq. (13) can be rewritten in the following form:

$$e_w = \frac{\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \bar{c}_{piw}^{\circ\mathrm{I}} \left(T_f - T_w\right)}{\left[1 - \exp\left[\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \bar{c}_{piw}^{\circ\mathrm{I}} \left(\frac{z_2 - z_1}{\lambda_w^{\mathrm{I}}}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}^{\mathrm{I}} \Delta_{\mathrm{II}\to\mathrm{I}} h_{iw}^{\circ}\left(T_w\right)\right] = e_{sen} + e_{lat} \qquad (14)$$

expressing the energy flux passing through the interfacial area. The term $z_2 - z_1$ expresses the thickness of the thermal boundary layer (i.e., $\delta_T = z_2 - z_1$) which is not known for the turbulent flow of the fluid. The same applies in principle for the quantity λ_w (more precisely λ_w^{I}) when the fluid flows turbulently. For this reason, the ratio of these quantities, when the fluid flows, is replaced by a new quantity (heat transfer coefficient), the value of which is determined on the basis of the experimental measurements (i.e., $1/\alpha^{\mathrm{I}} = (z_2 - z_1)/\lambda_w^{\mathrm{I}} = \delta_T/\lambda_w^{\mathrm{I}}$). The analogous situation occurs also in the case of mass transfer through the interfacial area when there is a turbulent fluid flow in the system. In such a case, the expression $E_{tAmw}/(z_2 - z_1)$ is replaced by the symbol F [(15), (18)]. Usually, the superscript I is not given for the quantities appearing in Eq. (14). In such a case, and in the light of the above, it is now possible to write the equation:

$$e_w = \frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(T_f - T_w\right)}{\left[1-\exp\left[\frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}}{\alpha}\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}}h^{\circ}_{iw}\left(T_w\right)\right] = e_{sen} + e_{lat} \tag{15}$$

resulting from Eq. (14). It is customary to present this equation in the following form:

$$e_w = \left[\frac{\Phi_T}{1-\exp\left(\Phi_T\right)}\right]\left(T_f - T_w\right)\alpha + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}}h^{\circ}_{iw}\left(T_w\right)\right] = e_{sen} + e_{lat} \tag{16}$$

in which expression:

$$\Phi_T = \frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}}{\alpha} \tag{17}$$

represents the rate factor.

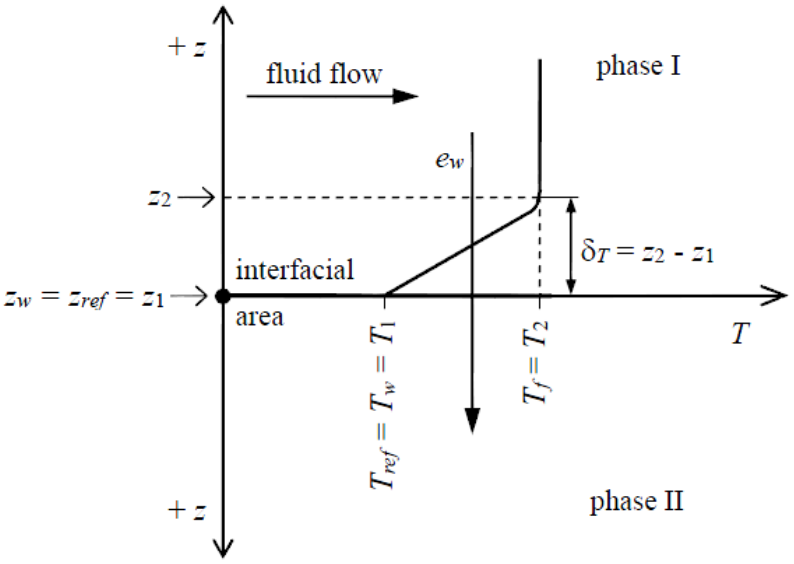


Fig. 5 Simultaneous heat and mass transfer (assuming the unidirectional mass transfer of substance *i*) through the interfacial area when one of the phases flows.

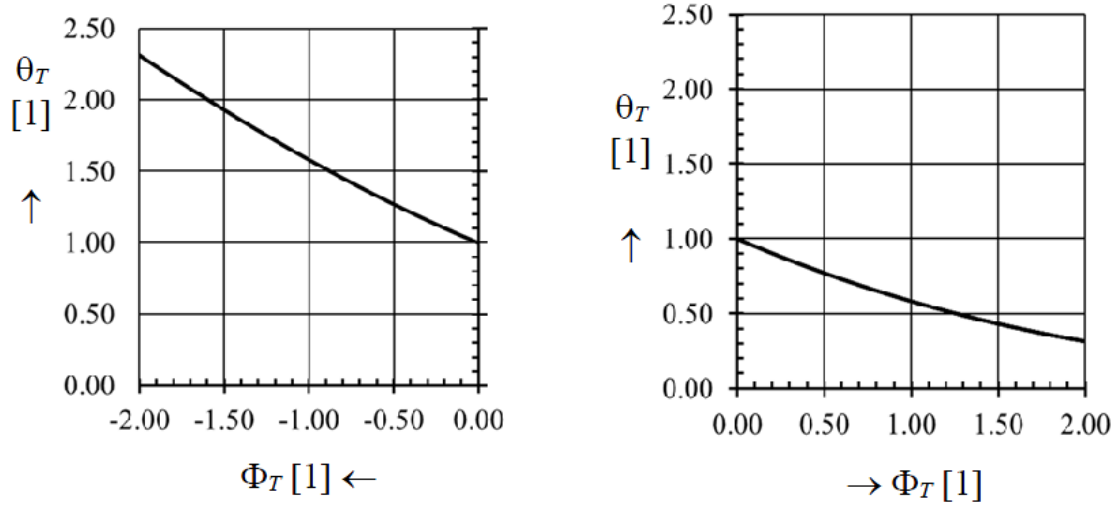
The expression $(T_f - T_w)\alpha$ mentioned in Eq. (16) can be modified to the form representing Newton's law of cooling (i.e., $q_w = -\alpha(T_f - T_w) = \alpha(T_w - T_f)$). Therefore, it is possible to write an equation in the form:

$$\begin{aligned} e_w &= \theta_T \alpha(T_w - T_f) + \left[\sum_{i=A}^{n} N_{iw} \Delta_{\text{II}\to\text{I}} h_{iw}^{\circ}(T_w)\right] = \\ &= -\theta_T \alpha(T_f - T_w) + \left[\sum_{i=A}^{n} N_{iw} \Delta_{\text{II}\to\text{I}} h_{iw}^{\circ}(T_w)\right] = e_{sen} + e_{lat}, \end{aligned} \tag{18}$$

in which expression:

$$\theta_T = \frac{\Phi_T}{e^{\Phi_T} - 1} = \frac{\Phi_T}{R_T} \tag{19}$$

represents a correction factor (the so-called Ackermann correction factor) respecting the effect of mass transfer on heat transfer when there is simultaneous heat and mass transfer in the system [(14)]. It follows from the equation that the relation $e^{\Phi_T} - 1 = R_T$ (or $\Phi_T = \ln(R_T + 1)$) also applies and thus the equation $\theta_T = \ln(R_T + 1)/R_T$ as well. The dependence of θ_T on Φ_T is shown in Fig. (6).



a) The dependence of θ_T on Φ_T if the mass transfer of the substance *i* is out of the given phase (the substance *i* leaves the phase, Φ_T is a negative).

b) The dependence of θ_T on Φ_T if the mass transfer of the substance *i* is into the given phase (the substance *i* enters to the phase, Φ_T is a positive).

Fig. 6 The dependence of θ_T on Φ_T by Eq. (19) when there is simultaneous heat and mass transfer in the system.

In terms of Eq. (18), or according to the comment on Eq. (12), it is clear that the following equation is also valid:

$$e_{sen} = \theta_T \alpha(T_w - T_f) = \theta_T q_w \tag{20}$$

which represents the corrected form of Newton's cooling law applicable to simultaneous mass and heat transfer. If there is only heat transfer through the interfacial area in the system, the following equations apply: $\theta_T = 1$ and $e_{sen} = q_w$.

Based on Eq. (17), Eq. (19), and Eq. (20) another relation can be written for calculation of the quantity R_T, i.e., the equation:

$$R_T = \frac{\left(\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\right)\left(T_w - T_f\right)}{e_{sen}} \tag{21}$$

For the sake of completeness, it can also be stated that the value of the quantity R_T can also be determined from the relation: $R_T = \left(\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\right) \Big/ \left(\theta_T \alpha\right)$. This relation results from the combination of Eq. (17) and Eq. (19). It is evident that when this relation is compared with Eq. (21), the obtained equation represents Eq. (20).

It can also be stated that these equations are presented in the literature in other forms as well, in more compact forms. For example, Eq. (18) may also be written in the following form:

$$\begin{aligned} e_w &= \theta_T \alpha\left(T_w - T_f\right)\left[1 + \frac{\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}} h^{\circ}_{iw}\left(T_w\right)}{\theta_T \alpha\left(T_w - T_f\right)}\right] = \\ &= \theta_T \alpha \frac{e_w}{e_{sen}}\left(T_w - T_f\right) = -\theta_T \alpha \frac{e_w}{e_{sen}}\left(T_f - T_w\right), \end{aligned} \tag{22}$$

possibly also in the form:

$$e_w = \theta_T \alpha\left(T_w - T_f\right)\left(1 + \frac{\alpha'}{\theta_T \alpha}\right) = \left(T_w - T_f\right)\left(\theta_T \alpha + \alpha'\right) = -\left(\theta_T \alpha + \alpha'\right)\left(T_f - T_w\right) \tag{23}$$

in which $\alpha' = \left(\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}} h^{\circ}_{iw}\left(T_w\right)\right) \Big/ \left(T_w - T_f\right)$. The most compact form of the equation representing the energy flux is possible to obtain when it will apply in Eq. (23) that $\alpha'' = \left(\theta_T \alpha + \alpha'\right)$ what gives the equation in form $e_w = -\alpha''\left(T_f - T_w\right)$.

5 Another form of Ackermann correction factor

In the literature [(15)], it can also be seen that the Ackermann correction factor is given by the equation:

$$\theta_T = \frac{\Phi_T}{1 - \mathrm{e}^{-\Phi_T}} = \frac{\Phi_T}{R_T} \tag{24}$$

which differs from Eq. (19). This form of the equation is also correct, however, it results from a different choice of the coordinate system orientation. Taking into account the situation shown in Fig. (7), then in terms of Eq. (12) it is obtained the following equation:

$$e_w = \frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(T - T_2\right)}{\left[1 - \exp\left[\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(\frac{z - z_2}{\lambda_w}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\alpha\to\beta} h^{\circ}_{iw}\left(T_2\right)\right] = e_{sen} + e_{lat}, \tag{25}$$

because now $T_{ref} = T_w = T_2$ and $z_{ref} = z_w = z_2$. In terms of the comment to Eq. (14), $T = T_f = T_1$ and $z = z_1$ can be further written, which means that Eq. (25) takes the form:

$$e_w = \frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(T_1 - T_2\right)}{\left[1-\exp\left[\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(\frac{-\left(z_2 - z_1\right)}{\lambda_w}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}}h^{\circ}_{iw}\left(T_2\right)\right] =$$
$$= \frac{\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(T_f - T_w\right)}{\left[1-\exp\left[\sum_{i=A}^{n} N_{iw}\bar{c}^{\circ}_{piw}\left(\frac{-\delta_T}{\lambda_w}\right)\right]\right]} + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}}h^{\circ}_{iw}\left(T_w\right)\right] = e_{sen} + e_{lat}\,. \tag{26}$$

Taking into account Eq. (17), as well as a comment on Eq. (13), the final form of Eq. (26) is obtained:

$$e_w = \left[\frac{\Phi_T}{1-\exp\left(-\Phi_T\right)}\right]\left(T_f - T_w\right)\alpha + \left[\sum_{i=A}^{n} N_{iw}\Delta_{\mathrm{II}\to\mathrm{I}}h^{\circ}_{iw}\left(T_w\right)\right] = e_{sen} + e_{lat} \tag{27}$$

from which follows another form of the equation representing the Ackermann correction factor (i.e., Eq. (24)).

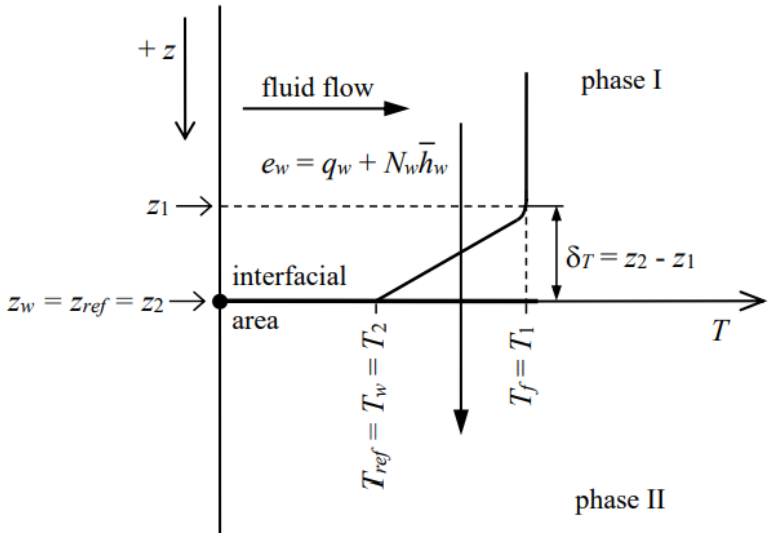


Fig. 7 Representation of the conditions near the interfacial area in the case of heat and mass transfer through the interfacial area resulting from a different choice of the coordinate system orientation.

Finally, it is necessary to know the value of the quantity N_{iw} to work with Eq. (27). In the case of the mass transfer, e.g., the transfer of the substance A (see Fig. (8)), the value of N_{iw} is determined from the following equation:

$$N_{Aw} = \frac{-E_{tAmw}}{\left(z_2 - z_1\right)\varphi}\left(c_{Aw} - c_{Af}\right) = k_c\left(c_{Af} - c_{Aw}\right) \tag{28}$$

in which $k_c = \frac{F}{\varphi}$, $F = \frac{E_{tAmw}}{z_2 - z_1} = \frac{E_{tAmw}}{\delta_D}$, $\varphi = \left(\frac{c_{Af}}{c\gamma} - \frac{c_{Aw}}{c\gamma}\right) \Big/ \ln\left(\frac{1-\frac{c_{Aw}}{c\gamma}}{1-\frac{c_{Af}}{c\gamma}}\right)$, and $\frac{1}{\gamma} = \frac{\sum_{i=A}^{n} N_{iw}}{N_{Aw}}$.

Eq. (28) takes into account the fact that follows from a different choice of the coordinate system orientation.

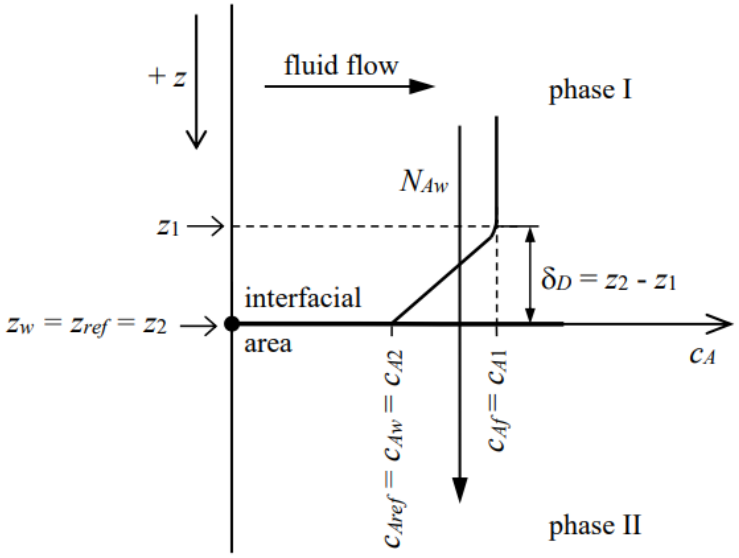


Fig. 8 Representation of the conditions near the interfacial area in the case of the unidirectional mass transfer of a substance *A* through the interfacial area resulting from a different choice of the coordinate system orientation.

Conclusion

When describing processes in which simultaneous heat and mass transfer occurs through the interfacial area, it is true that the mass and heat transfer are interrelated. This subsequently affects the method of calculations of the equipment in which such processes occur. From the point of view of the description of the processes taking place in such equipment, it is necessary to know the amount of energy that passes through the interfacial area from one phase to another. The article summarizes the equations used to describe such processes, mainly from the point of view of technical practice. It introduces the original equation commonly used to describe technical problems, as well as the equation for calculating the total flux of energy through the interfacial area valid for the assumptions given here.

In the article, various forms of equations determining the total energy flux are given, because the different forms of these equations can be commonly encountered in the literature. The form of the equation presented here, including its various modifications, can be considered as general for the purposes of solving technical problems. It follows from the presented equations that the aim is always to write the simplest possible form of the equation to describe the given problem. Such a form can be achieved by introducing correction factors (e.g., Ackermann correction factor) into the calculations, which are used to correct the equations, sometimes called laws (in this case, Newton's law of cooling), applicable for another case (in this case, when there is no mass transfer through the interfacial area).

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Nomenclature

Latin letters

A - interfacial area [m^2]

c - total molar concentration [mol/m^3]

c_i - molar concentration of a substance i [mol/m^3]

E_{tim} - total diffusivity coefficient for a multicomponent system [m^2/s]

e - energy flux [W/m^2]

F - mass transfer coefficient based on the molar concentration differences in the case of bidirectional mass transfer [m/s]

h - enthalpy [J/mol]

k_c - mass transfer coefficient based on the molar concentration differences [m/s]

N - molar flux of a substance [$mol/(m^2.s)$]

N_i - total molar flux of a substance i [$mol/(m^2.s)$]

n - number of substances in the mixture passing through the interfacial area [1]

p - total pressure [Pa]

$\dot{Q}$ - heat flow [W]

q - heat flux [W/m^2]

R - fluxes ratio [1]

T - absolute temperature [K]

v - molar volume [m^3/mol]

z - distance from the interfacial area or the distance in the direction of the z coordinate [m]

Greek letters

α - heat transfer coefficient related to all substances in the mixture [$W/(m^2K)$]

α' - hypothetical heat transfer coefficient related to only substances passing through the interfacial area [$W/(m^2K)$]

α'' - hypothetical total heat transfer coefficient [$W/(m^2K)$]

Δh^{mix} - mixing enthalpy [J/mol]

$\Delta_{\mathrm{II}\to\mathrm{I}}h(T_{\mathrm{II}\to\mathrm{I}})$ - enthalpy change during the phase change of the substance from phase II to phase I at the temperature $T_{\mathrm{II}\to\mathrm{I}}$ [J/mol]

$\Delta_{\alpha\to\beta}h(T_{\alpha\to\beta})$ - enthalpy change during the phase change of the substance from phase α to phase β at the temperature $T_{\alpha\to\beta}$ [J/mol]

δ_D - mass boundary layer thickness [m]

δ_T - thermal boundary layer thickness [m]

Φ - rate factor [1]

γ - the ratio of molar fluxes [1]

φ - factor taking into account the type of concentration diffusion [1]

λ - thermal conductivity [W/(mK)]

θ - correction factor [1]

Mathematical operators

∇ - nabla or del operator [1/m]

Overlines

$\vec{X}$ - vector

$\overline{X}$ - average value or partial quantity

$X|_{z=w}$ - near the interfacial area

Superscripts

X° - quantities belonging to pure substance

X^{I} - quantities belonging to the substance in phase I, in state I

X^β - quantities belonging to the substance in phase β, in state β

Subscripts

A, B - substances A and B

f - in the volume of fluid

i - substance i in mixture, $i = A, B, C, \ldots$

lat - latent

ref - reference state

w - near the interfacial area

z - Cartesian coordinate, in the z-axis direction

1, 2 - related to distances 1 and 2, related to cross-sections 1 and 2

sen	- sensitive	$II \rightarrow I$	- change of the state (phase) *II* to state (phase) *I*
T	- quantities related to the heat transfer taking into account the simultaneous heat and mass transfer	$\alpha \rightarrow \beta$	- change of the state (phase) α to state (phase) β

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