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QUADCOPTER PROTOTYPE STABILITY ASSESSMENT WITH PID CONTROLLER AND EULER-LAGRANGE APPROACH

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Abstract. ***The increasing use of drones in various fields has led to their popularity in developed countries due to their ease of use and manufacture. This Miniature Pilotless Aircraft has numerous beneficial usages such as express shipping, gathering information, crop monitoring, cargo transport, storm tracking, geographic mapping of inaccessible terrain, search and rescue operations, among others. This study aims to investigate the stability of a quadcopter through simulations based on the mathematical model that describes the quadcopter's dynamic and flight mechanics, using the Euler-Lagrange approach. It conducts simulations in MATLAB and present the principles that govern quadcopter stability, focusing on setting the PID coefficients to achieve optimal stability. This study provides insights into the principles of drone mechanics and stability, enabling us to better understand the quadcopter's behavior and performance.***

Keywords: simulation, quadcopter, command, stability, PID

1. INTRODUCTION

Quadcopters have spread quickly across a variety of sectors because to their adaptability and simplicity of usage, making them a popular alternative for many companies looking for effective solutions. Quadcopters may carry out a wide range of tasks as an Unmanned Aerial Vehicle (UAV), including package delivery, crop monitoring, search and rescue missions, and aerial videography. Quadcopters are essential in current operations because of their small size and agility, which allow them to negotiate difficult terrain and reach remote areas. As a result, companies and organizations all over the world have embraced this technology as a practical way to increase the effectiveness, speed, and accuracy of their operations.

The civilian drone market has seen a recent influx of new models, with many of these multi-rotors utilizing advanced and sophisticated technologies previously unexplored in the industry. The incorporation of high-precision technologies, particularly in the areas of tracking, recognition, and obstacle avoidance, has allowed for greater functionality and efficiency in drone operations. These cutting-edge technologies have opened up new possibilities for a wide range of applications, from aerial surveying and inspection to precision agriculture and emergency response. The introduction of these advanced features has further expanded the potential uses of civilian drones, making them an increasingly popular choice for various industries seeking innovative solutions. Quadrotors are highly maneuverable aerial vehicles that need complex modeling approaches for control and optimization. Since it allows for the introduction of nonlinear dynamics and external forces, the Euler-Lagrange technique is often employed for modeling quadrotors [1-2]. The Newton-Euler technique is also employed since it is based on Newton's principles of motion and gives a thorough knowledge of the quadrotor's motion [3-4]. The Hamiltonian technique is used to derive the quadrotor's equations of motion in an energy-efficient manner [5-6]. The state space technique may be used to model and control a system using linear equations [7]. The linearization method is frequently used to simulate the nonlinear dynamics of a quadrotor around an operational point [8]. Finally, multibody system approach serves as a crucial tool in accurately modeling the quadrotor's dynamic behavior and its intricate interactions with the environment, taking into consideration the complex movements of its various parts. This approach is applied in different software such as GAZEBO, Webots, SimMechanics, and ADAMS [9-10].

The present article aims to examine the stability of a novel prototype of a quadcopter by employing the Euler language approach and utilizing the Proportional-Integral-Derivative (PID) regulator on MATLAB (Figure 1). The study will go through the fundamental movements of the quadcopter, namely roll, pitch, yaw, and altitude, to obtain a comprehensive understanding of the dynamics and performance of the device. Overall, this study represents an important step towards improving the stability and performance of quadcopters, which have become increasingly important for various applications, including aerial photography, surveillance, and transportation. By gaining a deeper understanding of the quadcopter's dynamics and behavior, we can develop more effective control strategies and improve the safety and reliability of these devices.

The objective of this study is to investigate the stability of a new quadcopter prototype using the Euler language technique and the Proportional-Integral-Derivative (PID) regulator on MATLAB. The research will go through the fundamental movements of the quadcopter, including: roll, pitch, yaw, and altitude, in order to understande the quadrotors dynamics and performance. Overall, this work offers a significant step in improving the stability and performance of quadcopters, which have become more relevant for a variety of applications. However, Increasing the safety and dependability of these aircrafts can be achieved by better understanding the dynamics and behavior of quadcopters.

Figure 1. Quadrotor prototype.

2. QUADROTOR DYNAMICS AND REFERENCES

Quadcopters utilize rotor speed variation to execute fundamental movements. Tilting the quadcopter in the direction of a slower rotor results in translation along the corresponding axis, which is the basis of pitch and roll movements. Additionally, quadcopters are capable of vertical movement and rotation around the Z-axis, known as yaw, as depicted in Figure 2. These four movements, namely roll, pitch, vertical movement, and yaw, are controlled by the torque applied by the motors and are sufficient to manipulate the quadcopter's six degrees of freedom.

The quadcopters are governed by the laws of physics and aerodynamics. These unmanned aerial vehicles rely on the variation of rotor speeds to produce the basic movements required for their operation, namely roll, pitch, yaw, and altitude control. To describe the flight dynamics of quadcopters mathematically, two references are used: a fixed reference linked to the Earth and a mobile reference with its origin at the drone's center of gravity. The transformation matrix R is used to convert between these references and contains information about the orientation and position of the movable reference relative to the fixed reference. By modeling the quadcopter's flight dynamics, we can gain insight into its behavior and performance under different conditions, which can be used to improve its design and control strategies. The mathematical model can also be used to simulate the quadcopter's behavior and test various control algorithms and maneuvers in a virtual environment before applying them to real-world scenarios.

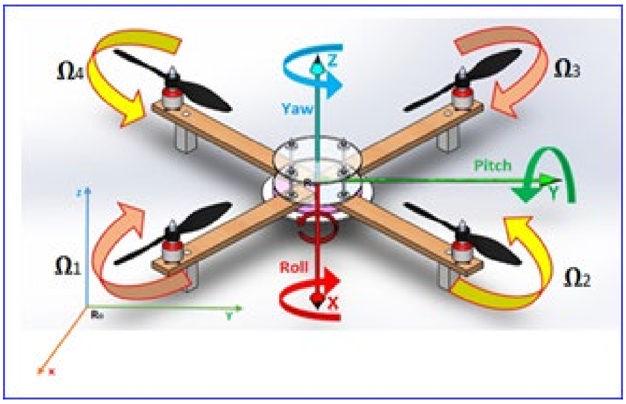

Figure 2. Quadrotor references and movements.

3. QUADROTOR EULER LAGRANGE MODEL

The rotation matrix, which describes the orientation of the quadcopter's movable reference frame relative to the fixed reference frame, can be obtained using Euler angles. Specifically, the rotation matrix is constructed by performing rotations around the X, Y, and Z axes, each by a respective angle of ϕ, θ, and ψ. These rotations correspond to the quadcopter's roll, pitch, and yaw movements, respectively, and are fundamental to controlling its position and orientation in three-dimensional space. The Euler angle approach is a powerful mathematical tool for simulating and analyzing dynamic systems, and its application to quadcopter flight dynamics enables us to study and optimize the performance of these devices. By understanding the relationship between the Euler angles and the quadcopter's movements, we can develop more effective control strategies and improve the safety and reliability of quadcopters in various applications.

Rotation matrix:

$R = R(\phi, \theta, \psi) =$

$$\begin{bmatrix} c\theta c\psi & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ c\theta s\psi & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (1)$$

Lift is the force that allows the quadcopter to rise if it at least equals drag. It creates in the direction of the X and Y axes, the following two moments.

$$\tau_x = bl(\Omega_1^2 + \Omega_4^2 - \Omega_2^2 - \Omega_3^2) \quad (2)$$

$$\tau_y = bl(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2) \quad (3)$$

Thrust coefficient: to calculate the thrust coefficient we use this equation:

$$b = \frac{F_{moteur}}{\Omega_{moteur}^2} \quad (4)$$

The drag is the result of the friction of the air on the quadcopter, it is opposed to the lift. She creates a vertical moment.

$$\tau_z = d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \quad (5)$$

To calculate the drag coefficient of a quadcopter, it is necessary to fix the quadcopter at its center of gravity and apply rotation to two opposing motors to initiate rotation around the vertical axis. By measuring the complete cycle time t, the drag coefficient can be determined using the following equation.

$$d = \frac{\pi I_z}{2\Omega^2 t^2} \quad (6)$$

When the quadcopter is rotating on two axes, this rotation generates a force that appears on the third axis and tends to resist the movements of the quadcopter (gyroscope effect).

$$\tau_{gx} = I_{rotor}\omega_y(\Omega_1 + \Omega_4 - \Omega_2 - \Omega_3) \quad (7)$$

$$\tau_{gy} = I_{rotor}\omega_x(\Omega_1 + \Omega_2 - \Omega_3 - \Omega_4) \quad (8)$$

To generate the transfer equations for a motor, it can be represented as a simple RLC circuit. By neglecting losses, we can derive the following equation. This equation is important for understanding the motor's behavior and response to various inputs, which is essential for designing control systems that effectively regulate the quadcopter's movements.

$$H(s) = \frac{K}{K^2 + RJs} \quad (9)$$

To determine the moments of inertia of a quadcopter, we can treat it as a solid body with a fixed mass and its axes parallel to the main axes of inertia. This allows us to model it as a rectangular parallelepiped with mass M and dimensions L, W, and H. The motors can be modeled as cylinders with mass m, height h, radius R, and located at a distance of l from the center of gravity. By accurately calculating the moments of inertia, we can better understand the quadcopter's rotational behavior and design control algorithms that ensure stable and precise movements.

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (10)$$

The moments of inertia of a rectangular parallelepiped are modeled by the following equations:

$$I_X = \frac{m_{pr}}{12}(W^2 + H^2) + m_c\left(R^2 + \frac{h^2}{3}\right) + 2m_c l^2 \quad (11)$$

$$I_Y = \frac{m_{pr}}{12}(L^2 + H^2) + m_c\left(R^2 + \frac{h^2}{3}\right) + 2m_c l^2 \quad (12)$$

$$I_Z = \frac{m_{pr}}{12}(L^2 + W^2) + 2m_c R^2 + 2m_c l^2 \quad (13)$$

The Lagrange formula can be used to obtain the angular accelerations of a quadrotor by using the equations above. These equations take into account the forces acting on the quadrotor, such as thrust and drag, as well as the moments that cause it to rotate.

$$L = T - U \quad (14)$$

With:

$$T = \frac{1}{2} mV^2 \quad (15)$$

$$U = \int[-g\sin\theta]x\,dm + \int[g\sin\phi\cos\theta]y\,dm + \int[g\cos\phi\cos\theta]z\,dm \quad (16)$$

Angular accelerations:

$$\ddot{\phi} = \frac{I_{rotor}\dot{\theta}(\Omega_1+\Omega_4-\Omega_2-\Omega_3)}{I_x} + \frac{1}{I_x}U_2 + \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} \quad (17)$$

$$\ddot{\theta} = \frac{I_{rotor}\dot{\phi}(\Omega_1+\Omega_2-\Omega_3-\Omega_4)}{I_y} + \frac{1}{I_y}U_3 + \frac{I_z - I_x}{I_y}\dot{\psi}\dot{\phi} \quad (18)$$

$$\ddot{\psi} = \frac{1}{I_z}U_4 + \frac{I_x - I_y}{I_z}\dot{\theta}\dot{\phi} \quad (19)$$

Linear accelerations:

$$\ddot{x} = \frac{1}{m}(\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi)U_1 \quad (20)$$

$$\ddot{y} = \frac{1}{m}(\cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi)U_1 \quad (21)$$

$$\ddot{z} = \frac{1}{m}(\cos\phi\cos\theta)U_1 - g \quad (22)$$

With:

$$U_1 = \sum_{i=1}^{4} b * T_i = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \quad (23)$$

$$U_2 = b.l(\Omega_1^2 - \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \quad (24)$$

$$U_3 = b.l(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2) \quad (25)$$

$$U_4 = d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \quad (26)$$

Control algorithmes :

$$M_1 = T + R + P - Y \quad (27)$$

$$M_2 = T - R + P + Y \quad (28)$$

$$M_3 = T - R - P - Y \quad (29)$$

$$M_4 = T + R - P + Y \quad (30)$$

4. SIMULATION

To model a quadcopter using MATLAB (Simulink), it's important to understand its behavior and movements. With six degrees of freedom, but only four motors, we can control four of the six degrees, including altitude, roll, pitch, and yaw. Understanding the relationships between equations, including motor thrust, angular acceleration, gyroscopic effect, control, and displacement equations, is crucial in establishing a reliable model. Through the use of MATLAB (Simulink) in order to determine the constants of the PID and ensure stabilization of the quadcopter on all three axes of roll, pitch, yaw, and altitude. Figure 3 and 4 illustrates the quadrotor model in matlab simulink. Table 1 presents the quadrotor constates used for the simulation.

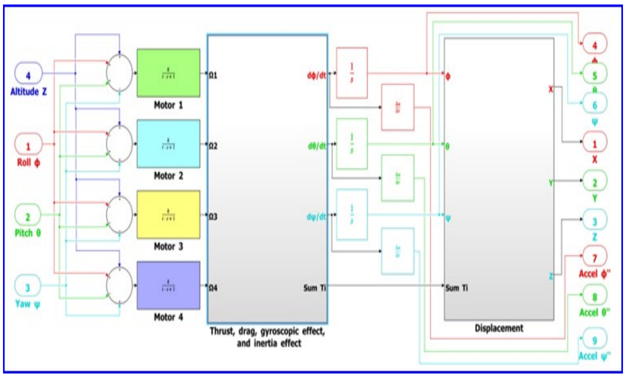

Figure 3. Quadrotor Simulink model

Table 1. Quadrotor parameters

Param	Value	Param	Value
Ix	$4.8 * 10^{-2}$ (Kg.m^2)	*l*	0.275 (m)
Iy	$4.8 * 10^{-2}$ (Kg.m^2)	*d*	$7.5 * 10^{-6}$ (Kg.m.rad^{-2})
Iz	$8.5 * 10^{-2}$ (Kg.m^2)	*b*	$3.1 * 10^{-6}$ (Kg.m.rad^{-2})
m	1.34(kg)	*K*	230.36 (rad/s)
I_{rotor}	$3.1 * 10^{-5}$ (Kg.m^2)	*t*	0.15 s

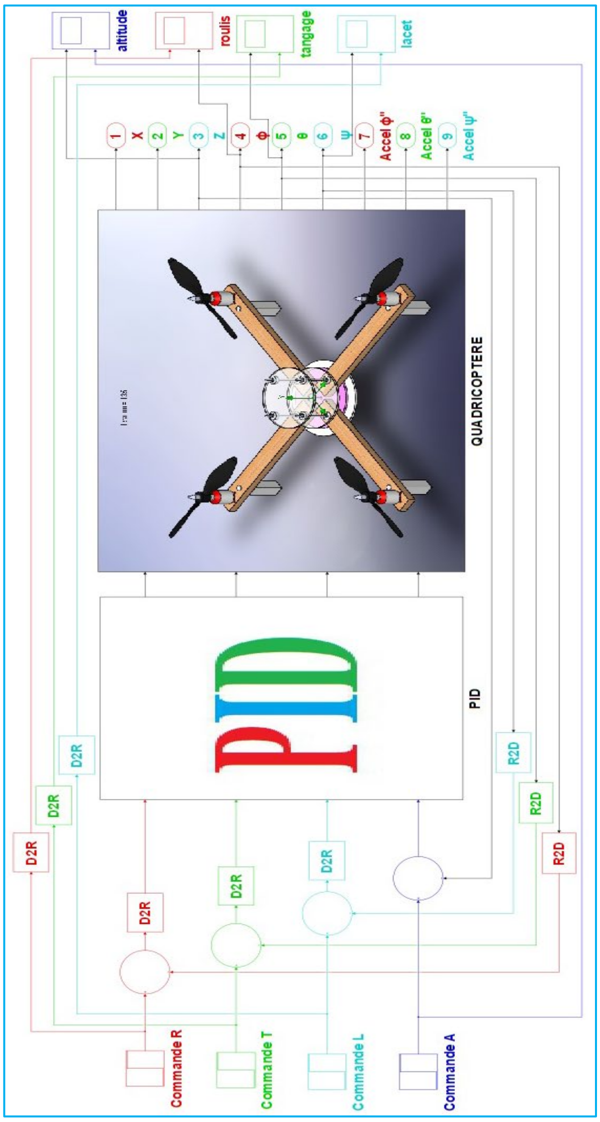

Figure 4. Quadrotor Simulink model with PIDs.

Manually adjusting the coefficients of the PID is a challenging task as it requires adjusting three coefficients at the same time with numerous possible combinations. The process starts with adjusting the Kp coefficient to improve the response time of the system, followed by adjusting the Ki coefficient to eliminate errors and ensure a quick and accurate response. Lastly, the Kd coefficient is adjusted to increase system stability by minimizing oscillations. The optimal values for each PID, including roll, pitch, yaw, and altitude, are provided in Table 2.

Table 2. PID parameters.

Roll		Pitch		Yaw		Altitude	
Param	Value	Param	Value	Param	Value	Param	Value
Kp	1	Kp	1	Kp	0.8	Kp	3
Ki	0.01	Ki	0.01	Ki	0.01	Ki	0.85
Kd	1.2	Kd	1	Kd	1.1	Kd	3

5. RESULTS AND DISCUSSION

Following the simulation of the quadrotor using MATLAB Simulink, we were able to obtain valuable results that depict the system's stability across all four movements, including roll, pitch, yaw, and altitude. In addition, the quadrotor's movements along the X and Y axes, based on its roll and pitch movements, were also evaluated. The results presented in Figures (5, 6, 7 and 8), offer comprehensive visual representations of the quadrotor's dynamic behavior and provide significant insights for the purpose of further analysis and control optimization.

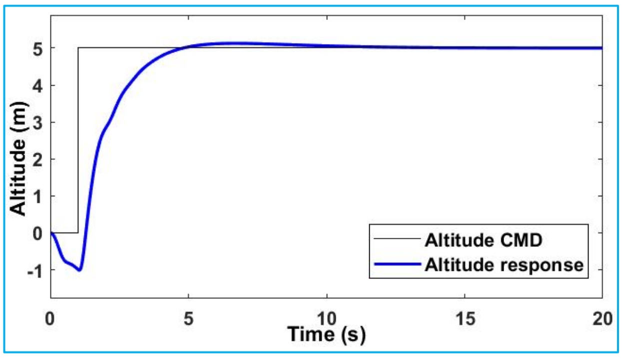

Figure 5. Altitude results.

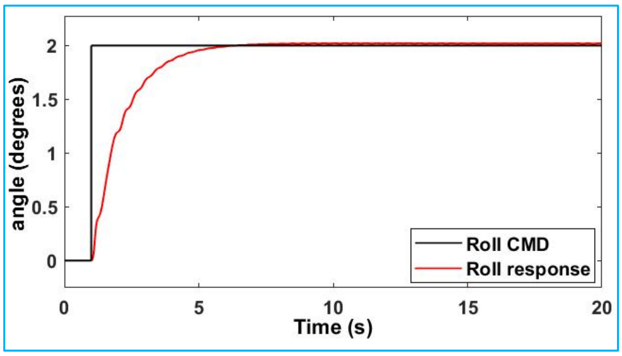

Figure 6. Roll results.

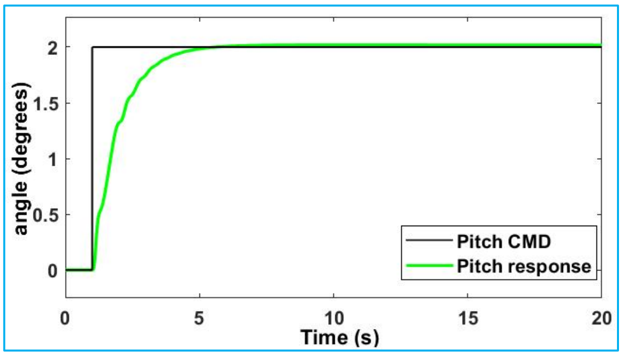

Figure 7. Pitch results.

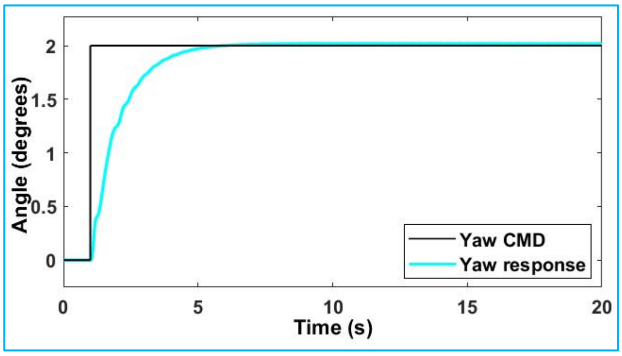

Figure 8. Yaw results.

6. CONCLUSION

In this study, The quadrotor model was simulated using the Euler-Lagrange technique in MATLAB Simulink, and its stability was tested using a Proportional-Integral-Derivative (PID) controller. The simulation results confirmed the PID controller's usefulness in controlling the quadrotor's motion and preserving its stability throughout flight. The study also emphasizes the significance of the Euler-Lagrange technique in adequately describing the complicated dynamics of the quadrotor. Overall, this work gives useful insights for the development and improvement of quadrotor control systems and emphasizes the need of using modern simulation tools like MATLAB to explore the dynamics of complex systems.

7. NOMENCLATURE

ENGLISH LETTERS

I_x: The moment of inertia along the X axis (Kg.m²).
I_y: The moment of inertia along the Y axis in (Kg.m²).
I_z: The moment of inertia along the Z axis in (Kg.m²).
I_{rotor}: The moment of inertia around the motor in (Kg.m²).
d: The drag coefficient (kg. m. rad^{-2}).
b: The thrust coefficient (kg. m. rad^{-2}).
l: The distance between the motor and the centre of gravity of the quadcopter (m).
Kp, Ki, Kd: The gains of proportional, integrals, derivatives.
b: The thrust coefficient in (kg.m/rad^2).
m_{pr}: The weight of the rectangular parallelepiped (kg).
m_c: The mass of the cylinder (kg).
W: The width of the rectangular parallelepiped (m).
h: The height of the cylinder (m).
H: The height of the rectangular parallelepiped (m).
K: The gain of the motor in (V.s / rad).
R: The internal resistance of the motor in (ohm).
J: The inertia of the rotor in (g.cm²).
Mi: Motors
T: Thrust
R: Roll
P: Pitch
Y: Yaw
C: cos
S: sin

GREEK SYMBOLS

ϕ: The angle of rotation around the 'X' axis (Roll) in (rad).
$\boldsymbol{\theta}$: The angle of rotation around the 'Y' axis (Pitch) in (rad).
$\boldsymbol{\Psi}$: The angle of rotation around the 'Z' (Yaw) axis in (rad).
$\boldsymbol{\Omega_r}$: The otors speed in (rad / s).
τ: The time constant of the motors (s).

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