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How to Read a Clock

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Abstract

In this paper we present several binary clocks. Using different geometric figures, we show how one can devise various novel ways of displaying time. We accompany each design with the mathematical background necessary to understand why these designs work.

1 Introduction

One of the classical ways for keeping time is to use a clock. Some clocks show time using a dial with fixed numbers and clock hands which indicate hours, minutes, and sometimes seconds. Some use an electronic display that specifies time using numerals. Others, use painted sheets of material that are mounted like the pages of a book.

In this paper we are concerned with clocks based on binary displays. These clocks have several lamps and when a lamp is lit it indicates that a certain period of time has passed. The simplest way to display time like this is to either represent each digit in binary or to represent the whole number in binary [bin]. A different approach was taken by the designer of the Mengenlehreuhr (German for "Set Theory Clock"), who used non-equal partitions of time [ber]. Another design, introduced in [Pre16, Pre06], uses a triangular arrangement to display time. Starting from this idea, we will further introduce new geometric arrangements that can be used to indicate time.

Structure of the paper. In Section 2 we present previous binary clock design. Our designs are presented in Section 3, together with the mathematical arguments needed to show that they work.

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2 Previous Designs

2.1 Binary Coded Clocks

The simplest way to display time is to directly encode numbers into binary. Therefore, there are two ways to do that. The first method, shown in Figure 1, encodes each digit into binary. The display is divided into three parts, one for hours, one for minutes and one for seconds. This method works since to represent 2 we need 2 bits, for 9 we need 4 bits and for 5 we need 3 bits. Remark that these numbers suffice to have a 24-hour display.

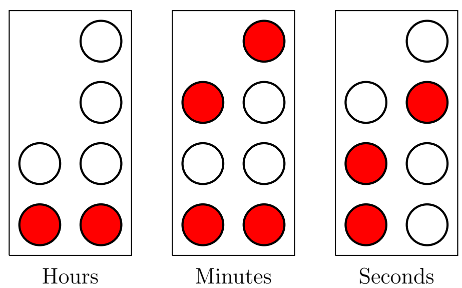

Figure 1: Digit-wise binary clock. The time displayed is 11:59:34.

The second method encodes whole numbers into binary. We can easily see that to encode 24 we need 5 bits and for 60 we need 6 bits. We present a 24-hour display example in Figure 2. Note that we can also display 24 hours using 4 bits to encode 12 hours and two colors, one for AM and one for PM.

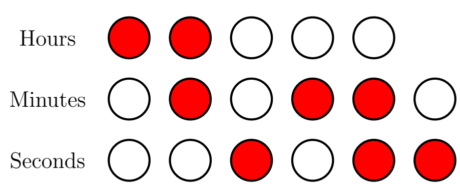

Figure 2: Number-wise binary clock. The time displayed is 03:26:52.

2.2 Mengenlehreuhr

The Berlin clock or the Mengenlehreuhr [ber] was designed in 1975 by Dieter Binninger on behalf of the Berlin Senate. Initially installed in Kurfürstendamm, the clock was decommissioned in 1995 and was later moved to the Europa-Center.

The clock is divided into four rows and the lamps in each row represent the same value (see Figure 3). Each lamp in the first row represents 5 hours, while the ones from

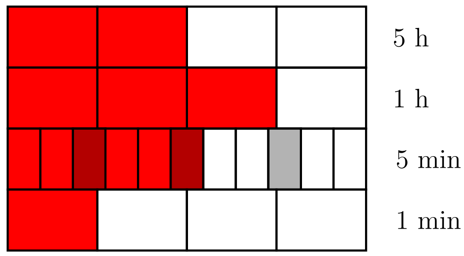

Figure 3: Berlin clock. The time displayed is 13:31:00.

the second row depict 1 hour. The lamps from the third and fourth row mean 5 and 1 minute(s). For better readability, every third lamp from the third row has a different color. The current time can be obtained by adding all the lit lamps.

Using Figure 3 as an example, we obtain

$$\begin{aligned} 2 \times 5 \text{ h} &= 10 \text{ h} \\ 3 \times 1 \text{ h} &= 3 \quad \text{h} \\ 6 \times 5 \text{ min} &= 30 \text{ min} \\ 1 \times 1 \text{ min} &= 1 \quad \text{min} \\ \hline &= 13 \text{ h } 31 \text{ min} \end{aligned}$$

2.3 Triangular Clock

The triangular design was introduced in [Pre06,Pre16] and consists of five rows. The lamps from the first and second row represents 6 and 1 hour(s), while the ones from rows three, four and five depict 30, 6 and 1 minute(s). The clock is designed to display 12 hours, but can be extended to 24 hours by using, for example, red for AM and green for PM.

Using Figure 4 as an example, we obtain

$$\begin{aligned} 0 \times 6 \quad \text{h} &= 0 \text{ h} \\ 1 \times 2 \quad \text{h} &= 2 \text{ h} \\ 2 \times 30 \text{ min} &= 1 \text{ h} \\ 1 \times 6 \quad \text{min} &= 6 \text{ min} \\ 3 \times 1 \quad \text{min} &= 3 \text{ min} \\ \hline &= 3 \text{ h } 9 \text{ min} \end{aligned}$$

To see why this design works, we have to compute the number of lamps needed to obtain a triangular display. Therefore, let the bottom row be the nth row, while the top

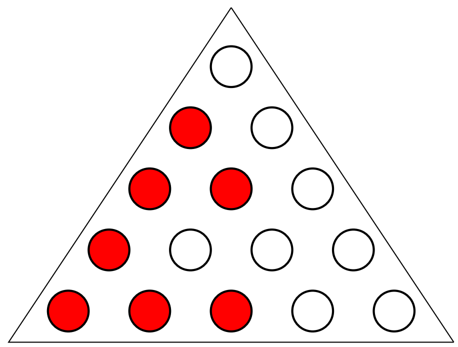

Figure 4: Triangular clock. The time displayed is 03:09:00.

row is the first row. Also, we denote by $\ell(i)$ the number of lamps in the ith row, where $i \leq n$. Note that we always impose the following restrictions[1]

- if a lamp is lit, then the lamp on the left must be lit,
- the lamps from the $i-1$ level represent $\ell(i)+1$ times the value in the ith row.

With this restrictions in place, we can see that the nth row can represent $n+1$ states, the second to last $(n-1)+1$ and so on. Therefore, for n rows we obtain that the total number of states that can be shown is

$$(n+1) \times ((n-1)+1) \times ((n-2)+1) \times \ldots \times (1+1) = (n+1)!$$

For a 12 hour display with a precision of 1 minute we obtain that we need $12 \times 60 = 6!$ states, and thus we can display them in a triangular fashion using six rows.

3 Novel Designs

3.1 Reverse Triangular Clock

The triangular design can also be reversed. We can see that if we start with a single lamp in the bottom row, an n row triangle can represent

$$2 \times 3 \times \cdots \times (n+1) = (n+1)!$$

states. Therefore, if we use a precision of 1 minute we obtain that the lamps from the fifth, fourth, third and second row represent 1, 2, 6 and 24 minute(s). The ones from the first row depict 2 hours.

[1]Note that the same restrictions hold for the Mengenlehreuhr.

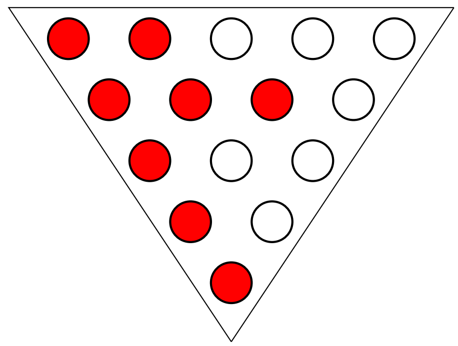

Figure 5: Reverse triangular clock. The time displayed is 05:21:00.

Using Figure 5 as an example, we obtain

$$\begin{aligned} 2 \times 2 \quad \text{h} \quad &= 4 \text{ h} \\ 3 \times 24 \text{ min} &= 1 \text{ h } 12 \text{ min} \\ 1 \times 6 \quad \text{min} &= 6 \text{ h} \\ 1 \times 2 \quad \text{min} &= 2 \text{ min} \\ 1 \times 1 \quad \text{min} &= 1 \text{ min} \\ \hline &= 5 \text{ h } 21 \text{ min} \end{aligned}$$

3.2 Rectangular Clock

Using the Mengenlehreuhr as a starting point, we wanted to design a rectangular clock that evenly divides the total time displayed in a row between the lamps. Let m be the number of rows, u be the unit of time depicted by a lamp in the mth row and we assume that each row contains n lamps. A rectangular designs can depict

$$u \times (n+1) \times (n+1) \times \cdots \times (n+1) = u \times (n+1)^m$$

states. If we want to represent the number of seconds in half a day, we simply write $12 \times 60 \times 60 = 25 \times 12^3$, and thus a unit of 25 seconds suffices for our purpose. Therefore, we can use 3 rows to represent 12 hours. More precisely, the lamps from the first row depict 1 hour, from the second one 5 minutes and the from the last one 25 seconds (see Figure 6).

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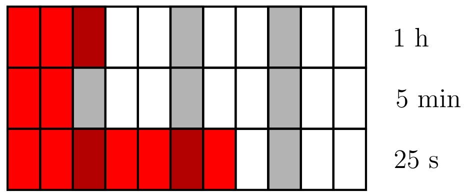

Figure 6: Rectangular clock. The time displayed is 03:12:55.

Using Figure 6 as an example, we obtain

$$\begin{array}{ll} 3 \times 1 \ \text{h} & = 3 \ \text{h} \\ 2 \times 5 \ \text{min} & = 10 \ \text{min} \\ 7 \times 25 \ \text{sec} & = 2 \ \text{min} \ 55 \ \text{sec} \\ \hline & = 3 \ \text{h} \ 12 \ \text{min} \ 55 \ \text{sec} \end{array}$$

3.3 Square Clock

Using square numbers as an inspiration, we can design a clock that is suitable for displaying 12 hours. Therefore, if we start with one lamp that represents u units of time, then the lamps from the second layer will represent $2 \times 1 \times u$ units, the ones from the third layer depict $4 \times 2 \times u$ units, the fourth layer displays $6 \times 4 \times 2 \times u$ units and so on. Hence, for n layers we obtain

$$u \times (2 \times 1) \times (2 \times 2) \times \cdots \times (2 \times n) = u \times 2^n \times n!$$

states. If we want to represent the number of minutes in half a day, we must observe that $12 \times 60 = 15 \times 2^3 \times 3!$. Therefore, 3 layers with a unit of time of 15 minutes suffice to represent 12 hours (see Figure 7). More precisely, the lamp from the first layer depicts 15 minutes. The three lamps from the second layer each represent 30 minutes. The five lamps from the last layer each correspond to 2 hours.

Using Figure 7 as an example, we obtain

$$\begin{array}{ll} 4 \times 2 \ \text{h} & = 8 \ \text{h} \\ 0 \times 30 \ \text{min} & = 0 \ \text{min} \\ 1 \times 15 \ \text{min} & = 15 \ \text{min} \\ \hline & = 8 \ \text{h} \ 15 \ \text{min} \end{array}$$

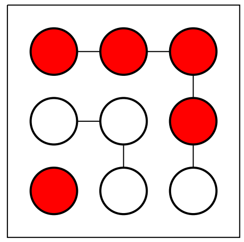

Figure 7: Square clock. The time displayed is 08:15:00.

3.4 Reverse Square Clock

As in the case of the triangular design, we can also reverse the square clock. Again, using 3 layers we have that the lamp from the first layer represents 6 hours (see Figure 8). The three lamps from the second layer each represent 1 hour and 30 minutes. Finally, the five lamps from the last layer each correspond to 15 minutes.

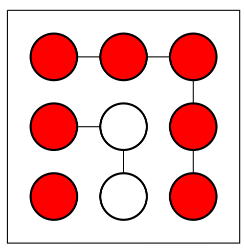

Figure 8: Reverse square clock. The time displayed is 08:45:00.

Using Figure 8 as an example, we obtain

$$\begin{array}{ll} 1 \times 6 \text{ h} & = 6 \text{ h} \\ 1 \times 1 \text{ h } 30 \text{ min} & = 1 \text{ h } 30 \text{ min} \\ 5 \times 15 \text{ min} & = 1 \text{ h } 15 \text{ min} \\ \hline & = 8 \text{ h } 45 \text{ min} \end{array}$$

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