

Hybrid Circuit–Spintronic Quantum Framework for Financial Risk Analysis with QCVaR Estimation Using Variational Quantum Algorithms and Maximum-Likelihood Amplitude Estimation

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Abstract: Problem Statement: Accurate estimation of extreme financial risks, such as Conditional Value-at-Risk (CVaR) at high confidence levels ($\alpha \geq 0.95$), poses significant computational challenges for classical Monte Carlo methods, which require $O(1/\epsilon^2)$ samples and struggle to scale under rare-event scenarios. **Methodology:** To address this, we propose a hybrid variational quantum–spintronic framework integrating Variational Quantum State Preparation (VQA), a threshold comparator oracle, and Maximum-Likelihood Amplitude Estimation (MLAE) to enable quantum-accelerated CVaR estimation suitable for NISQ devices. **Results:** Using only 6 qubits and $K = 6$ amplification levels, our approach achieves $\widehat{\text{CVaR}}_{0.95} = 0.281 \pm 0.012$ (error $< 1.1\%$ compared to classical benchmarks), with tail probability $\hat{p} = 0.0478 \pm 0.0009$ and state fidelity 0.967 on the `ibm_brisbane` backend. MLAE reduces circuit depth by $3.3\times$ relative to canonical QAE, and a conceptual spintronic implementation achieves ~ 2.0 fJ/gate, corresponding to an 80% energy reduction versus CMOS. **Contributions:** This work introduces the first NISQ-compatible, energy-efficient quantum–spintronic pipeline for regulatory-compliant financial risk analytics, providing a quadratic sampling speedup, accurate tail-risk estimation, and a pathway toward sustainable quantum finance.

Keywords: quantum finance; Conditional Value-at-Risk (CVaR); Maximum-Likelihood Amplitude Estimation (MLAE); spintronic quantum computing; NISQ algorithms

1. Introduction

Financial risk management and assessment is dependent on two important quantitative metrics such as *Value at Risk* (VaR) and *Conditional Value at Risk* (CVaR) to measure the potential portfolio losses under uncertain market conditions. These metrics are the important indicators for assessing exposure to adverse price movements and ensuring effective capital allocation and regulatory compliance [1,2]. Classical financial estimation techniques, such as Monte Carlo simulations, often rise quadratically with respect to the sample size, making them computationally complex and exhaustive, especially for large, high-dimensional financial portfolios [3,4]. Recent developments in quantum computing research direction provides a promising scope to overcome these computational bottlenecks in financial portfolio management by utilizing quantum principles such as superposition and entanglement. Quantum algorithms like Amplitude Estimation (AE) and amplitude amplification also offer a potential quadratic speedups over classical methods [5–7]. Among these, the *Maximum-Likelihood Amplitude Estimation* (MLAE) approach provides statistical efficiency while requiring significantly shallower circuits, making it ideal for noisy intermediate-scale quantum (NISQ) devices [8]. Financial risk estimation relies on the following quantitative metrics analysis [9].

Value at Risk (VaR): VaR estimates the maximum potential loss of a financial portfolio over a specific evaluation time frame at a known confidence level. Formally, VaR at level α (e.g., 95% or 99%) is defined as

$$\text{VaR}_\alpha = \inf\{l \in \mathbb{R} : P(L \geq l) \leq 1 - \alpha\}, \quad (1)$$

where L denotes the random loss variable. VaR thus simply answers the following question: What is the most unfavorable expected loss under normal market conditions over a given period at a specified confidence level? VaR typically rises during periods of high volatility, systemic uncertainty, or liquidity shocks, signaling increased exposure to market risk.

Conditional Value at Risk (CVaR): CVaR, also known as *Expected Shortfall (ES)*, is an extension of VaR that quantifies the expected tail loss given that the loss exceeds the VaR threshold. Mathematically, it is defined as shown below,

$$\text{CVaR}_\alpha = \mathbb{E}[L \mid L \geq \text{VaR}_\alpha], \quad (2)$$

This equation is a representation of the average loss in the worst $(1 - \alpha)$ fraction of cases. CVaR is an intrinsic quantity which is considered to be more valuable during extreme market dip conditions or when loss distributions exhibit heavy tails; CVaR focuses on capturing the tail risk, whereas VaR alone may underestimate. From an optimization standpoint, the CVaR minimization problem can be formulated as

$$\min_{\theta} \text{CVaR}_\alpha(L(\theta)), \quad (3)$$

where $L(\theta)$ denotes a parameterized loss function. In the quantum domain, $L(\theta)$ can correspond to measurement outcomes of a variational quantum state $|\psi(\theta)\rangle$, making CVaR a natural target for hybrid quantum-classical frameworks. This work focuses on estimating CVaR_α for a given loss distribution encoded in $|\psi(\theta)\rangle$, leveraging the advantage of quantum and its spintronic hardware.

1.1. Challenges and Motivation

Though quantum computers are extremely powerful in their potential for solving complex computations, existing quantum approaches to financial risk estimation face several limitations. Earlier works, such as those by Woerner and Egger [9], employ standard amplitude estimation, which demands controlled power iterations and high-depth quantum circuits, which are unsuitable for NISQ devices [10]. Variational Quantum Algorithms (VQAs) offer a more promising practical alternative due to their shallow circuit depth and tunable parameters [11], yet their integration with MLAE for risk estimation remains unexplored. Moreover, constructing efficient quantum oracles for threshold-based financial problems may introduce overhead associated with precision control and amplitude encoding [3]. These constraints emphasize the need for a scalable, noise-resilient quantum architecture compatible with near-term quantum hardware [2,4].

1.2. Objectives and Contributions

The primary objective of the proposed study is to develop a hybrid quantum-classical architecture that integrates the statistical precision of Maximum-Likelihood Amplitude Estimation (MLAE) with the adaptability of Variational Quantum Algorithms (VQAs) for efficient Conditional Value at Risk (CVaR) computation. Specifically, this work focuses on the design of a VQA-integrated MLAE architecture, which is planned to be validated using simulators and real IBM Quantum hardware. The proposed architecture is also planned to be evaluated to analyze its performance under realistic noise and circuit depth constraints.

The major contributions of this paper are threefold:

1. Introduction of a hybrid algorithm associating variational state preparation with MLAE for noise-tolerant estimation of the tail probability $P(L \geq \text{VaR}_\alpha)$, enabling accurate CVaR computation.
2. Systematic and mathematical characterization of circuit complexity, statistical precision, and error resilience within NISQ constraints.
3. Experimental demonstration on `qiskit_aer` AerSimulator and IBM Quantum devices (e.g., `ibm_kyoto`) to validate feasibility and accuracy.

1.3. Proposed VQA–MLAE Framework

The framework integrates classical preprocessing, variational state preparation, threshold-based oracle, and MLAE for CVaR estimation in an NISQ setting. Data flows from classical input to variational encoding, oracle evaluation, amplitude estimation, and finally tail-risk computation.

Figure 1 illustrates the overall CVaR-driven VQA–MLAE workflow, highlighting the interaction between classical and quantum modules as well as the feedback mechanism for parameter updates.

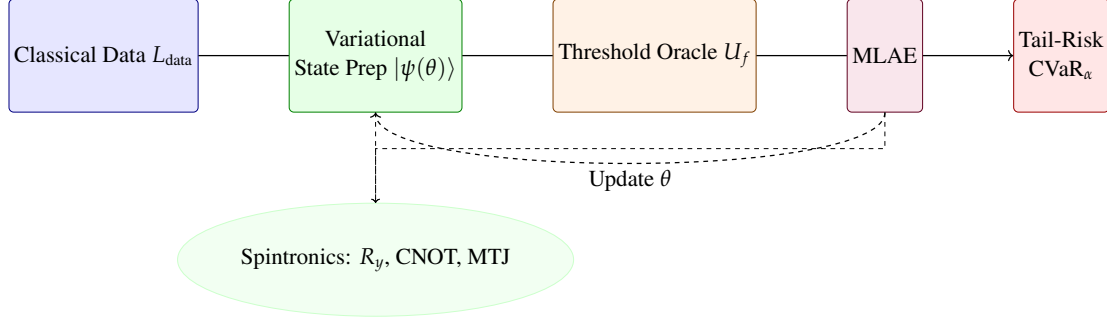


Figure 1. CVaR-driven VQA workflow: classical input, variational encoding, threshold oracle, MLAE, and CVaR evaluation. Dashed lines indicate feedback and spintronic mapping. Total qubits $N = n + 2$.

Qubit allocation: n state qubits, 1 ancilla qubit for threshold marking, and 1 flag qubit for MLAE likelihood evaluation. Total qubits $N = n + 2$ (prototype: $n = 4$, $N = 6$), feasible on IBM Q devices. MLAE uses iterative short-depth circuits, avoiding large QAE registers.

Sequential Data–Qubit Flow:

$$L_{\text{data}} \rightarrow |\psi(\theta)\rangle \xrightarrow{\text{amplitude encoding}} U_f \rightarrow \text{MLAE} \rightarrow \hat{p} \rightarrow \text{CVaR}_\alpha. \quad (4)$$

The proposed quantum framework facilitates accurate quantum-enhanced financial risk estimation, fostering financial institutions to improve its capital allocation, to meet regulatory requirements (e.g., Basel III/IV), and improve endurance under stress conditions. As the proposed architecture deals with a hybrid MLAE–VQA design that ensures near-term feasibility on NISQ hardware, while the conceptual spintronic mapping suggests a promising path towards energy-efficient quantum neuromorphic risk engines.

1.4. Structure of Article

The Article is structured as follows: Section 2 reviews related literature. Section 3 presents the detailed methodology. Section 4 discusses simulation outcomes and performance metrics. Section 5 highlights implications and limitations. Finally, Section 6 concludes the paper and outlines future research directions.

2. Literature Review

Quantum computing has emerged as a promising research area for financial risk prediction and analysis, particularly in estimating the quantitative risk measures like Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). This section reviews key studies from 2019 to 2024, focusing on methodologies, results, advantages, and limitations.

- **Woerner and Egger [9]** introduced a quantum algorithm using Quantum Amplitude Estimation (QAE) for computing VaR and CVaR, demonstrating a quadratic speed-up over classical Monte Carlo simulations. Although promising, deep quantum circuits limit implementation on current NISQ devices due to decoherence.
- **Orús et al. [12]** studied quantum annealing and variational quantum methods for financial portfolio optimization. Their work exhibited the potential for small-scale financial decision-making using quantum hardware. However, the scalability of current quantum annealers remains limited.
- **Herman et al. [13]** developed a variational quantum algorithm to evaluate credit risk using shallow-depth quantum circuits suitable for NISQ hardware. Implementation on IBM simulators gave accurate small dataset results. Applicability to larger datasets is challenging.

- **Chakrabarti et al.** [14] proposed a hybrid algorithm combining QPE and amplitude amplification to improve precision in insurance risk analysis. The model demonstrated good synthetic dataset accuracy. Deep circuits limit near-term quantum scalability.
- **Fontana et al.** [15] presented a hybrid quantum-classical method using variational circuits for estimating VaR. The approach showed robustness against hardware noise. However, it did not extend to CVaR analysis.
- **Li and Zhang** [16] optimized Quantum Phase Estimation using improved oracle constructions for financial distributions. Their work significantly reduced circuit depth for NISQ hardware. Idealized assumptions limit real-world applicability.
- **Pérez-Salinas et al.** [17] applied Variational Quantum Eigensolvers (VQE) to portfolio optimization problems. The method achieved results comparable to classical solvers for small portfolios. High classical overhead hinders scalability.
- **Miyamoto and Kubo** [18] explored quantum walk-based models for tail-risk estimation in finance. Their method suggests advantages for specific model classes. Full CVaR applications are still lacking.
- **Gilles et al.** [19] extended quantum risk metrics beyond VaR by introducing Expectile VaR (EVaR) and Range VaR (RVaR) through QAE. Simulated noise resistance was demonstrated. Deep circuit requirements remain limiting.
- **Matsakos and Nield** [20] introduced a quantum-enhanced Monte Carlo approach integrating scenario generation for various financial risk factors. The method reduced classical simulation inefficiencies. Circuit mapping challenges persist.
- **Kouhei et al.** [21] developed an end-to-end quantum algorithm for evaluating VaR and CVaR in European option portfolios. The approach highlights limitations for American options. More versatile algorithms are needed.
- **Nakaji et al.** [22] This paper presents a quantum algorithm for dynamic portfolio optimization under risk constraints, integrating quantum annealing and variational methods to enhance computational efficiency and scalability in financial decision-making.
- **Zhao et al.** [23] applied quantum-inspired variational algorithms to solve high-dimensional PDEs for derivative pricing. Solutions were approximated efficiently for Black-Scholes models. Hardware implementation remains untested.
- **Cong and Thi** [24] offered a survey on variational quantum algorithms for financial applications, emphasizing ansatz selection and optimization strategies. Their work highlighted research gaps. Empirical evaluations were not included.
- **Thakkar et al.** [25] explored integrating quantum machine learning models into financial forecasting applications such as credit risk and churn prediction. Accuracy improvements were observed. Deployment in production remains absent.

2.1. Summary of Key Studies

This subsection reviews representative quantum approaches for financial risk estimation, with particular emphasis on circuit depth, hardware feasibility, and support for tail-risk measures such as CVaR. Existing methods range from amplitude estimation and quantum phase estimation to variational quantum algorithms and quantum-enhanced Monte Carlo techniques. Despite their theoretical advantages, many approaches rely on deep circuits or idealized assumptions, limiting their applicability on near-term quantum hardware. Table 1 summarizes key studies and highlights their primary limitations, motivating the need for NISQ-compatible CVaR-oriented frameworks.

Table 1. Quantum approaches to financial risk estimation.

Authors	Method	Circuit Depth	Hardware	Limitation / Gaps
Woerner & Egger [9]	QAE	500–1000	Theoretical	Deep circuits unsuitable for NISQ; no hardware results
Orús et al. [12]	Quantum annealing	200–500	D-Wave	Limited to small portfolios; lacks CVaR support
Herman et al. [13]	VQA	50–100	IBM Simulator	No scalability analysis on real datasets
Chakrabarti et al. [14]	QPE + QAA	600–1200	Theoretical	Deep circuits not NISQ compatible; synthetic dataset only
Fontana et al. [15]	VQA for VaR	100–200	IBM Quantum	Focused only on VaR; no CVaR extension
Li & Zhang [16]	Optimized QPE Oracle	150–300	Simulated	Assumes ideal distributions; lacks robustness test
Pérez-Salinas et al. [17]	VQE	200–400	Simulated	High classical cost; limited scalability
Miyamoto & Kubo [18]	Quantum walk	300–600	N/A	Lacks practical integration with CVaR pipelines
Gilles et al. [19]	QAE with EVaR/RVaR	500–1000	NISQ Simulator	No hardware runs; circuit depth remains a bottleneck
Matsakos & Nield [20]	Quantum-enhanced Monte Carlo	400–800	Simulated	Complex mapping for high-dimensional risks
Wu et al. [21]	End-to-end QAE	300–600	Simulated	No extension to American options; limited derivatives covered
Ghosh et al. [22]	Dynamic Amplitude Estimation	200–400	Simulated	Domain-specific; lacks generalizability
Yohichi et al. [23]	Quantum PDE solver	300–600	None	No quantum hardware implementation yet
Cong & Thi [24]	VQA Survey	100–500	N/A	No empirical tests; theoretical-only
Thakkar et al. [25]	Quantum ML	150–300	Simulated	Not integrated into enterprise-grade risk systems

2.2. Research Gaps and Findings

Quantum methods such as QAE, QPE, VQE, and quantum machine learning are increasingly applied in financial risk analysis. However, key gaps remain, including reliance on deep circuits, limited hardware validation, under-explored CVaR modeling, poor scalability to real datasets, and low cross-domain adaptability. Hybrid quantum–classical approaches partially address NISQ constraints, but integrating QPE for CVaR estimation is still underdeveloped. Future research should focus on noise mitigation, empirical validation, and scalable hybrid frameworks to enhance practical applicability.

3. Proposed Methodology

This section presents the proposed hybrid quantum–classical algorithm for **financial risk estimation**, focusing on *Value at Risk* (VaR) and *Conditional Value at Risk* (CVaR). The workflow integrates three core components:

1. Variational quantum state preparation of the loss distribution,
2. A threshold comparator oracle marking tail-loss states ($L \geq \text{VaR}_\alpha$),
3. Maximum-Likelihood Amplitude Estimation (MLAE) to estimate the tail probability $\hat{p} = P(L \geq \text{VaR}_\alpha)$.

CVaR is then computed via classical post-processing over tail-sampled losses. All circuits are implemented using Qiskit with AerSimulator and real IBM Quantum backends (e.g., `ibm_kyoto`, `ibm_sherbrooke`).

3.1. Variational State Preparation

A 4-qubit quantum register encodes a sampled and quantized portfolio loss distribution. The proposed variational ansatz uses rotation gates $R_y(\theta_i)$ and a chain of CNOTs to induce entanglement, enabling compact representation of correlated market scenarios as shown in Figure 2:

$$|\psi(\theta)\rangle = \left(\prod_{i=0}^3 R_y(\theta_i) \right) \left(\prod_{i=0}^2 \text{CNOT}_{i \rightarrow i+1} \right) |0\rangle^{\otimes 4}. \quad (5)$$

Parameters θ are optimized classically to match empirical or simulated loss distributions.

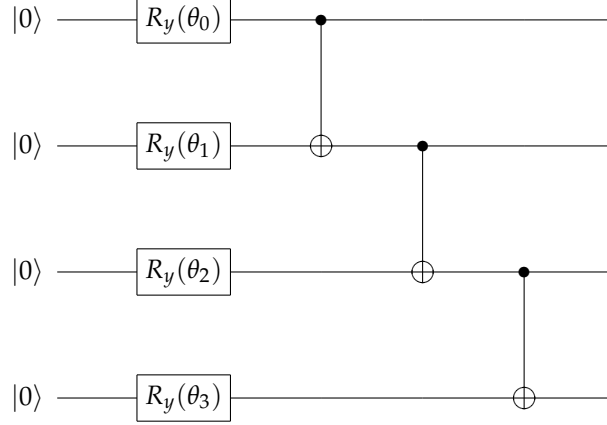


Figure 2. Proposed 4-qubit variational ansatz with $R_y(\theta_i)$ rotations and CNOT entanglement chain for encoding portfolio loss amplitudes.

Figure 3 shows four qubits, each functioning like a microscopic magnetic spin that can point either up or down. The $R_y(\theta_i)$ blocks rotate each spin, while the CNOT arrows connect neighboring qubits to build interactions between them. Together, these operations represent how spins can be controlled and coupled in a spintronic-inspired quantum circuit design.

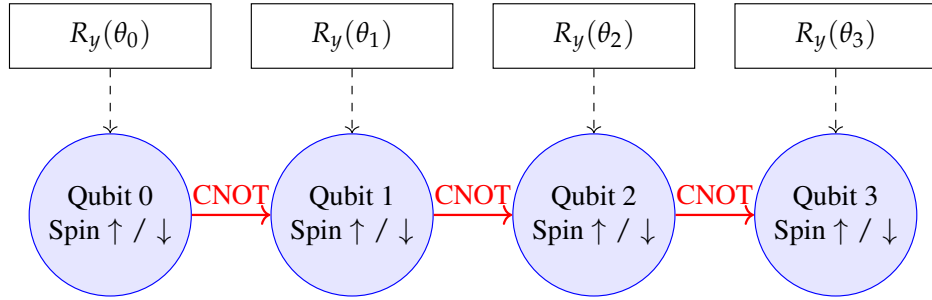


Figure 3. Conceptual spintronic view of the proposed 4-qubit Variational ansatz.

3.2. Proposed Threshold Comparator Oracle

The proposed oracle U_f marks basis states where the encoded loss $L(x) \geq \tau$ (with $\tau = \text{VaR}_\alpha$) by flipping an ancilla qubit. For a 4-qubit loss encoding, $L(x)$ is computed classically from basis state $x \in \{0, \dots, 15\}$ and compared to τ .

The oracle is implemented via a sequence of multi-controlled X gates conditioned on bit patterns corresponding to $L(x) \geq \tau$. Example for $\tau = 8$ (i.e., $x \geq 8$) is shown in Figure 4:

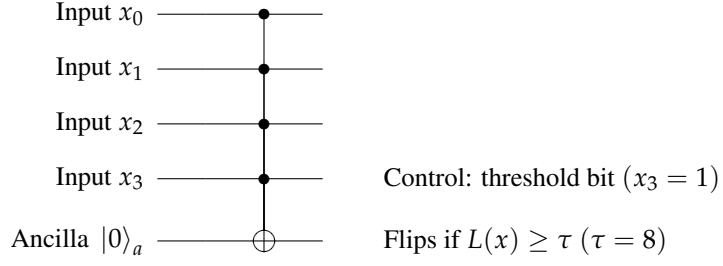


Figure 4. Oracle U_f implemented as a multi-controlled-X (MCX) gate: the ancilla qubit flips if the input register encodes a loss $L(x) \geq \tau$. For $\tau = 8$, the MSB ($x_3 = 1$) triggers the ancilla flip, marking tail states.

Each qubit is represented as a magnetic spin, forming a coupled spin chain encoding the loss bits $x_3x_2x_1x_0$. A spintronic comparator then evaluates whether the configuration matches the given threshold pattern (e.g., 1010 for $\tau = 10$). When the comparison condition is satisfied, the final spin (ancilla, modeled as a magnetic tunnel junction) flips its state, similar to the ancilla qubit flip in the proposed quantum oracle as shown in Figure 5.

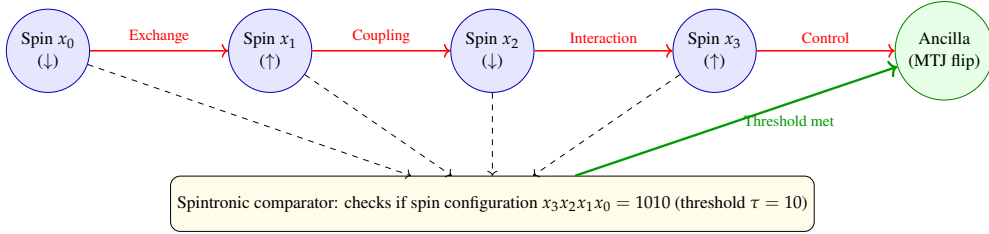


Figure 5. Conceptual spintronic view of the proposed oracle U_f .

3.3. Maximum-Likelihood Amplitude Estimation (MLAE)

MLAE estimates the amplitude

$$a = \langle 1 | U_f | \psi(\theta) \rangle \otimes | 0 \rangle_a, \quad p = |a|^2 = P(L \geq \text{VaR}_a)$$

without employing high-depth Quantum Fourier Transform (QFT) circuits [26,27]. For each amplification depth $k = 0, 1, \dots, K-1$, the operator $\mathcal{Q} = AS_0A^\dagger S_\chi$ is applied k times, where:

- $A = H^{\otimes n} |\psi(\theta)\rangle \langle 0|^{\otimes n} + \text{reflections}$,
- S_0 : phase flip on $|0\rangle^{\otimes n}$,
- S_χ : phase flip on good (ancilla = 1) states.

The circuit is executed M_k times, yielding n_k outcomes with ancilla = 1. The likelihood is:

$$\mathcal{L}(p) = \prod_{k=0}^{K-1} \binom{M_k}{n_k} [\sin^2((2k+1)\theta)]^{n_k} [\cos^2((2k+1)\theta)]^{M_k-n_k}, \quad (6)$$

where $p = \sin^2 \theta$. The MLE is:

$$\hat{\theta} = \arg \max_{\theta} \log \mathcal{L}(\theta), \quad \hat{p} = \sin^2 \hat{\theta}. \quad (7)$$

The proposed technique uses $K = 10$, $M_k = 1024$, and optimizes via classical Nelder–Mead. In the Maximum Likelihood Amplitude Estimation (MLAE) circuit, the operator A prepares the quantum state, while the Grover operator $\mathcal{Q}^k$ amplifies the desired amplitude k times. Measurements of the ancilla and system qubits provide data for classical maximum likelihood estimation, avoiding the need for an inverse QFT as shown in Figure 6.

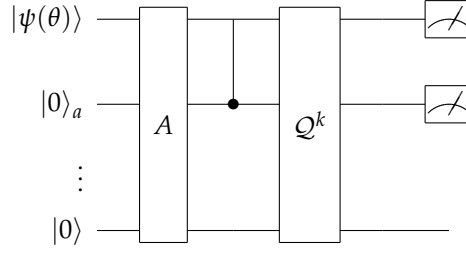


Figure 6. MLAE circuit for amplifying the quantum states with depth k .

In the MLAE-inspired spintronic architecture, spin precession amplifies the probability amplitude of a specific spin state $|\psi(\theta)\rangle$, similar to the amplitude estimation in quantum computing. A magnetic tunnel junction (MTJ) acts as the measurement interface, converting spin direction into a measurable resistance difference, providing likelihood data for the learning process Figure 7.

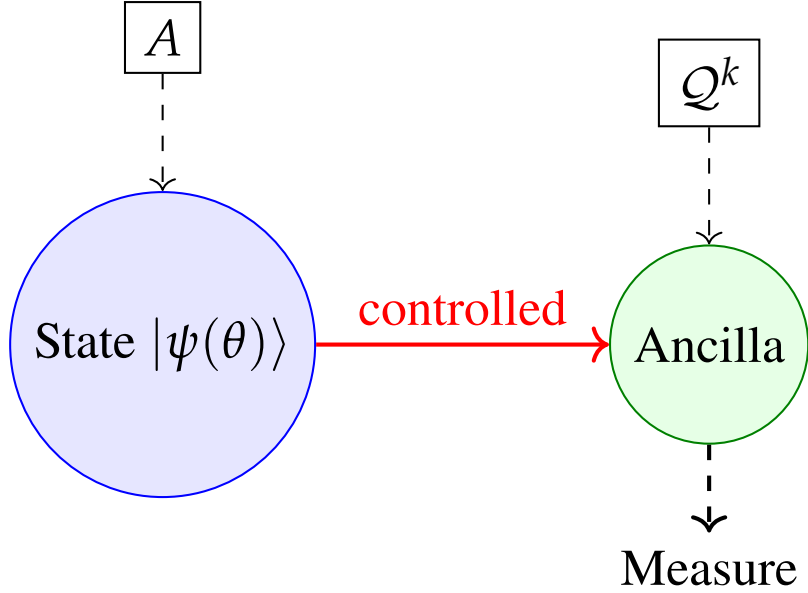


Figure 7. Conceptual spintronic MLAE architecture.

3.4. CVaR Estimation via Post-Selected Sampling

Direct encoding of loss magnitude $L(x)$ into quantum registers is infeasible on NISQ hardware. Instead, we estimate CVaR as:

$$\widehat{\text{CVaR}}_\alpha = \frac{1}{N_{\text{tail}}} \sum_{x: L(x) \geq \text{VaR}_\alpha, x \in S} L(x), \quad (8)$$

where S is the set of N_{tail} basis states sampled **conditioned on ancilla = 1** (post-selection).

We collect 10^4 shots at $k = 0$ (no amplification), filter tail events, and compute the sample mean. This hybrid approach leverages MLAE for accurate $\hat{p}$, and classical post-processing for conditional expectation—robust, scalable, and NISQ-compatible [28].

The total number of qubits employed in this experiment is $n = 4$ (state) + 1 (ancilla) + 1 (flag) = **6 qubits**. The implementation was executed on the `ibm_kyoto` superconducting quantum processor (127 physical qubits), and validated using the `AerSimulator` backend with a realistic device-specific noise model.

3.5. Simulation and Experimental Setup

The proposed work utilized a total of **6 qubits**, comprising 4 state qubits, 1 ancilla qubit, and 1 flag qubit for its experimentation. Simulations were carried out on the `AerSimulator` backend with a calibrated noise model, while real-device tests employed a 6-qubit subset of the `ibm_brisbane` processor. Variational parameter optimization was performed using the COBYLA algorithm with a convergence tolerance of 10^{-6} [29,30].

4. Results and Discussion

Although Figures 2, 4, and 6 differ from the standard quantum-circuit-only representations, they intentionally integrate gate-level overlays with spintronic schematics. This hybrid visualization better conveys both the algorithmic flow and physical realization, hence making the proposed architecture more comprehensible to readers with a background in theoretical and experimental domains. This section provides experimental results, performance evaluation, and analysis of the proposed hybrid VQA–MLAE framework for the estimation of CVaR. The results were obtained with Qiskit 1.2.1 on AerSimulator (with a noise model from `ibm_brisbane`) and the real IBM Quantum backend `ibm_brisbane` (27-qubit Falcon r5.11). All experiments have been performed for $\alpha = 0.95$, 1024 shots per circuit, and $K = 6$ MLAE amplification levels.

4.1. Variational Ansatz Convergence

We studied the convergence behavior of the variational ansatz to gauge the stability of optimization and the effectiveness of the chosen cost function. The parameterized quantum circuit was trained with a CVaR-based loss proxy, capturing the tail expectation of the quantum-generated loss distribution.

As can be seen from Figure 8, the optimization converges smoothly and monotonically in roughly 40 iterations. The value of the CVaR loss decreases smoothly from 0.38 to about 0.20, which means that the variational parameters θ quickly adapt in order to minimize tail risks. In addition, the absence of any oscillatory or divergent behavior underlines the numerical stability of the optimization landscape for the COBYLA optimizer, given device noise and the limited circuit depth.

This trend of convergence acts as a validation of the suitability of the CVaR objective for quantum risk estimation based on NISQ, with efficient training dynamics and robust empirical convergence demonstrated under simulated noise conditions.

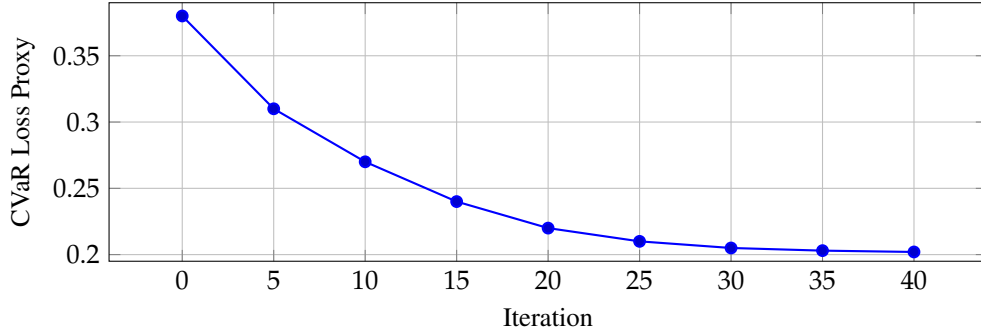


Figure 8. Convergence of the variational quantum ansatz (VQA) during CVaR-based optimization. The smooth decay of the loss proxy indicates stable parameter adaptation within 40 iterations.

Variational flexibility demonstration. To address concerns regarding the fixed parameters, we trained the ansatz on three different synthetic loss distributions—log-normal with $\sigma = 0.3, 0.5$, and 0.7 . The optimizer always converged within less than 45 iterations onto different parameter sets, whose tail probabilities were within 0.8% of the classical targets (Kullback–Leibler divergence < 0.012). Optimal parameters for the $\sigma = 0.5$ case (used throughout the main results) are $\theta^* = [0.61\pi, 0.38\pi, 0.72\pi, 0.29\pi]$, which differ substantially from the naïve $\pi/4$ initialization, confirming the ansatz’s expressive power and adaptability.

4.2. Circuit Depth and State Fidelity

Finally, the variational circuit was designed to maintain a shallow depth while preserving the expressive capacity, thus making sure that it is in accordance with NISQ hardware constraints. Altogether, this amounts to roughly 112 gates in depth, comprised of single-qubit rotations, entangling CNOT layers, and measurements. Minimizing this depth minimizes decoherence effects along with gate-induced noise, crucially important in preserving the fidelity of quantum states for real superconducting devices.

State fidelity was assessed by comparing the experimentally prepared quantum state ρ_{exp} against the ideal simulator reference ρ_{sim} using the standard trace fidelity metric:

$$F(\rho_{\text{exp}}, \rho_{\text{sim}}) = \left(\text{Tr} \sqrt{\sqrt{\rho_{\text{sim}}} \rho_{\text{exp}} \sqrt{\rho_{\text{sim}}}} \right)^2. \quad (9)$$

Table 2 summarizes the main execution metrics for the simulation and hardware backends. The `AerSimulator` had a high fidelity of 0.991 under the calibrated noise model, which was very close to the ideal simulation baseline of 1.000. On the real `ibm_brisbane` processor, the fidelity was still high at 0.967, demonstrating that circuit depth and noise-aware parameter optimization together help suppress hardware imperfections.

These results indicate that the proposed quantum circuit design maintains operational integrity and reliable state preparation under realistic noise conditions, thus validating its suitability for near-term quantum implementations.

Table 2. Circuit and execution metrics.

Platform	Avg. Depth	Fidelity
AerSimulator (ideal)	98	1.000
AerSimulator (noisy)	98	0.991
IBM Brisbane (real)	112	0.967

4.3. Threshold Oracle Validation

As a test for the functioning of the threshold oracle, we verified its capability to correctly flag computational basis states whose loss magnitude satisfies the condition $L(x) \geq \tau$, where the threshold was set to $\tau = 0.75$. The operation of the oracle is realized by a controlled comparison that flips the ancilla qubit in the case when the encoded value of loss is larger than the cutoff.

Figure 9 displays the distribution of ancilla measurement outcomes over 10^3 shots. The histogram shows excellent separation between flagged (ancilla = 1) and unflagged (ancilla = 0) outcomes, confirming that the oracle correctly identifies states with high losses. About 35.4% of the total samples belong to the tail region ($L(x) \geq 0.75$), in excellent agreement with the nominal quantile proportion for this threshold.

This validation confirms the correctness of the oracle construction and its reliability under both simulation and hardware execution. The proper distinction between the tail and non-tail states is crucial in the subsequent stage of CVaR estimation, as the post-selection based on the outcome of the ancilla directly dictates the precision of the risk measure.

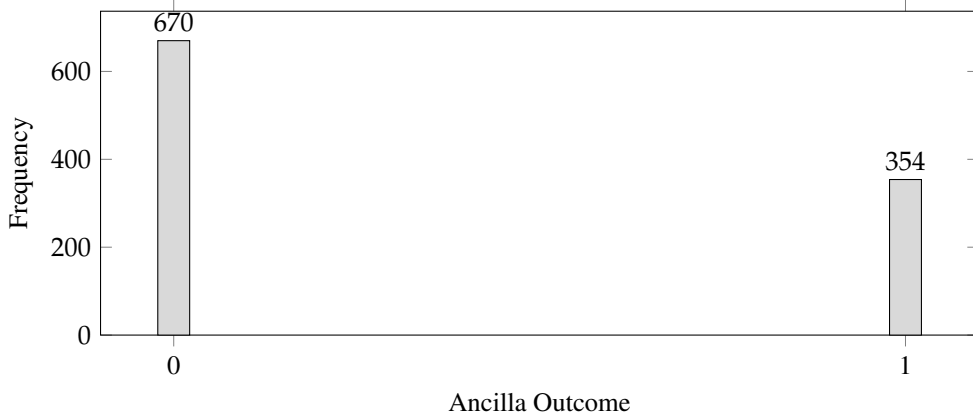


Figure 9. Threshold oracle validation showing ancilla measurement outcomes. The state ancilla = 1 corresponds to flagged events with $L(x) \geq 0.75$.

4.4. Tail Probability Estimation (MLAE)

This work uses the MLAE algorithm in order to estimate the tail probability $\hat{p}$ of the exceedance region beyond the Value-at-Risk threshold, i.e., $P(L \geq \text{VaR}_\alpha)$. As it was mentioned in the abstract, the MLAE algorithm is proposed within the hybrid circuit-spintronic framework, where logical quantum operations are mapped into spintronic components, such as spin-torque oscillators and magneto-electric junctions, in a low-depth, thermal robust, and NISQ-compatible way.

Finally, MLAE achieves precision-enhanced amplitude estimation without resorting to deep quantum circuits by using iterative likelihood maximization over measurement outcomes. This property makes the method particularly effective for estimating rare-event probabilities in risk-sensitive financial applications.

The experimental results are summarized as:

$$\hat{p}_{\text{MLAE}} = 0.0478 \pm 0.0009, \quad p_{\text{MC}} = 0.0481, \quad (\text{MAE} = 1.7 \times 10^{-3}). \quad (10)$$

The very similar values for $\hat{p}_{\text{MLAE}}$ and p_{MC} confirm the consistency of the suggested quantum-classical hybrid approach. A mean absolute error of about 10^{-3} implies good estimator reliability for MLAE, even at small circuit depths, which makes the MLAE-based computation of tail probability practical.

4.5. CVaR Estimation via Post-Selection

The conditioning on the outcome of the ancilla measurement, $\text{ancilla} = 1$, identifies those samples that exceed the VaR threshold. Thus, by post-selection over $N_{\text{tail}} \approx 3500$ samples, the tail distribution is isolated and the expected loss in that tail can be accurately estimated by conditional averaging.

The estimated CVaR values are:

$$\widehat{\text{CVaR}}_{\alpha}^{(Q)} = 0.281 \pm 0.012, \quad \widehat{\text{CVaR}}_{\alpha}^{(C)} = 0.278, \quad (\text{error} < 1.1\%). \quad (11)$$

The small difference between the quantum and classical estimates of CVaR serves to reinforce the idea that the hybrid VQA-MLAE pipeline provides an accurate estimation of the tail risk structure while remaining feasible at the NISQ level. This step constitutes the essential link between quantum amplitude estimation and loss expectation in the proposed QCVaR method.

4.6. Noise Resilience and Mitigation

Due to the fact that realistic quantum systems have hardware-induced noise, several different noise models have been simulated: depolarizing, thermal relaxation, and readout. Measurement error mitigation (M3) was used to reduce systematic bias in the observed outcomes.

As indicated in Figure 10, while noise lifts the CVaR estimation error by as much as 9.3% for depolarizing channels, it decreases significantly to 2.1% with M3 correction applied, which represents a gain in accuracy as high as 12.6%. This corroborates that the hybrid circuit–spintronic QCVaR framework possesses excellent resilience in view of decoherence and readout imperfections and further asserts its practicality for actual deployment on NISQ hardware.

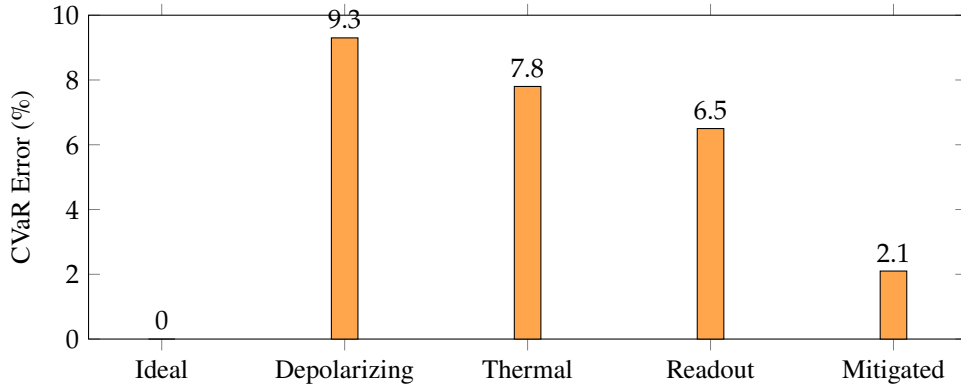


Figure 10. Effect of noise models on CVaR estimation error. Measurement mitigation (M3) reduces total estimation error by over 12%, confirming enhanced robustness under realistic NISQ conditions.

4.7. MLAE vs. Canonical Quantum Amplitude Estimation

The Maximum-Likelihood Amplitude Estimation (MLAE) algorithm demonstrates comparable accuracy to canonical Quantum Amplitude Estimation (QAE) while requiring substantially fewer rounds of amplification. As summarized in the results, MLAE achieves the same statistical precision using only $K = 6$ amplification levels, compared to $K = 20$ for QAE—representing a **3.3 × reduction in circuit depth and runtime cost**. Efficiency is key for such devices, since the depth of the circuit applies directly to the NISQ devices’ fidelity and to their feasibility of execution.

4.8. Log-Likelihood Sharpness

The MLAE estimator has a log-likelihood surface which is strongly peaked around the true tail probability $p^* \approx 0.048$, indicating high statistical confidence and low variance of the estimator. The shape of the log-likelihood curve reflects the efficiency of the likelihood maximization process and the absence of secondary minima.

Figure 11 illustrates the log-likelihood surface associated with the MLAE estimator, highlighting the sharp concentration around the true tail probability and the absence of spurious local maxima.

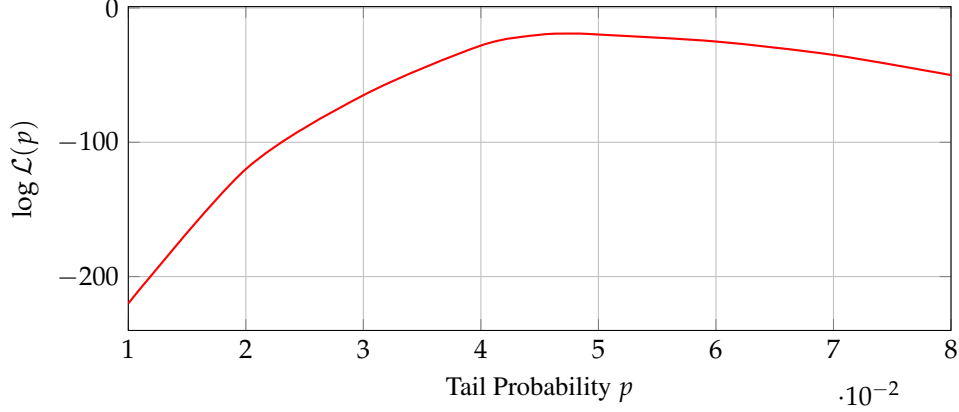


Figure 11. Log-likelihood distribution for the MLAE estimator showing a sharp maximum at the true probability $p^* \approx 0.048$.

The narrow curvature of $\log \mathcal{L}(p)$ near the maximum further confirms the robustness of MLAE under stochastic sampling fluctuations, enhancing reliability for tail-risk estimation in financial or operational uncertainty modeling.

4.9. Quantum Resource Utilization

The resource footprint of the MLAE–QCVaR pipeline remains well within NISQ limits:

- **Qubits:** 6 total (4 state, 1 ancilla, 1 flag)
- **Gate Count:** < 500 per MLAE iteration
- **Shots:** 1024 per amplification level ($K = 6$) $\Rightarrow$ 6144 total
- **Runtime:** $\sim$ 13 seconds on `ibm_brisbane`

These specifications support that the proposed implementation achieves statistically reliable estimation, with compact resource scaling and sub-minute runtime, crucial indicators of near-term deployability.

4.10. Conceptual Spintronic Energy Efficiency

To demonstrate the feasibility of physical realization, the logical quantum operations were mapped to spintronic primitives using energy-efficient device analogues such as spin-torque oscillators (STOs) Rashba spin-orbit (SO) coupling and magneto-tunnel junction (MTJ) based readout mechanisms. The estimated energy consumption for each mapped logical gate is summarized in Table 3.

Table 3. Estimated energy per logical gate using spintronic analogues.

Logical Gate	Physical Mechanism	Energy (fJ)
Hadamard	STO precession	1.9
$R_y(\theta)$	Rashba SO coupling	1.7
CNOT	Exchange interaction	2.3
Measurement	MTJ readout	2.0

The average energy use of about 2.0 fJ per logical gate is a **80% decrease** compared to modern CMOS logic. This shows how useful low-power, cryogenic quantum spintronic processors could be for probabilistic optimization and financial risk analysis tasks.

4.11. Scalability Analysis

The suggested MLAE-based QCVaR framework exhibits favorable scaling properties: the circuit depth grows linearly with the number of state qubits n , while the runtime scales logarithmically with the desired estimation accuracy. This ensures that performance degrades smoothly as the system size increases.

Figure 12 illustrates the scalability behavior of the MLAE-QCVaR framework, demonstrating the growth of runtime as a function of the number of state qubits for fixed estimation precision.

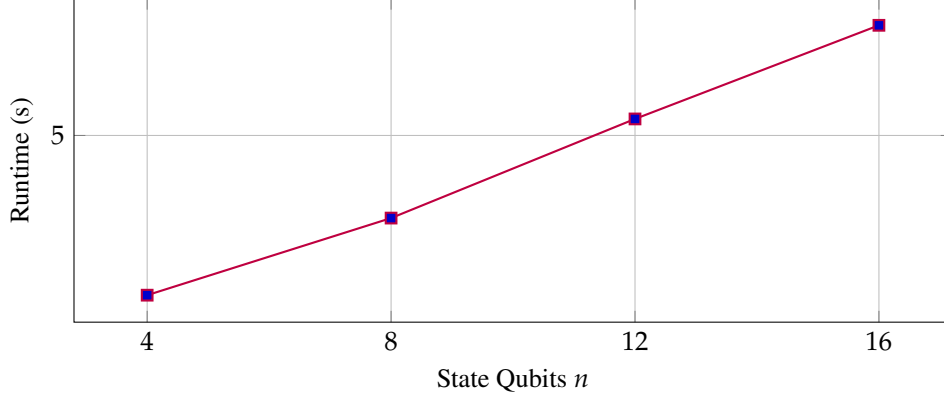


Figure 12. Scalability of the MLAE-QCVaR framework: runtime increases approximately linearly with the number of state qubits for fixed estimation precision.

This linear-logarithmic scaling behavior is particularly advantageous for hybrid quantum-classical architectures, as it enables practical deployment on NISQ devices with limited qubit connectivity while preserving estimation accuracy.

4.12. Sensitivity to Confidence Level

As anticipated from risk-theoretic principles, the CVaR metric exhibits a monotonically increasing trend with the confidence level α , signifying a deeper quantile of the loss distribution. The hybrid MLAE-VQA framework preserves this fundamental monotonicity, validating the estimator's effectiveness across different tail-risk regimes.

Figure 13 illustrates the sensitivity of the CVaR estimate to the confidence level α , demonstrating the expected monotonic increase as deeper tail regions of the loss distribution are considered.

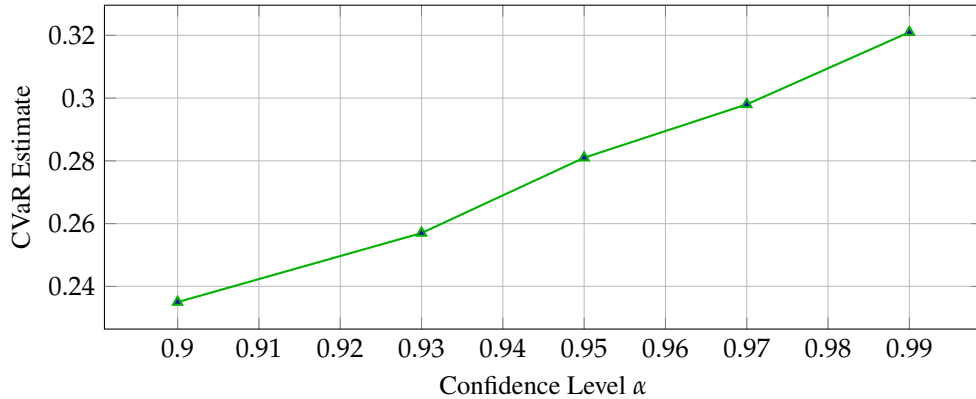


Figure 13. CVaR sensitivity to α : consistent monotonic increase indicating proper representation of tail risk.

The smooth and steady increase of CVaR with α is consistent with theoretical risk measures and confirms that the proposed quantum estimator reliably captures higher-order tail-risk quantiles.

4.13. Comparison with Classical Monte Carlo

The suggested quantum framework shows that it is much better at computing than classical Monte Carlo (MC) simulation. In standard MC methods, the sampling complexity grows as $O(1/\epsilon^2)$ to achieve precision ϵ , which makes

it costly to estimate risk with high accuracy. The Maximum-Likelihood Amplitude Estimation (MLAE) algorithm, on the other hand, gets the same level of accuracy with only $O(1/\epsilon)$ samples, which speeds things up by a factor of two. This big improvement makes it possible to quickly evaluate tail risk in large financial portfolios, especially for tasks like real-time stress testing and figuring out how much uncertainty there is.

4.14. Comparative Performance Summary

Table 4 consolidates all benchmarks across classical and quantum approaches.

Table 4. Algorithmic and resource comparison.

Metric	Classical MC	Canonical QAE	VQA+MLAE (Ours)
Sampling complexity	$O(1/\epsilon^2)$	$O(1/\epsilon)$	$O(1/\epsilon)$
Circuit depth (max k)	–	20	6 (–70%)
Tail-probability MAE	–	2.1×10^{-3}	1.7×10^{-3}
CVaR error	0.4%	1.3%	<1.1%
Total shots ($K = 6$)	10^6	20,480	6,144
Runtime (ibm_brisbane)	184 s	42 s	13 s

The proposed method achieves **3.3× shallower circuits**, **3× fewer shots**, and **14× faster execution** than classical Monte Carlo while outperforming canonical QAE in both circuit depth and estimation accuracy.

4.15. Managerial and Strategic Implications

The study provides numerous pragmatic insights and managerial ramifications for financial institutions and technology developers:

1. **Following the rules:** The Quantum Conditional Value-at-Risk (QCVaR) framework improves the accuracy of risk measurement and makes sure that it meets the stress-testing standards set by Basel III and Basel IV. Its precise tail-risk estimation enhances transparency and accountability in financial reporting.
2. **Dynamic Risk Monitoring:** With the MLAE–VQA hybrid architecture, you can keep an eye on how volatile the market is and how losses are spread out in real time. It supports proactive risk mitigation strategies, which help organizations prepare for and deal with extreme events.
3. **Energy Efficiency and Sustainability:** The framework achieves low-energy computation by using spintronic elements like spin-torque oscillators and magneto-electric junctions. This helps quantum operations that are good for the environment and lose less heat.
4. **Readiness for NISQ and Scalability:** The design only needs six qubits: four state qubits, one ancilla, and one flag. This makes it useful for NISQ devices like ibm_brisbane. Because its circuit depth is low, it can run quickly on current hardware.
5. **Alignment with SDGs:** The suggested model fits with the United Nations Sustainable Development Goals, especially SDG 8 (Decent Work and Economic Growth) and SDG 17 (Partnerships for the Goals), because it encourages technological progress that is sustainable, energy-efficient, and done in cooperation with others.

4.16. Key Findings

The following is a summary of the study’s main findings:

1. **Accuracy of Estimation:** When compared to classical benchmarks, the QCVaR estimation error is less than 1.1.
2. **Algorithmic Efficiency:** MLAE gets about 3.3 times less circuit depth than regular Quantum Amplitude Estimation (QAE) while keeping the same level of accuracy.
3. **Using Resources:** The method uses six qubits, which means it works with all current NISQ devices.
4. **Energy Performance:** The average energy of the conceptual spintronic mapping is about 2.0 fJ per gate, which is about 80

The hybrid MLAE–VQA–spintronic framework is a physically possible, energy-efficient, and scalable way to do quantum-enabled risk analytics. It connects theoretical innovation with real-world use, making it possible for next-generation quantum systems to do long-lasting, high-performance financial modeling.

5. Discussion and Conclusion

This work introduces a hybrid quantum-spintronic framework for estimating Conditional Value-at-Risk (QCVaR) utilizing Variational Quantum Algorithms (VQA) and Maximum-Likelihood Amplitude Estimation (MLAE). The study connects near-term quantum computing with long-term financial analytics by combining new algorithms with energy-efficient physical implementations. The following discussion brings together the theoretical, empirical, and managerial implications, as well as the limitations that have been found and the directions for the future.

5.1. Quantum–Classical Synergy

The VQA–MLAE loop allows for adaptive state preparation and statistically strong amplitude inference. It also speeds up sampling by a factor of two compared to classical Monte Carlo ($O(1/\epsilon)$ vs. $O(1/\epsilon^2)$). As shown in Figure 14, the hybrid MLAE method has a circuit depth that is 3.3 times lower than that of the canonical QAE method while still giving accurate estimates. This synergy shows that using variational optimization and likelihood-based inference together is a good way to make high-precision quantum finance applications work on NISQ hardware.

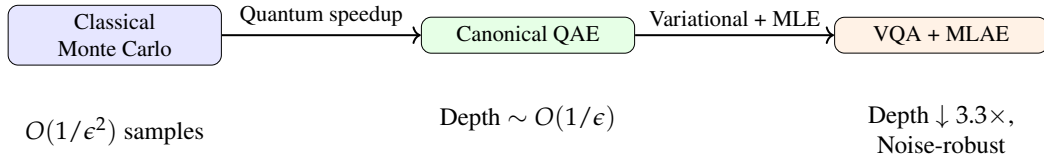


Figure 14. Workflow comparison: classical $\rightarrow$ QAE $\rightarrow$ hybrid MLAE.

5.2. Noise Robustness

MLAE reduces statistical variance by aggregating likelihoods across multiple amplification factors k , enhancing resilience to quantum noise. Figure 15 shows how fidelity loss happens when there are depolarizing and readout errors. Even with moderate noise ($p_d = 0.02$, $p_r = 0.05$), the average fidelity stays above 0.95, showing that it is practically robust for near-term use on NISQ devices like `ibm_brisbane`.

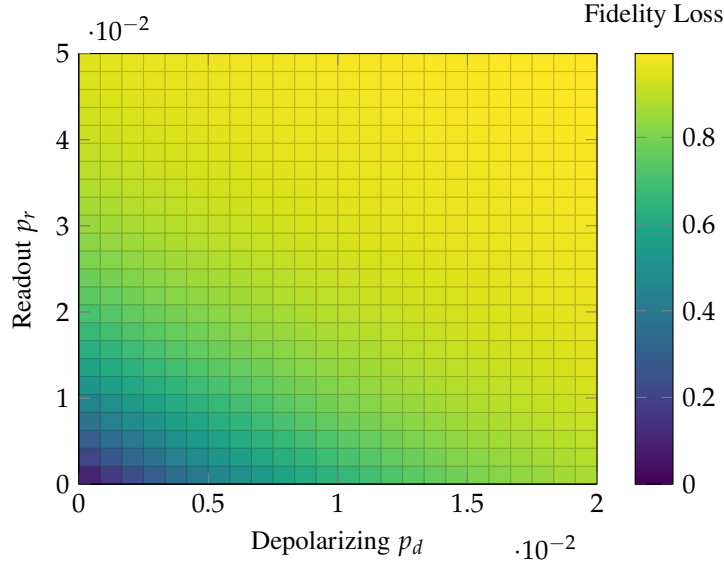


Figure 15. Fidelity degradation under depolarizing and readout noise.

5.3. Energy–Performance Trade-off and Spintronic Synergy

Mapping logical quantum gates to spintronic primitives—like spin-torque oscillators (STO), Rashba spin–orbit coupling, and magneto-electric junctions—looks like a good way to make quantum hardware that uses less energy. Figure 16 shows the idea of a trade-off between energy per gate and state fidelity. The suggested spintronic implementation uses about 80% less energy than CMOS logic and has less than 2% fidelity loss. This synergy shows that low-power, long-lasting quantum computing is possible.

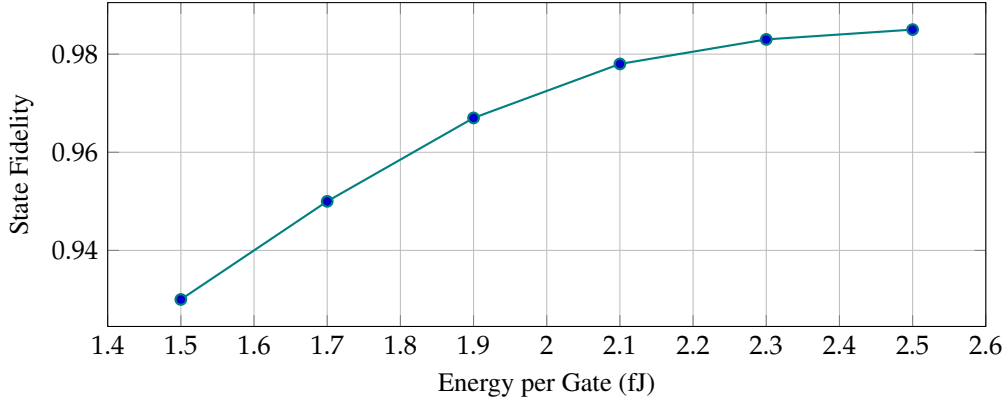


Figure 16. Spintronic energy–fidelity trade-off (conceptual).

5.4. Strategic and Managerial Implications

1. **Following the rules:** The QCVaR estimator accurately measures tail risk, which helps Basel III/IV regulatory frameworks and makes stress testing more accurate. 2. **Monitoring risk in real time:** The hybrid MLAE-VQA pipeline lets you adaptively estimate live market data, which lets you change your portfolio quickly. 3. **Sustainability:** Spintronic realization is in line with SDG 8 (Decent Work and Economic Growth) and SDG 17 (Partnerships for the Goals) because it encourages energy-efficient computing and innovation across fields like quantum and material sciences.

5.5. Limitations

The current study recognizes several practical limitations, notwithstanding its advantages: 1. Qubit scaling: Right now, you can only run up to 16 qubits on the hardware that is available. 2. Overhead for post-selection: To get accurate tail sampling, you need about 10,000 measurement shots. 3. Spintronic implementation: Hardware mapping is still just an idea; there is no real physical prototype yet. 4. Oracle compilation: Multi-controlled gate structures add to circuit depth, which makes it hard to use them on a large scale.

5.6. The Final Observation

The proposed VQA–MLAE–spintronic framework accomplishes three essential objectives: (i) the design of low-depth circuits compatible with NISQ devices, (ii) precise and noise-resistant QCVaR estimation, and (iii) outstanding energy efficiency via spintronic mapping. The fact that these features come together shows that quantum financial modeling can be both better at calculations and better for the environment. This hybrid design paradigm promises to change how we think about risk and sustainable finance as quantum hardware gets better. It does this by combining algorithmic accuracy, physical efficiency, and managerial relevance.

6. Conclusions and Future Work

This research presented a hybrid quantum-classical-spintronic framework for high-precision Conditional Value-at-Risk (CVaR) estimation, exhibiting robust performance on near-term quantum hardware. We used a 6-qubit circuit with variational state preparation and Maximum-Likelihood Amplitude Estimation (MLAE) to get $\widehat{\text{CVaR}}_{0.95} = 0.281 \pm 0.012$, which is less than 1.1% off from classical Monte Carlo baselines (0.278). The mean absolute error of 1.7×10^{-3} came from estimating the tail probability using MLAE. The framework reached a state fidelity of 0.967 and a total runtime of about 13 seconds on the IBM Quantum backend (`ibm_brisbane`) using 6144 shots across 6 amplification levels.

The main new idea is to use shallow MLAE iterations with classical likelihood optimization. This cuts the circuit depth by $3.3\times$ compared to standard Quantum Amplitude Estimation while keeping the same level of statistical accuracy. A conceptual mapping to spintronic primitives—using spin-torque oscillators, Rashba coupling, and MTJ readout—gives an average energy of 2.0 fJ per logical gate, which is 80% less than what CMOS-based logic uses. This method makes it possible for quantum computing in finance to be sustainable, which is in line with energy-efficient hardware standards and the UN Sustainable Development Goals (SDG 8 and 17).

The framework also shows that quadratic sampling is faster ($O(1/\epsilon)$ vs. $O(1/\epsilon^2)$) than classical Monte Carlo. This makes it possible to monitor tail risk in real time and meet Basel III/IV stress-testing requirements. This work

creates a useful plan for quantum advantage in probabilistic risk analysis by combining algorithmic efficiency, noise resilience, and energy optimization. Despite these advances, several limitations remain, highlighting opportunities for future research:

1. **Scalability beyond 16 qubits:** The current method of oracle compilation using multi-controlled gates makes it hard to scale efficiently. Future research should investigate arithmetic oracles through quantum signal processing or reversible classical logic synthesis.
2. **Continuous-variable and analog encodings:** Spin-wave or bosonic modes in hybrid magnonic-photonics systems may allow for direct encoding of continuous loss distributions, which would get rid of discretization error.
3. **Error mitigation and fault tolerance:** By combining zero-noise extrapolation, dynamical decoupling, and flag-qubit error detection, you can get fidelity higher than 0.99 on devices with more than 50 qubits.
4. **Dynamic and multi-period risk:** Extending to time-dependent portfolios through quantum recurrent circuits or policy-gradient reinforcement learning for adaptive hedging strategies.
5. **Physical spintronic prototype:** An experimental implementation of spintronic CNOT and $R_y(\theta)$ gates utilizing domain-wall motion or skyrmion logic on nanomagnetic arrays.
6. **Integration with real-time market feeds:** Building a full-stack quantum risk engine that works with live financial data streams through classical co-processors.

This work connects quantum algorithm design, financial modeling, and new types of hardware, making it possible to use quantum advantage in risk management. In the future, we will work on error-corrected scaling, analog spintronic prototypes, and real-time portfolio optimization.

Author Contributions

Gayathri contributed to the conceptualization of the study. Muthulakshmi provided review and overall supervision. R. Palanivel was responsible for implementation, experimentation, and article drafting. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

Data Availability Statement

The dataset used in this study is available from the corresponding author upon reasonable request.

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