

Hierarchical Model for an AUV Swarm with a Leader

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ABSTRACT

This paper introduces an innovative hierarchical model with a leader to facilitate navigation of a swarm of underwater robots, inspired by the collective behaviours observed in natural animal groups, such as schools of fish and flocks of birds. In this model, the leader robot carries a comprehensive set of navigation information, while the other robots are stratified based on the relative distances between them and follow the leader during the navigation process. The model incorporates repulsion and attraction forces to enable clustering and collision avoidance among the robots. Initial simulation results confirm the scalability of the model and its robustness against noise, while further simulations demonstrate that the proposed layered strategy effectively manages polyline and circular trajectory navigation and guides the robotic group around obstacles while maintaining the group's structural stability and efficiency. In addition, the decentralised nature of the model and its minimal communication requirements make it highly suitable for practical underwater tasks. This research not only provides an effective and deployable solution for the cooperative synchronisation of underwater robots but also offers valuable insights for understanding and designing other types of robotic swarm systems.

Keywords: multiple underwater robots, self-organisation, swarm, hierarchical structure, leader

INTRODUCTION

Demand in the areas of ocean exploration and resource development has been a significant driver of advancements in underwater robotics technology [1,2]. Multi-robot systems have been shown to have extensive application prospects, particularly in large-scale and time-sensitive missions [3]. Biomimetic swarm technology involving multiple underwater robots, inspired by the behaviour mechanisms of biological groups, enables efficient area coverage and task allocation,

which are of paramount importance in environmental monitoring, ocean exploration, and disaster rescue. However, the challenges associated with deploying complex swarm strategies, coupled with limitations on communication, continue to make large-scale multi-underwater robot swarm navigation and collision avoidance a significant challenge [4,5].

Although considerable progress has been made in the study of swarm robotics in aerial and terrestrial environments [6-8], research in underwater environments remains relatively

limited. Distributed interaction in underwater environments is limited by the communication range, an effect that is pronounced in aerial and terrestrial environments. Aerial robots can communicate over long distances using high-frequency radio signals or visual sensors, while terrestrial robots can use more reliable wireless communication technologies. Traditional methods for studying underwater robot swarms have primarily focused on global path planning based on centralised control and distributed algorithms [9,10]. Centralised control methods rely on global information from a central node, but under the constrained conditions of underwater communication, the feasibility of this approach is greatly reduced. Although distributed algorithms can partially alleviate issues related to communication [11], the majority of traditional distributed algorithms focus primarily on formation maintenance and trajectory planning [12]. There is a notable lack of research on scalable swarm systems that can accommodate increasing group sizes, particularly under conditions involving noise and obstacles.

Although some typical distributed algorithms have made certain advancements in the control of multi-underwater robot swarms, numerous scientific challenges remain that urgently require investigation. Stability and scalability in complex marine environments remain critical issues, particularly when facing noise and obstacles, as current methods exhibit suboptimal performance in these scenarios [13]. Current research on multi-underwater robot swarms also mainly focuses on improving traditional bio-inspired swarm algorithms, and there has been less research interest in maintaining overall cohesion in distributed control swarms, which is a key focus of this paper.

This article presents a novel cooperative algorithm for multi-underwater robots based on swarm control algorithms, with the aim of addressing the challenges of computational complexity and scalability when handling large-scale swarms with traditional distributed algorithms. The swarm algorithm draws inspiration from the Vicsek model in the area of statistical physics [14,15], and fully leverages both distributed and self-organising characteristics. Inspired by biological systems, the concept of a swarm with a leader is introduced to provide more explicit directionality and objectives for biomimetic swarm robots in terms of navigation and task execution [16,17]. In this framework, the leader not only sets the direction of movement but also promotes the global consistency of the swarm through limited-range interactions [18].

In the Vicsek model, the interactions between individuals are of equal strength, rendering each individual's status effectively equivalent. However, biologists have demonstrated that hierarchical mechanisms are prevalent in nature, and can effectively maintain group stability and cohesion [19,20]. Inspired by this finding, we propose a hierarchical mechanism in which a leader with path information constitutes the first layer, underwater robots that directly interact with the leader form the second layer, and the remaining underwater robots form the third layer. This model achieves compactness and safe distances between robots by combining attraction,

repulsion, and artificial potential fields [21].

To validate the feasibility of the proposed model, we conducted numerical simulations to test the navigation of underwater robots in groups of varying scales, under both noise-free and noisy conditions, using polyline and circular trajectories. The results indicate that swarms of different scales can quickly self-organise and converge to create a quasi-lattice formation, without collisions, thus demonstrating the scalability and robustness to noise of the swarm algorithm. Our research not only provides a new approach for multi-underwater robot swarms but also lays the foundation for future swarm algorithms and practical sensor tests in three-dimensional underwater environments.

The structure of this paper is as follows. Section 2 describes the underwater robot swarm model. Numerical simulations are reported in Section 3, which validate the effectiveness and advantages of the proposed model. Section 4 provides a summary of the research presented in this paper.

MODEL DESCRIPTION

Unlike some existing complex and algorithmically challenging swarm algorithms, our approach is rooted in the Vicsek model [14], a classical self-driven particle system that is used to construct a basic kinematic framework for underwater robots and simulate their movement behaviour within a swarm.

KINEMATIC MODEL OF UNDERWATER ROBOTS

This section introduces a new kinematic model of underwater robots, which is constructed and optimised based on the Vicsek model. As a paradigm of self-driven particle systems, the Vicsek model provides a crucial theoretical foundation for understanding and simulating the collective behaviour of underwater robots. Similarly to the environmental conditions described in the Vicsek model, we have constrained our study to a fixed-depth water environment, which is essentially two-dimensional.

Vicsek model

This model, which was proposed by T. Vicsek in 1995, is a self-driven particle model characterised by the dynamic behaviour of complex systems and local interactions. These particles can be regarded as autonomous underwater vehicles (AUVs). The Vicsek model describes the movement of N AUVs within an $L \times L$ area with periodic boundaries, at a constant speed v , in discrete time steps. Each AUV is initially randomly positioned, and its heading angle is uniformly distributed within the range $[0, 2\pi]$. The position and speed of each AUV are adjusted at regular intervals according to specific rules, as follows:

$$\begin{bmatrix} x_i(t+1) \\ y_i(t+1) \end{bmatrix} = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} + V_i \cdot \Delta t \cdot \begin{bmatrix} \cos \psi_i(t) \\ \sin \psi_i(t) \end{bmatrix} \quad (1)$$

where $[x_i(t) \ y_i(t)]^T$ represents the position of underwater robot i , Δt represents the discrete time interval, and V_i represents the velocity of underwater robot i , with a constant magnitude. $\psi(t)$ represents the heading angle of robot i , and the angle update algorithm adopts an improved Vicsek model strategy [17,22], with specific updating rules as follows:

$$\psi_i(t+1) = \psi_i(t) + e^{\frac{1}{2}[1-\phi_i^\alpha(t)]} [\hat{\psi}_i(t) - \psi_i(t)] + \Delta\psi_i \quad (2)$$

$$\hat{\psi}_i(t) = \tan^{-1} \frac{\sum_{j \in D_i(t)} (\sin(\psi_j(t)))}{\sum_{j \in D_i(t)} (\cos(\psi_j(t)))} \quad (3)$$

where α is the exponential adjustment parameter, and $\hat{\psi}_i(t)$ represents the heading angle update algorithm based on the Vicsek model. In previous work, we verified that choosing an appropriate value for α in Eq. (2) is highly efficient in terms of achieving group alignment. In this work, we choose $\alpha = 1$. $\Delta\psi_i$ is a random noise term, with a value in the range $[-\eta, \eta]$ with an amplitude greater than zero. $D_i(t)$ is the set of all neighbours of an individual i , and $\phi_i(t)$ represents the local alignment degree of underwater robot i with its neighbours, as follows:

$$\phi_i(t) = \frac{|\sum_{j \in D_i(t)} \vec{v}_j|}{\sum_{j \in D_i(t)} |\vec{v}_j|}, j = 1, 2, \dots, N \quad (4)$$

In the above equation, the “neighbours” refer to the set of all individuals within a circle of radius R centred on underwater robot i , which can be denoted as:

$$D_i(t) = \{j \mid \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} < R\} \quad (5)$$

where $[x_j(t), y_j(t)]$ represent the positions of underwater robots j at time t .

To measure the alignment effectiveness of the entire underwater robot swarm system, Vicsek et al. proposed a global alignment order parameter, which is expressed as follows,

$$\phi = \frac{|\sum_{i=1}^N \vec{v}_i|}{\sum_{i=1}^N |\vec{v}_i|}, 0 \leq \phi \leq 1 \quad (6)$$

In the Vicsek model, N represents the number of underwater robots in the swarm. However, although the system achieves directional alignment, the overall direction of the group remains uncertain. We therefore introduce a leader and a hierarchical structure for navigation, which incorporates repulsive **components** to avoid collisions between robots within the swarm and attractive **components** to maintain the compactness of the swarm.

HIERARCHICAL SWARM NAVIGATION STRATEGY

In the process of multi-underwater robot cooperation, if each underwater robot possesses navigation route information, the data transmission load will become heavy and difficult to manage. Hence, in practical navigation situations, it is important that only a few underwater robots obtain accurate target information and select the route to complete the navigation task. The proposed hierarchical

algorithm is specifically designed for underwater robot swarm technology, and uses a limited interaction range for distributed control, which is not an issue that needs to be considered for aerial drones or ground robots.

Hierarchical structure with leader

Fig. 1 shows the proposed hierarchical mechanism with the interaction topology for a multi-underwater AUV swarm. The entire swarm system includes only one leader with precise navigation capabilities; this individual is shown in yellow, is situated in the first layer, and is unaffected by any neighbours. The blue individuals are the second-layer underwater robots within the leader’s perception and interaction radius, which are primarily influenced by the leader’s direction. The red individuals in the third layer cannot directly perceive the leader, but have second-layer robots within their perception radius, as shown in Eq. (5).

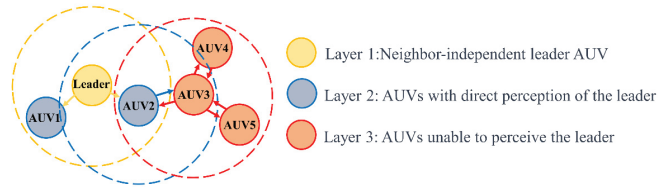


Fig. 1. Interaction topology for a multi-underwater AUV swarm with a hierarchical mechanism (arrows represent the direction of information flow, and circular dashed lines indicate the interaction range of the central individual; in this hierarchical structure, the leader belongs to the first layer, AUV1 and AUV2 are in the second layer, and AUV3, AUV4, and AUV5 are in the third layer)

It can be concluded that the leader is not influenced by any other underwater robot, and that its state changes are only affected by commands from the base station or environmental factors (such as obstacles). Based on this hierarchical relationship, three matrices are defined: the adjacency matrix, the information flow matrix, and the contribution matrix.

The adjacency matrix, $A_{ij} = [a_{ij}]_{N \times N}$ is used to describe the mutual perception among the N underwater robots. Each row and column of the matrix represents a robot in the swarm, and the elements within the matrix indicate whether the robots can directly perceive each other. If two robots can mutually perceive each other, the corresponding element of the matrix has a value of one; if they cannot, the element is zero. This range of perception can be measured using Eq. (5). The specific elements of the adjacency matrix are expressed as follows:

$$a_{ij} = \begin{cases} 1, i = 1, \dots, N, j \in D_i \\ 0, \text{otherwise} \end{cases} \quad (7)$$

The information flow matrix, $H_{ij} = [h_{ij}]_{N \times N}$ describes the direction of information transfer among the N underwater robot. The elements of the matrix have the following meaning: for a given underwater robot i , if another underwater robot j can transfer information to i , then $h_{ij} = 1$; otherwise, $h_{ij} = 0$. The information flow matrix h_{ij} can be described as follows:

$$h_{ij} = \begin{cases} 0, & \text{if } i \in \Lambda_1 \\ 1, & \text{if } i \in \Lambda_2 \text{ or } \Lambda_3 \end{cases} \quad (8)$$

where Λ_i represents the set of all individuals in the i th layer of the multi-underwater robot hierarchical system. Based on the aforementioned description and expressions, it is evident that a robot cannot transfer information to itself, i.e., $h_{ij} = 0$. Moreover, only when the information from one underwater robot flows to a given underwater robot can it contribute to the adjustment of the navigation direction of the given robot.

The contribution matrix, $K_{ij} = [k_{ij}]_{N \times N}$ is used to measure the weights of the contributions made by underwater robots to the angle update strategy. Each element k_{ij} in the matrix represents the weight of the contribution made by underwater robot j to the heading update of underwater robot i . In the proposed hierarchical relationship, the influence weight of the first layer on the second layer is denoted by K_1 , the influence weight of the second layer on the third layer is denoted by K_2 , and the influence weight of the third layer on other underwater robots (excluding the leader) is denoted by K_3 , as shown in Eq. (9):

$$k_{ij} = \begin{cases} K_1, & \text{if } j \in \Lambda_1 \\ K_2, & \text{if } j \in \Lambda_2 \\ K_3, & \text{if } j \in \Lambda_3 \end{cases} \quad (9)$$

If the contributions between individuals in different layers are the same, as described by the Vicsek model, it is evident that $K_1 = K_2 = K_3$. However, in the hierarchical mechanism proposed in this paper, the contributions of individuals to angle updates can vary across layers. The contribution strategy proposed in this paper is given by the following expression:

$$k_{ij} = \begin{cases} K_1 = k_\alpha * e^{(1 - \text{length}(\Lambda_2)/N)} & \text{if } j \in \Lambda_1 \\ K_2 = k_\beta * e^{(1 - \text{length}(\Lambda_2)/N)} & \text{if } j \in \Lambda_2 \\ K_3 = 1 & \text{if } j \in \Lambda_3 \end{cases} \quad (10)$$

where N is the total number of underwater robots in the swarm, and we set $k_\alpha = 5$, $k_\beta = 2$.

The interactions between the underwater robots should be heterogeneous and follow a hierarchical relationship. Hence, the angle update for a single underwater robot is modified from Eq. (2) to the following expression:

$$\psi_i(t+1) = \psi_i(t) + e^{\frac{1}{2}[1-\phi_i^\alpha(t)]} [\tilde{\psi}_i(t) - \psi_i(t)] + \Delta \psi_i(t) \quad (11)$$

$$\tilde{\psi}_i(t) = \tan^{-1} \frac{\sum_{j \in D_i^1(t)} (C_{ij} \times \sin(\psi_j(t)))}{\sum_{j \in D_i^1(t)} (C_{ij} \times \cos(\psi_j(t)))} \quad (12)$$

The principal result derived from Eq. (11) is predicated on a layered strategy for velocity alignment, where $C_{ij} = A_{ij} \cdot H_{ij} \cdot K_{ij}$ is ascertained through the multiplication of three contributive matrices. Furthermore, in the context of the leader acting as the navigator, a linear trajectory $\tilde{\psi}_i(t) = \psi_i(t)$ is maintained in the absence of navigational directives from the ground station or obstacles. The receipt of course-altering commands or encounters with obstacles will necessitate a change in direction.

Attractive and repulsive components

To realise comprehensive multi-underwater robot cooperative control, it is imperative to incorporate both attractive and repulsive components into the swarm dynamics, in addition to the navigational hierarchy. The inclusion of an attractive component serves to maintain the compactness of the robot swarm, while the introduction of a repulsive component ensures that each robot maintains a safe distance from the others during navigation, thereby preventing collisions during transit. The formulations for these components are presented in Eqs. (13) and (14), respectively:

$$\vec{v}_i^{att}(t+1) = \sum_{j \in D_i^1(t)} \left(\frac{r_{rep} - \|\vec{x}_{ij}(t)\|}{r_{att} - r_{rep}} \cdot \frac{\vec{x}_{ij}(t)}{\|\vec{x}_{ij}(t)\|} \right) \quad (13)$$

$$\vec{v}_i^{rep}(t+1) = \sum_{j \in D_i^2(t)} \left(\frac{r_{rep} - \|\vec{x}_{ij}(t)\|}{r_{rep}} \cdot \frac{\vec{x}_{ij}(t)}{\|\vec{x}_{ij}(t)\|} \right) \quad (14)$$

where $\vec{x}_{ij}(t)$ defines the directional vector from underwater robot i to its neighbouring underwater robot j . The parameter r_{rep} signifies the safe distance maintained by the underwater robots to avoid collisions, while r_{att} represents the distance over which robots in a swarm attract each other to maintain a cohesive grouping. $D_i^1(t)$ represents the set of all neighboring robots j of an individual i , where the distances are between the values of r_{rep} and r_{att} . This set is defined as the attraction zone. Conversely, $D_i^2(t)$ denotes the set of robots that are within the distance range from $[0, r_{rep}]$ from robot i , which is designated as the repulsion zone. At moment, $t+1$, $\vec{v}_i^{att}(t+1)$ represents the sum of attractive components affecting the speed $\vec{v}_i$ of a given robot. Similarly, $\vec{v}_i^{rep}(t+1)$ at moment $(t+1)$ quantifies the cumulative repulsive components impacting the speed $\vec{v}_i$.

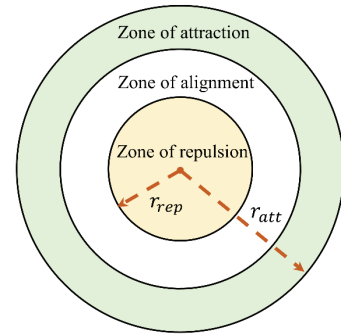


Fig. 2. Schematic representation of the attraction and repulsion zones

As illustrated in Fig. 2, the area is divided into zones of attraction, alignment, and repulsion. When an underwater robot j , a neighbour of robot i , is located within the repulsion zone of robot i , it exerts a repulsive force on robot i to prevent collisions. Conversely, when robot j is within the attraction zone of robot i , it generates an attractive force to maintain group cohesion. In addition, when robot j resides in the alignment zone of robot i , robot i adjusts its direction using the designed hierarchical navigation system to update its course.

COLLISION AVOIDANCE STRATEGY

The influence of a circular obstacle on the navigation of underwater robots is mathematically represented in Eq. (15) as follows:

$$v_i^{obs} = \sum_{j=1}^{card(S^{obs})} \frac{S_j - P_i}{|S_j - P_i|} \mu(|P_i - S_j| - (R_o + R_k)) \quad (15)$$

$$S^{obs} = \{S_1, S_2, S_3, \dots, S_n\} \quad (16)$$

In the model presented here, S^{obs} represents the set of coordinates of all obstacles in the underwater environment. S_j denotes the centre coordinates of a circular obstacle j , while P_i specifies the position coordinates of an underwater robot i . In addition, R_o represents the safety distance maintained between the robot and the obstacle, and R_k is the radius of the circular obstacle. The function $\mu(|P_i - S_j| - (R_o + R_k))$ is defined such that if $|P_i - S_j| > (R_o + R_k)$, then $\mu(|P_i - S_j| - (R_o + R_k)) = 0$; otherwise, $\mu(|P_i - S_j| - (R_o + R_k))$ is set to 1, signifying that no circular obstacle is encountered that can affect the navigation of the underwater robot.

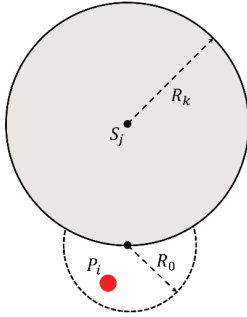


Fig. 3. Schematic diagram of a circular obstacle

In Fig. 3, the schematic shows a circular obstacle, with a grey circular area representing the space occupied by the obstacle and a red dot indicating the projection point of the underwater robot at the obstacle's boundary. To effectively avoid collisions, a dashed-line sectorial region, centred at the obstacle boundary projection point and with a radius equivalent to the safety distance R_o , is designated as the manoeuvring zone for obstacle avoidance.

Hence, the directional update strategy derived from Eq. (11) is primarily utilised for alignment, while a combination of Eqs. (13) and (14) imparts both repulsive and attractive components to the swarm of underwater robots. In the event that an obstacle is encountered, Eq. (15) serves as the velocity adjustment component for avoidance. Consequently, the speed direction strategies for the underwater robots, other than the leader, are adjusted for the subsequent moment as follows:

$$\begin{aligned} \vec{v}_i(t+1) = & c^{align} \cdot \vec{v}_i^{align}(t+1) + c^{rep} \cdot \vec{v}_i^{rep}(t+1) + \\ & + c^{att} \cdot \vec{v}_i^{att}(t+1) + c^{obs} \cdot \vec{v}_i^{obs}(t+1) \end{aligned} \quad (17)$$

In the hierarchical strategy outlined in Eq. (17), $\vec{v}_i^{align}(t+1)$ represents the directional alignment component, which is primarily used to ensure coordinated alignment of the robotic swarm. $\vec{v}_i^{rep}(t+1)$ and $\vec{v}_i^{att}(t+1)$ correspond to the repulsive and attractive speed components derived from Eqs. (13) and (14), respectively. $\vec{v}_i^{obs}(t+1)$ is the avoidance component derived from Eq. (16). The coefficients c^{align} , c^{rep} , c^{att} , and c^{obs} are used to adjust the alignment, repulsion, attraction, and avoidance components of the robots, respectively.

NUMERICAL CALCULATIONS AND ANALYSIS

To validate the effectiveness of the proposed hierarchical relationship and swarm control algorithms in terms of the collaborative navigation of multiple underwater robots, two simulation experiments for cooperative navigation were designed involving sets of 15 and 24 underwater robots, respectively. These numbers of robots were chosen to strike a balance between the available experimental resources and the need for validation of the algorithm, while also enabling an assessment of scalability. The simulation parameters were as follows, $L = 200$, $r = 8$, $r_{rep} = 6$, $r_{att} = 9$, $v_o(t) = 0.5$, $c^{align} = 1$, $c^{rep} = 1$ and $c^{obs} = 1$ with the initial positions of the underwater robots randomly generated within a specified square area $L \times L$ and the initial angles randomly generated within the range $[-\pi/2, \pi/2]$. In the following, the leader is represented in red, while the other underwater robots are shown in blue.

COOPERATIVE NAVIGATION

Simulations were conducted without obstacles, and the leader was provided with predefined polyline or circular path information to test the feasibility of the proposed swarm algorithm under conditions where only the leader was aware of the set path.

Polyline cooperative navigation

Initially, a simulation of polyline cooperative navigation involving 15 underwater robots was conducted. Fourteen of these robots (depicted as blue circles) were randomly distributed within the square area $\{(x, y) | x \in [30, 45], y \in [30, 45]\}$, with the leader starting at coordinates (45, 38). The followers then navigated along a polyline path, following the leader (represented by a red circle). The coordinates of the polyline path were (150, 38), (150, 150), (50, 150), and (50, 50), and the simulation was performed under noise-free conditions. Fig. 4 displays the distribution of underwater robots at four time points shortly after the start of the simulation, to illustrate their positions over a segment of the journey.

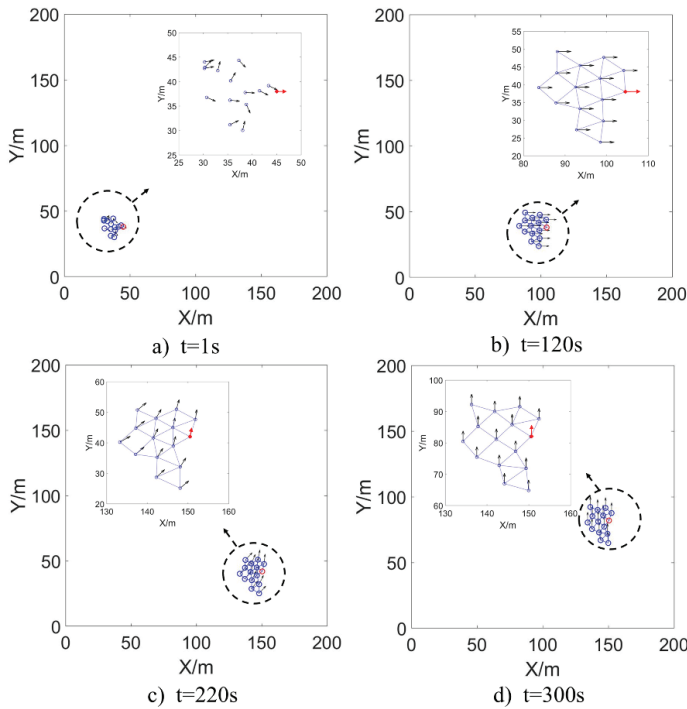


Fig. 4. Polyline navigation of 15 underwater robots

As depicted in Fig. 4, the 14 underwater robots were initially randomly positioned, and navigated along a polyline path under the guidance of the leader. Fig. 4(a) shows that at the outset, the spacing between the robots was uneven: some were too close, and others too far apart. However, under the influence of the swarm control algorithm, they gradually moved into a quasi-lattice formation, as shown in Fig. 4(b), maintaining consistent and approximately equal distances within their respective interaction radii, and continued to navigate in this formation.

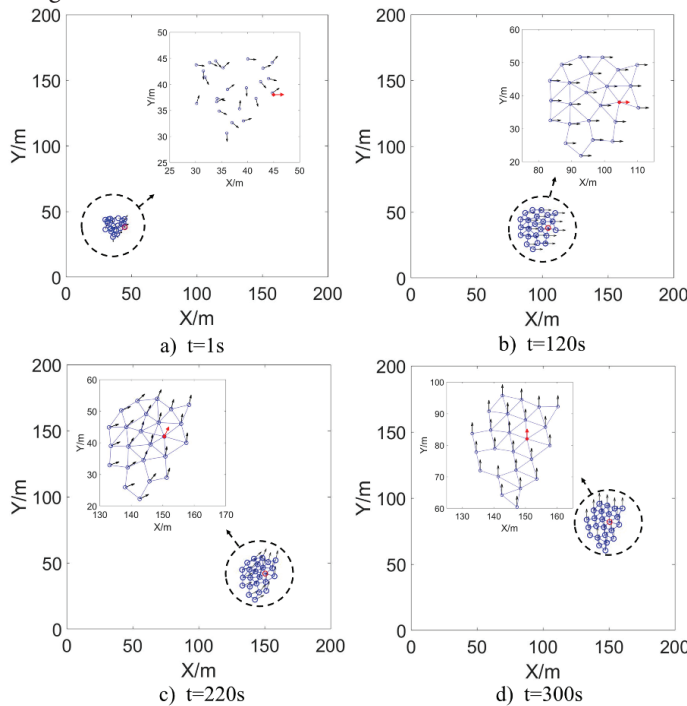


Fig. 5. Polyline navigation of 24 underwater robots

In particular, as illustrated in Fig. 4(c), the inclusion of a right-angle turn in the navigation process did not significantly disrupt the cohesive state of the swarm. After this turn, the formation remained relatively stable, as can be seen in Fig. 4(d). Even with the introduction of noise, with a value of $\eta = 0.2$, the robots were still able to complete the polyline navigation task (further details will be provided in the subsequent sections), thus demonstrating robustness of the multi-underwater robot swarm strategy under these operational conditions.

Next, a simulation experiment involving the coordinated polyline navigation of 24 underwater robots was conducted. The parameters and scenario were consistent with those in Fig. 4. The difference, however, lay in an increase in the number of underwater robots from 14 to 23, which then navigated based on a hierarchical relationship following the leader.

Fig. 5 shows the 24 underwater robots, starting from randomly distributed initial positions, gradually forming a quasi-lattice formation and successfully completing the polyline navigation task. The two experiments described above show that the simulations of collaborative navigation by underwater robot swarms demonstrated significant scalability, without the need to alter any other parameters of the algorithm. This has important implications for large-scale underwater swarm tasks.

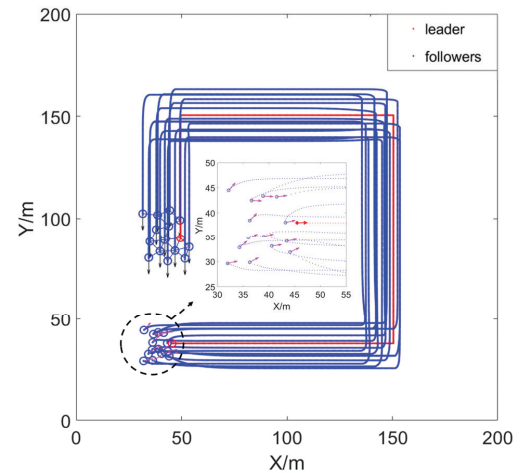


Fig. 6. Polyline cooperative navigation trajectories for multiple underwater robots

The simulation experiment with 15 underwater robots is taken as an example, and Fig. 6 illustrates the polyline navigation trajectory of the entire collaborative group.

Circular cooperative navigation simulation

Based on the polyline navigation experiments, a further simulation experiment involving circular cooperative navigation of 15 underwater robots was conducted. In this experiment, the leader's deflection angle for each navigation was fixed at $\pi/10$ to approximate circular motion. Fig. 7 shows the trajectories of multiple underwater robots performing circular motion under the leader's

guidance, with the noise parameter set to $\eta = 0.2$.

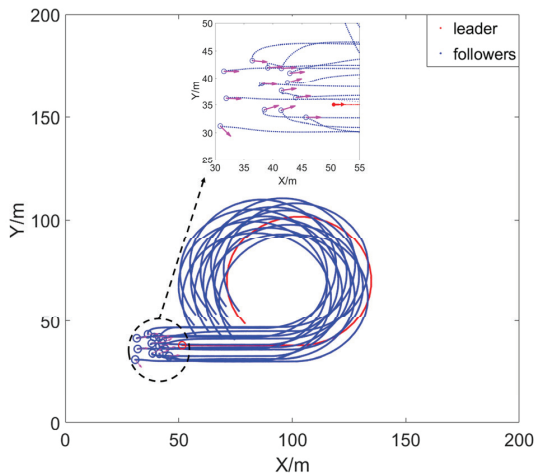


Fig. 7. Circular cooperative navigation trajectories for multiple underwater robots

It can be seen from Fig. 7 that initially, the positions of the underwater robots within the swarm were randomly distributed and their directions varied. Under the influence of the proposed navigation strategy based on hierarchical relationships, the entire swarm was able to quickly self-organise into a stable state and follow the leader in a circular trajectory. To better understand the stability and effectiveness of this self-organising behaviour, we conducted a systematic analysis of the swarm’s collective dynamics over time. This approach allowed us to examine how quickly and effectively the swarm achieved a cohesive movement pattern.

Based on a qualitative analysis of the different trajectory navigation tasks and the various sizes for the underwater robot swarms, we selected global consistency as an evaluation metric to quantitatively analyse the collective behaviour of the swarm at key time points. In this case, $t = 1$ s represents the initial state, $t = 2$ s corresponds to the result after the first iteration of the algorithm, $t = 220$ s marks the turning point in the polyline navigation, while $t = 120$ s and $t = 300$ s were the chosen steady-state time points. The performance for a swarm size of $N = 15$ is shown in Table 1.

Table 1. Global consistency of 15 underwater robots over time

		Time(s)				
ϕ		1	2	120	220	300
N=15 Polyline	Without noise	0.6606	0.9305	1	0.9055	1
	With noise	0.6606	0.9269	0.9983	0.8855	0.9992
N=15 Circular	Without noise	0.6606	0.9527	0.9997	0.9989	0.9999
	With noise	0.6606	0.9501	0.9984	0.9912	0.9993

As shown in Table 1, the alignment effectiveness of the proposed algorithm is further validated, as there is a significant improvement in global consistency within a short period. This improvement is largely due to the incorporation of the artificial potential field and hierarchical strategy into the original model. At the polyline turning point ($t = 220$ s), the global consistency drops significantly, but it quickly recovers to almost one after the turn. In contrast, for circular

trajectory navigation, the swarm system constantly adjusts its heading angle, making it impossible to achieve a global consistency of one, even in the absence of noise or under steady-state conditions.

Table 2. Global consistency of 24 underwater robots over time

		Time(s)				
ϕ		1	2	120	220	300
N=24 Polyline	Without noise	0.6875	0.9809	1	0.9336	1
	With noise	0.6875	0.9762	0.9979	0.9117	0.9981
N=24 Circular	Without noise	0.6875	0.9801	0.9993	0.9982	0.9998
	With noise	0.6875	0.9749	0.9970	0.9947	0.9979

The results shown in Table 2 are similar to those in Table 1, for a swarm size of $N = 24$. Although the noise amplitude in Table 2 is also set to $\eta = 0.2$, the initial state and noise distribution have been reallocated, which is why the global consistency at the initial moment differs from that in Table 1. In addition, after $t = 1$ s, the overall global consistency generally increases with the number of underwater robots, as the greater number of nodes in the complex network enhances connectivity. This increased connectivity provides more opportunities for mutual influence among individuals, which in turn improves the efficiency of alignment across the swarm.

Several simulation experiments were designed for polyline and circular navigation involving multiple underwater robots, including two sets of simulations with 15 and 24 underwater robots, respectively, to verify the scalability of our approach.

COOPERATIVE OBSTACLE AVOIDANCE

In real marine environments, obstacles such as buoys may exist. We therefore explored the swarm performance of multiple underwater robots under conditions with obstacles, using the proposed strategies. The initial positions of the underwater robots were randomly generated within a square area $\{(x, y) | x \in [20, 55], y \in [20, 55]\}$, and their directions were randomly determined within the range $[-\pi/2, \pi/2]$. In the diagrams below, red represents the leader, while blue indicates followers. For the circular obstacle avoidance simulation, to optimise the interaction range and obstacle avoidance performance, the obstacle-related parameters were selected as $R_k = 4$, $R_o = 3$, and multiple obstacle coordinates were set at (80, 50), (60, 20), (80, 24), (100, 29), with noise still at $\eta = 0$. Simulation experiments involving obstacle avoidance with multiple obstacles were conducted using 15 underwater robots.

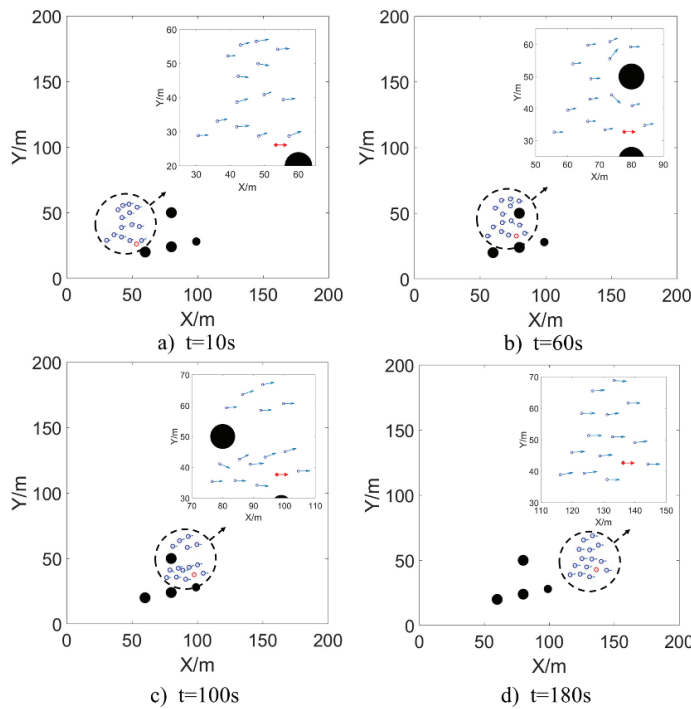


Fig. 8. Cooperative obstacle avoidance by 15 underwater robots with multiple obstacles

From Fig. 8, it can be seen that the underwater robots were initially randomly distributed within a designated area and gradually formed a stable swarm under the influence of the swarm control algorithm. Under the control of the hierarchical cooperative navigation algorithm, they followed the leading underwater robot. During navigation, multiple obstacles were encountered, but the swarm of underwater robots was able to pass safely without impact. Influenced by an artificial potential field, they successfully navigated around obstacles while reforming into a stable swarm and maintaining the necessary distances.

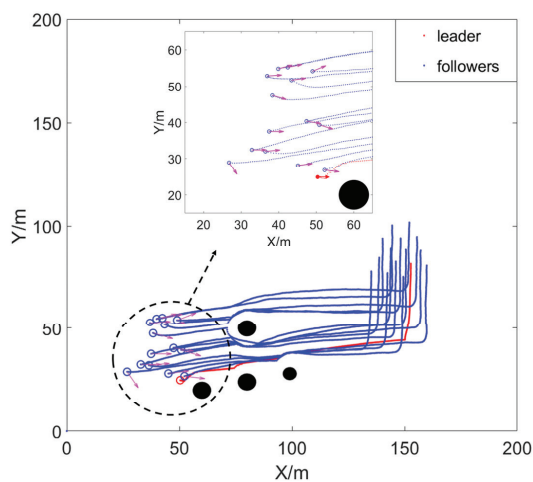


Fig. 9. Cooperative obstacle avoidance trajectories for 15 underwater robots with multiple obstacles

As shown in Fig. 9, the underwater robots were randomly

distributed within the initialisation area. The red trajectory represents the navigation route of the swarm leader, while the blue trajectories represent the navigation routes of the other follower robots in the swarm. The purple arrows indicate their initial navigation directions. From Fig. 9, it can be seen that after introducing noise into the swarm system, the trajectories exhibited some disturbances, and were not very smooth. Particularly after passing obstacles, there was a tendency for the robots to gradually converge and then to form a stable swarm, before proceeding with the turning navigation movements.

In order to further test the applicability of the proposed swarm algorithm for dynamic obstacle avoidance, a simulation experiment was conducted in which 15 underwater robots avoided dynamic obstacles. The parameters R_k and R_o were consistent with the settings applied in the static obstacle simulations. The other simulation parameters were consistent with those in the obstacle-free polyline cooperative navigation section, with the circular obstacle moving westward. In the dynamic obstacle avoidance simulation experiment, the initial coordinates of the obstacle were set to (150, 32), and it moved westward at a constant speed of 0.4 m/s. The results are shown in Fig. 10 below.

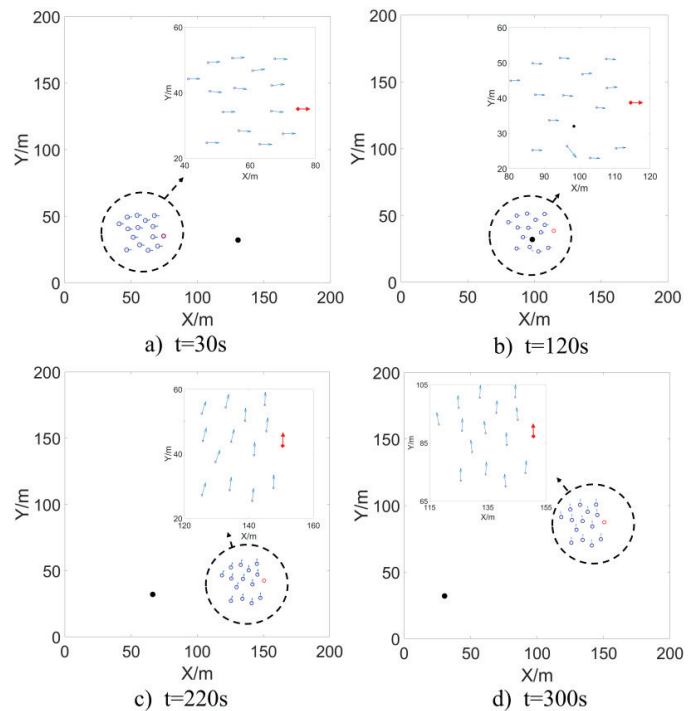


Fig. 10. Cooperative navigation by 15 underwater robots with a dynamic obstacle

As shown in Fig. 10(b), at around $t = 120$ s, the obstacle was located in the middle of the swarm system, and some individuals changed their heading angles to avoid it. Fig. 10(c) shows the heading situation at the turning point, where the followers experienced a slight phase lag relative to the leader. Finally, the dynamic obstacle moved farther away from the swarm, as shown in Fig. 10(d).

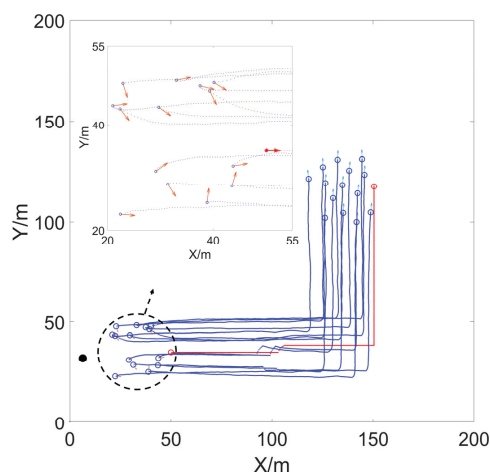


Fig. 11. Cooperative navigation trajectories of 15 underwater robots with a dynamic obstacle

Although the dynamic obstacle increased the distance between individuals in the swarm system, as shown in Fig. 11, as long as they remained within the interaction range, the swarm system could still maintain its integrity and continue moving as a cohesive unit after making a right-angle turn. This further validates the effectiveness of the proposed swarm algorithm.

CONCLUSION

In this paper, we have presented a hierarchical model with a leader, inspired by biological systems, to facilitate swarm navigation of multiple underwater robots. In this swarm system, the leader possesses the path information and constitutes the first layer; the underwater robots that directly interact with the leader form the second layer; and the remaining underwater robots make up the third layer. The motion states of the follower robots and the intensity of the interactions between layers are solely dependent on their neighbours, rather than on global information. The proposed hierarchical model incorporates both attraction and repulsion components to maintain compactness and a safe distance between each robot. In addition, an artificial potential field is specifically integrated to endow the swarm with obstacle avoidance capabilities.

To validate the feasibility of our approach, numerical simulations were conducted for swarms of different sizes under various operating conditions. We performed simulation experiments with 15 and 24 underwater robots, navigating along polyline and circular trajectories under both noise-free and noisy conditions, to verify the scalability and robustness of the model. The findings showed that swarms of different sizes could quickly self-organise and converge to a quasi-lattice navigation state without collisions, even when initially distributed randomly. In the obstacle avoidance navigation problem, scenarios with multiple obstacles were set up, and obstacle avoidance simulation experiments were carried out. The results demonstrated that the proposed algorithm enabled the entire multi-underwater robot system

to effectively avoid obstacles and to resume a stable state to continue navigation.

In summary, this study presents a new method for multi-underwater robot swarms that is easily deployable. This scheme draws inspiration from biological swarm phenomena and combines flexible biomimetic algorithms with hierarchical control strategies; it not only provides new perspectives for understanding complex systems in nature, but also opens up new possibilities for designing and implementing efficient and robust underwater robot swarm systems. This interdisciplinary research has strong scientific importance and offers substantial potential and value for engineering applications. Building on this foundation, future work will focus on swarm algorithms for three-dimensional underwater environments and the practical testing of underwater swarms using visual sensors.

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