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Fractional optimal control problem for a mathematical modeling of African swine fever virus transmission

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Abstract. To have a more realistic model, in this paper, This manuscript is devoted to investigating a fractional-order mathematical model of Kouidere et al. That describes the dynamics of spread of African swine fever virus (ASFV). The aim of this work is to protect susceptible pigs from the virus, In our model, by including three controls which represent: the iron fencing and spraying pesticides and get rid.

The aims of this paper is to reduce the number of infected pigs and ticks by using optimal control strategy and fractinal order derivation.

Pontryagin's maximal principle is used to describe optimal controls with Caputo time-fractional derivative and the optimal system is resolved in an iterative manner. Numerical simulations are presented based on the presented method. We finished tis article with a conclusion.

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1. Introduction

China has confirmed the outbreak of African swine fever in two major pork-producing provinces, Sichuan and Hubei, and 38 pigs have been infected so far, a new crisis added to China in light of the outbreak of the new epidemic of the Corona virus, according to what German radio reported, so what do we know about this fever.

China is the largest producer and consumer of pork, and a fever that broke out earlier in 2019, killed half of the pig herds across the country.

An outbreak of African swine fever wiped out nearly half of China's pig wealth in 2019. A virus that infects pigs and causes severe fever and internal bleeding. More than 90 per cent of sick pigs die within one week. The virus has a long life cycle. The disease is transmitted through contact with infected blood, body fluids or corpses and remains in the corpses from months to years. The current strain of the virus does not infect humans, but China announced that there is a new strain that is more deadly, and therefore it was not revealed whether it was able to transmit to humans or would it be like the other strain?.

The European Union has not approved any vaccine against African swine fever, but China has announced that it has developed a vaccine. The spread of African swine fever is often through humans themselves due to non-compliance with hygiene and disposal of leftover meat. The first diagnosis of this epidemic was in 1910 during the British occupation of Kenya [40, 8, 22].

Number of mathematical models have been published in a related topics ([37, 7, 16, 23, 24, 19, 20, 12, 39]).

Fractional derivatives plays a important role to monitor the effect of memory on the system dynamics by replacing the normal ordinary equation with a fractional derivative equation ([2, 42, 33, 21, 5, 32, 4, 38, 3, 41, 34, 35]).

After studied the mathematical model of Kouidere et al. [26], by using Routh–Hurwitz criteria, the local stability of ASFV-free equilibrium and ASFV equilibrium were obtained. We also studied the sensitivity analysis of the model parameters to know the parameters that have a high impact on the reproduction number R_0, and proposed several strategies to reduce the number of infected pigs and infected ticks.

Considering the above, they did not take into account the memory to reducing the spread of the African swine fever virus between pigs between them, as well as the transmission of the virus through ticks and flies.

The main contributions of this work are as follows:

(1) Developing an epidemic model with fractional derivative equations to better capture the dynamics of African swine fever virus and evaluate the impact of control strategies.
(2) maintaining the mathematical modol of kouidere et al, Y_1, Y_2, Y_3, Y_4 and (Y_5) respectively Pig susceptible, pig Infected, Recovered pigs, ticks susceptible and ticks Infected, respectively.

The rest of the paper is organized as follows. In Section 2, we give some basis definitions and proprieties on fractional order integral and derivatives. For the fractional order differentiation, we will use the Caputo definition due to its convenience for initial conditions of the differential equations.In Section 3, we present the optimal control problem based on the proposed

model and we characterize the optimal control terms using Pontryagin's maximum principle. Numerical simulations are given in Section 5. Finally, we concluded the paper in Section 6 .

2. Definitions and preliminaries

We will follow the same methodology adopted in the paper (see [1, 30, 29]).

3. Mathematical model

We used the mathematical model of kouidere et al $S_P I_P R_P S_T I_T$, that described the spread of ASFV in pig population.

In the rest of this paper, we replace $S_P = Y_1, I_P = Y_2, R_P = Y_3, S_T = Y_4$ and $I_T = Y_5$.

Hence, we present the fractional optimal control mathematical model of ASFV which is governed by the following system of differential equation :

$$\begin{cases} D^{\alpha}Y_1(t) = \Lambda_P - \mu_P Y_1(t) - \beta_1 \frac{Y_1(t)Y_2(t)}{N} - \beta_2 \frac{Y_1(t)Y_5(t)}{N} \\ D^{\alpha}Y_2(t) = \beta_1 \frac{Y_1(t)Y_2(t)}{N} + \beta_2 \frac{Y_1(t)Y_5(t)}{N} - (\mu_P + \alpha + \delta)Y_2(t) \\ D^{\alpha}Y_3(t) = \alpha Y_2(t) - \mu_P Y_3(t) \\ D^{\alpha}Y_4(t) = \Lambda_T - \mu_T Y_4(t) - \beta_3 \frac{Y_4(t)Y_2(t)}{N} \\ D^{\alpha}Y_5(t) = \beta_3 \frac{Y_4(t)Y_2(t)}{N} - \mu_T Y_5(t) \end{cases} \tag{3.1}$$

where $Y_1(0) \geq 0$, $Y_2(0) \geq 0$, $Y_3(0) \geq 0$, $Y_4(0) \geq 0$ and $I_R(0) \geq 0$ are the initial state.

With
- Λ_P: the recruitment rate of pigs susceptible.
- Λ_T: the recruitment rate of ticks susceptible.
- μ_P : Natural mortality rate of pigs susceptible.
- μ_T : Natural mortality rate of ticks susceptible.
- β_1: The rate of pigs which were infected with the virus by contact with the infected pigs.
- β_2: The rate of pigs which were infected with the virus by contact with the infected ticks.
- β_3: The rate of ticks which were infected with the virus by contact with the infected pigs.
- α: The rate of pigs which recovered from the virus.
- δ: Mortality rate due the ASF virus.

3.1. Model Basic Properties.

3.1.1. *Positivity of Solutions.*

Theorem 3.1. *If* $Y_1(0) \geq 0$, $Y_2(0) \geq 0$, $Y_3(0) \geq 0$, $Y_4(0) \geq 0$ *and* $Y_5(0) \geq 0$, *the solutions* $Y_1(t)$, $Y_2(t)$, $Y_3(t)$, $Y_4(t)$ *and* $Y_5(t)$ *of system (3.1) are positive for all* $t \geq 0$.

Proof. It follows from the first equation of system (3.1) that

$$\begin{aligned} \frac{dY_1(t)}{dt} &= \Lambda_P - \mu_P\, Y_1(t) - \beta_1 \frac{Y_1(t)Y_2(t)}{N} - \beta_2 \frac{Y_1(t)Y_5(t)}{N} \\ &\geq -\mu_P\, Y_1(t) - \beta_1 \frac{Y_1(t)Y_2(t)}{N} - \beta_2 \frac{Y_1(t)Y_5(t)}{N}. \end{aligned}$$

$$\frac{dY_1(t)}{dt} + \left(\mu_P + \beta_1 \frac{Y_2(t)}{N} + \beta_2 \frac{Y_5(t)}{N}\right) Y_1(t) \geq 0.$$

where $F(t) = \mu_P + \beta_1 \frac{Y_2(t)}{N} + \beta_2 \frac{Y_1(t)}{N}$. The both sides in last inequality are multiplied by $\exp(\int_0^t F(s)ds)$.

We obtain

$$\exp(\int_0^t F(s)ds).\frac{dY_1(t)}{dt} + F(t)\exp(\int_0^t F(s)ds).Y_1(t) \geq 0.$$

then $\frac{d}{dt}\left(Y_1(t)\exp(\int_0^t F(s)ds)\right) \geq 0$.

Integrating this inequality from 0 to t gives:

$$\int_0^t \frac{d}{ds}\left(Y_1(s)\exp(\int_0^t (\mu_P + \beta_1 \frac{Y_2(t)}{N} + \beta_2 \frac{Y_5(t)}{N})ds)\right) ds \geq 0.$$

then

$$Y_1(t) \geq Y_1(0)\exp(-\int_0^t \left(\mu_P + \beta_1 \frac{Y_2(t)}{N} + \beta_2 \frac{Y_5(t)}{N}\right) ds).$$

$\Longrightarrow Y_1(t) \geq 0$

.

Similarly, we prove that $Y_1(t) \geq 0$, $Y_2(t) \geq 0$, $Y_3(t) \geq 0$, $Y_4(t) \geq 0$ and $Y_5(t) \geq 0$.

3.1.2. *Invariant region.*

Theorem 3.2. *The set* $\Omega = \left\{(Y_1, Y_2, Y_3, Y_4, Y_5) \in \mathbb{R}^6_+ / \ 0 \leq Y_1 + Y_2 + Y_3 + Y_4 + Y_5 \leq \frac{\Lambda}{\mu}\right\}$ *positively invariant under system* (3.1) *with initial conditions* $Y_1(0) \geq 0$, $Y_2(0) \geq 0$, $Y_3(0) \geq 0$, $Y_4(0) \geq 0$ *and* $I_R(0) \geq 0$.

Proof. Also, one assumes that:

$$\begin{aligned} \frac{dN}{dt} &= \Lambda - \mu N - \delta I \leq \Lambda - \mu N \\ &\Longrightarrow \quad N(t) \leq \frac{\Lambda}{\mu} + N(0)e^{-\mu t}. \end{aligned}$$

with $\Lambda = \Lambda_P + \Lambda_T$, and $\mu = \mu_P + \mu_T$.

If we take limit $t \to \infty$ we have $0 \leq N(t) \leq \frac{\Lambda}{\mu}$.

and $N(0)$ represents the initial values of the total population.

Thus, $\lim_{t\to\infty} \sup N(t) = \frac{\Lambda}{\mu}$. It implies that the region Ω is a positively invariant set for system ((3.1)).

3.1.3. *Exictence of solutions.*

Theorem 3.3. *The system* (3.1) *that satisfies a given initial condition* $(Y_1(0), Y_2(0), Y_3(0), Y_4(0), Y_5(0))$ *has a unique solution.*

Proof. Let $X = \begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \\ Y_4(t) \\ Y_5(t) \end{pmatrix}$ and $\varphi(X) = \begin{pmatrix} \frac{dY_1(t)}{dt} \\ \frac{dY_2(t)}{dt} \\ \frac{dY_3(t)}{dt} \\ \frac{dY_4(t)}{dt} \\ \frac{dY_5(t)}{dt} \end{pmatrix}$

so the system (3.1) can be rewritten in the following form:

$$\varphi(X) = AX + B(X) \tag{3.2}$$

where

$$A = \begin{pmatrix} -\mu_P & 0 & -0 & 0 & 0 \\ 0 & -\mu_P + \alpha + \delta) & 0 & 0 & 0 \\ 0 & \alpha & -\mu_P & 0 & 0 \\ 0 & 0 & 0 & -\mu_T & 0 \\ 0 & 0 & 0 & 0 & -\mu_T \end{pmatrix}$$

and

$$B(X) = \begin{pmatrix} \Lambda_P - \beta_1 \frac{Y_1(t)Y_2(t)}{N} - \beta_2 \frac{Y_1(t)Y_5(t)}{N} \\ \beta_1 \frac{Y_1(t)Y_2(t)}{N} + \beta_2 \frac{Y_1(t)Y_5(t)}{N} \\ 0 \\ \Lambda_T - \beta_3 \frac{Y_4(t)Y_2(t)}{N} \\ \beta_3 \frac{Y_4(t)Y_2(t)}{N} \end{pmatrix}$$

The second term on the right-hand side of (8) satisfies $|B(X_1) - B(X_2)| \leq M(|X_1(t) - X_2(t)|)$.

then

$$\|\varphi(X_1) - \varphi(X_2)\| \leq V. \|X_1 - X_2\|.$$

Where $V = \max(M, \|A\|) < \infty$, and M is constant.

Thus, it follows that the function φ is uniformly Lipschitz continuous, and the restriction on $Y_1(t) \geq 0$, $Y_2(t) \geq 0$, $Y_3(t) \geq 0$, $Y_4(t) \geq 0$ and $Y_5(t) \geq 0$, we see that a solution of the system exists [9].

4. Fractional optimal control problem

4.1. Formulation of the optimal control problem. The strategy of control that has been adopted consists of a program to minimize the spread of ASF virus in pigs. Our main aim is to minimize the number of pigs infected and ticks infected. In this model, we included three controls $u_1(t)$, $v(t)$ and $w(t)$ for $t \in [0, T]$. The first control u representing the iron fence at a given time t. The second control v represented the praying pesticides at a given time t. The third control w represents the get rid at a given time t.

$$\begin{cases} D^{\alpha}Y_1(t) = \Lambda_P - \mu_P Y_1(t) - \beta_1(1-u(t))\frac{Y_1(t)Y_2(t)}{N} - \beta_2\frac{Y_1(t)Y_5(t)}{N} \\ D^{\alpha}Y_2(t) = \beta_1(1-u(t))\frac{Y_1(t)Y_2(t)}{N} + \beta_2\frac{Y_1(t)Y_5(t)}{N} - (\mu_P+\alpha+\delta)Y_2(t) - w(t)Y_2(t) \\ D^{\alpha}Y_3(t) = \alpha Y_2(t) - \mu_P Y_3(t) \\ D^{\alpha}Y_4(t) = \Lambda_T - \mu_T Y_4(t) - \beta_3\frac{Y_4(t)Y_2(t)}{N} - v(t)Y_4(t) \\ D^{\alpha}Y_5(t) = \beta_3\frac{Y_4(t)Y_2(t)}{N} - \mu_T\, Y_5(t) \end{cases} \tag{4.1}$$

with $Y_1(0) \geq 0$, $Y_2(0) \geq 0$, $Y_3(0) \geq 0$, $Y_4(0) \geq 0$ and $Y_5(0) \geq 0$ are the initial state.

$$J(u,v,w) = \int_0^T \left[Y_2(t) + Y_5(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \right] dt \tag{4.2}$$

Where $A > 0$, $B > 0$, and $G > 0$ are the cost coefficients, they are selected to weigh the relative importance of $u(t)$, $v(t)$, and $w(t)$ at time t. T is the final time.

In other words, we seek the optimal controls u^*, v^*, and w^* such that

$$J(u^*,v^*,w^*) = \min_{(u,v,w)\in U} J(u,v,w) \tag{4.3}$$

Where U is the set of admissible controls defined by
$U = \{(u,v,w)/\ 0 \leq u_{min} \leq u_i(t) \leq u_{max} \leq 1,\ i = \{1;2;3\}\ /\ t \in [0,T]\}$.

The main point in FOCPs is to find the optimal control (u,v,w) which minimizes the following objective function:

$$J(u,v,w) = \int_0^T \left[Y_2(t) + Y_5(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t) \right] dt \tag{4.4}$$

Subject to the constraint

$D^{\alpha}Y_1(t) = f_1(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w)$
$D^{\alpha}Y_2(t) = f_2(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w)$
$D^{\alpha}Y_3(t) = f_3(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w)$
$D^{\alpha}Y_4(t) = f_4(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w)$
$D^{\alpha}Y_5(t) = f_5(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w)$

The following expression defines a modified objective function:

$$\hat{j}(u,v,w) = \int_0^T \left[H^*(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t)\right] dt - \int_0^T \left[\sum_{i=1}^{5} \lambda_i(t)\, f_i(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t) \right] dt \tag{4.5}$$

where $H(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t)$

$$H(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t) \;=\; Y_2(t) + Y_5(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t)$$

$$+\sum_{i=1}^{5}\lambda_i\,(t)\,f_i(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t)$$

From (4.4) and (4.6), we can derive the following

$$\begin{aligned} D^{\alpha}Y_1(t) &= -\tfrac{\partial H^*(t)}{\partial \lambda_1}(t) \qquad D^{\alpha}Y_2(t) = -\tfrac{\partial H^*(t)}{\partial \lambda_2}(t) \\ D^{\alpha}Y_3(t) &= -\tfrac{\partial H^*(t)}{\partial \lambda_3}(t) \qquad D^{\alpha}Y_4(t) = -\tfrac{\partial H^*(t)}{\partial \lambda_4}(t) \\ D^{\alpha}Y_5(t) &= -\tfrac{\partial H^*(t)}{\partial \lambda_5}(t) \end{aligned} \tag{4.6}$$

With the transversality conditions at time T :

$$\lambda_1(T)=0,\ \lambda_2(T)=0,\ \lambda_3(T)=0,\ \lambda_4(T)=0 \text{ and } \lambda_5(T)=0 \tag{4.7}$$

.

Equations (4.6) and (4.7) describe the necessary conditions in terms of a Hamiltonian for the FOCP defined previously.

These conditions result in a set of fractional differential equations, in terms of the variables state Y_1, Y_2, Y_3, Y_4, Y_5, controls u, v, w and Lagrange multiplier λ_i, to be solved analytical or numerically or even both.

4.2. Characterization of optimal control. To derive the necessary conditions for optimal control, the Pontryagins maximum principle [31], given in was used. This principle converts into a problem of minimizing a Hamiltonian $H(t)$ at time t defined by

$$\begin{aligned} H(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t) &= Y_2(t)+Y_5(t)+\frac{A}{2}u^2(t)+\frac{B}{2}v^2(t)+\frac{G}{2}w^2(t) \\ &\quad +\sum_{i=1}^{5}\lambda_i\,(t)\,f_i(Y_1,Y_2,Y_3,Y_4,Y_5,u,v,w,t) \end{aligned}$$

where f_i is the right side of the differential equation of the i^{th} state variable at time t.

Theorem 4.1. *Given the optimals controls (u^*,v^*,w^*) and the solutions Y_1^*, Y_2^*, Y_3^*, Y_4^* and I_R^* of the corresponding state system (4.1), there exists adjoint variables $\lambda_1(t)$, $\lambda_2(t)$,$\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ satisfying:*

$$D^{\alpha}\lambda_1(t) = -\lambda_1(t)\left[-\mu_P-\beta_1(1-u(t))\frac{Y_2(t)}{N}-\beta_2\frac{Y_5(t)}{N}\right]-\lambda_2(t)\left(\beta_1(1-u(t))\frac{Y_2(t)}{N}-\beta_2\frac{Y_5(t)}{N}\right)$$

$$\begin{aligned} D^{\alpha}\lambda_2(t) &= -\lambda_1(t)\left[-\beta_1(1-u(t))\frac{Y_2(t)}{N}\right]-\lambda_2(t)\left[\beta_1(1-u(t))\frac{Y_1(t)}{N}-(\,\mu_P+\alpha+\delta)-w(t)\right] \\ &\quad -\lambda_3(t)\alpha+\lambda_4(t)\beta_3\frac{Y_4(t)}{N}-\lambda_5(t)\beta_3\frac{Y_4(t)}{N} \end{aligned}$$

$$D^{\alpha}\lambda_3(t) = \lambda_3(t)\mu_P$$

$$D^{\alpha}\lambda_4(t) = \lambda_4(t)(\mu_T\ Y_4(t)+\beta_3\frac{Y_2(t)}{N}+v(t))-\lambda_5(t)(\beta_3\frac{Y_2(t)}{N})$$

$$D^{\alpha}\lambda_5(t) = \lambda_1(t)(\beta_2\frac{Y_1(t)}{N}) - \lambda_2(t)(\beta_2\frac{Y_1(t)}{N}) + \lambda_5(t) \times \mu_T$$

With the transversality conditions at time T: $\lambda_1(T) = 0,\ \lambda_2(T) = -1,\ \lambda_3(T) = 0,\ \lambda_4(T) = 0$ *and* $\lambda_5(T) = -1$.

Furthermore, for $t \in [0, T]$, *the optimal controls* u^*, v^* *and* w^* *are given by*

$$u^* = \min\left(u_{1\max}, \max\left(u_{1\min}, \frac{(\lambda_1(t) - \lambda_2(t))}{A} \times \beta_1\frac{Y_1^*(t)Y_2^*(t)}{N}\right)\right) \tag{4.8}$$

$$v^* = \min\left(u_{2\max}, \max\left(u_{2\min}, \frac{\lambda_4(t)}{B} \times Y_4^*(t)\right)\right) \tag{4.9}$$

$$v^* = \min\left(u_{2\max}, \max\left(u_{3\min}, \frac{\lambda_2(t)}{G} \times Y_2^*(t)\right)\right) \tag{4.10}$$

Proof. We use the Pontryagins maximum principle [31], to characterized optimal controls. Therefore, we define the Hamiltonian H as follows:

$$\begin{aligned}
H^*(t) \;=\;& Y_2(T) + Y_5(T) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) + \frac{G}{2}w^2(t)\\
&+\lambda_1(t)\left[\Lambda_P - \mu_P\, Y_1(t) - \beta_1(1-u(t))\frac{Y_1(t)Y_2(t)}{N} - \beta_2\frac{Y_1(t)Y_5(t)}{N}\right]\\
&+\lambda_2(t)\left[\beta_1(1-u(t))\frac{Y_1(t)Y_2(t)}{N} + \beta_2\frac{Y_1(t)Y_5(t)}{N} - (\,\mu_P + \alpha + \delta)Y_2(t) - w(t)Y_2(t)\right]\\
&+\lambda_3(t)\left[\alpha Y_2(t) - \mu_P Y_3(t)\right]\\
&+\lambda_4(t)\left[\Lambda_T - \mu_T\, Y_4(t) - \beta_3\frac{Y_4(t)Y_2(t)}{N} - v(t)Y_4(t)\right]\\
&+\lambda_5(t)\left[\beta_3\frac{Y_4(t)Y_2(t)}{N} - \mu_T\, Y_5(t)\right]
\end{aligned}$$

In this case, it is considered that N is constant. For $t \in [0, T]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's maximum principle [31, 25, 17, 27] such that

$$\begin{aligned}
D^{\alpha}\lambda_1(t) \;=\;& -\frac{\partial H(t)}{\partial Y_1(t)} = -\lambda_1(t)\left[-\mu_P - \beta_1(1-u(t))\frac{Y_2(t)}{N} - \beta_2\frac{Y_5(t)}{N}\right]\\
&-\lambda_2(t)\left(\beta_1(1-u(t))\frac{Y_2(t)}{N} + \beta_2\frac{Y_5(t)}{N}\right)
\end{aligned}$$

$$\begin{aligned}
D^{\alpha}\lambda_2(t) \;=\;& -\frac{\partial H(t)}{\partial Y_2(t)} = -\lambda_1(t)\left[-\beta_1(1-u(t))\frac{Y_2(t)}{N}\right]\\
&-\lambda_2(t)\left[\beta_1(1-u(t))\frac{Y_1(t)}{N} - (\,\mu_P + \alpha + \delta) - w(t)\right]
\end{aligned}$$

$$-\lambda_3(t)\alpha + \lambda_4(t)\beta_3\frac{Y_4(t)}{N} - \lambda_5(t)\beta_3\frac{Y_4(t)}{N}$$

$$D^\alpha\lambda_3(t) = -\frac{\partial H(t)}{\partial Y_3(t)} = \lambda_3(t)\mu_P$$

$$D^\alpha\lambda_4(t) = -\frac{\partial H(t)}{\partial Y_4(t)} = \lambda_4(t)(\mu_T\ Y_4(t) + \beta_3\frac{Y_2(t)}{N} + v(t)) - \lambda_5(t)(\beta_3\frac{Y_2(t)}{N})$$

$$D^\alpha\lambda_5(t) = -\frac{\partial H(t)}{\partial Y_5(t)} = \lambda_1(t)(\beta_2\frac{Y_1(t)}{N}) - \lambda_2(t)(\beta_2\frac{Y_1(t)}{N}) + \lambda_5(t) \times \mu_T$$

For, $t \in [0, T]$ the optimal controls u, v and w can be solved from the optimality condition

$$\begin{aligned}
-\frac{\partial H(t)}{\partial u(t)} &= 0 \ \Rightarrow -Au(t) - (\lambda_2(t) - \lambda_1(t))\beta_1\frac{Y_1(t)Y_2(t)}{N} = 0\\
-\frac{\partial H(t)}{\partial v(t)} &= 0 \Rightarrow -Bv(t) - \lambda_4(t)Y_4(t) = 0\\
-\frac{\partial H(t)}{\partial w(t)} &= 0 \Rightarrow -Gw(t) - \lambda_2(t)Y_2(t) = 0
\end{aligned}$$

We have

$$u(t) = \frac{(\lambda_1(t) - \lambda_2(t))}{A} \times \beta_1\frac{Y_1(t)Y_2(t)}{N}$$

$$v(t) = \frac{\lambda_4(t)}{B} \times Y_4(t)$$

$$w(t) = \frac{\lambda_2(t)}{G} \times Y_2(t)$$

By the bounds in U of the controls, it is easy to obtain u^*, v^* and w^* are given by (4.10, 4.9, 4.8) in the form of system(4.6).

5. Numerical simulation

In this section, we shall solve numerically the optimal control problem for our $Y_1Y_2Y_3Y_4Y_5$ as a fractional order model. We investigate and compare numerical results of the following control strategies. The strategies are
(i) Strategy 1: The iron fence strategy.
(ii) Strategy 2: The iron fence, Get rid and spraying pesticide strategies.

The adjoint system is solved by using the backward in time finite-difference method with terminal conditions $\lambda_k(T) = 0$, where $T = 150$ days and initial conditions $Y_1(0) = 10000$, $Y_2(0) = 8000$, $Y_3(0) = 3000$, $Y_4(0) = 30000$, $Y_5(0) = 10000$. The controls u, v and w are considered bounded and the weights in the objective functional are estimated to be $A = 100$, $B = 100$ and $G = 100$. With the initial value of control and the initial condition $X(0) = X^0$, the state solution system is solved forward in time using the finite-difference method of order four. The update of the controls is done using a convex combination of the current and previous controls

to obtain the new solution for X and λ. The method continues by using these new updates aiming at finding a fixed point (X, λ, u'). This iterative process terminates when the last and preceding iterations are negligible close and the last iteration is the solution of the optimal problem. The parameter values of the state system and the objective function are obtained from different literatures and others are estimated depending on the dynamics of population as shown in Table 1. In this paper, all plots for state variables are in the logarithmic form. The code is written and compiled in Matlab using the following data.

Paramter	Description	value in d^{-1}
Λ_P	recruitment rate of susceptible of pigs.	0.02
Λ_T	recruitment rate of susceptible of Ticks.	0.2
μ_P	Natural mortality rate in pigs	0.1
μ_T	Natural mortality rate in ticks	0.8
β_1	The rate of pigs become infected by cont with the infected pigs	0.4
β_2	The rate of pigs become infected by cont with the infected ticks.	0.2
β_3	The rate of ticks become infected by cont with the infected pigs.	0.1
α	The rate of pigs which recovered from the virus.	0.2
δ	Mortality rate due to complications	0.3

Table2: Parameter values used in numerical simulation

Since control and state functions are on different scales, the weight constant value is chosen as follows: $A = 100$, $B = 100$ $G = 100$.

For This strategy, we use only the optimal control $u(t)$.

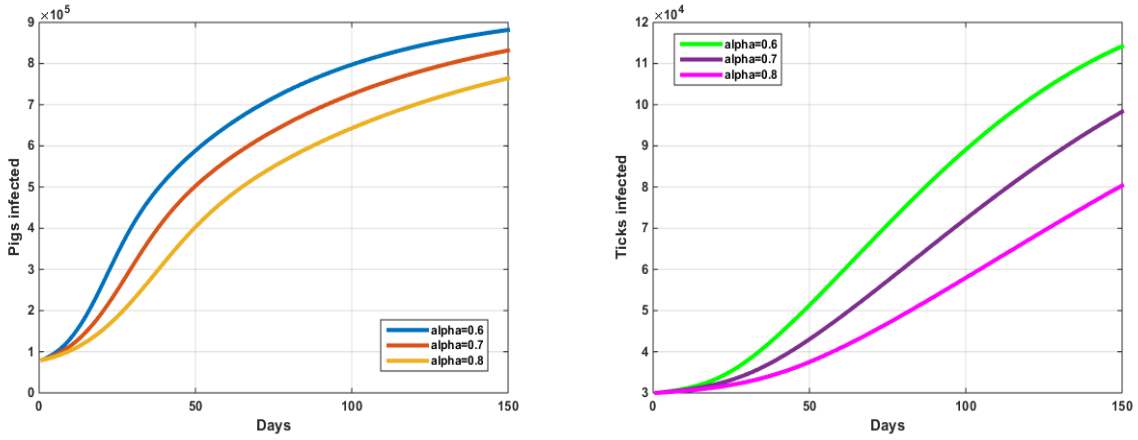


FIGURE 1. The evolution of the number of pigs and ticks infected without control

Based on Fig 1 and the parameter values in Table 1, we noted an increase in the number of pigs infected with ASFV, which leads to an increase in the number of deaths pigs, in addition to an increase in the number of ticks infected with the virus, which contribute negatively to transmitting the virus to pigs. This increase is attributed to the lack of implementation of strategies to limit the spread of the virus within urban swine.

5.1. Strategy A: Iron Fence strategy. For This strategy, we use only the optimal control $u(t)$. In figure 2, we note a decrease in the number of pigs infected with the ASFV, which led to

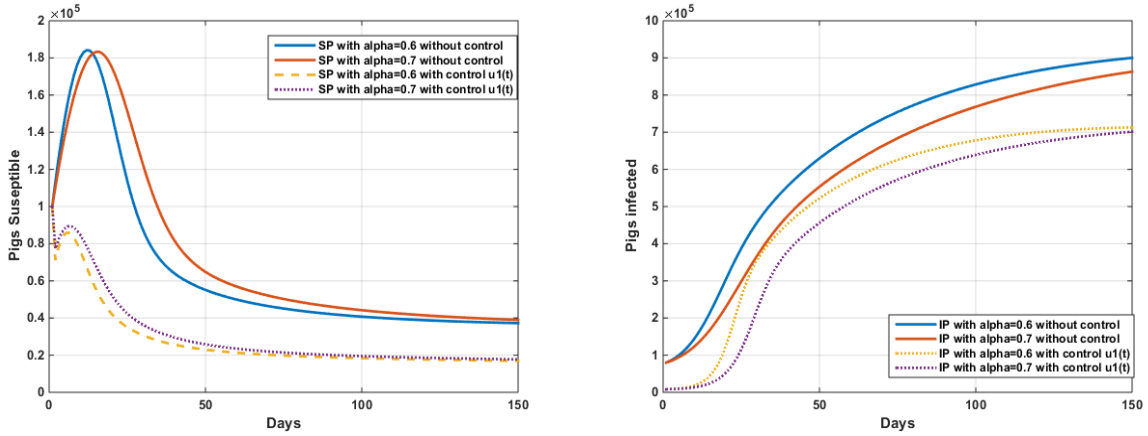


FIGURE 2. The evolution of the number of pigs and ticks infected without and with control $u(t)$

a relative balance in the number of non-solidified pigs after the application of the iron fence strategy to isolate the uninfected pigs, thus reducing the relative spread of the virus. the number of pigs infected without control is 9.10^5 $(\alpha = 0.6)$ to $7.10^5(\alpha = 0.6)$ with control and $8, 6.10^5(\alpha = 0.7)$ to $6, 8.10^5(\alpha = 0.7)$ with control.

On the other hand, the fractional derivatives play an important role in describing the effects of memory in dynamic systems. As α limits to 1, the memory effects are reduced. Furthermore, the fractional derivative order α plays the role of time delay in ordinary differential models. We see from Figure 2 that when the derivative order α is reduced from 1, the memory effect of the system increases, and therefore the number of pigs infected decreases for a long time.

5.2. Strategy B: The iron fence, get rid and spraying pesticide strategies. In This strategy, we combined three controls $u(t)$, $v(t)$ and $w(t)$.

In Fig 3, we notice an important decrease in the number of pigs infected, that after 150 days, the number of pigs infected without control is $9.10^5(\alpha = 0.6)$ to $0, 2.10^5(\alpha = 0.6)$ with control, and the number of ticks infected without control is $8, 6.10^5(\alpha = 0.7)$ to $0, 2.10^5(\alpha = 0.7)$ with control. after applying the three strategies, which are the iron fence, which is to get rid of the infected pigs, which will have important results in limiting the spread of the virus, and the results were positive.

6. Conclusion

Using fractal fractional-order derivative, we have successfully established theoretical and computational analysis for a ASFV mathematical model. a fractional mathematical model $S_P I_P R_P S_T I_T$. that described the transmission of ASFV in pig population. which included three controls.

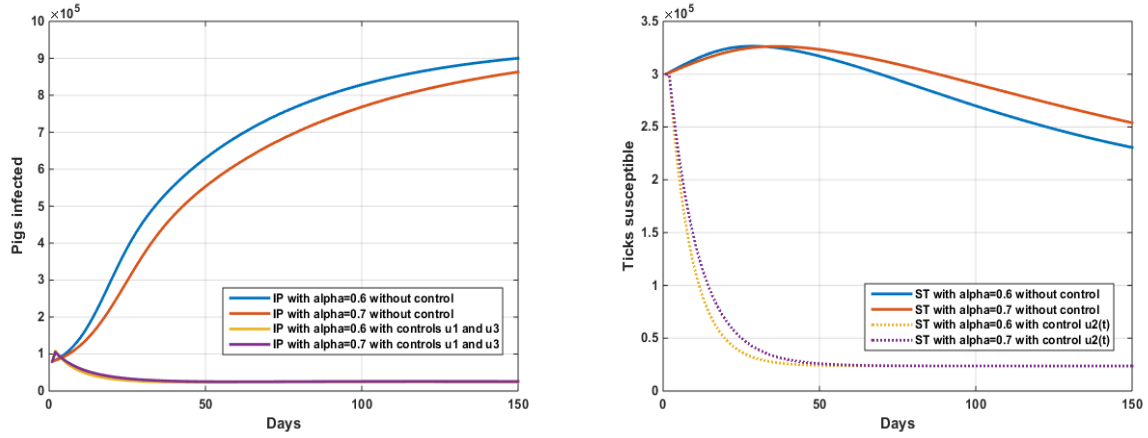


FIGURE 3. The evolution of the number of of pigs infected and ticks susceptible with and without controls $u(t)$, $v(t)$ and $w(t)$

We also proposed three controls which, respectively, represent the iron fence, spraying pesticides, and get rid as we proposed in Kouidere et al. We also studied the optimal control, led to reduce the spread of the ASF virus, by reducing the number of pigs infected and the number of ticks infected. We applied the results of the control theory and we managed to obtain the characterization of the optimal control. The numerical simulation of the obtained results showing the effectiveness of the proposed control strategies. The entire investigation of this work revealed that control of the dynamic spread rate is vital for stopping the transsmision of the spreading ASFV epidemic.

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