sciendo

Mathematical Modelling in Civil Engineering
Vol. 16-No. 4 : 1-12-2021
Doi: 10.2478/mcee-2021-0016

RELIABILITY ANALYSIS OF STONE COLUMNS BEARING CAPACITY

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Abstract: Reliability analysis and stone columns techniques are widely used, with an increasing trend, in different geotechnical engineering problems. Design methods of stone columns is still based on deterministic approach, in spite some degree of uncertainties is associated with soil and stone column parameters. In the present work, reliability analysis of bearing capacity of stone columns is examined; using three methods of reliability analysis, First Order Second Moment FOSM, Point Estimate Method PEM, and Monte Carlo simulations MCs. Parametric analysis using all variables involved in stone columns design has been conducted. Distributions of random variables involved in stone columns design are considered either as normal or lognormal. The random variables correlation effect is examined. The study has allowed to evaluate parameter uncertainty effect on reliability index of stone columns, and has permitted to figure out reduced set of parameters that must be considered as random in reliability analysis of stone columns beside sensitivity analysis.

Keywords: Reliability analysis; Stone columns; Bearing capacity; Design methods; Sensitivity analysis

1. Introduction

Stone columns inclusions are one ground improvement techniques, broadly speaking, constructed to meet various project demands and have been intensively utilised and studied in recent years, for example analytically and experimentally. But all these studies methods attracted the attention of researchers with in traditional deterministic approach merely, in the present study we will studied by probabilistic approach, with the use of reliability methods presented in [2], [5] and [12], the effects of uncertainties in stone columns bearing capacity design. Stone columns are uses with success in large number of case. However, their uses are not a hundred percent successful. In some cases their use without precautions ended with failure [1]. In the absence of reliable in situ data this can be a major source of error, but likely always to be on the conservative side [10].

The safety of civil work constitutes one of the main subjects under development, the application of partial safety factors by means semi probabilistic approach, based on Solicitation – Resistance concept, is appeared recently with the contemporary design codes of construction in ultimate limit state as the *BAEL* for concrete constructions types and the *Eurocodes* for generalizations in all kinds of civil engineering constructions.

However, the traditional approach used to design was based on global and unique predetermined safety factor; this factor does not give any information about the importance of the design parameters, and took the same value for all design conditions, not consider any variation in uncertainties; reliability analysis methods provide as an rational safety measured and give parameters necessary for a rational decision as the failure probability, reliability index and the parametric sensitivities.

In the last years, significant progress has been made in the application of reliability theory based above probabilistic approach of calculation in geotechnical engineering, for assumed the effects of uncertainties in different dimensioning parameters and obtained failure probability. This application is considered as well as a rational and economical approach of safety design.

The recent geotechnical books of Baecher and Christian[2], Griffiths and Fenton [6] and Phoon [12], contains presentations of the reliability analysis methods in the field of geotechnical

engineering design, these methods have been well reviewed, documented and largely applied. This paper has endeavoured to the reliability of stone columns bearing capacity.

The purpose of this study to reach two major goals: firstly, to identify the effects of uncertainties on parameters; secondly, to select method of reliability analysis, type of distribution probability and correlation with parameters more adapted for stone columns design.

2. Stone Columns Bearing Capacity Design

There are many different practical applications of stone columns in the field of ground improvement, there are used to: increase the bearing capacity of weak and soft soils; decrease the excessive settlements by accelerating the time of consolidation; and also reduce even eliminate the liquefaction risk in susceptible prone regions; generally the primordial goal is to assure global stability. A comprehensive discussion and evaluation of all these roles is given by Barksdale and Bachus [3] via numbers of real projects and cases histories. In this review we bases above this document of Federal High Way Administration, who represent an excellent source of references and with the contemporary literature as [4] and [6].

This article emerge as in pioneering work for study of stone columns, and the choice of in analysis of bearing capacity of stone columns by probabilistic approach is due to the following reasons: in the weak and soft soils the bearing capacity is considered as a problem of great significant; the practice great importance into uses of stone columns to improves soil characteristics; the increase popular in present day geotechnical engineering; the economical interests and benefits provide as support in compressible and cohesive soils; and especially the high variability and their significant uncertainties in geotechnical parameters of soft soils.

When the defined of the performance function is the first and crucial step in reliability analysis, the deterministic study is necessary. A variety of method for calculation of bearing capacities of stone columns have been proposed since the previous decades of ago century, the purpose of this section is to determine the functions of performances for two models of bearing capacity: single isolated and group, and we are interested only in these functions in each models.

2.1 Bearing Capacity Of Single Isolated Stone Column

The bearing capacity of a single isolated stone column surrounded by a soft soil was studied by a number of theories presented for predicting the improved ultimate bearing capacity. The bulging failure is usually developed in the single isolated stone column, and the cavity of expansion of Gibson and Anderson and Vesic are widely used by all methods of design stone column improved bearing capacity in frictionless and friction cohesion soils respectively.

The analysis of the behaviour of isolated stone columns proposed by Hughes and Withers provided the basis of the prediction [10] of the bearing capacity, it is based above Gibson and Anderson cavity expansion theory, when an expanding cylindrical cavity was considered similar to the cavity developed during the pressuremeter test, and the stone column material (stones) of internal friction angle (φ_c) is assumed in a plastically state with an coefficient of passive pressure (K_p) equal to:

$$K_p = \frac{1+\sin\varphi_c}{1-\sin\varphi_c} \tag{1}$$

The ultimate undrained lateral stress σ_3 can be derived as follows:

$$\sigma_3 = \sigma_{ro} + C_u\left[1+\ln\frac{E_s}{2C_u(1+\nu_s)}\right] \tag{2}$$

where σ_{ro} : the total initial lateral stress in situ, it is possible to measured directly in situ or is calculated as follows: $\sigma_{ro} = \gamma z + 2C_u$, (C_u) unconsolidated undrained cohesion and (z) is the average depth of the bulge.

The ultimate bearing capacity is expressed as:

$$q_{ult} = \left\{\sigma_{ro} + C_u\left[1 + \ln\frac{E_s}{2C_u(1+\upsilon_s)}\right]\right\}\left(\frac{1+\sin\varphi_c}{1-\sin\varphi_c}\right) \quad (3)$$

Where E_s and υ_s Elastic modulus and Poisson's ration of the soil, (C_u) unconsolidated undrained cohesion

In which this bearing capacity is appeared as the ultimate stress that can be applied in soil improved by stone column technique.

2.2 Bearing Capacity Of Stone Columns Group

An isolated single column compared to a stone column group has a slightly smaller ultimate load capacity per column than in the group (the effect of group) [3]. A simple and conservative analysis was developed by Bell, extended by Terzaghi, and further modified by Sowers [14].

The ultimate bearing capacity of the group can be determined by approximating the failure surface by two straight rupture lines [3], and formed a triangular block as represented by figure 1:

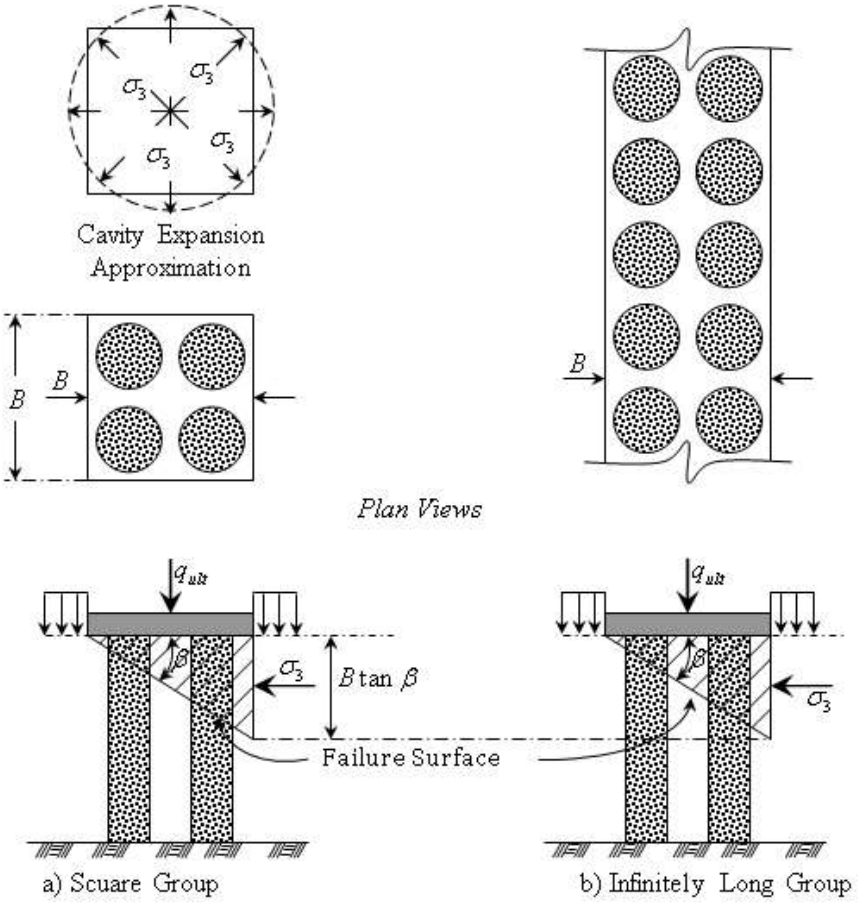


Fig.1 - Stone columns group analysis.

The average shear strength parameters within this block are given as follows:

$$\left[\tan\varphi\right]_{avg} = \mu_c a_c \tan\varphi_c \quad (4)$$

and:

$$C_{avg} = (1-a_c)C_u \quad (5)$$

The planer rupture surface is inclined at an angle β given by:

$$\beta = 45 + \frac{\varphi_{avg}}{2} \tag{6}$$

The average lateral confining pressure σ_3 equal:

$$\sigma_3 = \frac{\gamma_{sat} B \tan\beta}{2} + 2C_u \tag{7}$$

$$q_{ult} = \sigma_3 \tan^2\beta + 2C_{avg} \tan\beta \tag{8}$$

This last equation, according to Barksdale and Bachus [3], gives results reasonably close to the Terzaghi local bearing failure theory.

3. Reliability Analysis

When a wide range of civil works practice use stone columns as technique of ground improvement for weak and soft soils, the estimate of safety for conception of these columns is important topics of research for geotechnical engineers and appeared as a very important problem of design. The current methods for assessing conception and design of stone columns based above the deterministic approach, recognizing the limitation of this approach, reliability analysis methods based above the probabilistic approach was recently used by number of authors, and has been employed with boost for several geotechnical problems. In this reliability analysis, the effect of uncertainties of a wide range of such parameters as the geotechnical properties, geometrical variables and loads parameters are considered, and result from this reliability analysis are provided in graphical. This reliability analysis is examined via two model of single isolated and group of stone column using probabilistic approach; this approach is able to take into the uncertainties effects. Calculation of bearing capacity using both deterministic and probabilistic approaches is shown through examples of singular isolated and group of stone columns.

In reliability analysis for characterizing the designing parameters (soil properties, load effects, and geometry parameters) a statistical description is requires, this it is given in Table 1. All random variables are assumed to be characterized by normal and lognormal distributions, these distributions are defined by means values, defined by table 1, and standards deviations, equal to: μ / COV, and for each one of these cases, random variables are regarded as independent in first, and correlated according correlation matrix (R) defined below. Traditionally, the stone columns bearing capacity is designed via a deterministic approach; this approach has been widely used the notion of safety factor for the reduction of uncertainties associated with the evaluation of parameters. The safety factor for bearing capacity is determined by comparing Resistance against applied action and calculated by the following formula:

$$Fs = q_{ult} / q \tag{9}$$

In which: q_{ult} represent the ultimate bearing capacities of soil improved by the use of stone columns, calculated by (3) and (8) equations, and q is the allowable bearing capacity, who can be obtained as: (1) $q = Q_v / \left(B - 2\delta\right)^2$ (B represent foundation width) , for single isolated stone column subjected to vertical (Q_v) and returned moment of rigid shallow strip foundation (raft) (M) combined loading of pad foundation, where δ is the eccentricity of the load, and equal to $\psi M / Q_v$, when ψ is an coefficient of increasing of 1.5 for an single and isolated column, for this combined loading the repartition of stress is bring to rectangular equivalent repartition of Meyerhof solution, and defined clearly effective contact width, and (2) $q = Q_v / B^2$ for stone columns group submissive a vertical load uniform repartition assured by rigid shallow strip foundation (raft).

Substituting the formulas (3) or (8) in (9) gives the limit states design of stone columns in deterministic study (Fs), and were used for formulating the performances functions in reliability analysis to calculate the reliability index (β) as defined by Hasofer and Lind [8], the performance function (limit state) usually was defined as:

$$g(X) = Fs - 1 \tag{10}$$

The case of $g(X) = 0$ separated the stable ($g(X) > 0$) and unstable ($g(X) < 0$) states, or the safe and failure regions.

Calculation of the deterministic safety factors are makes by values of fourth column of Table 1, gives values of 2.00 for single isolated and group of columns modes, typically these values are very accepted by the recommendations of [4].

The uncertainties associated with designing parameters are not incorporated in a systematic way, and the safety factor is presented as not excellent factor of reliability evaluation because it is chosen on the basis of subjective decision. The probability of failure (p_f) is appeared as an objective measure of reliability and given then by the following integral:

$$p_f = P\left[g(X < 0)\right] = \int_{g(X)<0} f_X(X)dX \tag{11}$$

The reliability index (β) is a measure of the reliability of an engineering system that reflects both the mechanics of the problem and the uncertainty in input variables [11], it is instead of the failure probability by the standard normal distribution function as:

$$\beta = -\Phi^{-1}(P_f) \tag{12}$$

In the equation (11), $f_X(X)$ is the common probability density function of the vector formed by the vector of continuous random variables (X), and (Φ^{-1}) is the standard normal cumulative distribution inverse function. These two measures of reliability assured a consistent level of safety, and incorporate the uncertainties associated with designing parameters in rational framework of probability theory.

Table 1

Values of parameters used in the analysis

Parameter	Symbol	Variable	Mean value
Commons parameters			
Undrained shear strength	C_u	Random	22 kPa
Angle of internal friction of stone	φ_c	Random	40°
Unit weight of soil	γ_s	Random	19 kN/m^3
Diameter of the stone column	D_c	Random	1 m
Young's modulus of the surrounding soil	E_s	Random	2000 kPa
Height of stone column	H	determinist	15 m
Single isolated stone column model			
Vertical load	Q_v	Random	300 kN
Poisson's ratio of the surrounding soil	ν_s	Random	0,45
Turned moment	M	Random	230 kN.m
Isolated shallow foundation width	B	determinist	1 m
Stone column group model			
Spacing of stone columns "centre to centre"	S	determinist	2 m
Vertical load	Q_v	Random	3500 kN
Young's modulus of the stone column	E_c	Random	60 MPa
Strip foundation width	B	determinist	4 m

Some of the random variables are assumed to be correlated, as follows. The diameters of columns and the Undrained shear strength are reasonably negative correlated with coefficient of correlation equal to $\rho_{C_u D_c} = -0,5$ [this value indicate that there is a linear correlation and the D_c value decreases as the C_u increase (The diameter of the column may vary over its length, depending on the resistance of the different layers penetrated)]. The diagonal terms of the correlation matrix (R) are equal to one, and the rest of the correlation properties are set to zero. Foundation dimensions and height of stone columns and depth of improved soil layer can be regarded as deterministic values with no variance.

The determining of probability of failure is one of the topics of intensive research in reliability analysis, in practice as the determining is not easily possible task, thus approximate techniques have been developed; FOSM, PEM and MCs are efficient methods in geotechnical engineering, so that they can be used and examined now. The above two deterministic models of single isolated and group column were studied with probabilistic methods: First Order Second Moment (FOSM), numerical PEM and simulations MCs. For PEM method the calculations were performed using EXCEL spreadsheet, and MATLAB computer software was chosen in this study for programming FOSM and MCs. These three probabilistic methods (methods FOSM, PEM and MCs) are present the II and III levels (fully probabilistic with approximations and fully probabilistic) of reliability analysis.

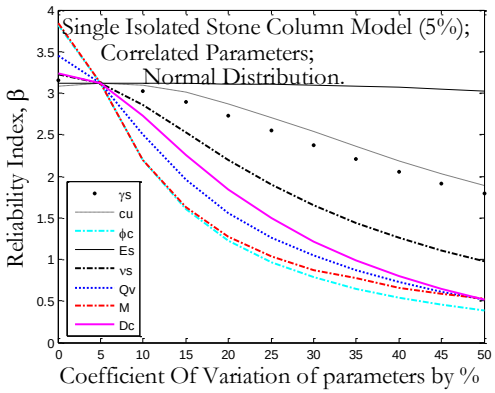
Referring to Appendix figures, the FOSM, PEM and MCs results are very similar. Consequently the use of PEM is sufficient, and in this reliability analysis on the basis of numerical PEM.

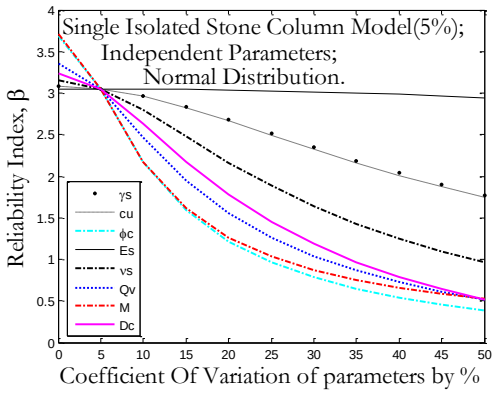
The application of a reliability analysis based on the probabilistic approach applied to quantification the weight of uncertainties in design and conception of stone columns, is presented in this section. The analysis employed herein takes into account the effects of uncertainties in geotechnical, geometrical and loading parameters. Figures 1 and 2 presents the results of the variations in the reliability indices versus the coefficient of variation of a selected random variable are obtained by varying of a selected random variable are obtained by varying the (COV) % at constant mean value of the selected random variable. The (COV) of all other random variables are arbitrarily fixed at 5%, from these figures, it can be seen that, for any given cases, a decrease in reliability index results in an increase in the (COV) (uncertainties of parameters).

From Figures 2 and 3, we can notice that the reliability indices values are widely different (range from the order of four in the model of single column but they are from the order of ten in the model of group), just as that the safety factor of two models are equivalent (Fs=2).

3.1 Reliability Analysis Of Single And Isolated Stone Column

The figure 1 summarizes the results of parametric reliability index, for the case of single isolated stone column model using eight random variables.





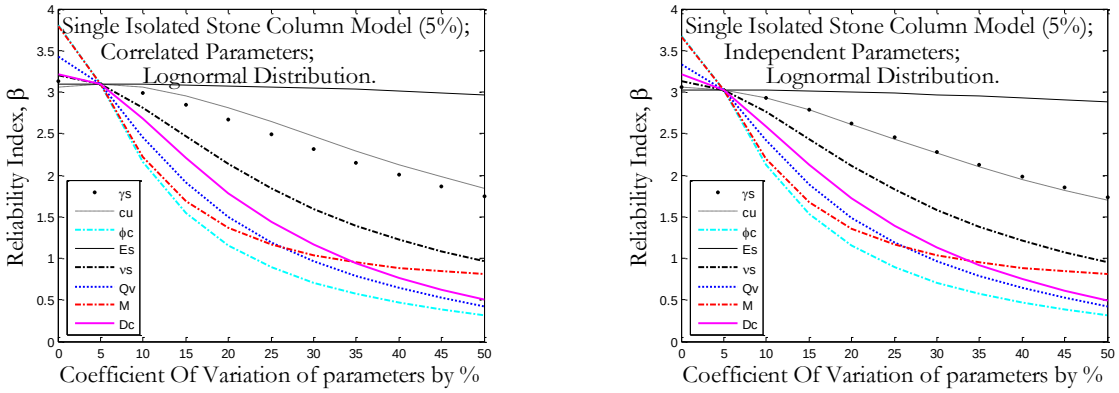


Fig.2 - Variation of reliability index versus parameters COV single and isolated

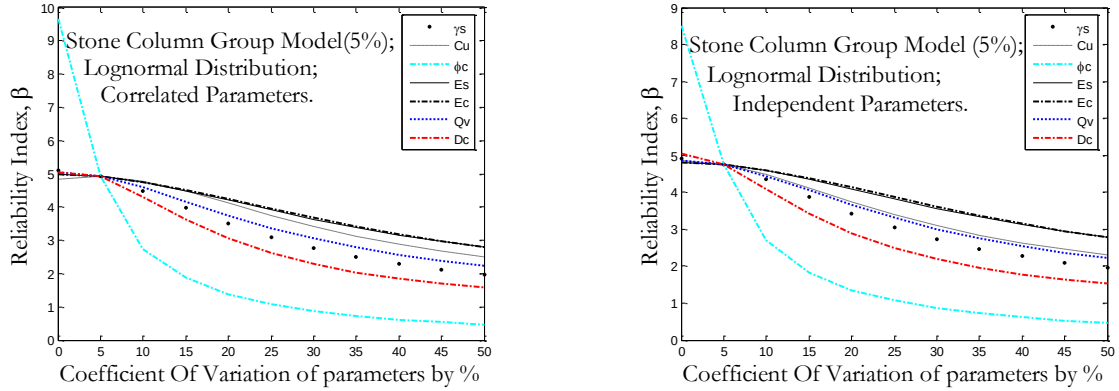
For study the influence of different parameters on the design of stone columns, changing the amount of uncertainties in these parameters is expected to have a considerable impact on the reliability. From the graphs of figure 2, it is clear that the reliability index is highly sensitive to the uncertainties in the friction angle of the stones, φ_c, the loading parameters, M and Q_v, and the diameter of column, D_c; this index is comparatively less sensitive to the uncertainties in the Young's modulus of the surrounding soil, E_s, the undrained shear strength, C_u, Unit weight of soil, γ_s, and Poisson coefficient ν_s.

Comparison between normal and lognormal distributions in calculation of reliability indices appeared that only the turned moment variable is especially affected by the distribution law, also for evaluate the impact of the correlation between parameters, it can be seen that the reliability indices calculated with correlated or independents parameters, theirs are very little difference in all cases and parameters. For evaluate the impact of the correlation assumptions on the effects of stone column model.

Reliability analysis of group of stone columns

According to results of parametric reliability analysis in this Figure 3, the following remarks are drawn:

- The reliability index depends strongly on the stone friction angle, loading parameters (Q_v and M) and diameter of column;
- The reliability index is slightly sensitive to changes in the probability distribution law, and also the effect of correlation matrix is not many affecting.



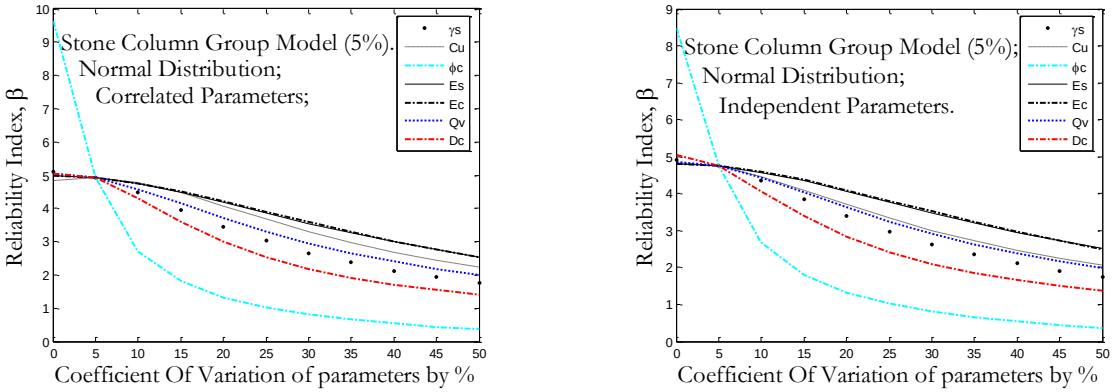


Fig.3 - Variation of reliability index versus parameters COV stone column group model.

4. Sensitivity Analysis And Reduction Of Random Variables

A sensitive analysis improves the quality of any study. Under a sensitivity analysis, not only the solution of the problem is sought but also how sensitive it is to data changes [5]. In reliability analysis of stone columns bearing capacity the number of random variables is large, the sensitivity analysis used to reduce the number of random variables, and to determine the dominate parameters. Based on this and the obtained result of the previous section, a question is arisen. Can replace all these random variables by few limited and more influential? Toward answering this question, it can be considered the example of Table 2.

Table 2 summarizes the parametric studies necessaries to sensitivity analysis, the mean friction angle and mean diameter column have been changed in case 1 to case 2, the (COV) of all parameters are varied systematically according to the two last columns of the Table 2.

Table 2

Parameters values considered in the sensitivity analysis

Parameter	Symbol	Mean value		Cov (%)	
		Case 1	Case 2	Case 1	Case 2
Commons parameters					
Undrained shear strength	C_u	22 kPa	22 kPa	50	70
Angle of internal friction of the stone	φ'_c	40°	42°	0-50	0-50
Unit weight of soil	γ_s	19 kN/m^3	19 kN/m^3	10	15
Diameter of column	D_c	1 m	1,2 m	0-50	0-50
Young's modulus of the soil	E'_s	2000 kPa	2000 kPa	40	40
Single isolated stone column model					
Vertical load	Q_v	300 kN	300 kN	10	15
Poisson's ratio	ν_s	0.45	0.45	5	5
Returned moment	M	230 kN.m	230 kN.m	10	15
Stone column group model					
Vertical load	Q_v	3500 kN	3500 kN	10	10
Young's modulus of the stone column	E'_c	60 MPa	60 MPa	10	15

This section is devoted to the reduction of the number of random parameters necessary for the reliability analysis of stone columns bearing capacity, by the identification of important and less signification parameters in order to reduce the computational effort.

Using the (USArmy Corps of Engineers) document [11], the sensitivity analysis conveys the relative importance of the random variables in affecting reliability. The omission sensitivity coefficient is used in this paper. It gives the relative error in the reliability index when a random variable is replaced by its deterministic mean value.

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4.1 Reduction parameters of single and isolated stone column

A summary of sensitivity analysis results for independents parameters normal distributed are shown in graphs of Figure 4 for single isolated model of stone column. Comparison have been run with considering all five variables as random, used as reference, and compared with those combinations when the number of random variables is limited at two, three, four or five. The results show that the reliability index is more affected by the number of random variables, and it is estimated adequately using the combination of five principal random parameters mentioned in legends of graphs.

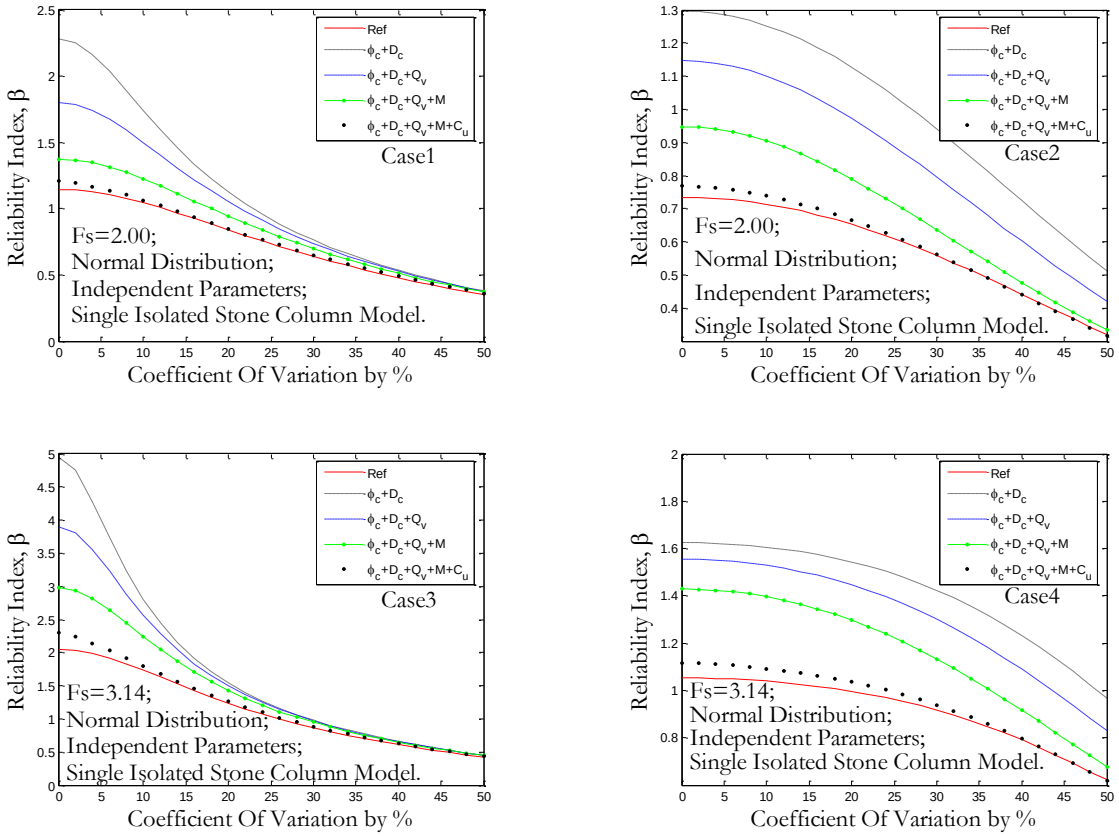


Fig. 4 - Reduction of random parameters - single and isolated stone column model.

The data from Table 3 and the data from graphs reported in this Figure 4 suggest that the reduction of random parametres to five is possible and considered as safe reduction in all cases

4.2 Reduction parameters of group of stone column

Quintessential of sensitive analysis is the reducing of the random variables number involved in the reliability analysis. In the stone columns group model, the reliability index variations computed by considering the entire eight variables as random is named “ref” and compared with the reliability index variations computed with a limited number of random variables; the results are

shown in Figure 4. Where, it is observed that the reduction is considered acceptable if the entire five variables mentioned in legend are taken random, the other remain are taken as deterministic.

Table 3

Sensitivity analysis summary

Random parameters combination used	Reduction				Appropriate random parameters combination
	Case 1	Case 2	Case 3	Case 4	
Single and isolated stone column model					
$D_c + \varphi_c$	▲	▲	▲	▲	Angle of internal friction of the stone + Diameter of column + Vertical load + Lumped moment + Undrained shear strength
$D_c + \varphi_c + Q_v$	▲	●	▲	▲	
$D_c + \varphi_c + Q_v + M$	●	●	▲	●	
$D_c + \varphi_c + Q_v + M + Cu$	●	●	●	●	
Group of stone columns model					
$C_u + \varphi_c$	▲	▲	▲	▲	Angle of internal friction of the stone + Diameter of column + Vertical load + Unit weight of soil + Undrained shear strength
$C_u + \varphi_c + Q_v$	▲	▲	▲	▲	
$C_u + \varphi_c + Q_v + \gamma_s$	▲	▲	▲	▲	
$C_u + \varphi_c + Q_v + \gamma_s + C_u$	●	●	●	●	
● :acceptable reduction ▲ : not acceptable reduction					

As shown in these two last Figures 4 and 5, it is clear that the graphs of the combination of five random variables has coincides with references graphs for the different (COV) values, therefore the parameters of these reduction combination are needed judge random in any reliability analysis of stone columns bearing capacity.

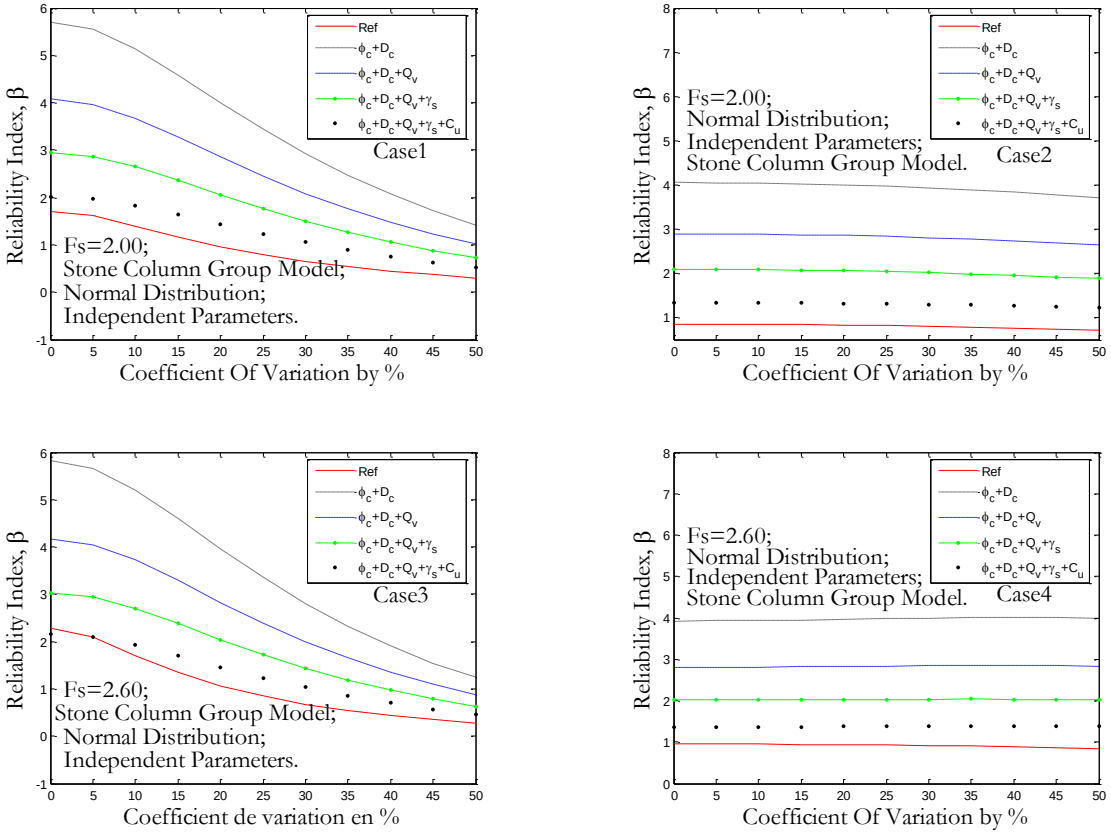


Fig. 5 - Reduction of random parameters - group of stone column model.

Conclusion

The effect of the uncertainties in geotechnical design of stone columns was addressed in this paper by reliability analysis under probabilistic approach. As epilogue, and on the basis of the results and analysis presented, the following point's conclusions can be drawn from this research study:

- for safe and economical designs of bearing capacities of stone columns, an probabilistic approach for representation of uncertainty effects is recommended;
- the reliability level decreases with increasing range of uncertainties, thus for reliable design an decrease of uncertainties as possible is needed;
- the form of probability distribution does not have a significant effect on the calculated reliability, the considering of the correlation increase the reliability levels;
- a great influence of the uncertainties in the friction angle of stones, diameter columns, and loading parameters, affected the reliability level of stone columns bearing capacity design;
- based above a sensitivity analysis, certain parameters as (E_s, ν_s, E_c) were treated as deterministic values, since the sensitivity analysis indicated that the variation of reliability index with coefficient of variation of these elasticity parameters is unimportant in reliability design.

From the probabilistic analysis of the bearing capacity problem it is possible to compare advantages and disadvantages of well-known probabilistic methods. In spite of some limitations, the Point Estimate Method shows to be a simple, but powerful technique as it matches Monte Carlo simulations extremely well. The Point Estimate Method will be used for a further study as it is much faster than Monte Carlo simulations. As an alternative to the Point Estimate Method, the First Order Second Moment method was applied, which gave results similar to those of the Point Estimate Method.

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APPENDIX

This appendix presents comparisons between the results of three reliability analysis methods, the aim of this appendix is to compare the results of FOSM and PEM with those obtained using 200000 sampling for MCs, it is seen from Figure 6 that the results of these methods are very agree.

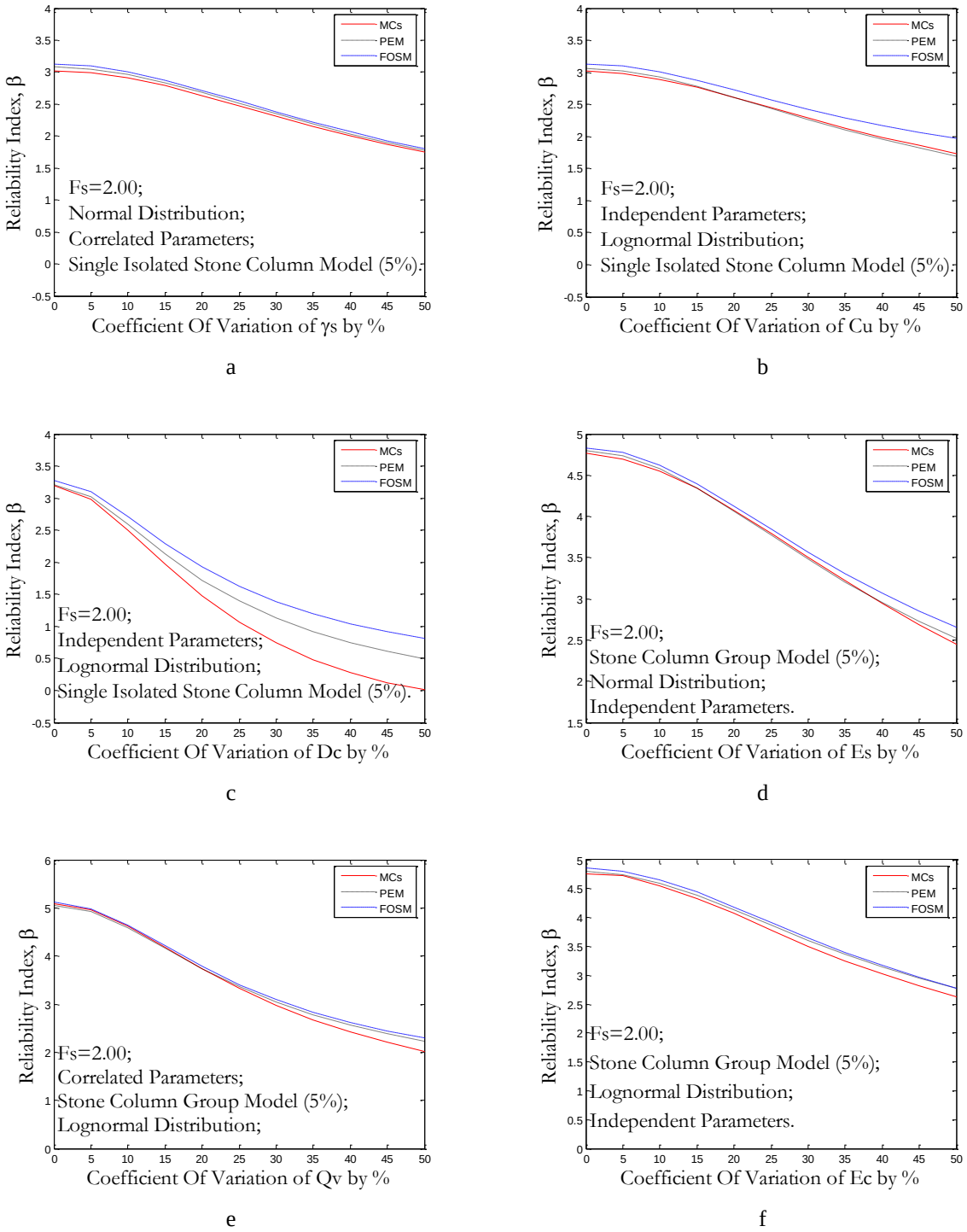


Fig. 6 - Comparison between reliability indices results computed by FOSM, PEM and MCs reliability analysis methods.