sciendo International Conference KNOWLEDGE-BASED ORGANIZATION
Vol. XXVI No 3 2020

MEASUREMENT OF GEOMETRIC FEATURES OF SPHERICAL OBJECTS

Branko SOTIROV, Danko TONEV

University of Rousse, Bulgaria
bsotirov@uni-ruse.bg

Abstract: *Spherical shaped objects are some of the most interesting and difficult to measure objects in dimensional metrology. Made of steel, alloys, ceramics, glass, rubber or plastics, they are important elements of precision bearings, pumps, valves, flow meters, medical, measuring, automotive, aerospace and other equipment, used for polishing, grinding, etc. Quality requirements, including their geometric accuracy, are high. A spherical object is defined geometrically by a centre, radius, and spherical spatial surface. There are no uniform rules and definitions for measuring these characteristics, and there is no standard definition of a form deviation. A number of solutions and publications are known to address the problem in practice, which, for the time being, do not allow a uniquely substantiated definition of the derived geometric elements and methods for their measurement. Full measurement of the deviation from sphericity, for example, is impossible. The question of the type of its associated element satisfying certain standard criteria remains open. The accuracy and reliability of measuring spherical objects is related to finding a reasonable solution to the outlined issues.*

Keywords: form error, sphericity evaluation, minimum material condition, CMM measuring

1. Introduction

The objects having spherical form are among the most interesting and hard to measure objects in dimensional metrology. Made out of steel, alloys, ceramics, glass, rubber or plastic materials, they are key elements of precise bearings, pumps, valves, flow meters, medical, measurement, automotive, aerospace and other equipment, used also for polishing, grinding, etc. Very often, the requirements for their quality characteristics, including their geometric accuracy, are high. In order to satisfy such requirements known contact and contactless methods are used. They are realized by using various measurement devices – spherometers, goniometers, roundness gages, specialized stands, laser trackers, etc.[2, 3, 11, 14]. These methods and devices provide solutions to specific problems but at the same time are subject to a number of limitations and compromises which hinder an unambiguous and justified solution of the complex requirements for the geometric accuracy of the controlled spherical objects. The most widely used and the most productive method used in dimensional metrology is the coordinate method which is implemented in coordinate-measuring machines (CMM).

2. Description of the problem

It is known that the sphere is a spatial surface obtained by rotating a circle with a radius and a centre, called radius and centre of the sphere. The geometric accuracy of real spherical objects is determined by the deviations of the radius/diameter and/or

DOI: 10.2478/kbo-2020-0133
© 2015. This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 3.0 License.

form deviation of the spherical surface.
The complete measurement of the latter, for example, similar to the complete measurement of the cylindricity deviation is impossible because it is practically not possible to obtain measurement information about all countless points and sections of the spherical surface that define its diameter and form deviation. This is why, among all compromises it is most acceptable and convenient to use CMMs, especially 4-coordinate CMMs with a rotary table. This approach also has certain shortcomings that impact the measurement accuracy. The most important among them are:

- Instrumental error of the CMM;
- Error of the indirect measurement method, the limited number of measurements and the applied assessment algorithms;
- Deviations of the spherical surface, that is used simultaneously as a measurement datum and as a source of measurement data;
- The traditional set-up errors, the errors caused by the forces acting during measurement, and thermal deformations.

The measurement and assessment of the geometric features of the sphere is often related to the need to practically determine the virtual centre of the sphere. Even the single measurement of the diameter, defined in some documents as 'the distance between two planes that are parallel and tangential to the spherical surface'[6], which is easily realised in practice, is not representative for a dimension obtained as the distance between two random, albeit opposite points. For such points, there is no guarantee that the radii which are perpendicular to the tangential surfaces, lie on line. Even more crucial is the issue to define the deviation from sphericity which is not even standardised for some objects. In[6] and several renowned publications[1, 7, 10]this deviation is defined based on the „minimum condition principle" (an associated element of the minimal zone MZ) – Chebyshev criterion, and is related to mandatory determining the centre of two homocentric(concentric) associated spheres, limiting the smallest zone of the extracted points of the real spherical surface (minimum zone sphere MZS [7, 10]i.e.:

$$MZS = R_{MAX\,(MZC)} - R_{MIN\,(MZC)} \qquad (1)$$

where, $R_{MAX(MZC)}$ and $R_{MIN(MZC)}$ are the radii of the two homocentric spheres, determining the minimal zone MZ, and MZC (X_0, Y_0, Z_0) is the unknown centre of the minimal zone.
Considering the limited number of measurements and the abovementioned objective sources of uncertainty it can be argued that every extracted centre of the sphere C_i (X_i, Y_i, Z_i) differs from the geometric centre, and is positioned in a kind of centroid which contains n centres of extracted spheres which would be obtained as a result of repeated measurements (Fig.1). For all sections passing through the centre of the sphere (meridian sections) the width of this zone is identical [2, 10], equal to the minimal zone of the points of the real spherical surface, and can be considered a section of this zone with any meridian plane. It can be expected that the centroid has a nearly spherical shape [7]. How to determine the centre of both homocentric (concentric) spheres is the research purpose in a number of publications that also present various algorithms [7,9,10], but the issue remains debatable. The main shortcoming of the proposed solutions is that their real application is subjected to a number of conditions such as a big data base (measurements), including the measurement of angles, complex algorithms, etc. which increase measurement accuracy, but at the same time significantly decrease productivity.

2. Method of successive approximations

The recommended minimum number of

points necessary for assessing the geometric accuracy of a sphere is 4, 5 or 6, equally spaced along the largest (equatorial) circle[4]. Initially, two groups of consecutive measurements are made for 4, 5 or 6 points, positioned at equal central angles from one another, in two relatively perpendicular

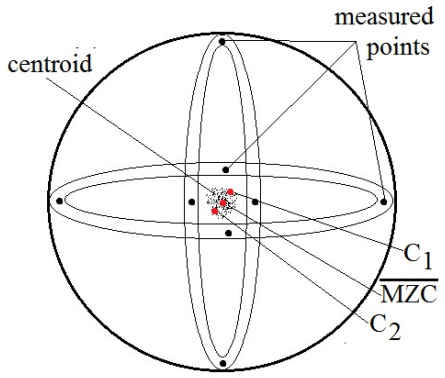


Figure 1: Measurement scheme

meridian planes (Fig. 1). Also, the method described in [11] could be applied, where the optimal configuration of 4 points on the sphere is determined, which ensures the minimum dispersion of the estimate of the radius, and the standard deviation of the estimation error is calculated. However, such an approach would create certain calculation problems, combined with assessment of the form error [1].

The two centres C_1 (X_1, Y_1, Z_1) and C_2 (X_2, Y_2, Z_2) of the two extracted spheres are determined. The resulting values are two independent random variables, obtained as a result of independent random measurements. Geometrically they belong to the zone of the centroid, and statistically they conform to the equal probability law(uniform distribution). This allows us to determine the coordinated of the extracted centre of the minimal zone as the average of the coordinated of $C_1$and C_2, and then to correct the radii R_i and to recalculate the coordinates Xi, Yi and Zi and the measured points in relation to the obtained $\overline{MZC}$, that lies halfway on the segment C_1C_2:

$$R_i = \sqrt{[(X_i - X_{\overline{MZC}})]^2 + [(Y_i - Y_{\overline{MZC}})]^2 + [(Z_i - Z_{\overline{MZC}})]^2} \quad (2)$$

where, $X_{\overline{MZC}}$,$Y_{\overline{MZC}}$, $Y_{\overline{MZC}}$are the coordinates of $\overline{MZC}$, i = 1…n, n is the number of the measured points.

Obviously, this operation will improve the positioning accuracy of$\overline{MZC}$ 1.41 times, which now presents the centre of a new coordinate system. The positioning accuracy of $\overline{MZC}$would be further improved upon possible subsequent iterations.

The issue is substantially different for the initial steps when using CMMs with rotary table. Here the first two groups of measurements have to be done in a continuous mode, in two mutually perpendicular meridian planes, following the scheme for measurement of roundness deviation. The number of scanned points has to be as large as reasonably feasible, and the centres C_1 and C_2 determined upon measurement represent physically the centres of two circular diagrams (polar profiles or graphs). The obtained displacement due to the change of datum's of the sphere for the second measurement needs to be corrected based on the two common intersecting points of the large circles of the perpendicular planes (Fig. 1), after which is determined the dimension of the segment C_1C_2. The middle of this segment is determined, and following dependence (2) the radii of the measured points of the two large circles are determined. Then, they are adapted to the coordinate system with a centre $\overline{MZC}$.

The issue with measurement precision cannot be ignored and as a rule it is related to the number of measured points. In this case, the necessary minimal number of measured points n_{min} can be determined using dependence:

$$n_{min} \geq \frac{t^2_{\alpha;n-1}}{[\delta]^2} S^2 \quad (3)$$

where, S is the unbiased estimate of the standard deviation of the corrected radii R_i of the measured points, $t^2_{\alpha;n-1}$–the value of Student's t-test coefficient for significance level$\alpha$and degrees of freedom n-1, n – the number of initially measured values, and $\delta$is the permissible measurement uncertainty.

By means of using the so defined centre and the coordinates of points recalculated against this centre, we determine the minimal zone of the extracted points, which is limited by both homocentric (concentric) circumscribed and inscribed spheres with radii $R_{MAX(MZC)}$ and $R_{MIN(MZC)}$. The numerous methods used for this purpose can be attributed to the algebraic methods (including white-box and black-box methods) and computational geometry-based methods [1].

The determined radii R_{MAX} *(MZC)*and R_{MIN} *(MZC)* represent two random normally distributed variables, whose distribution follows the difference modulus law:

$$x = |R_{MAX} - R_{MIN}| \quad (0 \leq x \leq \infty)$$

It is known that this law is applicable to the distributions of a range of form deviations of any type of surface, assessed as the difference between the measured limit values of dimensions, the parallelism deviation of two surfaces, the deviation of an axis in relation to a surface, the thread pitch deviations, etc.

This law has two parameters:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\left[e^{-\frac{(x-a)^2}{2\sigma^2}} + e^{-\frac{(x+a)^2}{2\sigma^2}}\right] \quad (4)$$

where, $\sigma$is the standard deviation, and $a = x_{max} - x_{min}$ is determined experimentally.

Dependence (4) allows determining the probability of occurrence of specific values of the deviation from sphericity.

3. Discussion

It is known that CCMs most widely apply the least-squares technique. The least-squares method is robust and fast. However, when assessing sphericity, the method is non-linear and as such it require sequal angular spacing of the measured points.

Several methods are proposed to improve the results obtained by the least squares method, including those that result in a minimum zone tolerance value.[1, 13].This issue is beyond the scope of this paper, but can be solved relatively easy in the context of the processed volume of data, the necessary speed of calculation and, of course, the functional purpose of the measured object [8]. A useful solution in this case also may be to realize the requirement presented in [6] to determine the deviation from sphericity D was: 'the largest radial distance in any equatorial plane between the surfaces of the smallest circumscribed and the biggest inscribed spheres whose centres coincide with the sphere of the minimal standard deviation'.

The issue of determining the diameter D of the measured sphere remains debatable. Aside from the existing traditional practices and algorithms, D may be determined by using one of the following two ways:

- As the diameter of the average circle of the measured points having a centre $\overline{MZC}$determined by using the least-squares method;
- As the least radius of the sphere circumscribed around the same points (minimum covering sphere) [12].

Similar to the measurement and assessment of the other two geometric deviations of form– roughness and waviness, here also may be given only some general recommendations related to the operational (working) behaviour and associating of controlled objects. It is known that using average associated elements is appropriate for interference fits or air cushion bearing,

while the circumscribed (contact) associated elements are appropriate for fits having zero or vary small clearance [8].

Figure 2 presents a scheme of the algorithm of the proposed method for measuring geometric features of spherical objects.

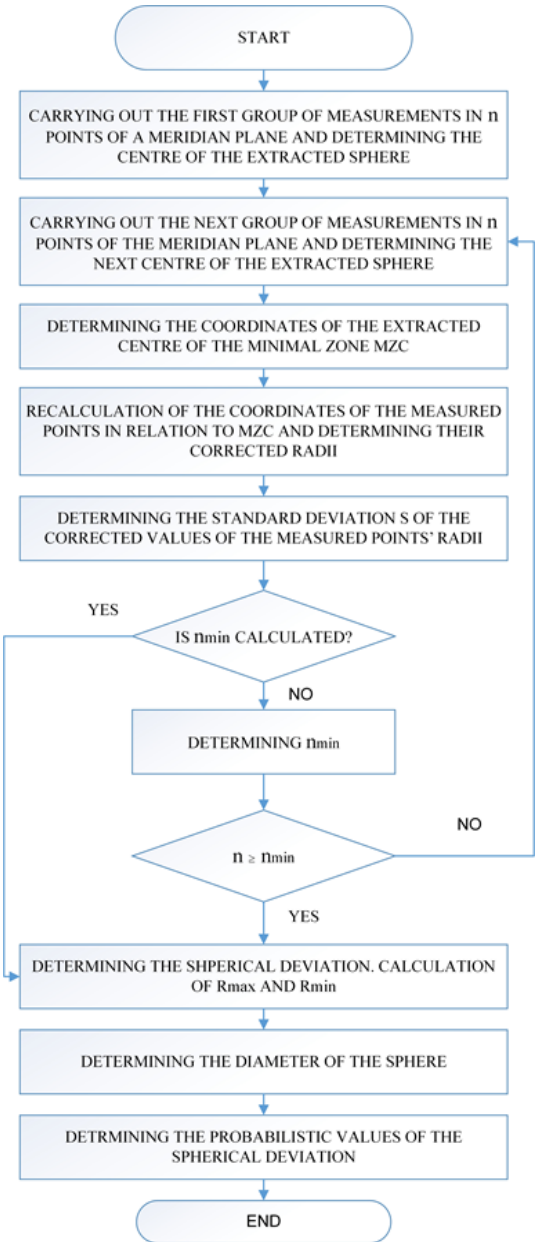


Figure 2: Scheme of the algorithm of the method

4. Conclusion

The paper presents a fast, simple and effective algorithm of a method for measuring and assessing geometrical features of spherical objects using CMM. It is based on a minimal number of standard successive measurements which aim to determine the centre of the minimal zone, formed between two homocentric spheres, while maintaining the necessary measurement accuracy. A method for determining the probabilistic values of the deviation from sphericity based on the difference modulus law is also proposed.

References

[1] Abhijit Gosavi, *Form errors in Precision Metrology: A survey of Measurement Techniques*, Quality Engineering 24(3):369-380, July 2012.

[2] Diakov D, Kalamanova I et al, *Investigation of the form of femoral head articulating surfaces, Proceedings of the symposium "Metrology and Metrology Assurance 2012"*, pp. 133-139, Sozopol, Bulgaria, ISSN 1313-9126, September 10-14, 2012 (in Bulg.).

[3] Diakov D, Kalamanova I et al, *Method for measuring of sphericity of the femoral head articulating surfaces, Proceedings of the symposium "Metrology and Metrology Assurance 2012"*, pp. 140-145, Sozopol, Bulgaria, ISSN 1313-9126, September 10-14, 2012 (in Bulg.).

[4] Hans Joachim Neumann, *Coordinate Metrology - Technology and Application*, Verlag modern industrie AG&Co, 1990.

[5] ISO 1101:2017 (E) – Geometrical Product Specification (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out.

[6] ISO 3290-1:2014 Rolling bearings — Balls — Part 1: Steel balls.

[7] Jian Mei, Qiang Huang et al, *A simple asymptotic search method for estimation of minimum zone sphericity error*, AIP Advances 10, 015322 (2020).

[8] Metrologiyaiizmervatelnatehnika: kniga-spravochnik v tri toma. Tom 2, Pod obsht. rad. na prof. Hr. Radev.-Sofiya: Softreyd, ISBN 978-954-334-093-4, 2010, 987 str.

[9] Prisco, U., Polini, W., *Least squares assessment of flatness, cylindricity and sphericity through surface classification based on continuous symmetry of geometric object*, Journal of Manufacturing Engineering, Vol.5, Issue 3, pp 153-165, September 2010.

[10] Rossi A, Stefano Chiodi S, Lanzetta M, *Minimum centroid neighborhood for minimum zone sphericity*, Precision Engineering, 38(2), 337-347, 2014.

[11] Sukhovilov B. M, *Method for Determining the Radius Spherical Surface*, Bulletin of the South Ural State University. Ser. Computer Technologies, Automatic control, Radio Electronics, vol.18, no. 2, pp. 164-171, 2018 (in Russ.).

[12] Theodore H. Hopp, Charles P. Reeve, *An algorithm for computing the minimum covering sphere in any dimension*, NIST, doc. NISTIR 5831, May 1996.

[13] Weber T, Motavalli S, et al, *A unified approach to form error evaluation,* Precision Engineering, Vol 26, pp 269-278, 2002.

[14] https://www.faro.com.