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Small Area Estimates of Poverty Incidence in Costa Rica under a Structure Preserving Estimation (SPREE) Approach

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Obtaining reliable estimates in small areas is a challenge because of the coverage and periodicity of data collection. Several techniques of small area estimation have been proposed to produce quality measures in small areas, but few of them are focused on updating these estimates. By combining the attributes of the most recent versions of the structure-preserving estimation methods, this article proposes a new alternative to estimate and update cross-classified counts for small domains, when the variable of interest is not available in the census. The proposed methodology is used to obtain and up-date estimates of the incidence of poverty in 81 Costa Rican cantons for six postcensal years (2012–2017). As uncertainty measures, mean squared errors are estimated via parametric bootstrap, and the adequacy of the proposed method is assessed with a design-based simulation.

Key words: Extreme poverty; intercensal updating; small area estimation; log-linear models.

1. Introduction

The estimation and monitoring of socio-economic indicators is relevant for decision-making and the development of public policies aimed at improving the conditions of the citizens. Among other characteristics, high-quality statistics must be relevant, accurate, and reliable to use them in the design, development and assessment of programs of social interest (Eurostat 2017). The success of these plans depends on how they are formulated and oriented, but in many cases, the information available is not enough to achieve this objective. Traditionally, national surveys are carried out every year in many countries to produce an up-to-date status of important topics such as poverty, inequality, and unemployment. This information, which is obtained periodically, usually satisfies the quality requirements, for instance of national statistical offices only at bigger domains. In other words, due to lack of resources, the sample sizes are not large enough to study the problems of interest in detail. For example, in the case of poverty: Where is the most vulnerable population located? Which areas have been improved through the years and which areas have been stagnated? Which other conditions (e.g., sex, age, disabilities) are associated with this phenomenon, and in which local areas?

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Small area estimation (SAE) methods have the goal of producing reliable estimates in smaller domains, that is, with adequate precision. Most of these methodologies, usually classified as unit-or area-level models, provide efficiency gains if the correlation between existing auxiliary information and the survey data is sufficient. (Pfeffermann 2013; Rao and Molina 2015; Tzavidis et al. 2018). In middle-income countries, administrative records are usually not sound enough and therefore censuses are the most important auxiliary source of information for the entire population, with the limitation that it is usually collected every ten years.

The time gap between annual surveys and population censuses is usually ignored in SAE methods. The use of covariates from an earlier period may lead to less reliable indicators than what would be expected from more solid auxiliary information. Academic literature on updating estimates in small areas is limited. Post censal population estimates have been obtained, for example from traditional procedures in demography, like the component method (United Nations 1956) and vital rates (Rao 2003). Emwanu et al. (2006) use panel survey data to obtain small area estimates of welfare in post-census years by regressing recent income (or expenditure) data on household characteristics that are available in both survey and census data. The best-known tool in this field is the structure-preserving estimation (SPREE) method, which is also the focus of this article and will be described in detail in Subsection 3.1. This technique was originally introduced by Purcell and Kish (1980) to obtain post-census estimates (counts or proportions), arranged by small domains and categories of interest. SPREE have been especially applied for updating demographic information and socio-economic indicators including employment (Hidiroglou and Patak 2009; Berg and Fuller 2009; Luna-Hernández et al. 2015) and poverty (Isidro et al. 2016).

Thereafter, several versions of SPREE have been proposed with the aim of improving the method by adding flexibility and reducing bias, namely the Generalized-SPREE (GSPREE) (Zhang and Chambers 2004) and most recently, the Multivariate-SPREE (MSPREE) (Luna-Hernández 2016). These SPREE-techniques have specific assumptions and requirements for their implementation. For example, the variable of interest must be categorical, and it must be not only in the survey (most recent) but also in the census data, which for indicators like the poverty rate are usually not available for variables based on income or expenditure. An alternative version called Extended-SPREE (ESPREE) (Isidro et al. 2016) solves this problem by applying a small area estimation technique (the Elbers, Lanjouw and Lanjouw (ELL) method (Elbers et al. 2003), as a previous step to obtain the required information for the census year. Once the estimated census information is obtained, Isidro et al. (2016) perform the original SPREE (Purcell and Kish 1980) to compute post-censal estimates. Moreover, Luna-Hernández (2016) showed that MSPREE is more efficient compared to SPREE (in terms of lower mean squared errors). Therefore, the current article extends the framework of Isidro et al. (2016) by allowing for the MSPREE in the updating process. In particular, the article aims to provide a modern methodology to (a) estimate and (b) update counts or proportions of relevant indicators in small areas when the information of interest is not available in the census data.

The motivation of the proposal is to offer updated reliable income-based poverty estimates of Costa Rican cantons in three mutually exclusive categories: “extreme poor”, “poor” (not extreme), and “not poor”. Due to its political stability and good performance in

general macroeconomic aspects, Costa Rica has been for several years an example among other economies in the region (OECD 2016). Despite this, a point that draws attention and has been the object of study in recent years is the stagnation of relative poverty that the country has had for more than two decades, unlike other Latin American countries that have achieved greater reductions in their poverty rates (CEPAL and MIDEPLAN 2016). As well as in the international agenda, previously through the Millennium Development Goals (MDGs) (United Nations 2019a) and currently, the Sustainable Development Goals (SDGs) (United Nations 2019b), one of the main concerns specifically in this country is the extreme poverty. Traditionally, and for international comparison, the National Institute of Statistic and Censuses (INEC, Instituto National de Estadıstica y Censos) of Costa Rica measures poverty based on the poverty line method. With this approach, a person or household is considered poor (or extreme poor) if its monthly per capita income is equal or below a specific poverty line. The idea of defining a threshold or line is to set the minimum amount in the per capita income that a person or household requires to satisfy food and non-food needs, included in a basket of goods and services (INEC 2015). In Costa Rica, extreme poverty had a reduction from 2004 to 2013 of only 0.8 percentage points in accordance with a diagnosis of structural gaps. Research showed that this status of poverty has three determinants: the adverse home and social environment, the insufficient scope of social programs and the exclusive labor market (CEPAL and MIDEPLAN 2016). Because of a lack of data, this kind of studies can only be conducted in census years or for larger areas, limiting the possibility of applying targeted policy interventions for specific groups or domains.

As well as other middle-income economies, Costa Rica faces several limitations to obtain small area estimates of poverty: administrative records at the unit level are not available, the census does not contain income or expenditure information to compute poverty estimates via the poverty line method, and the census is carried out only every ten years which can reasonably lead to outdated poverty estimates. The main study previously conducted in Costa Rica to obtain estimates of poverty in local areas was carried out for the same year as the census, using the ELL method. Although this work, developed by Méndez and Bravo (2011), certainly allowed to obtain more detailed information about poverty in local areas (classified as poor and not poor), two aspects can be improved with the proposal presented in this article: (a) provide poverty estimates for non-censal years, and (b) produce estimates on extreme poverty as a specific group of interest. The methodology proposed in this article can also be applied to many other countries that share similar conditions and extended to other relevant demographic and socio-economic (categorical) indicators.

This article has the following structure: Section 2 describes the data sets and explains the definition of poverty used in the application. Section 3 introduces the SPREE methods as small area estimation and updating techniques. The strategy proposed to obtain and update poverty estimates in Costa Rican cantons is also explained in this section, as well as the methodology to produce uncertainty measures. Application results are shown in Section 4. The results of a simulation study to validate the proposed method are presented in Section 5. The last section is dedicated to the conclusions and recommendations for further research.

2. Data Description and Definition of Poverty

As will be explained in detail in Subsection 3.1, the basic approach of the SPREE techniques requires one complete composition (usually a cross-classification from a census gathered in a previous time) and updated, reliable population totals (margins) for the variable of interest and for the area population sizes. Extensions to this methodology including the one implemented here require of an updated estimate of the cross-classification of interest that can be obtained from survey data. The aim of this section is to describe the data sources available and explain the definition of poverty considered in the application. Population and housing census, as well as the National Household Survey data sets, were provided by the INEC of Costa Rica, under specific confidential agreements.

2.1. Population and Housing Census 2011

In Costa Rica, the Population and Housing Censuses are carried out every ten years by the INEC. The most recent census was conducted in 2011 (data collection from 30th, May to 3rd, June) and it collected information of people, households, and dwellings necessary for the planning, execution, and evaluation of public policies (INEC 2012). With the information collected, it is possible to identify the relevant characteristics of the population such as access to education, employment, social security, technology, and health centers. Although the census 2011 includes questions to compute the unsatisfied basic needs (UBNS) index (Feres and Mancero 2001), it did not produce information about income or expenditures of the persons and households, which are necessary to calculate the incidence of poverty via the poverty line method. The sampling frame which is needed to conduct national surveys and other statistical operations is constructed based on this population and housing census. With this census, 10,461 primary sampling units (PSUs) and 1,359,168 dwellings were identified.

2.2. National Household Surveys 2011–2017

The National Household Survey (Encuesta Nacional de Hogares (ENAHO)) is the primary source for poverty and inequality measures in Costa Rica. In this aspect, this survey collects information about housing characteristics, education, social security and employment of the household members. This study is carried out annually (data collection during the month of July). Surveys from 2011 to 2014 used the sampling framework from the previous census 2000, the following surveys used the sampling framework updated with the census 2011. The sampling design used in the ENAHO is a two-stage stratified random sampling where census segments are the first stage units selected with probability proportional to size, and dwellings are defined as the final stage units. Administratively, Costa Rica has four disaggregation levels: two zones, six planning regions, 81 cantons and 473 districts (municipalities). The sampling design specifies twelve strata – each planning region divided by urban and rural areas. In this case, the strata coincide with the study domains. Smaller domains are not considered to guarantee a coefficient of variation less than 15% for the main poverty measure (percentage of household under poverty) (INEC 2017). For 2011, the ENAHO selected 1,120 PSUs and 13,440 dwellings (10.7% and 9.9% of the sampling framework, respectively).

There are two main differences across the survey rounds as well as regarding the census data. On the one hand, the last surveys collected more variables than the previous ones and the census, and the definition of some of the variables also changed. On the other hand, there were land reforms in the period of study: in 2011 there were only 81 cantons but in 2017 another canton was created. To deal with these obstacles, only variables that exist with the same definition in both the census and survey data are included for the analysis. Also, for the last survey, cantons were grouped exactly the same as in the census 2011. This straightforward-aggregation is possible because the new canton was created by dividing one of the existing cantons. In this article, the objective is to obtain quality poverty information of households for the third administrative level, that is, the 81 cantons, which are defined as the target small areas.

2.3. Demographic Projections

In order to apply a SPREE technique, it is necessary to provide reliable and up-to-date area totals as it will be explained in detail in Subsection 3.1. Since survey data usually does not produce trustworthy population sizes for small areas (due to sample sizes and areas out of sample), demographic projections are used instead in this article. In Costa Rica, population projections are calculated with the cohort component method (Preston et al. 2001) which considers changes in three components: mortality, fertility and migration. The mortality projection was carried out with an autoregressive integrated moving average (ARIMA) random walk model with drift, and for the fertility and migration components, functional data analysis models were implemented. Further details can be found in INEC and CCP (2013). Because of population projections in Costa Rica consider persons at an aggregate (e.g., canton) level, but in this application the aim is to update the total of households according to their status of poverty, the headship rate (United Nations 1973) by sex and age groups is applied in order to get the household projections. A previous implementation of this methodology in this country can be found in Sáenz (2002).

2.4. Definition of Poverty

In Costa Rica, poverty is measured under different uni-and multidimensional proaches. One of the most important, and that is the focus in this article, is the (monetary) poverty rate which is based on the poverty line method (using non-equivalized household per capita income). The INEC defines two types of lines or thresholds:

1. The indigence or extreme poverty line: set by the per capita costs of a basic food basket. The composition of this basket is defined from the National Survey on Income and Expenditure of the Households (Encuesta Nacional de Ingresos y Gastos de los Hogares, ENIGH) which is carried out every five years. The value of the basket is updated every month based on the consumer price index. If the monthly per capita income of a household is below this line, it is considered under "extreme" poverty. For the census time (July 2011), the indigence line was 39,428 colones (Costa Rican currency) (INEC 2011) which was 27.9% of the median per capita income at that time.

2. The poverty line: considers non-food basic needs. A household is classified in this category if the monthly per capita income is equal to or below this value but higher than the indigence line. The poverty line in July 2011 was 84,006 colones which is 60.7% of the median per capita income (Méndez and Bravo 2011).

In this article, the number of households grouped in three categories of poverty ("extreme poor", "poor" (not extreme) and "not poor") is estimated and updated for six post-censal years (2012–2017) in 81 cantons.

3. Methodology

This section describes the methodology for estimating and updating counts and proportions for small areas. First, SPREE techniques are introduced because they are the basis of this proposal. Second, the recommended strategy for obtaining and updating estimates is explained. Finally, the steps to produce uncertainty measures are described.

3.1. Structure Preserving Estimation (SPREE) Methods

As stated above, SPREE was originally proposed by Purcell and Kish (1980) as a tool to update counts or proportions of a categorical variable of interest according to study domains in intercensal years. The target information of interest in a recent time t_1 is shown as a multi-way contingency table Y_{aj,t_1} grouped by $a = 1, ..., A$ areas or domains (rows) and $j = 1, ..., J$ categories of the variable of interest (columns) (e.g., poverty status). In other words, for a population of size N, all the units i (e.g., individuals or households) are organized according to the area and category to which they belong. The structure of a two-way contingency table can be represented as a saturated log-linear model:

$$\log Y_{aj,t_1} = \alpha^Y_{0,t_1} + \alpha^Y_{a,t_1} + \alpha^Y_{j,t_1} + \alpha^Y_{aj,t_1}. \tag{1}$$

These four terms $\alpha^Y_{0,t_1}, \alpha^Y_{a,t_1}, \alpha^Y_{j,t_1}$ and α^Y_{aj,t_1} can be defined using a centred-constraint parametrization, see, for example, Luna-Hernández (2016). Notice that data from a census (time t_0) can also be arranged as a contingency table (Z_{aj,t_0}) and represented as a saturated log-linear model ($\log Z_{aj,t_0}$) with the same constraints.

For intercensal years, the production of official statistics relies in many cases on survey data. However, due to sampling size limitations, trustworthy results are only available for big areas. The SPREE method provides a solution when updated estimates of frequency characteristics are required in smaller domains. The terms $\alpha^Y_{0,t_1}, \alpha^Y_{a,t_1}$ and α^Y_{aj,t_1} represent the *allocation* structure which are benchmarked to totals or current margins (A row and J column totals), usually provided by direct survey estimates and/or demographic projections. In this article, for simplicity, the allocation structure is also referred to as the survey margins, although it is made up of both demographic projections and survey data. It is assumed that these totals are reliable and updated for postcensal years. Furthermore, the method supposes that the interactions between rows and columns of the census Z_{aj,t_0} (inner cells in the contingency table) remain unchanged for the target years. Therefore, the structural assumption is;

$$\alpha^Y_{aj,t_1} = \alpha^Z_{aj,t_0}. \tag{2}$$

This interaction term provided by the census is usually known as the *association* structure.

Following the proposal of Purcell and Kish (1980), the updated estimates are obtained via the iterative proportional fitting (IPF) algorithm (Deming and Stephan 1940) (also found in literature as *raking* or *contingency table standardization*). A detailed description of the IPF is available for example, in Bishop et al. (2007) and Zaloznik (2011). This algorithm fits a census table by keeping reliable survey margins fixed. The process to obtain the updated SPREE of Y_{aj,t_1} is represented by Luna-Hernández (2016) as:

$$\hat{Y}^{S}_{aj,t_1} = \text{IPF}\left[\exp\left(\hat{\alpha}^{Y}_{aj,t_1}\right), Y_{a,t_1}, Y_{j,t_1}\right], \tag{3}$$

with Y_{a,t_1} and Y_{j,t_1} representing the reliable survey margins (rows and columns) and $\hat{a}^{Y}_{aj,t_1} = a^{Z}_{aj,t_0}$.

As it will be later described, new versions of SPREE estimators have been proposed and each of them define different assumptions on their association structure $\hat{\alpha}^{Y}_{aj,t_1}$. The process in Equation 3 is applied with the defined association structure of each SPREE-type estimator.

In order to apply a fitting strategy via IPF, Koebe et al. (2022) summarise some basic requirements that should be considered:

1. the data to fit must be arranged in categories (e.g., contingency tables),
2. the margins of the census and survey structures must have the same length (same number of rows and columns),
3. totals by rows and columns must be equal,
4. the census data (association structure) must contain the indicator of interest (e.g., poverty status) with the same definition or a highly correlated indicator (Green et al. 1998).

In practice, requirements two and four are not met in the Costa Rican scenario. Due to administrative reforms that occurred in postcensal years (e.g., merge or split domains), the number of local areas in the census and in the surveys differs. Another situation where requirement two may not be fulfilled is when some areas were not selected in the sample, leading to an incomplete allocation structure. For these cases, several solutions can be considered: complementary information such as administrative registries or population projections could be used as reliable survey margins, rows that are not in the survey composition can be eliminated from the census, or adding missing rows with small values (e.g., 1) to re-construct the survey compositions in the same way as defined in the census. The first alternative is implemented in the current example.

Regarding the fourth requirement, income or poverty information is not obtained directly in the Costa Rican census. A solution for this kind of situation was proposed by Isidro et al. (2016). The so-called ESPREE considers the case when the indicator of interest is not present in the census data, and another small area estimation method (e.g., the ELL method) is applied as a first step with the aim of obtaining the required information for the census year. Thereafter, the original SPREE technique is conducted as a second step to proceed with the updating process. Considering the characteristics of the

case of study exposed in this article, the methodology of ESPREE is followed to obtain updated poverty estimates.

Another line of research within the SPREE framework is the bias reduction of the estimator. The GSPREE introduced by Zhang and Chambers (2004) makes the structural assumption of SPREE more flexible by adding a *proportionality coefficient* (constant obtained through direct estimates $\hat{Y}^{Dir}_{aj,t_1}$) adjusting Equation 2 to:

$$\alpha^{Y}_{aj,t_1} = \beta \alpha^{Z}_{aj,t_0}. \tag{4}$$

Note that the SPREE assumption defined previously in Equation 2 is the case of Equation 4 when $\beta = 1$. Moreover, Luna-Hernández (2016) proposes a version that aims to relax two restrictions of the former methods (SPREE and GSPREE): the relationship between the association structures (interaction terms) of the census and target compositions are controlled only by one parameter, and in the case of the GSPREE the proportionality parameter β, is assumed to be the same for all the categories. GSPREE and MSPREE mainly differ from the original SPREE because a multi-way contingency table of direct estimates is also necessary in order to update the association structure, meanwhile, the former version only requires the availability of suitable (total) margins. The novel method is called *multivariate* SPREE (MSPREE) because in this case, the target compositions are the interactions within each area. Then, the coefficient $\boldsymbol{\beta}$, similar to the proportionality coefficient specified by Zhang and Chambers (2004), now is represented by a $J \times J$ matrix (with $(J-1) \times (J-1)$ free parameters), and varies inside each area, from one category to another. The main benefit of this proposal is to be able to capture better relationships between categories, instead of assuming that these interactions remain identical over time. With this, the bias that can occur through changes in the association structure (which is not accounted in SPREE and GSPREE), is also reduced. Similarly as in Equations 2 and 4, the MSPREE structural assumption is expressed as:
which is equivalent to:

$$\alpha^{Y}_{a,t_1} = \boldsymbol{\beta} \alpha^{Z}_{a,t_0} \tag{5}$$

where $a = 1, ..., A$ areas, and for each area, the interaction terms are: $\alpha^{Y}_{a,t_1} = (\alpha^{Y}_{a1,t_1}, ..., \alpha^{Y}_{aJ,t_1})$ and $\alpha^{Z}_{a,t_0} = (\alpha^{Z}_{a1,t_0}, ..., \alpha^{Z}_{aJ,t_0})$. The target MSPREE composition Y^{M}_{aj,t_1} can also be obtained via IPF as in Equation 3. In the same way as in the original SPREE and GSPREE, the reliable total margins are preserved, but the same list of requirements aforementioned must be fulfilled.

Estimates of β can been obtained via Maximum Likelihood (ML) or Iterative Weighted Least Squares (IWLS), in both cases by using a log or a logit link. Because a target contingency table can be represented as a log-linear model, SPREE fits within the framework of generalized linear models (Marker 1999; Noble et al. 2002). Moreover, as Agresti (2002) indicates, a log link with a Poisson response is commonly applied to model cell counts in contingency tables, but he also shows that the Poisson expected frequencies μ_{aj,t_1} are equal to $n\pi_{aj,t_1}$, where π_{aj,t_1} are the cell probabilities used under multinomial sampling and n the sample size. For this reason, when working with log-linear models (as SPREE), the estimation of coefficients β can be obtained with a Poisson $\hat{Y}^{Dir}_{aj,t_1} | \alpha^{Z}_{a,t_0} \overset{\text{ind}}{\sim}$ Poisson(μ_{aj,t_1}) or Multinomial $\hat{Y}^{Dir}_{aj,t_1} | \alpha^{Z}_{a,t_0} \overset{\text{ind}}{\sim}$ Multinomial $(\sum_{j=1}^{J} \hat{Y}^{Dir}_{aj,t_1}, \pi_a, t_1)$

leading to similar results. Notice that, similarly to the GSPREE, the computation of β requires direct estimates (i.e., not only survey margins but a survey composition $\hat{Y}^{Dir}_{aj,t_1}$).

The second alternative to obtain estimates of β is using IWLS. This algorithm is implemented in Zhang and Chambers (2004) and Luna-Hernández 2016 suggests it when the sample is drawn using a complex sampling design because using ML for the parameter estimation can lead to misspecification if sampling design information is ignored. In the case of the survey data used in this article, the sample was gathered in a two-stage selection process considering unequal selection probabilities, and for this reason, a fully distributional approach should not be assumed. Consequently, in this article, the parameters of interest are estimated via IWLS. This method requires an estimate of the variance-covariance matrix of the target composition $\hat{Y}^{M}_{aj,t_1}$ which is usually not available. Luna-Hernández 2016 solves this issue by multiplying a design effect with the variance that corresponds to a simple random sample without replacement design. This proposal assumes that samples are independently selected in each area and there are no existing correlations among estimates from different areas. The estimate of the variance-covariance matrix is represented as:

$$\hat{V}_{aj,t_1} = \frac{\text{deff}_{j,t_1}\ \hat{\pi}^{M}_{aj,t_1}\left(1-\hat{\pi}^{M}_{aj,t_1}\right)}{n_{a,t_1}}, \tag{6}$$

with $\hat{\pi}^{M}_{aj,t_1} = \frac{\hat{Y}^{M}_{aj,t_1}}{\hat{Y}_{a,t_1}}$ and n_{a,t_1} the area sample sizes. Further details about the IWLS algorithm can be consulted in Jiang (2007) and Luna-Hernández (2016).

3.2. The Empirical Best Predictor (EBP) Method

The EBP is applied in order to obtain the information of poverty status in the census structure. The EBP methodology proposed by Rao and Molina (2015) implements a unit-level nested error regression model to get estimates of a specific variable of interest in the census, using (usually) survey data that contains this variable. This method has been extensively implemented in SAE problems and also specifically for poverty estimation (see e.g., Pratesi 2016 or Das and Haslett 2019). The process assumes a random effects model for a finite population of size *N:*

$$y_{ai} = \mathbf{x}^{T}_{ai}\beta + u_a + e_{ai},$$

where y_{ai} represents the target variable and $\mathbf{x}_{ai}$ the set of covariates for the ith individual or household in ath area, u_a indicates the area-specific random effects and e_{ai}, the unit-level error. The two last terms are assumed to be normal, independent and identically distributed. By using survey data, the following estimates are obtained: $\hat{\beta}, \hat{\sigma}^2_u, \hat{\sigma}^2_e$ and the weighting factors $\hat{\gamma}_a = \frac{\hat{\sigma}^2_u}{\hat{\sigma}^2_u + \frac{\hat{\sigma}^2_e}{n_a}}$, where n_a denotes the sample size in area a, and σ^2_u and σ^2_e indicates the between and within group variance respectively.

Then, $l = 1, ..., L$ Monte Carlo simulations to generate a pseudo population are conducted:

$$y_{ai} = \mathbf{x}^{T}_{ai}\hat{\beta} + \hat{u}_a + v^{(l)}_a + e^{(l)}_{ai}, \tag{7}$$

where $v_a^{(l)} \overset{iid}{\sim} N(0, \hat{\sigma}_u^2(1-\hat{\gamma}_a))$ and $e_{ai}^{(l)} \overset{iid}{\sim} N(0, \hat{\sigma}_e^2)$ and the predicted random effect $\hat{u}_a$ is defined as $\hat{u}_a = E(u_a|\mathbf{y}_{as})$. The final indicator of interest is obtained taking the mean over the L iterations.

Molina and Rao (2010) explain that the EBP estimator can be biased when model error terms depart from normality. When working with income data, a common practice to achieve Gaussian assumptions is by using a logarithmic transformation (Elbers et al. 2003; Molina and Rao 2010) which is a special case of the Box-Cox transformation (Box and Cox 1964). In this case study, as will be explained in detail in Subsection 5.1, the incidence of poverty is approximated by modeling income, for this reason, departures from normality are reduced with a Box-Cox transformation. Further details on the performance of the EBP under data-driven transformations can be found in Rojas-Perilla et al. (2020). The EBP was conducted using R (R Core Team 2018), specifically with the Package emdi (Kreutzmann et al. 2019).

3.3. *Strategy to Estimate and Update Poverty Estimates in Costa Rican Cantons*

The goal of this article is to obtain and update poverty estimates in three categories: "extreme poor", "poor" (not extreme), and "not poor". However, as noted previously, no poverty information is collected directly from the census, nor income or expenditure data. Thus, the applied methodology considers characteristics of some of the SPREE methods, specifically the ESPREE and the MSPREE. Since the census data does not contain poverty information, this article adjusts the ESPREE framework. Instead of applying an ELL model to estimate poverty in the census data followed by the original SPREE to update the counts as in Isidro et al. (2016), the empirical best predictor (EBP) (Rao and Molina 2015) is implemented followed by the MSPREE in this work (for simplicity, also referred as EBP-MSPREE). To the best of my knowledge, MSPREE is the most recent and complete technique mentioned in SPREE literature. This version provides more flexibility and bias reduction compared with the previous versions, therefore it is implemented as the main tool in the updating part of the process.

The estimation and updating strategy can be summarized in the following steps:

1. **Estimating the proxy association structure via EBP** Considering that census and survey data, both for the same year at individual level are available, the EBP explained in Subsection 3.2 is applied in order to obtain the information of poverty status in the census structure.
2. **Obtaining the allocation structure.** Household projections as mentioned in Subsection 2.3 provide the total of households in each canton (row margins) and survey data described in Subsection 2.2, the total of households by poverty status. Both sources are available for postcensal years (2012–2017).
3. **Updating estimates via MSPREE.** Intercensal EBP-MSPREE compositions $\hat{Y}_{aj,t_1}^{EM}$ are estimated considering the outputs from the two previous steps (association structure α_{aj,t_0}^{Z} from Step 1, and row Y_{a,t_1} and column Y_{j,t_1} margins from Step 2, that represent the allocation structure). Direct estimates from survey data and design effects are also required as explained at the end of Subsection 3.1. With these inputs, the procedure can be described as follows:

1. the matrix of coefficients β is estimated with an IWLS algorithm that requires a variance-covariance matrix. This matrix is approximated using design effects as showed in Equation 6 with EBP composition estimated in t_0 and the direct estimate obtained from the survey in t_1,
2. $\hat{\alpha}^{Y}_{aj,t_1}$ is estimated with Equation 5,
3. taking into account all these elements, the target estimate $\hat{Y}^{EM}_{aj,t_1}$ is finally obtained with Equation 3.

3.4. Uncertainty of the Updated Estimates

The benefits of two SPREE versions are used in this article. To motivate the proposed $\widehat{\text{MSE}}(\hat{Y}^{EM}_{aj,t_1})$, the procedures via bootstrap of the predecessors (MSE of ESPREE and MSPREE) are briefly described in this Section. Details about other approaches, for example, via linearization methods can be found in Isidro (2010), Isidro et al. (2016) and Luna-Hernández (2016).

Two sources of variation are considered when obtaining estimates via the ESPREE method: survey data (allocation structure) and pseudo-populations (association structure). Being $\hat{Y}^{E}_{aj,t_1}$ the ESPREE estimates, the uncertainty estimate is the sum of two variances: $\text{Var}(\hat{Y}^{E}_{aj,t_1}) = \text{Var}^{\text{survey}}(\hat{Y}^{E}_{aj,t_1}) + \text{Var}^{\text{census}}(\hat{Y}^{E}_{aj,t_1})$. The first variance term $\text{Var}^{\text{survey}}$ is obtained by generating $b = 1, ..., B$ independent bootstrap samples from the original survey data and computing $\hat{Y}^{E,b}_{aj,t_1}$ by keeping fixed the census data (i.e., association component) in every replication. The second term $\text{Var}^{\text{census}}$, is obtained in a similar way. In this case, B-ESPREE estimates $\hat{Y}^{E,b}_{aj,t_1}$ are computed based on $b = 1, ..., B$ bootstrap populations or pseudo-census to account for the uncertainty provided by the association structure, and the allocation structure (survey margins) will be held fixed over the replications. Because in ESPREE, the census values are estimated via ELL (or EBP), the $b = 1, ..., B$ pseudo-census generated from the ELL process can be also be used here. Both, $\widehat{\text{Var}}^{\text{survey}}$ and $\widehat{\text{Var}}^{\text{census}}$ are unconditional variances. As aforementioned, the final uncertainty estimation is only the sum of these two terms, meaning that the authors assume that there is no covariance between both estimators.

Regarding MSPREE uncertainty measures of $\hat{Y}^{M}_{aj,t_1}$, Luna-Hernández (2016) proposed three alternatives: an analytical approximation for the variance of the estimator, finite population MSE (FP-MSE) and an unconditional MSE (U-MSE). For simplicity and consistency, the last one is here described. From the point estimate $\hat{Y}^{M}_{aj,t_1}$, calculate the within-area proportions $\hat{\pi}_{a,t_1} = \frac{\hat{Y}^{M}_{aj,t_1}}{\hat{Y}_{a,t_1}}$ for each area $a = 1, ..., A$. Generate B-bootstrap populations Y^{*b}_{aj,t_1} under the assumption that the target estimate across areas has the following distribution: $\mathbf{Y}_a | \alpha^{Z}_{a,t_0} \sim \text{Multinomial}(Y_{a,t_1}, \hat{\pi}_{a,t_1})$. From each bootstrap population draw a sample and follow the steps of Subsection 3.2 to obtain $\hat{Y}^{M,b}_{a,j,t_1}$. Finally, compute $\widehat{\text{U-MSE}}(\hat{Y}^{M}_{aj,t_1}) = \frac{1}{B}\sum_{b=1}^{B}(\hat{Y}^{M,b}_{aj,t_1} - \hat{Y}^{*b}_{aj,t_1})^2$.

The two procedures to obtain the MSE of $\hat{Y}^{E}_{aj,t_1}$ and $\hat{Y}^{M}_{aj,t_1}$, were briefly described to better contextualize the MSE proposed in this article and to point out the differences and similarities between them. Thus, results from these two previously described MSEs are not produced in this article.

Because the estimation and updating process in this application makes use of the EBP followed by the MSPREE, (EBP-MSPREE) the sources of uncertainty in all of the steps

should be considered. The idea of including the variation of each element is taken from Isidro et al. (2016). The objective is to contemplate that the MSE of the EBP-MSPREE entails three sources of uncertainty: allocation structure, association structure, and the yearly association updating). The difference in this proposal is that instead of calculating each variability term and adding them ($\text{Var}^{\text{survey}} + \text{Var}^{\text{census}} + \text{Var}^{\boldsymbol{\beta}}$), a single bootstrap procedure will be performed, varying each of the required elements in each replication. This decision is made since it cannot be denied that covariances between the estimators exist. This aspect requires special attention and it should be studied in further investigations. The steps to obtain the MSE for the EBP-MSPREE are as follows:

1. From Equation 7, L Monte Carlo pseudo populations in t_0 were generated. Based on defined thresholds (poverty lines), L cross-classified population tables are created with dimension: A areas and J categories of poverty. Notice that the average across them was used to defined α^{Z}_{aj,t_0} and finally compute the point estimate $\hat{Y}^{EM}_{aj,t_1}$. Now, these L pseudo populations are used to create $\alpha^{z,b}_{aj,t_0}$ (with $L = B$), to account for the uncertainty that this structure provides, similarly as in Isidro (2010) and Isidro et al. (2016).
2. To take into consideration the uncertainty from the allocation structure, generate B pairs of margins $Y^{b}_{a,t_1}, Y^{b}_{j,t_1}$ from the point estimate $\hat{Y}^{EM}_{aj,t_1}$ assuming a multinomial distribution, e.g., $Y^{b}_{a,t_1} \sim \text{Multinomial}(\tilde{Y}, \tilde{\pi}_a)$, where $\tilde{\pi}_a = \frac{\hat{Y}_{a,t_1}}{\tilde{Y}}$ and $\tilde{Y} = \sum_{a=1}^{A} \hat{Y}_{a,t_1}$.
3. The uncertainty due to the estimation of β required in the MSPREE is obtained following the procedure of the U-MSE previously described: From the point estimate $\hat{Y}_{aj,t_1}$ calculate the within-area proportions $\tilde{\pi}_{a,t_1} = \frac{\hat{Y}^{M}_{aj,t_1}}{Y_{a,t_1}}$ for each area $a = 1, ..., A$. Generate B-bootstrap populations Y^{*b}_{aj,t_1} under the assumption that the target estimate across areas has the following distribution: $\text{Y}_a | \alpha^{Z}_{a,t_0} \sim \text{Multinomial}(Y_{a,t_1}, \hat{\pi}_{a,t_1})$. From each B-bootstrap population draw a sample to get $y^{EM,b}_{aj,t_1}$.
4. With the output of Step 1 and 3, specify the MSPREE structural assumption $\hat{\alpha}^{Y,b}_{a,t_1}$ as in Equation 5.
5. With $L = B$ and the output of Step 2 and 4, compute B EBP-MSPREE estimates

$$\hat{Y}^{EM,b}_{aj,t_1} = \text{IPF}\left[\exp\left(\hat{\alpha}^{Y,b}_{a,t_1}\right), Y^{b}_{a,t_1}, Y^{b}_{j,t_1}\right].$$

6. Finally, estimate the MSE:

$$\widehat{\text{MSE}}\left(\hat{Y}^{EM}_{aj,t_1}\right) = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{Y}^{EM,b}_{aj,t_1} - \hat{Y}^{EM}_{aj,t_1}\right)^2. \tag{8}$$

4. Results of the Application

In this section, the results of the estimation and the updating process are explained. First, the EBP model for obtaining the association structure is described, (i.e., poverty information in the census composition) with its corresponding model evaluation and descriptive statistics. Second, some relevant results in the practical sense are shown, for example, the evolution of (updated) poverty indicators from 2012 to 2017 for selected cantons, and the cantons with the highest poverty rates in the last year of study. Finally, uncertainty measures and validation results are presented.

4.1. Poverty Estimates for the Census

An EBP model to obtain poverty incidence by domain was conducted by setting income per capita of the household as the dependent variable and the socio-economic covariates described in Table 1 as predictors. In Costa Rica, urbanity plays an important role in socioeconomic topics. For this reason, the variable zone (urban/rural) is added to the model. The other groups of variables contain information on the head of the household, the household members, and the housing. Most of them have been used in previous studies as predictors of poverty or are part of indexes such as the UBNS and the multidimensional poverty index (MPI) (Alkire and Foster 2007). Under the model here specified, the sample design is assumed non-informative.

To select the model, several transformations were considered to reduce normality departures of the error terms, but the final version applies a Box-Cox transformation (Box and Cox 1964) with an optimal lambda (0.1585) using the Restricted maximum likelihood (REML) approach. Regarding the normality assumptions of this linear mixed model, graphical diagnostic of the residuals are used. Figure 1 shows that normality assumptions are rejected for the unit level (Skewness -0.0491 and Kurtosis 6.4179), but not for the random effects (canton-level). The latter is also confirmed with the Shapiro Wilk test ($W = 0.9860$ and $p = 0.5286$), and Skewness (-0.3584) and Kurtosis (3.0649) measures. For this example, normality for both, the unit level and the random effects, is assumed. Also, the marginal $R^2 = 0.5003$ and the conditional $R^2 = 0.5081$ were observed.

Table 2 shows the summary statistics of the population and sample domains. For 2011, all cantons are in-sample although it is not the case for some post-censal years. Domain sizes from the census data vary from 1705 (minimum) to 84,066 (maximum) households, and in the case of the ENAHO, it varies from 12 to 877 households.

Table 1. Covariables included in the EBP model to obtain poverty estimates in census.

Category	Variable
Geographical	1. Zone
head of the	2. Age
household	3. Highest degree of education completed
	4. Sex
	5. Labor condition
Household	6. Proportion of employees in the household
conditions	7. Equivalized size of the household
	8. Overcrowding
	9. At least one member without health insurance
	10. Quantity of economically dependent members
	11. At least one member not attending to formal education
	12. At least one member not with a educational lag
Housing	13. Poor condition of the floor or ceiling
conditions	14. Any member not used internet last 3 months
	15. No garbage disposal system
	16. No exclusive toilet for the household

Notes: Labor condition has three categories: employed, unemployed, out of Labor Force.
Variable 9 refers to population older than 17 years old.
Variables 11 and 12 refer to population between five and 19 years old.
Variable 14 refers to population older than four years old.

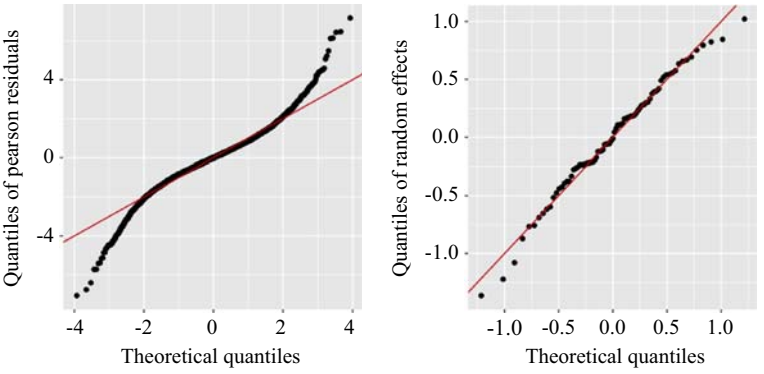


Fig. 1. Q-Q plots of the unit-level errors and the random effects.

Table 2. Summary statistics for sample and population sizes.

	Min	1st Q	Median	Mean	3rd Q	Max
Sample domains (In-sample: 100%)	12	62	103	144.37	191	877
Population domains	1705	5961	11032	15271.37	17148	84066

In order to study poverty in the three aforementioned interest groups, the two poverty lines described in Subsection 2.4 were implemented (as "customised indicators" in the R package `emdi`, for further details about this functionality see Kreutzmann et al. 2019) Descriptives of coefficients of variation (CV) for the directand model-based estimates obtained via EBP are presented in Table 3. As expected, the CVs reflect the lack of precision in the categories "extreme poor" and "poor" (not extreme) for the direct estimates. The improvement when the EBP model is conducted is clear, with a maximum CV of 29.9% and 18.2% for "extreme poor" and "poor" (not extreme) categories respectively. Notice also, that the third quartile of the CV in the category "extreme poor" is below 20%.

Figure 2 shows the proportion of households under extreme poverty based on the direct estimates and the model-based estimates obtained via the EBP model. The maps on the right side give a closer look at the metropolitan area which consists of 31 cantons, covering approximately 60% of the population. In four of these cantons, no households were identified as "extreme poor" with direct estimates (out-of-sample domains represented in

Table 3. Coefficients of variation of the direct and model-based estimates for poverty status.

	Direct			EBP		
CV	Extreme	poor	Not poor	Extreme	poor	Not poor
Min	0.068	0.071	0.033	0.085	0.045	0.012
1st Q.	0.232	0.164	0.086	0.127	0.065	0.023
Median	0.332	0.323	0.110	0.155	0.085	0.033
Mean	0.412	0.271	0.123	0.160	0.089	0.033
3rd Q.	0.524	0.234	0.145	0.181	0.106	0.040
Max	1.000	1.000	0.281	0.299	0.182	0.062

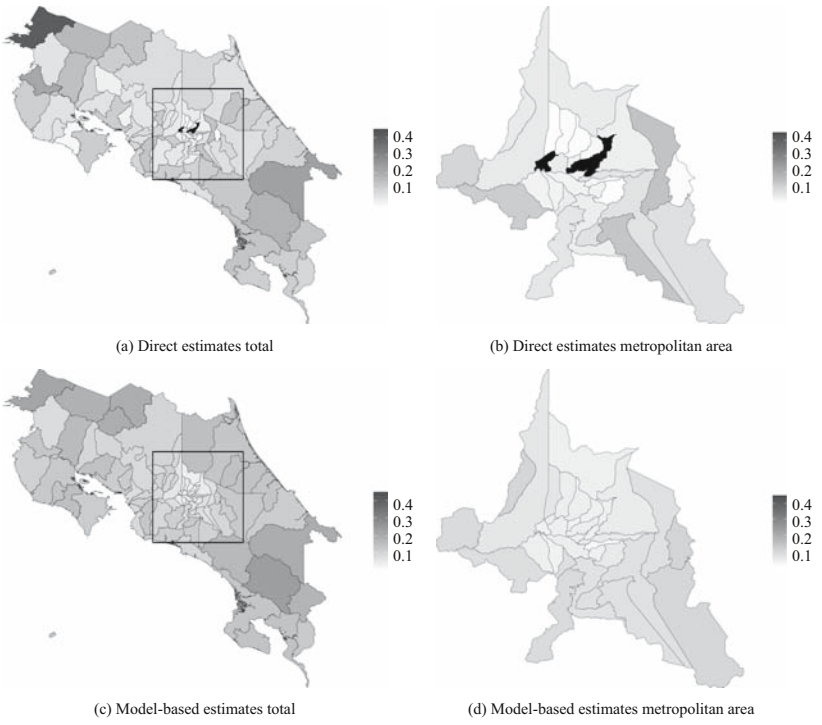


Fig. 2. Proportion of extreme poverty: direct and model-based estimates, 2011.

black in Figure 1 (a) and (b)). The results are consistent with a previous study (Méndez and Bravo 2011), where higher levels of poverty are found on the border with Nicaragua (e.g., La Cruz, located in the northwest of the country) and on the border with Panamá (e.g., Buenos Aires, and Talamanca, located in the southeast of the country).

4.2. Updated Poverty Estimates

To analyze major changes in the incidence of extreme poverty, estimates of 2011 and 2017 are compared using Z-scores:

$$Z = \frac{\text{Estimate}_{2011} - \text{Estimate}_{2017}}{\sqrt{(\text{Standard error}_{2011})^2 + (\text{Standard error}_{2017})^2}}$$

Figure 3 presents the three cantons with biggest change in this category of poverty. Among all of the 81 cantons, these are the cantons with Z-scores higher than two. Note that all of them show a reduction in the incidence of poverty between 2011 and 2017. It is also important to mention that none of the three cantons are among the poorest, which means that the biggest improvements are not observed in the areas most in need. The cantons Curridabat and Montes de Oca had the highest growth in extreme poverty (although small Z-score values: 0.2235 and 0.2052 respectively). However, both are among the cantons with the lowest incidence of extreme poverty in both years.

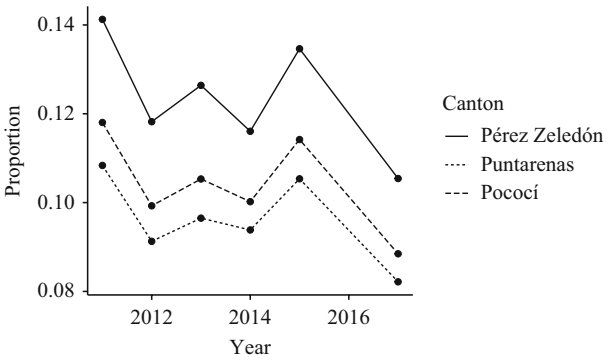


Fig. 3. Cantons with biggest change in the incidence of extreme poverty from 2011 to 2017.

Identifying the poorest cantons is also relevant in order to fight against this phenomenon in a more efficient way. Figure 4 shows the small areas with the highest incidence of extreme poverty (in proportions) in the last year (2017). Here, it is important to point out that for all the years of study (2011–2017) the same five cantons remain on this list, indicating that economic conditions of these areas have not been better in comparison with other areas in recent years.

The sources of uncertainty that were explained in Subsection 3.4, are displayed for the last year of study as a coefficient of variation in Figure 5. As expected, the category "not poor" is the one with the minimum CV and most of the values are under 20% which is considered "acceptable" according to the parameters for official publications of the National Statistical Office of Costa Rica, INEC (2015).

As explained in Subsection 2.2, the target areas in this article (81 cantons) are nested in six planning regions. Because the INEC of Costa Rica publishes official results on poverty only for these planning regions (gathered from the ENAHO), this is the only geographical level where it is possible to make the comparison with the updated estimates via EBP-MSPREE. For this reason, as a way to evaluate the updated estimates of poverty, model-based estimates of cantons are aggregated into the six planning regions and compared with

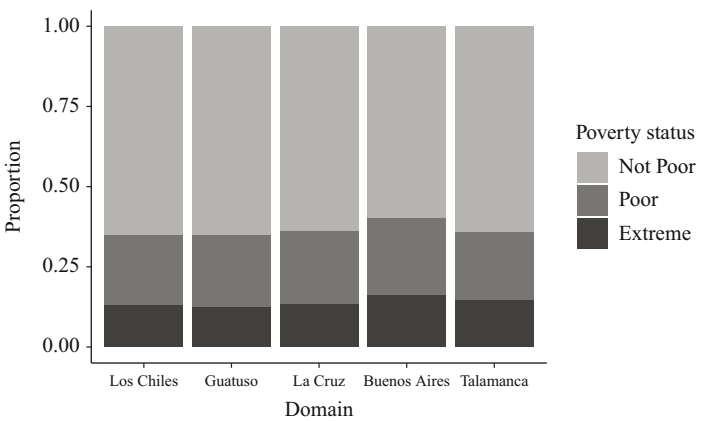


Fig. 4. Cantons with the highest incidence of extreme poverty, 2017.

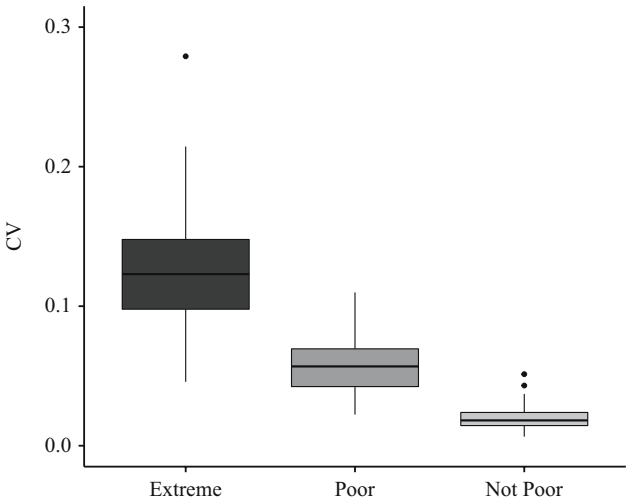


Fig. 5. Coefficients of variation for 2017.

Table 4. Direct and EBP-MSPREE estimates for poverty status (proportions) by planning region 2017.

	Direct			EBP-MSPREE		
Regions	Extreme	Poor	Not poor	Extreme	Poor	Not poor
Central	3.9	11.9	84.2	3.8	11.7	84.5
Chorotega	5.9	16.5	77.6	8.3	18.0	73.7
Pacífico Central	8.9	21.0	70.1	6.8	16.7	76.5
Brunca	10.4	19.1	70.5	10.7	20.7	68.6
Huetar Caribe	8.9	17.8	73.3	9.3	18.9	71.8
Huetar Norte	9.2	18.3	72.5	7.8	17.6	74.6
Total	5.7	14.3	80.0	5.7	14.3	80.0

the official publication. It is relevant to mention that three cantons overlap with two regions at the same time. This problem was solved by allocating the estimated counts in proportion to the respective population in each region. For a more practical comparison, proportions instead of counts are shown in Table 4. EBP-MSPREE results are satisfactory in terms of their similarity to the direct estimates. Most of the regions show close results to the published one, and the region with the highest discrepancies is the Pacífico Central. This region, however, is the domain with a smaller sample size, therefore it is expected to have less accurate results. The opposite is the case of the region Central which has the biggest sample size and results are very close to the ones in the official publication (results available in INEC 2017).

5. Design-Based Simulation

In this section, results from a design-based simulation study are presented. The objective is to evaluate the EBP-MSPREE procedure explained in Subsection 3.2 to estimate and update counts on poverty incidence in three categories: “Extreme poor”, “poor” not

extreme, and "not poor," as well as assessing the performance of the bootstrap MSE estimator described in Subsection 3.3. To conduct the evaluation, two census compositions are required. One census from time t_0 as the primary input to get EBP-MSPREE estimates but also a second census from time t_1 to compare the updated results. Due to a recent census is not available, survey data described in Subsection 2.2 is used in this experiment. Survey data set from 2011 is used as a census in t_0 (Z_{aj,t_0}) and survey data from 2012 as a census in $t_1(Z_{aj,t_1})$.

Samples are drawn from them with simple random sampling without replacement, with a sampling fraction of f = 0.2. Since survey data is used as a census, there are many domains with few observations. For this reason, the number of cantons in the simulation is reduced to $A = 23$, (instead of $A = 81$ as in the application) and the biggest domains were selected. This allows having all cells with a positive sample, in this case, with at least 15 observations in each category of poverty for all domains. For the first part of the procedure where point estimates are obtained, that is, with an EBP model, a Box-Cox transformation is chosen. Also, the income per capita is defined as the dependent variable and a reduced number of covariates are included in the model, namely: the proportion of employees in the household, the highest degree of education completed by the head of the household, zone, quantity of economically dependent members in the household, equivalized size of the household, at least one member without health insurance, and at least one member not with an educational lag.

In this simulation study, the performance of an EBP-SPREE (similarly as in Isidro et al. 2016) is compared with the proposal of this article, that is, EBP-MSPREE. A total of R = 500 Monte Carlo iterations are defined, with L = 100 Monte Carlo iterations for implementing the EBP, and B = 100 bootstrap iterations for MSE estimation. The performance of the estimated EBP-MSPREE ($\hat{Y}^{EM}_{aj,t_1}$) is evaluated with the relative bias (RB) and the root MSE (RMSE), defined as:

$$\mathrm{RB}\left(\hat{Y}^{EM}_{aj,t_1}\right) = \frac{1}{R}\sum_{r=1}^{R}\left(\frac{\hat{Y}^{EM,r}_{aj,t_1} - Z_{aj,t_1}}{Z_{aj,t_1}}\right)$$

and,

$$\mathrm{RMSE}\left(\hat{Y}^{EM}_{aj,t_1}\right) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(\hat{Y}^{EM,r}_{aj,t_1} - Z_{aj,t_1}\right)^2}, \tag{9}$$

which is treated as the empirical RMSE. A plot comparing the estimated RMSE (from Equation 8) and the empirical RMSE (from Equation 9) is used to validate the proposed MSE estimator $\widehat{\mathrm{MSE}}(\hat{Y}^{EM}_{aj,t_1})$. Relative bias and relative RMSE of the estimated RMSE for each area a and category j are also computed as follows:

$$\mathrm{rel.Bias.Est.RMSE} = \left(\frac{\mathrm{Est.RMSE} - \mathrm{Emp.RMSE}}{\mathrm{Emp.RMSE}}\right)$$

$$\text{rel.RMSE.Est.RMSE} = \frac{\sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(\text{Est.RMSE}^{(r)} - \text{Emp.RMSE}\right)^2}}{\text{Emp.RMSE}}.$$

5.1. Results of the Design-based Simulation

Table 5 summarizes the results of the evaluation of the EBP-MSPREE estimator in comparison with a previous version, namely EBP-SPREE. The values of RB and RMSE are averaged over 23 areas for each category of poverty. When looking at the median and mean for each category of poverty, the EBP-MSPREE procedure provides a smaller RMSE than the EBP-SPREE procedure for three out of the six cases, especially in the category "not poor". The absolute value of the RB of the EBP-MSPREE procedure is considerably lower than the absolute value of the RB of EBP-SPREE in four out of the six cases. This is observed in the categories "extreme" and "not poor".

Figure 6 displays the estimated and empirical RMSE over the domains and categories of poverty for the EBP-MSPREE and the EBP-SPREE methods. Based on this figure, it is possible to conclude that the estimated RMSE tracks the empirical RMSE better for the EBP-MSPREE procedure, and this can be observed for all of the categories of poverty. A closer look at the performance of the proposed MSE is provided in Table 6. The RB-RMSE for the EBP-MSPREE indicates a moderate underestimation in the mean and the median for the "extreme" and "poor" categories and an overestimation for the median of the category "poor". In terms of RB-RMSE and RRMSE-RMSE, the results on the performance of the MSE are favorable for the EBP-MSPREE.

6. Conclusions and Further Steps

Public policies require not only accurate and reliable information for decision- making but this information should also be timely. It is common that the production of official statistics faces challenges due to limitations of resources. In this article, a methodology to obtain reliable and updated estimates in small areas is presented and exemplified with a real-world application. For many developing countries, censuses are conducted every ten years and sample sizes of annual national surveys are not big enough to provide reliable results for small areas. An additional limitation that is considered in this work is that information of interest is not present in census data, as it is required for SPREE methods. The strategy proposed considers two well-known small area estimation techniques.

Table 5. RB and RMSE for the incidence of poverty by status under different SPREE approaches.

		Extreme		Poor not extreme		Not poor	
		Median	Mean	Median	Mean	Median	Mean
RB	EBP-SPREE	0.1965	0.1279	0.0185	0.0112	-0.0219	0.0337
	EBP-MSPREE	0.0195	0.0987	0.0322	0.0144	-0.0125	0.0098
RMSE	EBP-SPREE	0.3120	0.3468	0.1274	0.1514	0.0744	0.1060
	EBP-MSPREE	0.2781	0.3538	0.1479	0.1673	0.0705	0.0985

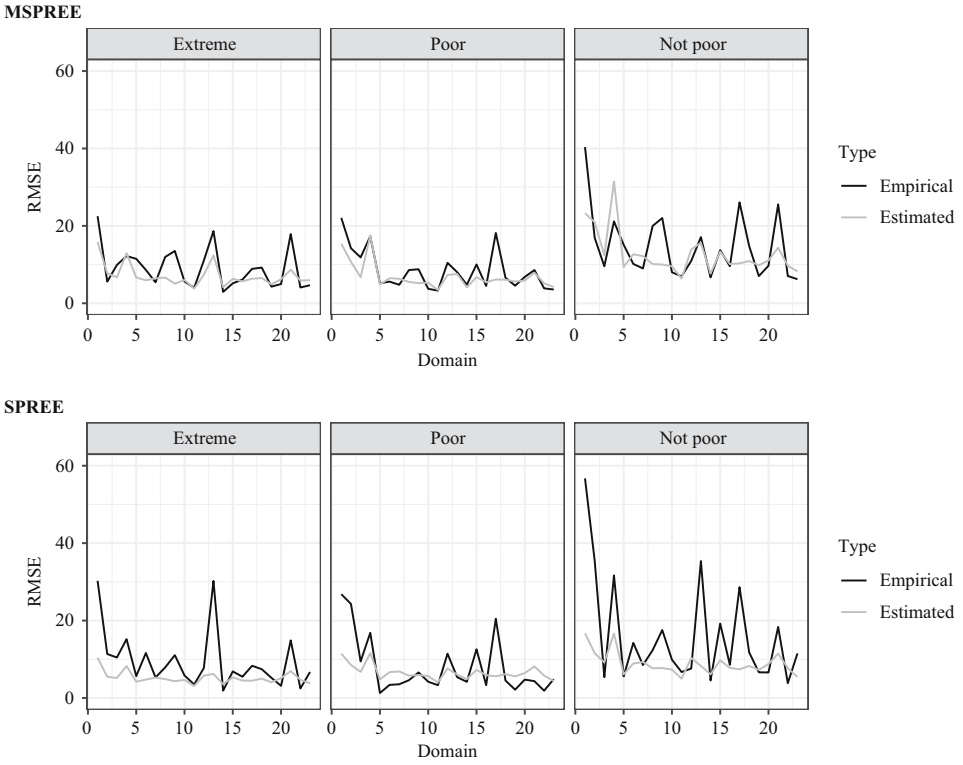


Fig. 6. Estimated and empirical domain-and category-specific RMSEs of the counts on poverty incidence.

Table 6. Performance of the MSE estimator: Mean and Median of RB and RMSE by poverty status.

		Extreme		Poor not extreme		Not poor	
		Median	Mean	Median	Mean	Median	Mean
RB-	EBP-SPREE	-0.3294	-0.2294	0.2107	0.3627	-0.2969	-0.1711
RMSE	EBP-MSPREE	-0.0752	-0.0812	-0.0765	-0.0714	0.1388	0.0080
RRMSE-	EBP-SPREE	0.4616	0.4595	0.3785	0.6769	0.3813	0.4477
RMSE	EBP-MSPREE	0.3812	0.3911	0.3586	0.3665	0.3678	0.3666

An EBP is conducted to get poverty estimates in the census data, and as a second step, the MSPREE of Luna-Hernández (2016) is applied to update the estimates in postcensal years. Based on the results of the application, it is possible to conclude that the strategy proposed delivers quality results in terms of CVs and compares favourably with direct estimates. The application showed that this methodology gives the opportunity to analyze specific groups of interest, areas, and years. For example, that the poorest cantons in Costa Rica have remained with little overall improvements for the period studied.

Although the methodology proposed allows to obtain the target estimates, there are several aspects that can be improved, especially in the uncertainty estimation. Original SPREE and two other versions (GSPREE and MSPREE) assume that the census has the

variable of interest and therefore no uncertainty from the association structure is required. However, when another small area estimation method is needed as a first step to get the census structure (in this case with an EBP), variability from it should be considered. In this methodology, a parametric bootstrap is implemented to get uncertainty from the allocation, the association structure, and the estimation of the β coefficients required in MSPREE. One potential topic for further research is to combine the MSE that is produced directly from the EBP with the one from MSPREE under an analytical approach. Furthermore, the impact that extreme values in the first part of the procedure (e.g., EBP) can have in the final updated estimates deserves also to be investigated.

SPREE methods have other disadvantages that require further study to get a more flexible technique. For example, a potential improvement in the method could be to allow updating more complex indicators or non-categorical indicators such as the Gini index or (mean and/or median) per capita income. The inclusion of associated variables as suggested by Purcell and Kish (1980) can also be beneficial in the estimation procedure, for instance, the inclusion of urbanity (urban and rural) can be relevant when working with poverty status.

The over-shrinking problem present in the context of small area estimation when the expected sample variance is smaller than the true parameter also deserves to be explored when implementing SPREE-type methods.

Understanding the benefits of SPREE-type methods in comparison with existing models in the small area estimation context requires also further research. Three alternative approaches to deal with the problem of obtaining updated counts or proportions in small domains have been identified and deserve a closer comparison with the SPREE-type methods: 1. The use of the EBP in each year of study with a final benchmark operation performed with MSPREE, 2. exploring potential advantages of using panel survey data or time-series models for example, with the extension of the Fay-Herriot model proposed by Rao and Yu (1994), or 3. implement measurement error models also in the context of area-level models (Ybarra and Lohr 2008). Finally, it is recommended to study the inclusion of non-traditional information sources (e.g., big data) as proposed in Koebe et al. (2022) since the structures of population censuses can quickly become obsolete. A clear example of this is the socio-economic effect that the COVID-19 pandemic generated in many countries, altering the living conditions of many people in a short period of time.

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