

Investigating the discharge characteristics over a piano key weir with varying key slope and key width ratios under varying discharges

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Abstract: The objective of the present study was to investigate the effect of inlet to outlet key slope ratios (S_i/S_o), inlet to outlet key width ratios (W_i/W_o) and head over the weir on the coefficient of discharge (C_{pkw}) of type-A trapezoidal piano key weir (TPKW). It was observed that the C_{pkw} of TPKW was found to be decreased as the S_i/S_o ratio reduced. The percentage of reduction in C_{pkw} was found to be 18.5%, 16%, 13.7%, 9.4%, 6.3%, and 3% at $S_i/S_o = 0.60, 0.75, 0.80, 1.00, 1.25$, and 1.33 , respectively in comparison with the $S_i/S_o = 1.67$. As far as the effect of W_i/W_o ratio on the C_{pkw} of TPKW is concerned, the C_{pkw} of TPKW was observed to be increased with increasing W_i/W_o ratios. Moreover, the C_{pkw} at $W_i/W_o = 1.32$ and $W_i/W_o = 1.15$ was found to be 9.4% and 7.5% higher than $W_i/W_o = 1$ respectively, while for the case of $W_i/W_o = 0.76$, its value was found to be 3.3% lower than $W_i/W_o = 1$. Furthermore, the hydraulic efficiency of TPKW was found to be decreased as head over the weir increased. The maximum value of C_{pkw} of TPKW was observed to be 3.2 corresponding to lowest h/P ratio for the model having $S_i/S_o = 1.67$ and $W_i/W_o = 1.32$, while the minimum value of C_{pkw} of TPKW was found to be 1.58 corresponding to highest h/P ratio for the model having $S_i/S_o = 0.60$ and $W_i/W_o = 0.76$. Additionally, sensitivity analysis results revealed that the C_{pkw} of TPKW is more sensitive with h/P ratio than other ratios.

Keywords: Coefficient of discharge; Inlet to outlet key slope ratios; Inlet to outlet key width ratios; Trapezoidal piano key weir; Type-A piano key weir.

1 INTRODUCTION

A weir is a hydraulic structure which is constructed across the river for various purposes: like to measure the flow rate in a river, navigation purposes, and to divert water from river to irrigation canals etc. One of the purpose of weir is to raise the water level at the upstream of the weir, so that the water can be drawn from river to irrigation canals during the low flow season in the river. During the high flood season in the river, the linear weir cannot pass the high flows due to its less discharge passing capacity. Labyrinth and piano key weir (PKW) have the ability to increase the discharge capacity by three to four times relative to linear weir at the same head and weir crest height. Labyrinth and PKW have a zigzag crest shape, and thus called a non-linear weir. The small footprint of PKW makes this weir an efficient and cost-effective solution over the classical labyrinth weir.

PKWs are one of the best solution to modify the existing weirs because they are economical and having the ability to pass more discharge during flooding in the river (Machiels et al., 2011; Ouamane and Lempérière, 2006). The first PKW was installed at the Goulours dam (France) in the year 2006 (Laugier, 2007). Lempérière and Ouamane (2003) performed an experimental study to investigate the coefficient of discharge of PKW and proposed the modified shape of free flow PKW and concluded that the discharge capacity of PKW is almost 4–5 times greater than the original discharge. PKWs are divided into four types based on their upstream and downstream nappes (Lempérière and Ouamane, 2003): Type A PKW consists of nappes on both upstream and downstream side, while type B and type C consists of nappes on one side; only upstream and downstream, respectively. While for type D PKW, there is no nappe both at the upstream as well as at downstream side (Lempérière et al., 2011). A naming convention and notations of the various

geometrical parameters of the PKW are presented at the end of the manuscript. The general 3-D view, plan, and cross-sectional view of PKW with geometric parameters can be seen in Fig. 1 and Fig. 2, respectively.

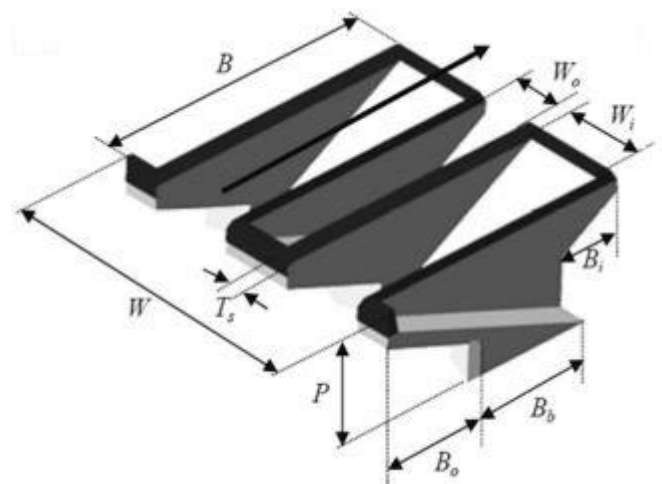


Fig. 1. 3-D view of PKW (Pralong et al., 2011).

In literature a number of studies have been reported to investigate the impact of geometric and hydraulic parameters on the discharge efficiency of rectangular PKW. (Kabiri-Samani and Javaheri, 2012; Leite Ribeiro et al., 2012; Oertel, 2015) performed a numerical and experimental study to investigate the discharge capacity of PKW. Machiels et al. (2011) performed a laboratory study to investigate the flow performance over the PKWs. (Pfister and Schleiss, 2013; Ribeiro et al., 2012; Sy and

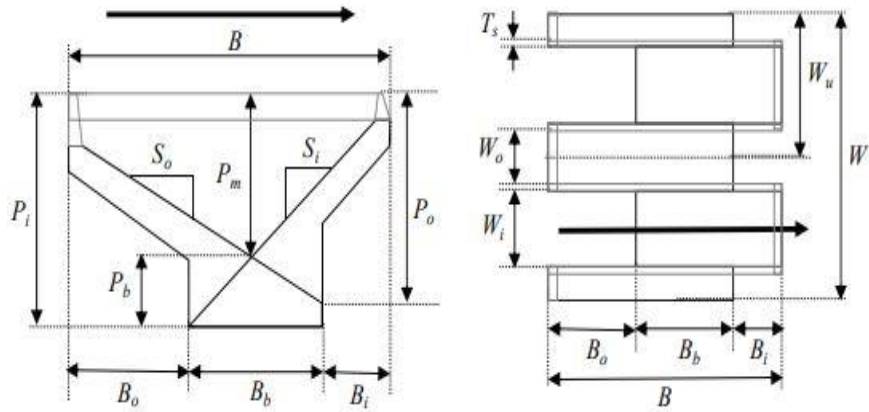


Fig. 2. Cross sectional (left) and plan (right) view of PKW (Pralong et al., 2011).

Manh, 2017) conducted various types of research regarding the hydraulic design of PKWs. (Anderson and Tullis, 2013; Karaeren and Bozku, 2015) performed a comparative study on the PKW and labyrinth weir and they concluded that PKW is more efficient as compared to labyrinth weir. Bekheet et al. (2022) performed a study to investigate the impact of shape and type of PKW on its discharge capacity. Kumar et al. (2019) performed a laboratory study to estimate the discharge equation of PKW. Kumar et al. (2020) performed a laboratory and numerical study to investigate the discharge capacity of trapezoidal and rectangular PKW. Eslinger and Crookston (2020) performed an experimental study to investigate the energy dissipation of type A PKW.

Mehboudi et al. (2017) performed a research work to investigate the flow behaviour over the trapezoidal PKW by varying geometric parameters. Li et al. (2020) carried out a research to investigate the influence of auxiliary geometric parameters on the discharge efficiency of PKW. Cicero et al. (2013) performed a numerical and experimental study to determine the discharge capacity of TPKW and it was found that discharge efficiency of TPKW can be increased by 5 to 20% by lowering the head of water over the weir. Ghasemzadeh et al. (2015) performed a research to investigate the discharge capacity of PKW. Bremer and Oertel (2017) performed a numerical study to investigate the effect of wall thickness on the discharge efficiency of PKW. Denys et al. (2017) conducted an experimental and numerical simulation work to investigate the fluid-structure interaction in PKW. Erpicum et al. (2016) conducted a study to investigate the scale effects in physical PKW models. Mehboudi et al. (2016) carried out an experimental study to investigate the coefficient of discharge of TPKW by varying the geometric parameters. Khassaf and Al-Baghdadi (2015) performed an experimental study to investigate the influence of side wall angle on the discharge efficiency of PKW. It was concluded that the changing of side wall angle to around 5° can increase the discharge capacity of weir and the changing of side wall angle to around 10° has a negative effect on the discharge capacity of weir. Mehboudi et al. (2016) performed an experimental study to investigate the discharge coefficient of TPKW by keeping the side wall angle in the range of 0.18° to 0.43° . Cicero et al. (2013) performed a study to investigate the effect of side wall angle on the flow efficiency using the computer program FLOW-3D. Iqbal and Ghani (2024a) predicted the discharge capacity of type-A TPKW using soft computing tools including artificial neural network (ANN), adaptive neuro-fuzzy interference system (ANFIS), support vector machine (SVM), and non-linear regression technique. The

results predicted by ANN model were satisfactory than other predicted models.

A few studies have also been reported in literature regarding the investigation of effect of inlet to outlet key width ratios (W_i/W_o) on the discharge capacity of PKW. Ich et al. (2022) performed a numerical study to investigate the effect of inlet to outlet key width ratios on the discharge capacity of PKW. Singh and Kumar (2023) investigated the hydraulic behaviour of PKW by varying the inlet to outlet key width ratios. Saghari et al. (2019) evaluated the effect of using one and two cycles on the discharge capacity of TPKW. Several numerical and CFD techniques have also been utilized to investigate the discharge capacity of PKW (Guo et al., 2019; Singhal et al., 2011).

Numerous studies and design projects have been done on the hydraulic behaviour and optimization of the PKWs with the goal of increasing the discharge capacity of such weirs. PKW involves a lot of geometric parameters and their effect are very crucial on the discharge capacity of weir. Currently very limited geometric parameters have been investigated previously so far. However, to the knowledge of the authors none of the past studies have investigated the effect of inlet to outlet key slope ratios (S_i/S_o) of TPKW. Few experimental studies have been carried out in the past to study the impact of W_i/W_o ratios on the discharge efficiency of PKW, but there is still insufficient information about the effect of this parameter on the discharge capacity of TPKW. The objective of the present study was to investigate the effect of S_i/S_o ratios, W_i/W_o ratios and head over the weir on the discharge capacity of type-A TPKW.

2 MATERIAL AND METHODS

2.1 Dimensional analysis

According to the geometric parameters defined by (Leite Ribeiro et al., 2012), the discharge over a TPKW is a function of the following parameters as given in Eq. (1).

$$Q = f(\rho, g, \mu, \sigma, h, y, V, L, P, W, W_u, W_i, W_o, B_i, B_o, B, N_u, S_i, S_o, T_s, R, \alpha) \quad (1)$$

Description of these parameters are defined in notation table of the manuscript. Applying the Buckingham π theorem and selecting L, ρ and V as repetitive variables. The obtained π -terms and modified π -terms are given in Eq. (2) and Eq. (3), respectively. Finally, the obtained dimensionless parameters are given in Eq. (4). The Eq. (3) was developed by the modification of various π -terms. Similarly, in Eq. (3) the factor $1.5/\sqrt{2}$ was multiplied on both side of the equation to make complete the equation of coefficient of discharge of PKW.

$$\begin{aligned} \Pi_1(Q) &= \frac{Q}{VL^2}, \Pi_2(g) = \frac{gL}{V^2}, \Pi_3(\mu) = \frac{\mu}{L\rho V}, \Pi_4(\sigma) = \frac{\sigma}{L\rho V^2}, \Pi_5(h) = \frac{h}{L}, \Pi_6(y) = \frac{y}{L}, \Pi_7(P) = \frac{P}{L}, \Pi_8(W) = \frac{W}{L}, \Pi_9(W_u) = \frac{W_u}{L}, \\ \Pi_{10}(W_i) &= \frac{W_i}{L}, \Pi_{11}(W_o) = \frac{W_o}{L}, \Pi_{12}(B_i) = \frac{B_i}{L}, \Pi_{13}(B_o) = \frac{B_o}{L}, \Pi_{14}(B) = \frac{B}{L}, \Pi_{15}(T_s) = \frac{T_s}{L}, \Pi_{16}(R) = \frac{R}{L}, \Pi_{17}(S_i) = S_i, \\ \Pi_{18}(S_o) &= S_o, \Pi_{19}(N_u) = N_u, \Pi_{20}(\alpha) = \alpha \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\Pi_7(P)}{\Pi_9(W_u)} &= \frac{P}{W_u}, \frac{\Pi_5(h)}{\Pi_7(P)} = \frac{h}{P}, \frac{\Pi_{16}(R)}{\Pi_7(P)} = \frac{R}{P}, \frac{\Pi_{15}(T_s)}{\Pi_7(P)} = \frac{T_s}{P}, \frac{\Pi_{16}(R)}{\Pi_{15}(T_s)} = \frac{R}{T_s}, \frac{\Pi_{12}(B_i)}{\Pi_{14}(B)} = \frac{B_i}{B}, \frac{\Pi_{13}(B_o)}{\Pi_{14}(B)} = \frac{B_o}{B}, \\ \frac{\Pi_{12}(B_i)}{\Pi_{13}(B_o)} &= \frac{B_i}{B_o}, \frac{\Pi_{10}(W_i)}{\Pi_{11}(W_o)} = \frac{W_i}{W_o}, \frac{\Pi_{17}(S_i)}{\Pi_{18}(S_o)} = \frac{S_i}{S_o}, \Pi_3\mu = \frac{1}{\Pi_3(\mu)} = \frac{\rho VL}{\mu}, \sqrt{\frac{\Pi_2'(g)}{\Pi_6(y)}} = \frac{V}{\sqrt{gy}} = F_r, \frac{1.5}{\sqrt{2}} \times \frac{\sqrt{\frac{\Pi_1(Q)}{(\Pi_2(g)\Pi_5(h)/\Pi_8(W))}}}{\Pi_5(h)\Pi_8(W)} = \\ \frac{1.5Q}{W\sqrt{2gh^2}} &= C_{pkw} \end{aligned} \quad (3)$$

$$f\left(\frac{P}{W_u}, \frac{h}{P}, \frac{R}{P}, \frac{T_s}{P}, \frac{R}{T_s}, \frac{B_i}{B}, \frac{B_o}{B}, \frac{B_i}{B_o}, \frac{W_i}{W_o}, \frac{S_i}{S_o}, \frac{W}{L}, \alpha, N_u, \frac{1.5Q}{\sqrt{2gh^2}}, \frac{V}{\sqrt{gy}}, \frac{\rho LV}{\mu}, \frac{\sigma}{L\rho V^2}\right) = 0 \quad (4)$$

By eliminating the constant parameters (i.e., the number of cycle, thickness of the side wall, weir crest height, parapet wall, base length, unit cycle width) and the effect of secondary parameters that varied by the variation of inlet to outlet key slope ratios (i.e., the ratio of the inlet to outlet overhang lengths, total overflowing crest length and side wall angle). So, Eq. (4) can be reduced to Eq. (5).

$$f\left(\frac{S_i}{S_o}, \frac{W_i}{W_o}, \frac{h}{P}, \frac{1.5Q}{\sqrt{2gh^2}}, \frac{V}{\sqrt{gy}}, \frac{\rho LV}{\mu}, \frac{\sigma}{L\rho V^2}\right) = 0 \quad (5)$$

here, $\rho LV/\mu$ is the Reynolds number (Re), $\sigma/L\rho V^2$ is the Weber number (We), $1.5Q/\sqrt{2gh^2}$ is the coefficient of discharge of TPKW (C_{pkw}), and $V/\sqrt{gy}$ is the upstream Froude number (F_r). The effect of surface tension is negligible for the water head over 3 cm as recommended by (Novák and Čabelka, 1981), and the effect of viscosity is not significant compared to the gravity effect in turbulent flow as suggested by (Ribeiro et al., 2012). Viscosity and surface tension are inherent fluid properties that cannot be easily scaled, leading to scale effects when dealing with small overflow depths on weir crests. According to Hager (2010), critical considerations arise for depths $h < 0.05$ m, while Novak et al. (2018) suggest $h < 0.03$ m. Therefore, in the present study the depths below 0.03 m were excluded in the development of the Eq. (8), as both PKW and free overfall models exhibit

scale effects. The water head over the weir was more than 3cm for all experimental tests and the flow was entirely turbulent throughout the experiments, so that the effect of We and Re is neglected. The finally dimensionless parameters could be obtained as Eq. (6).

$$C_{pkw} = f\left(\frac{S_i}{S_o}, \frac{W_i}{W_o}, \frac{h}{P}, F_r\right) \quad (6)$$

2.2 Description of laboratory setup

Experiments were performed in the hydraulic laboratory of Civil Engineering Department at University of Engineering and Technology Taxila, Pakistan. The rectangular channel having a length of 10 m, width 0.31 m and depth 0.5 m was used for experimentation as shown in Fig. 3. The channel side walls, and bed were made of glass and stainless steel respectively. The level gauge having ± 0.1 mm accuracy was used to measure the depth of water at various points along the length of the channel. A rectangular sharp-crested weir was used at the end of the channel for the measurement of flow rate. All the experimental tests were carried out under fully developed subcritical flow conditions at the upstream of the TPKW. To find the Froude number (F_r) at the upstream (u/s) of the TPKW the depth and velocity of water was measured at a distance of 1.2 m from the weir (Li et al., 2020).



Fig. 3. Laboratory channel setup.

2.3 Preparation of TPKW models

All the TPKW models were prepared using the wooden material. Silicon gel was used to fill the joints of models and to make these watertight. For each experiment, the TPKW model was installed at a distance of 5 m from the inlet of the channel. In this study a total of 224 test runs were performed to obtain the discharge coefficient of TPKW by varying the seven different cases of inlet to outlet key slope ratios ($S_i/S_o = 0.60, 0.75, 0.80, 1.00, 1.25, 1.33, \text{ and } 1.67$) and four different cases of inlet to outlet key width ratios ($W_i/W_o = 0.76, 1.00, 1.15, \text{ and } 1.32$) of type-A TPKW at a wide range of discharges from 16.01 l/s to 24.88 l/s. The other geometric parameters including the weir crest height ($P = 0.20$ m), number of cycles ($N_u = 3$), thickness of the side walls ($T_s = 0.01$ m) and the base width ($B_b = 0.25$ m) of the TPKW models were remained fixed throughout the experimentation. The range of side wall angle (α) was 0.44° to 0.64° and the ratio of L/W ranged from 9.52 to 13.39 in the present research work. The geometric parameters which were used for the current study are presented in Table 1.

The actual laboratory view of the model M7 is presented in Fig. 4.



(a)



(b)

Fig. 4. The actual laboratory view of the model M7 (a) side view; (b) top view.

Table 1. Geometric parameters of TPKW used in the present research work.

TPKW models	Inlet to outlet key slope ratios (S_i/S_o)	Inlet to outlet over hanged length ratios (B_i/B_o)
M1	0.60	0.20
M2	0.75	0.33
M3	0.80	0.60
M4	1.00	1.00
M5	1.25	1.67
M6	1.33	3.00
M7	1.67	5.00

2.4 Calculation of Inlet and outlet key slopes of TPKW

Typical representation and calculation of inlet to outlet key slope ratios of model M7 is given in Fig. 5. As the crest height of TPKW was kept constant ($P = 0.20$ m) throughout the experimentation, overhang length was varied for each case while changing the key slopes. Seven different S_i/S_o ratios were investigated in this research work at 8 different flow rates. For each S_i/S_o ratio, four different W_i/W_o ratios were also investigated to observe the impact of W_i/W_o ratio on the coefficient of discharge of TPKW.

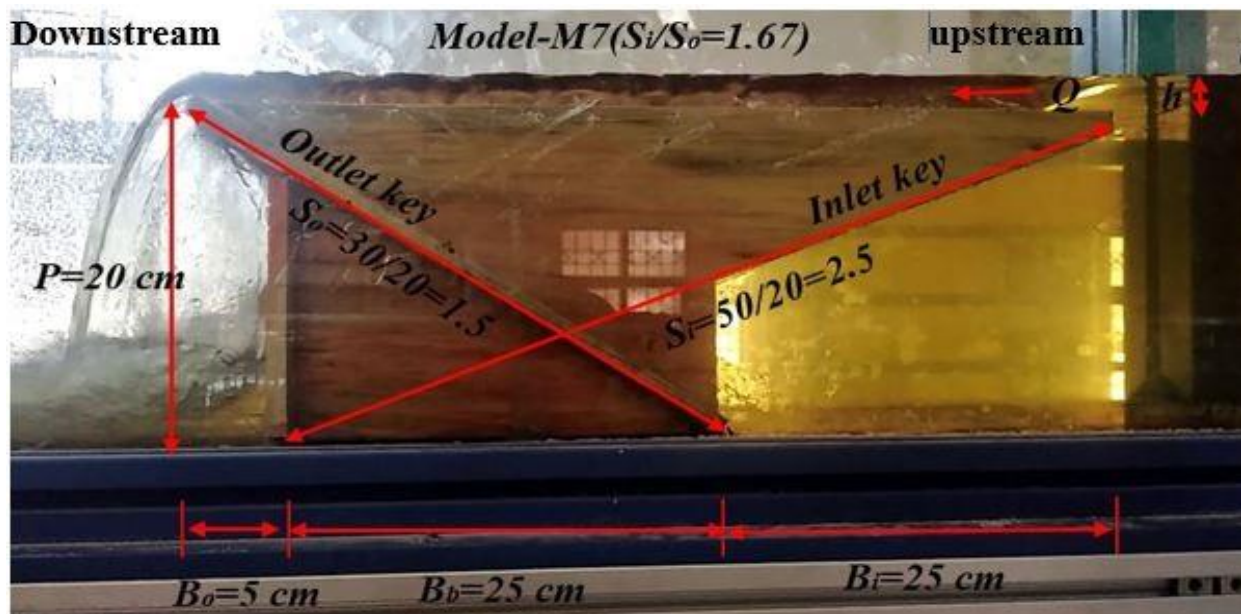


Fig. 5. Calculation and representation of inlet to outlet key slopes of model M7.

2.5 Measurement of discharge capacity of TPKW

The coefficient of discharge of TPKW was determined using the equation of traditional linear weir as used by (Kabiri-Samani and Javaheri, 2012; Mehboudi et al., 2016) as given in Eq. (7) to estimate the discharge capacity of TPKW.

$$Q_{Pk} = \frac{2}{3} C_{pkw} W \sqrt{2g} h^{\frac{3}{2}} \quad (7)$$

To reduce the scale effects (effect of surface tension and viscosity), researchers including: (Abhash and Pandey, 2021; Bekheet et al., 2022; Karimi et al., 2018; Novak et al., 2018; Pfister et al., 2013) suggested a limiting upstream head criteria of 3 cm over the weir. According to Tullis et al. (2020), proposed a criteria of $h/P \geq 0.09$ for which scale effect is negligible. Erpicum et al. (2016) recommended that a coefficient of discharge of a weir measured in a physical scale model can be applicable to the corresponding prototype if the head over the weir is more than 2 cm. Erpicum et al. (2016) suggested that the influence of scale effect between the model and prototype becomes negligible when head over the weir is sufficiently high to overcome the surface tension everywhere, resulting the entire crest of the weir will be engaged in conveying discharge. Surface tension effect is insignificant, if head over the crest of weir is much sufficient to initiate the entire crest length engaged in conveying discharge. If both the model and prototype utilize the same fluid, the impact of viscous forces can be ignored (Idrees and Al-Ameri, 2023). In the present study, the ratio of head over the TPKW to weir height (h/P) varied from $0.15 \leq h/P \leq 0.4$, and the head over the weir varied from $0.03 \leq h \leq 0.08$ m. So, both these effects can be ignored for the current study.

The present study has subjected to the following limitations regarding the laboratory channel used in present research work; (i) the maximum flow capacity of the pump that was installed with the laboratory channel was 25 l/sec. So, the test runs were not performed at higher flow rate, (ii) Digital flow measuring device was not attached to the laboratory channel, so the flow was measured using a sharp crested weir.

3 RESULTS AND DISCUSSIONS

To investigate the discharge characteristics of TPKW the results of the present study have been analysed in different ways which are provided in the following subsections.

3.1 Effect of inlet to outlet key slope ratios (S_i/S_o) on the discharge capacity of type-A TPKW

The coefficient of discharge of a weir basically represents the discharge capacity of the weir. Accurate knowledge of coefficient of discharge is essential while designing a weir, and it also facilitates accurate flow rate measurements. To understand the influence of inlet to outlet key slope ratios on the discharge efficiency of TPKW the results obtained from the present study have been plotted between the coefficient of discharge (C_{pkw}) and S_i/S_o ratios at different key width (W_i/W_o) ratios.

The experimental results show that inlet to outlet key slope ratio (S_i/S_o) has a considerable impact on the discharge capacity of TPKW. Fig. 6 shows that the coefficient of discharge (C_{pkw}) of TPKW increased with increasing inlet to outlet key slope ratios, while with increasing S_i/S_o ratios the change in C_{pkw} values for any particular discharge keep on decreasing. At fixed weir height, increasing of S_i/S_o ratio means the increasing of lateral crest length of the side wall (B) which results in increasing

the total crest length (L) of the weir. Consequently, the discharge passing capacity of the weir increased due to the increasing of effective crest length of the TPKW. The maximum efficiency of TPKW was observed corresponding to highest inlet to outlet key slope ratio ($S_i/S_o = 1.67$) and was found to be minimum at smallest inlet to outlet key slope ratio ($S_i/S_o = 0.60$). Moreover, the discharge capacity of TPKW at $S_i/S_o = 1.67$, 1.33, and 1.25 was found to be 10%, 7.6%, and 4.8% higher, respectively than $S_i/S_o = 1$, while its capacity observed to be lower by 3%, 6.5%, and 9.4% at $S_i/S_o = 0.80$, 0.75, and 0.60, respectively than $S_i/S_o = 1$. Further, the discharge capacity of TPKW was observed to be reduced with the increasing of flow rate (head) over the crest of the weir. Moreover, the dependence for a discharge of 16.68 l/s was little bit different from other discharges, the reason could be the fluctuations of pump that was attached with the channel.

3.2 Effect of h/P ratios on the discharge capacity of type-A TPKW

To further understand the discharge performance over the TPKW the graphs were plotted between the coefficient of discharge (C_{pkw}) and h/P ratios under different S_i/S_o and W_i/W_o ratios as shown in Fig. 7. It was observed that at constant weir height (P), the discharge efficiency of TPKW decreases with increasing h/P ratios. The maximum value of C_{pkw} was observed corresponding to lowest h/P ratio and minimum value of C_{pkw} was observed corresponding to highest h/P ratio irrespective of the S_i/S_o and W_i/W_o ratios. Furthermore, according to experimental results the value of C_{pkw} of type-A TPKW was found to be 3.2 corresponding to lowest h/P ratio ($h/P = 0.15$) for the model having $S_i/S_o = 1.67$ and $W_i/W_o = 1.32$, while the value of C_{pkw} of type-A TPKW was observed to be 1.58 corresponding to highest h/P ratio ($h/P = 0.33$) for the model having $S_i/S_o = 0.60$ and $W_i/W_o = 0.76$. Moreover, the value of C_{pkw} was found to be 3.2, 3.1, 2.95, 2.82, 2.64, 2.43, 2.3 and 2.18 at $h/P = 0.15$, 0.16, 0.178, 0.19, 0.21, 0.23, 0.25 and 0.27, respectively corresponding to hydraulically efficient model ($S_i/S_o = 1.67$ and $W_i/W_o = 1.32$). While, the value of C_{pkw} was observed to 2.3, 2.15, 2.07, 1.98, 1.82, 1.75, 1.63 and 1.58 at $h/P = 0.194$, 0.205, 0.222, 0.242, 0.27, 0.288, 0.315 and 0.33, respectively corresponding to hydraulically least efficient model ($S_i/S_o = 0.60$ and $W_i/W_o = 0.76$). Overall, the percentage of increasing in C_{pkw} of type-A TPKW was observed to be 22% more at lower heads than higher heads.

3.3 Effect of W_i/W_o ratios on the discharge efficiency of type-A TPKW

To demonstrate the effect of key width ratios (W_i/W_o) on the discharge capacity of type-A TPKW, the graphs are plotted between the C_{pkw} of type-A TPKW and h/P ratios at various inlet to outlet key width ratios ($W_i/W_o = 1.32$, 1.15, 1.00, and 0.76) under different S_i/S_o ratios as shown in Fig. 8. All tested TPKW models have performed hydraulically efficiently at $W_i/W_o = 1.32$ due to higher flow area and perimeter of the inlet key and all models provided hydraulically least efficient results at $W_i/W_o = 0.76$ for all cases of S_i/S_o ratios. Further, the coefficient of discharge of the weir was observed to be increased with increasing W_i/W_o ratios, but their contribution in increasing of C_{pkw} was somewhat different. However, the percentage of increasing of C_{pkw} was found to be 12.5%, 10.6%, and 3.3% more at $W_i/W_o = 1.32$, 1.15, and 1.00, respectively in comparison with the $W_i/W_o = 0.76$. Furthermore, Fig. 8 Shows that the discharge efficiency of type-A TPKW is decreasing with the increasing of h/P ratios irrespective of the S_i/S_o and W_i/W_o ratios.

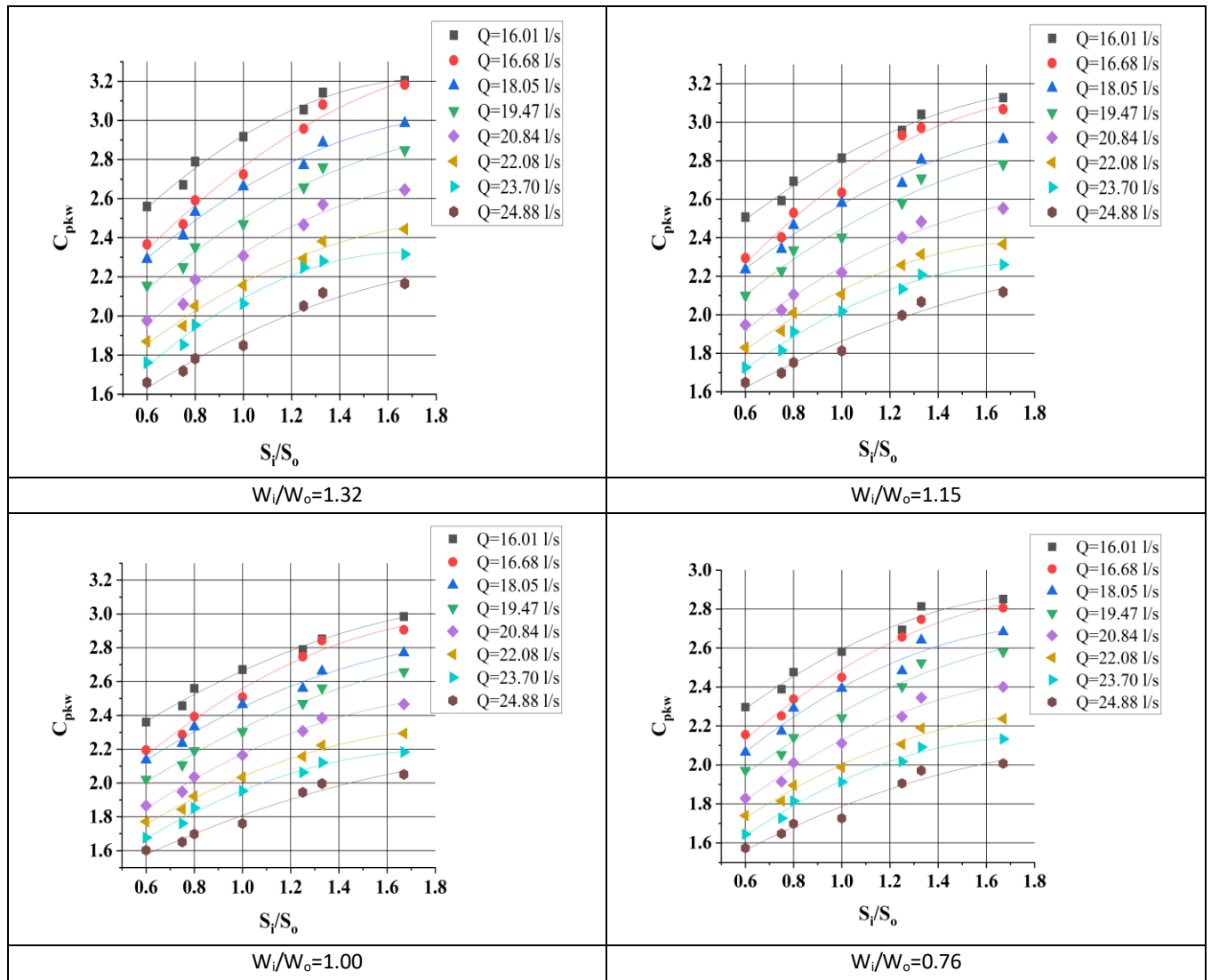
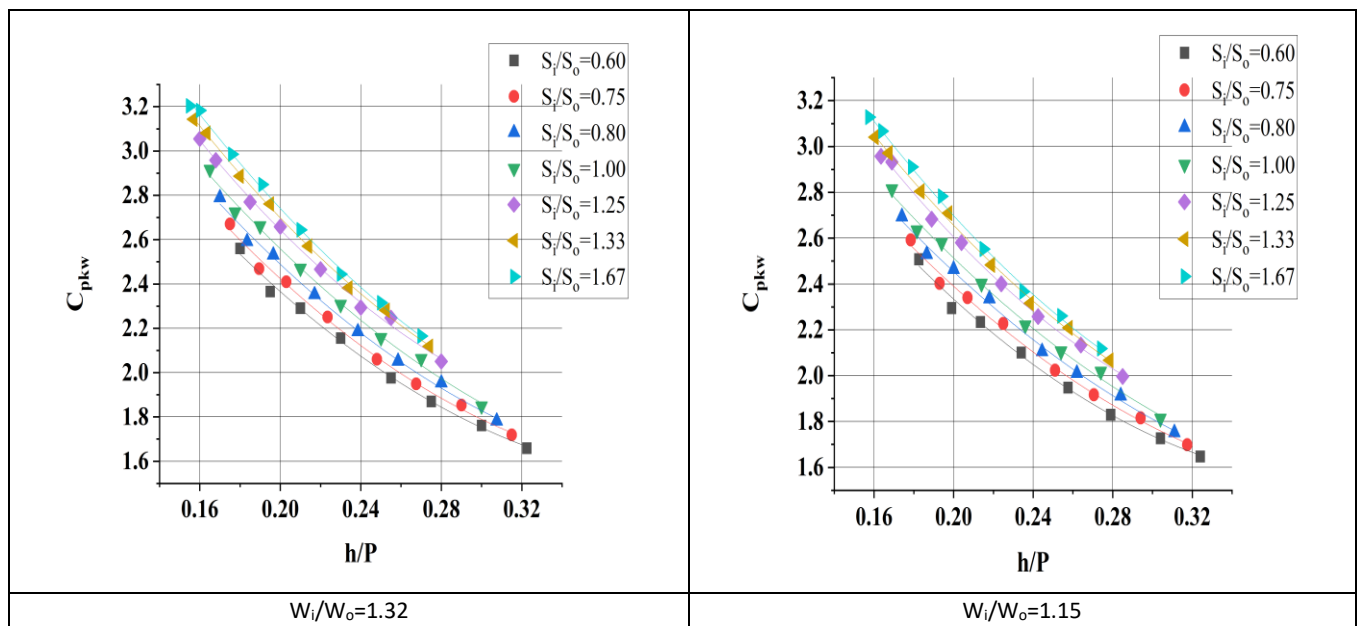


Fig. 6. Relationship between the coefficient of discharge of TPKW (C_{pkw}) and (S_i/S_o) ratios.



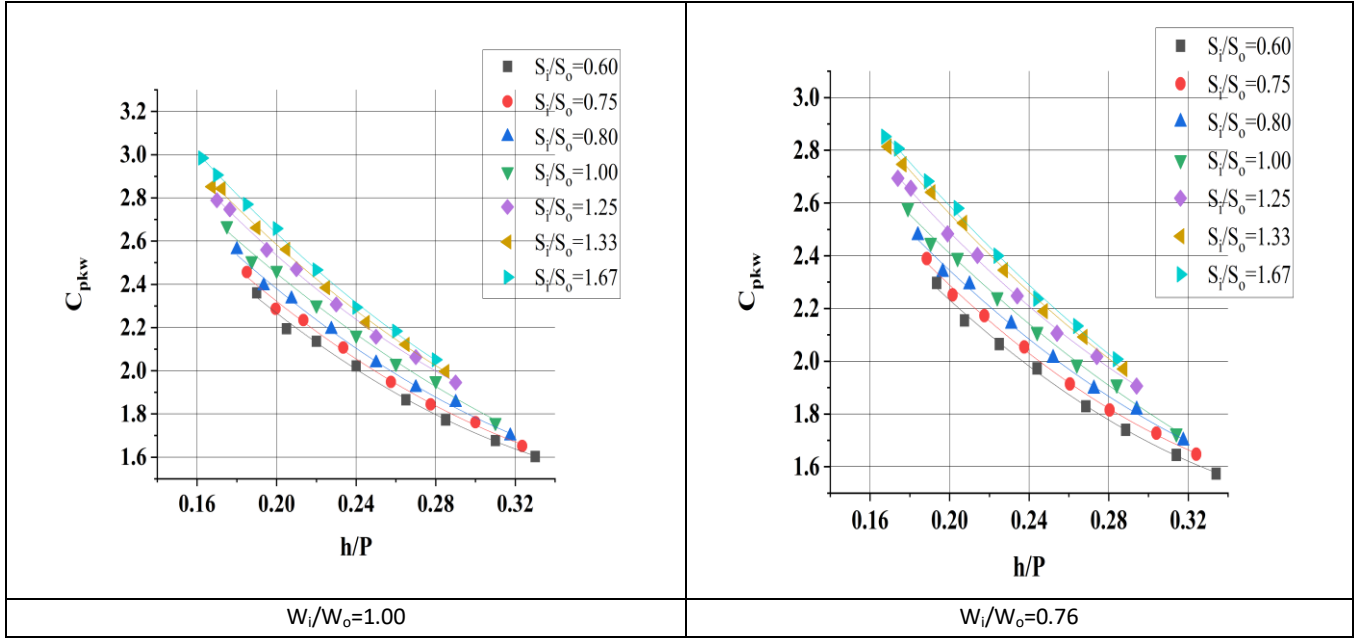
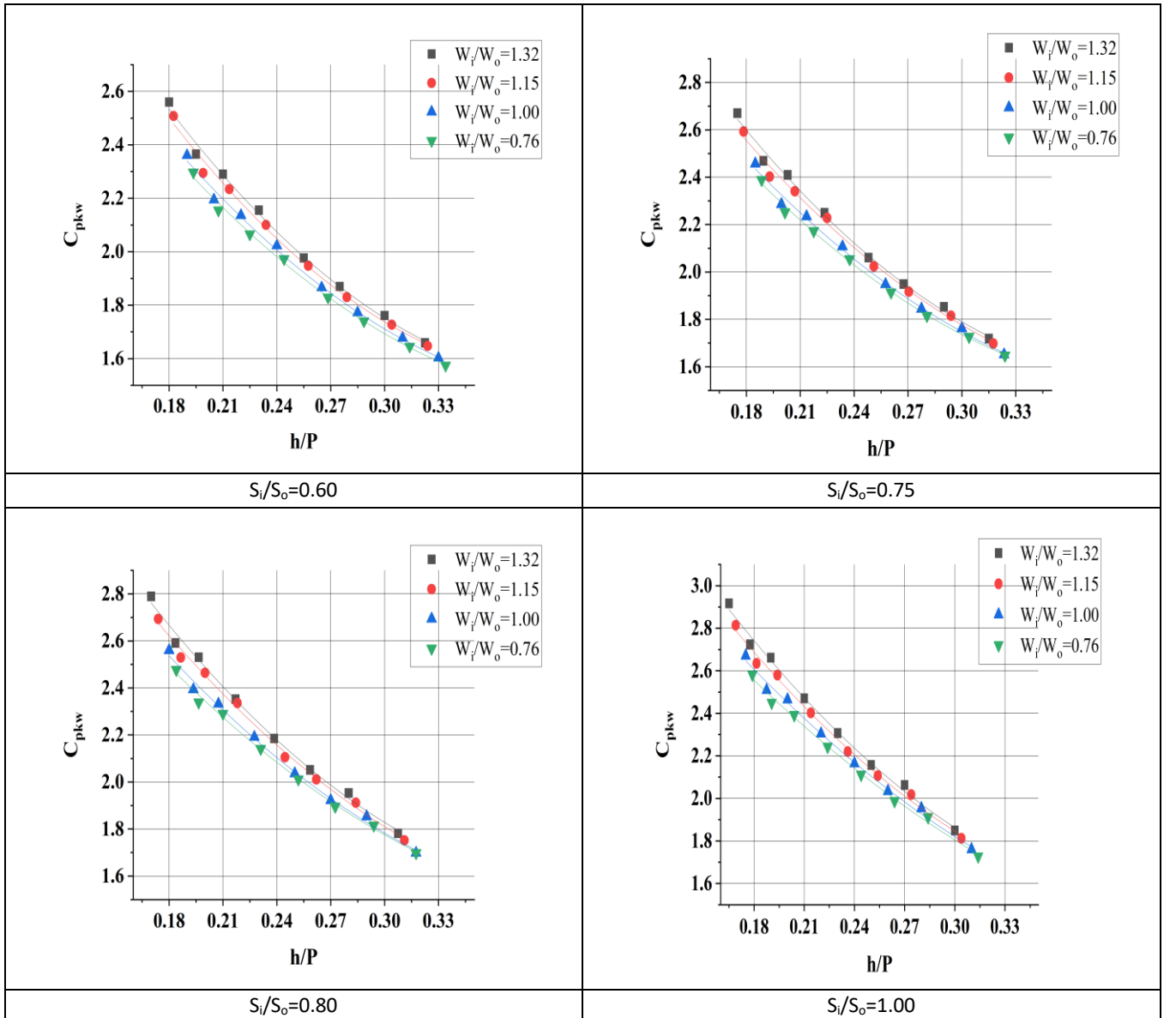


Fig. 7. Relationship between coefficient of discharge of type-A TPKW and h/P ratios.



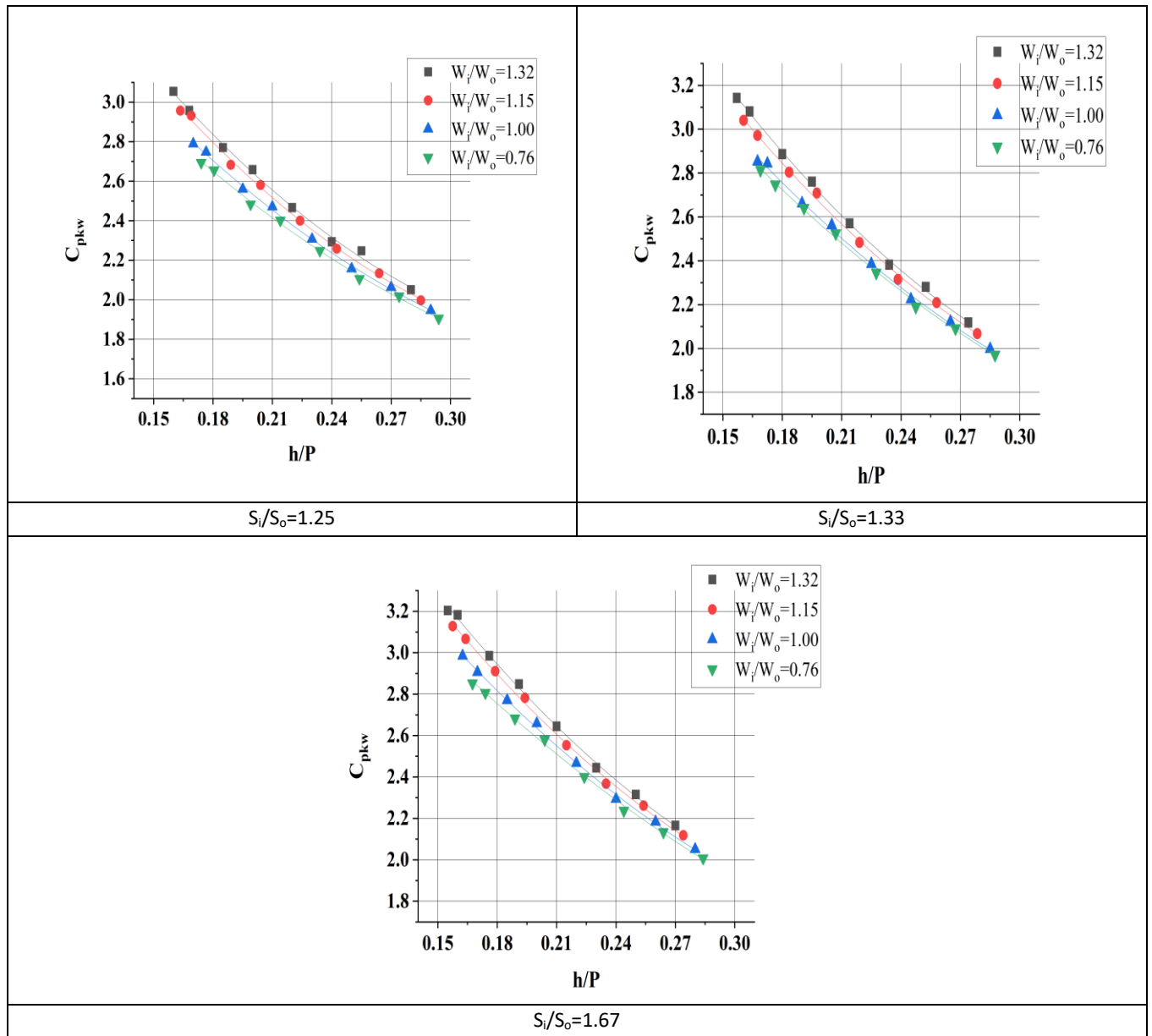


Fig. 8. Relationship of coefficient of discharge of TPKW and h/P ratios at various S_i/S_o , and W_i/W_o ratios.

3.3 Effects of upstream Froude number (F_r) on the discharge characteristics of TPKW

To check the effect of coefficient of discharge of TPKW on upstream Froude number, the graphs are plotted between the C_{pkw} and upstream Froude number (F_r) for all W_i/W_o ratios as represented in Fig. 9. It can be seen that the coefficient of discharge of TPKW decreases with the increasing of upstream Froude number at all W_i/W_o and S_i/S_o ratios. The reason could be the increasing of head above the crest of the weir, because at higher head the TPKW provided hydraulically less efficient results. In response to increasing of head, the upstream Froude number (F_r) was found to be decreased with the increasing of depth of water at the upstream of the weir, and reduction of velocity.

3.4 Regression analysis

Based on the dimensional analysis and the experimental results of present study a mathematical equation was developed using non-linear regression analysis as given in Eq. (8). The

equation was developed between the input parameters (S_i/S_o , W_i/W_o , h/P , F_r) and output parameter (C_{pkw}) obtained by performing a dimensional analysis. Randomly, 75% dataset of the present study was used for developing an equation for the estimation of C_{pkw} of TPKW. The remaining 25% of the dataset was used for the validation of Eq. (8). Usually, 70–80% dataset is used for developing an equation, and the remaining 20–30% datasets can be used for the validation of the equation (Ansari et al., 2019; Hussain et al., 2016; Iqbal and Ghani, 2024b; Shariq et al., 2022). After developing an equation, the comparison between the computed and observed values of C_{pkw} was drawn for both the training and validation datasets as shown in Fig. 10. All the predicted values of C_{pkw} fall within the $\pm 10\%$ error line, which represents a good relationship between the observed and predicted values of C_{pkw} .

$$C_{pkw} = 1.796 \left(\frac{h}{P} \right)^{-1.118} \left(\frac{S_i}{S_o} \right)^{0.027} \left(\frac{W_i}{W_o} \right)^{0.0179} (F_r)^{0.803} \quad (8)$$

Hence, Eq. (8) can be used to predict the coefficient of discharge of TPKW under free flow conditions.

3.5 Sensitivity analysis

Sensitivity analysis was also performed among the given input parameters (S_i/S_o , W_i/W_o , h/P , F_r) to find the most optimal input parameter that have a greatest influence on C_{pkw} . To perform the sensitivity analysis, first the average value of all the input parameters were calculated from the observed experimental datasets of present study, by considering errors in every input parameter to be independent as suggested by (Mohammadpour et al., 2013). The average value of the input parameters (X): S_i/S_o , W_i/W_o , h/P and F_r were found to be, 1.057, 1.0575, 0.242, and 0.166, respectively. For the sensitivity analysis, value of the each of four input parameters was varied individually by 10% (defined as ΔX), and the corresponding change in C_{pkw} was calculated using Eq. (8). The methodology was applied one by one to all the input parameters for both increment and reduction by 10% and keeping the remaining three input parameters as constant.

The following three indices; absolute sensitivity ($AS = \Delta C_{pkw}/\Delta X$), relative error ($RE = \Delta C_{pkw}/C_{pkw}$), and relative sensitivity ($RS = (X \times \Delta C_{pkw})/(\Delta X \times C_{pkw})$) were used to evaluate the performance of the sensitivity analysis. After performing sensitivity analysis, the obtained results are presented in Table 2. Based on the results of AS , RE , and RS , it is observed that h/P is the most significant or sensitive input parameter for the estimation of C_{pkw} than other input parameters, while W_i/W_o found to be least sensitive input parameter.

3.6 Design criteria for TPKW based on the experimental results of present study

In present research work, the hydraulic efficiency of the TPKW is categorized into three types: high efficiency ($C_{pkw} > 2.0$), medium efficiency ($1 < C_{pkw} < 2$) and low efficiency ($C_{pkw} < 1$) as proposed by (Mehboudi et al., 2016). The geometric design of TPKW is preferred based on the high efficiency.

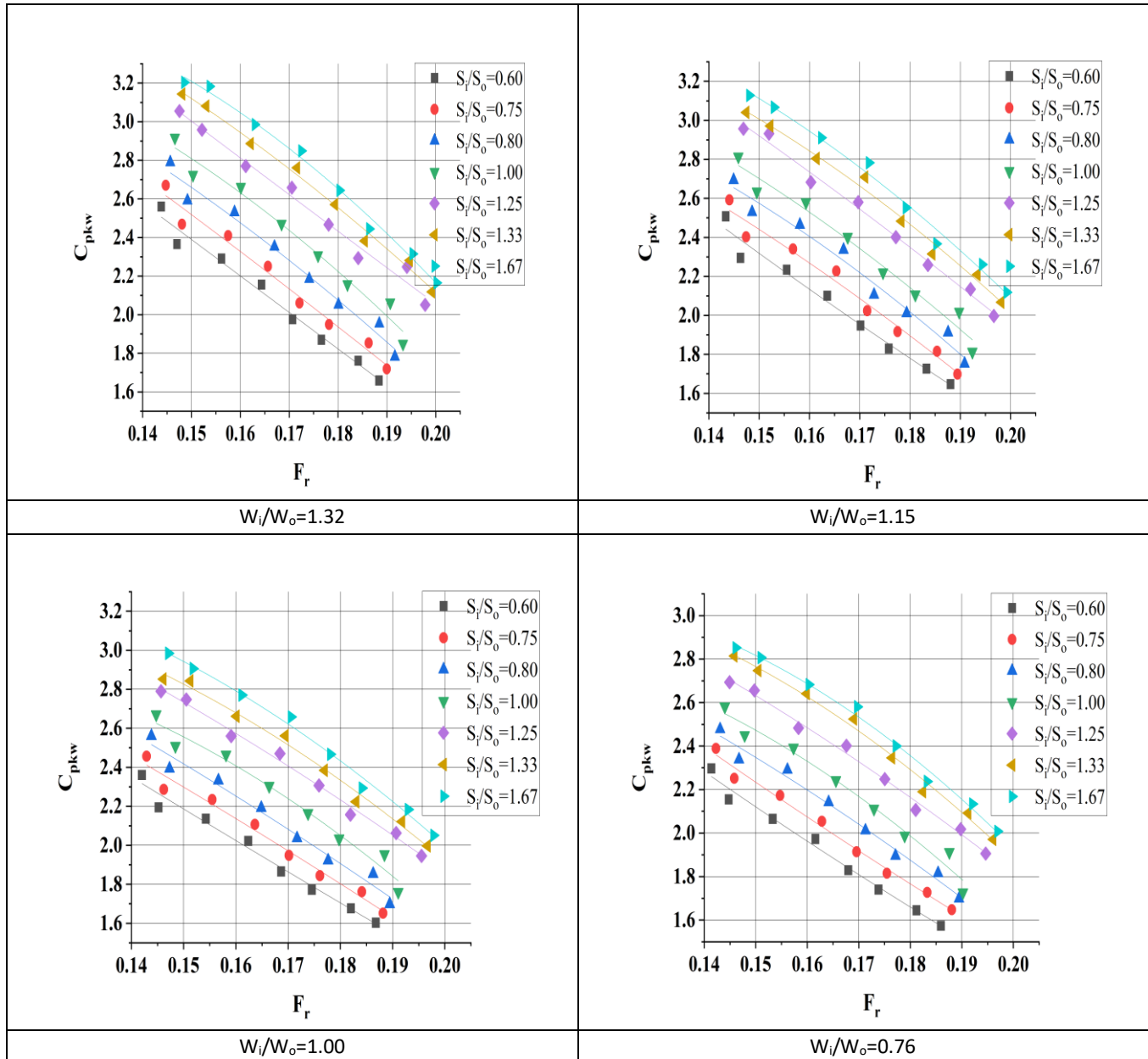


Fig. 9. Relationship between coefficient of discharge of TPKW and upstream F_r .

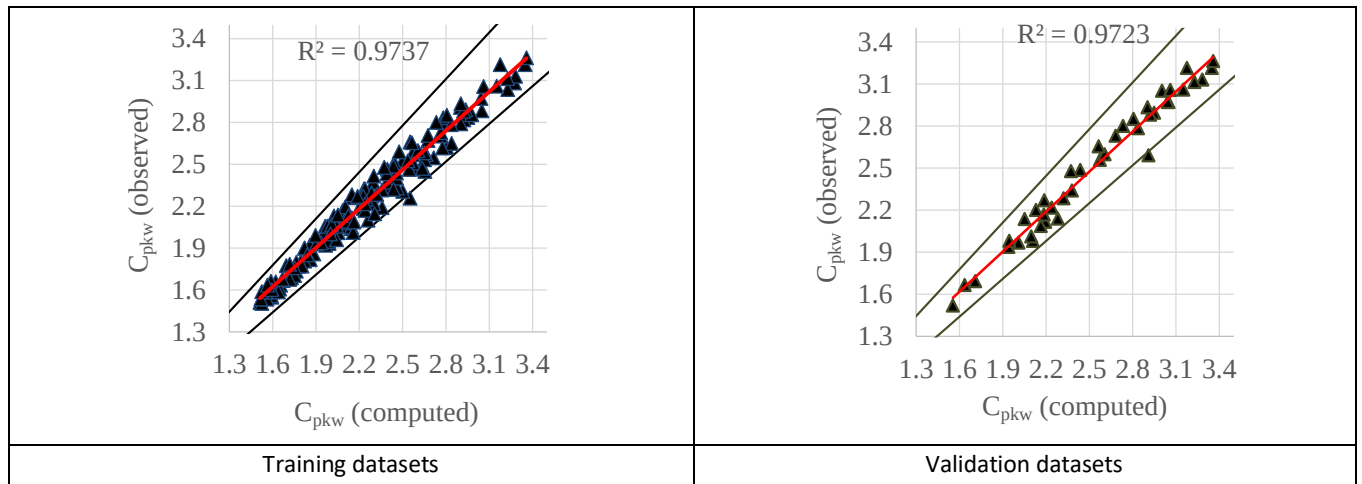


Fig. 10. Comparison between experimental C_{pkw} versus computed C_{pkw} .

Table 2. Sensitivity analysis for Eq. (8).

Input Parameters (X)	ΔX	Results of 10% increment in X				Results of 10% reduction in X			
		ΔC_{pkw}	AS	RE	RS	ΔC_{pkw}	AS	RE	RS
h/P	0.0242	0.27	11.16	0.117	1.174	-0.272	-11.23	-0.118	-1.18
S_i/S_o	0.1057	0.02	0.189	0.008	0.087	0.035	0.331	0.015	0.152
W_i/W_o	0.10575	0.025	0.236	0.011	0.109	0.033	0.312	0.014	0.143
F_r	0.0166	-0.152	-9.156	-0.066	-0.66	0.213	12.83	0.093	0.926

Table 3. Design criteria for TPKW.

Classification of hydraulic efficiency of the TPKW	h/P	S_i/S_o	W_i/W_o
High efficiency ($C_{pkw} > 2$)	< 0.25	> 1.25	> 1.15
Medium efficiency ($1 < C_{pkw} < 2$)	$0.25 - 0.40$	$0.60 \leq S_i/S_o \leq 1.25$	$0.76 \leq W_i/W_o \leq 1.15$
Low efficiency ($1 < C_{pkw}$)	> 0.40	< 0.60	< 0.76

Table 4. Comparison of geometric parameters between the present and past studies.

Bekheet A.A et al. (2022)	Kumar M. et al. (2020)	Mehboudi et al. (2016)	Present study (model M4)
$P = 20$ cm	$P = 18$ cm	$P = 20$ cm	$P = 20$ cm
$W_i/W_o = 0.80$	$W_i/W_o = 2.11$	$W_i/W_o = 1.33$	$W_i/W_o = 1.32$
$S_i/S_o = 1.00$	$S_i/S_o = 1.00$	$S_i/S_o = 1.00$	$S_i/S_o = 1.00$
$L/W = 5.00$	$L/W = 6.25$	$L/W = 6.54$	$L/W = 11.83$
$W = 30.3$ cm	$W = 40$ cm	$W = 49$ cm	$W = 30$ cm
$N_u = 2$	$N_u = 3.5$	$N_u = 2$	$N_u = 3$

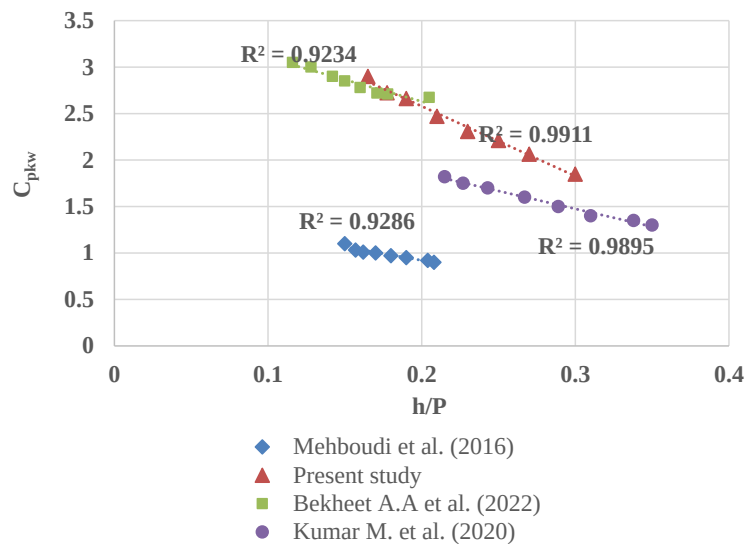


Fig. 11. Comparison between the present study and past studies.

By considering the hydraulic and geometric parameters of the weir, the design criteria of this type of weir can be proposed as given in Table 3. Based on the experimental results, generally the TPKW is considered to be highly efficient, if $h/P < 0.25$, $S_i/S_o > 1.25$ and $W_i/W_o > 1.15$; and medium efficient, if $0.25 \leq h/P \leq 0.40$, $0.60 \leq S_i/S_o \leq 1.25$ and $0.76 \leq W_i/W_o \leq 1.15$. The C_{pkw} of all the tested TPKW models were found to be more than 1, so all the tested TPKW models showed medium to high hydraulic efficiency.

3.7 Comparison of present research work with previous studies

The published experimental results of (Bekheet et al., 2022; Kumar et al., 2020; Mehboudi et al., 2016) were used to validate Eq. (8) of the present study. The geometric parameters of PKW used by (Bekheet et al., 2022; Kumar et al., 2020; Mehboudi et al., 2016) are given in Table 4.

To find the h/P ratios of past studies including (Bekheet et al., 2022; Kumar et al., 2020; Mehboudi et al., 2016) first stage-discharge relationship was developed by using their published experimental results. After establishing stage and discharge relationship, head (h) above the crest of weir was determined at the same discharges values which were used in the present study and finally h/P ratios and F_r were determined at known weir height (P). At last, the values of C_{pkw} were computed using Eq. (8) of the present study by using the experimental results of (Bekheet et al., 2022; Kumar et al., 2020; Mehboudi et al., 2016). The comparative analysis is presented in Fig. 11.

Despite the somewhat similar trends, the computed C_{pkw} values of the present study are higher than the Kumar et al. (2020) and Mehboudi et al. (2016) experimental results and the computed C_{pkw} values of the present study are slightly lower than Bekheet et al. (2022) experimental results. The reason for this difference could be the deviation of geometric parameters used in the present study and past studies.

4 CONCLUSIONS

From an extensive experimental study on the type-A TPKW, the following conclusions have been drawn from the present research work.

Based on the experimental results of present study, it is concluded that inlet to outlet key slope ratio (S_i/S_o) has a direct relation with the coefficient of discharge of TPKW. The hydraulic efficiency of TPKW was found to be 18.5%, 16%, 13.7%, 9.4%, 6.3%, and 3% more at $S_i/S_o = 1.67$, 1.33, 1.25, 1.00, 0.80, and 0.75, respectively in comparison with the $S_i/S_o = 0.60$. Overall, the discharge capacity of TPKW was increased to 18.5% when S_i/S_o reduced from 1.67 to 0.60. This study also concluded that the coefficient of discharge of TPKW was observed to be increased with increasing W_i/W_o ratios. Furthermore, the coefficient of discharge at $W_i/W_o = 1.32$ and $W_i/W_o = 1.15$ was found to be 9.4% and 7.6% higher than $W_i/W_o = 1$ respectively, while for the case of $W_i/W_o = 0.76$, its value was found to be 3.3% lower than $W_i/W_o = 1$. Overall, the discharge capacity of TPKW was reduced to 12.5%, when W_i/W_o reduced from 1.32 to 0.76.

It was also concluded that the maximum value of C_{pkw} was observed corresponding to lowest h/P ratio and minimum value of C_{pkw} was observed corresponding to highest h/P ratio. Moreover, the value of C_{pkw} was found to be 3.2, 3.1, 2.95, 2.82, 2.64, 2.43, 2.3 and 2.18 at $h/P = 0.15$, 0.16, 0.178, 0.19, 0.21, 0.23, 0.25 and 0.27, respectively corresponding to hydraulically efficient model ($S_i/S_o = 1.67$ and $W_i/W_o = 1.32$). While, the value of

C_{pkw} was observed as 2.3, 2.15, 2.07, 1.98, 1.82, 1.75, 1.63 and 1.58 at $h/P = 0.194$, 0.205, 0.222, 0.242, 0.27, 0.288, 0.315 and 0.33, respectively corresponding to hydraulically least efficient model ($S_i/S_o = 0.60$ and $W_i/W_o = 0.76$). Furthermore, the TPKW is hydraulically more efficient at lower heads as compared to higher heads. The discharge capacity of TPKW was found to be 22% more at lower heads than higher heads. Based on the sensitivity analysis, it is concluded that h/P is the more sensitive than F_r from hydraulic parameters.

Statements and declarations

Competing Interests. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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NOMENCLATURE

Parameter symbol	Explanation	Unit
α	Side wall angle of TPKW	degree
B	Total crest length of the side wall ($B = B_i + B_b + B_o$)	m
B_i	Over hanged length of inlet key	m
B_o	Over hanged length of outlet key	m
B_b	Base width of TPKW	m
C_{pkw}	Coefficient of discharge of TPKW	(-)
F_r	Froude number	(-)
g	Acceleration due to gravity	m/s^2
h	Head above the weir	m
L	Total overflowing crest length of TPKW	m
N_u	Number of cycles of TPKW	(-)
P	Crest height of TPKW	m
Q	Discharge passing over the TPKW	m^3/s
R	Height of parapet wall of TPKW	m
Re	Reynolds number	(-)
S_i	Inlet key slope of TPKW (length to height)	(-)
S_o	Outlet key slope of TPKW (length to height)	(-)
T_s	Thickness of the side wall of TPKW	m
μ	Dynamic viscosity of water	$kg\ m^{-1}\ s^{-1}$
V	Approach velocity of water	m/s
W	Width of the channel	m
W_i	Width of inlet key	m
W_o	Width of outlet key	m
W_u	Width of one cycle of TPKW	m
We	Weber number	(-)
y	Total upstream head	m
σ	Surface tension	N/m
ρ	Density of water	kg/m^3