

Flow over thin-plate weirs with a triangular notch – influence of the relative width of approach channel with a rectangular cross-section

Šimon Pospíšilík*, Zbyněk Zachoval, Pavel Gabriel

Brno University of Technology, Faculty of Civil Engineering, Institute of Water Structures, Veverí 331/95, Brno, 602 00, Czech Republic.

* Corresponding author. E-mail: simon.pospisilik@vutbr.cz

Abstract: The article deals with the determination of the influence of the relative width of an approach channel with a rectangular cross-section on the discharge of clean water flowing over thin-plated weirs with a triangular notch and zero height of the weir above the bottom of the approach channel. To evaluate the influence, the Kindsvater-Shen equation was modified by introducing the basic effective discharge coefficient and the coefficient of the relative width of the approach channel. The coefficient of the relative width of the approach channel was determined based on the evaluation of data from three extensive experimental research investigations. It is valid for the entire possible range of relative widths of the approach channel and for the range of notch angles from 5.25° to 91.17° . The relative error of discharge determination is approximately up to $\pm 2\%$ over the entire range of discharges used in the conducted experimental research. The evaluation made it possible to determine the boundary distinguishing the partially contracted weirs from the fully contracted weirs in terms of impact on discharge.

Keywords: Triangular notch (V-notch); Thin-plate weir; Kindsvater-Shen formula; Coefficient of the relative width.

INTRODUCTION

Weirs are structures over which liquid flows. During this overflow, the subcritical flow changes to supercritical flow via critical flow (Bos, 1989). The formation of critical flow at a free overflow ensures unambiguity in the mutual relationship between the discharge and the head. Thin-plated weirs (TPWs) are defined by weir crest lengths in the flow direction that range from 0.001 m to 0.002 m (ISO 1438, 2017). Triangular-notch TPWs (also referred as V-notch) are among the discharge measurement structures summarised by Bos (1989). The triangular notch is placed perpendicular to the sides and bottom of a straight channel. The line which bisects the angle of the notch should be vertical and at the same distance from both sides of the channel (Bos, 1989). The approach channel must ensure the formation of a developed velocity profile. In general, the approach channel should be smooth, straight, and rectangular. The upstream face of the plate must be smooth (in the vicinity of the notch) (ISO 1438, 2017).

The basic measured quantities when clean water flows over TPWs with a triangular notch (Fig. 1) are the notch angle α , the width of the approach channel B , the length of the approach channel L , the level of the approach channel bottom Z_b , the level of the weir crest (notch vertex) Z_p , the water surface level Z_h measured at the agreed distance upstream from the weir ($2h_{max}$ to $4h_{max}$, where h_{max} is the maximum head (ISO 1438, 2017)), and the water temperature T . The weir height is $P = Z_p - Z_b$ and the head is $h = Z_h - Z_p$.

According to the influence of the position of the side walls and the bottom of the approach channel on the formation of the flow contractions caused by TPWs with a triangular notch, Bos (1989) divides weirs into partially contracted weirs and fully contracted weirs. Partially contracted weirs produce flow contractions that are not fully developed, due to the proximity of the walls and/or bottom of the approach channel to the edges of the triangular notch. Fully contracted weirs produce flow contractions that are fully developed, due to the walls and/or bottom of the approach channel being sufficiently distant.

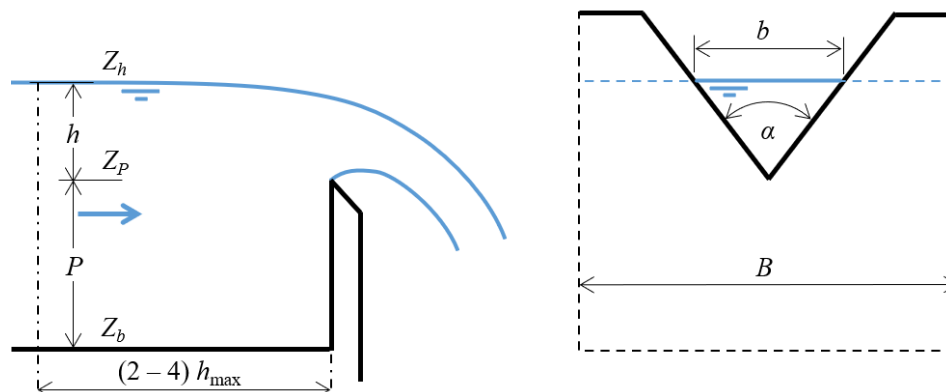


Fig. 1. Diagram of flow over TPWs with a triangular notch.

CURRENT STATE OF KNOWLEDGE

Previous research on the influence of approach channel width on overflow has focused mainly on TPWs with a rectangular notch, and less on TPWs with a triangular notch.

Research related to TPWs with a rectangular notch that concerns the influence of the width of the approach channel as well as its relative width has been carried out and published mainly by Bazin (1890), Carter (1956), Kindsvater and Carter (1957), and Zmitko (2020). The results of earlier research were summarised by Bos (1989), who divided weirs into fully contracted weirs, partially contracted weirs, and full width weirs. Discharge measurement with the aid of TPWs with a rectangular notch is covered in the ISO 1438 (2017) standard.

Research into water flow over TPWs with a triangular notch has been dealt with by several authors and summarised by Pospíšilík and Zachoval (2023). The research conducted by Thomson (1858; 1861), Barr (1910), Strickland (1910), King (1916), Cone (1916), Yarnall (1912; 1927), Pardoe (Schoder and Turner, 1929), Greve (1930; 1932; 1945), Barrett (1931), Allerton (1932), Lenz (1942), Numachi et al. (1940a; 1940b), Numachi and Hutizawa (1941a; 1941b; 1942a; 1942b), Numachi and Saito (1948; 1951), and Milburn and Burney (1988) can be considered pivotal. The results and data presented by these, and other authors were followed by comprehensive works describing flow over TPWs with a triangular notch: Shen (1981), Bos (1989), Hager (1990), and the ISO 1438 (2017) standard. Currently, research on triangular notch TPWs focuses on the geometry of the approach channel (Hattab et al., 2019), the shape of the notch (Ali et al. 2015; Martínez et al., 2005; Piratheepan et al., 2007), new approaches to determining the discharge coefficient (Bautista-Capetillo et al., 2013; Chanson and Wang, 2013) and extending the applicability of such approaches to include small notch angles and small discharges (Pospíšilík and Zachoval, 2023).

The influence of the position of the bottom of an approach channel with a rectangular cross-section on the flow over TPWs with a triangular notch was investigated by Pospíšilík and Zachoval (2023). Based on the evaluation, the authors found that the relative height of the weir has no effect on the effective discharge coefficient within the range of their measurements.

The influence of the position of the side walls of an approach channel with a rectangular cross-section on the flow over TPWs with a triangular notch was investigated for the notch angle 90° by Numachi et al. (1940a; 1940b) and Numachi and Hutizawa (1941a; 1941b; 1942a; 1942b), for the notch angle of 30.53° by Pospíšilík (2023), and for notch angles ranging from 5.25° to 91.17° by Gabriel (2023). A summary which expresses the effect of the proximity of the side walls has not yet been produced, and this is the aim of the presented article.

THEORY

The equation for the discharge of liquid flowing over a weir can be derived in three basic analytical ways. The first of these is to use the equation of mechanical energy (Bernoulli's equation), the second is to use the momentum equation, and the third is to use dimensional analysis.

Often, the preferred way to derive the discharge equation for triangular notch TPWs is to use Bernoulli's equation for an ideal fluid without surface tension. When deriving the equation, it is assumed that all the mechanical energy of the flow in the approach channel is converted into kinetic energy in the TPW crest profile. The equation then takes the form

$$Q_I = \frac{8}{15} \cdot \tan \frac{\alpha}{2} \cdot (2 \cdot g)^{0.5} \cdot H^{2.5}, \quad (1)$$

where α is the notch angle, g is the acceleration due to gravity ($9.80665 \text{ m} \cdot \text{s}^{-2}$), and H is the total head.

Kindsvater and Shen modified Equation (1) to a form suitable for the direct calculation of the water discharge $Q_{C,K-S}$

$$Q_{C,K-S} = C_e \cdot \frac{8}{15} \cdot \tan \frac{\alpha}{2} \cdot (2 \cdot g)^{0.5} \cdot h_e^{2.5}, \quad (2)$$

introducing the effective discharge coefficient C_e , which is a function of three variables

$$C_e = f_1 \left\{ \frac{h}{P}, \frac{P}{B}, \alpha \right\}, \quad (3)$$

and expressing the combined effect of viscosity and surface tension using an experimentally determined height $h_h = f_2 \{ \alpha \}$, which is part of the effective head

$$h_e = h + k_h. \quad (4)$$

Pospíšilík and Zachoval (2023) have shown that coefficient C_e is only a function of the notch angle α for a limited range of validity corresponding to the definition of a fully contracted weir (for $B = 0.5 \text{ m}$ in the range of α from 5.25° to 91.17° , P from 0.00 m to 0.20 m , h from 0.18 m to $h_l = f_3 \{ \alpha \}$ and T from 15°C to 45°C)

$$C_e = C_{e,0} = f_4 \{ \alpha \}. \quad (5)$$

Thus, the influence of the h/P ratio and the P/B ratio in the functional relation (5) is negligible in the given range, and therefore the effective discharge coefficient C_e in the given range can be considered to be the basic effective discharge coefficient for fully contracted weirs $C_{e,0}$.

In the case of partially contracted weirs, the influence of the width of the approach channel expresses the P/B ratio, which is linked to the h/P ratio (Shen, 1981). The P/B ratio can be converted to the ratio $h/B = (P/B) \cdot (h/P)$ (Novak and Čábelka, 1981), which separately describes the influence of the width of the approach channel for a given notch angle and $P = 0 \text{ m}$. The unification of the influence of the approach channel width for all notch angles can be obtained (based on the geometry of the triangular notch) by introducing the relative width of the approach channel b/B , where b is the width of the triangular notch at the measured level of the water surface

$$b = 2 \cdot h \cdot \tan \left(\frac{\alpha}{2} \right). \quad (6)$$

The expression of the influence of the relative width of the approach channel on the discharge flow over TPWs with any angle of the triangular notch and with the weir height $P = 0 \text{ m}$ can then be achieved by introducing the coefficient of the relative width of the approach channel

$$C_B = f_5 \left\{ \frac{b}{B} \right\}. \quad (7)$$

The modified form of the Kindsvater-Shen equation is then

$$Q_C = C_{e,0} \cdot C_B \cdot \frac{8}{15} \cdot \tan \frac{\alpha}{2} \cdot (2 \cdot g)^{0.5} \cdot h_e^{2.5}. \quad (8)$$

Function (7) can be determined based on data from experimental research or simulations. Data from experimental research were used to determine the C_B coefficient for discharge through a TPW with a triangular notch.

EXPERIMENT

Data from three experimental research investigations were used.

The earliest research was carried out by Numachi and Hutzawa (1941b), who took measurements to evaluate the influence of the approach channel width (with a rectangular cross-section) on the discharge of water flow over TPWs with a triangular notch of 90° . The measurement was carried out in a 7.92 m long and 1.18 m wide channel with a rectangular cross-section and inserted adjustable approach channel of length $L = 4.6$ m. The weir crest was 0.002 m long in the direction of flow. The water level was measured using a hook gauge and the discharge was measured via the volumetric method using measuring vessels. Measurements were taken for different weir heights, but only the data they obtained for the weir height $P \leq 0.003$ m were used for evaluation in this current article.

The following research was conducted by Pospíšilík (Pospíšilík and Zachoval, 2023). A short channel 1.23 m long with a rectangular cross-section 0.50 m wide and 0.40 m high was used. A frame structure was placed at the end of the channel, into which TPWs with a triangular notch were gradually attached. The angle of the triangular notch was determined using the two-cylinder method (ISO 1438, 2017; Pospíšilík, 2020). The frame structure prevented the TPWs from deflecting, allowing the weir to be centred in the channel and the location of the weir plates to be varied in the vertical plane. A flow straightener was placed in the channel to ensure the even distribution of longitudinal velocities along the cross-section of the channel (Fig. 2). To ensure the predefined width of the approach channel B was achieved, side walls of length $L = 0.82$ m made of polyvinyl chloride with a thickness of 0.010 m were inserted into the channel. The TPWs were made of stainless steel sheet with a thickness of 0.001 m. The water discharge Q was determined by calibrated electromagnetic flowmeters with the diameters DN10 ($0.000016 \text{ m}^3 \cdot \text{s}^{-1} \leq Q \leq 0.00050 \text{ m}^3 \cdot \text{s}^{-1}$), DN32 ($0.00033 \text{ m}^3 \cdot \text{s}^{-1} \leq Q \leq 0.0034 \text{ m}^3 \cdot \text{s}^{-1}$) and DN50 ($0.00051 \text{ m}^3 \cdot \text{s}^{-1} \leq Q \leq 0.0086 \text{ m}^3 \cdot \text{s}^{-1}$). Measurement of Z_b and Z_h was carried out using a digital point gauge with a resolution of 0.01 mm placed on a cross beam in the longitudinal plane of symmetry of the flume. The Z_h value for the determination of h was measured at 0.38 m upstream from the weir plate ($2h_{\max}$). The width B was measured using a calibrated metal flat gauge with a resolution of 1 mm. The temperature T was measured with a digital thermometer with a resolution of 0.1 °C. More information about the experiment is presented by Pospíšilík and Zachoval (2023).

The most recent research was conducted by Gabriel (Gabriel,

2023). Gabriel used the Pospíšilík flume (Pospíšilík and Zachoval, 2023) and modified it. The flume was intentionally extended in its central part to a total length of 3.50 m (Fig. 3) to evaluate the possible influence of the length of the approach channel. Unlike the side walls inserted into the channel used by Pospíšilík (Pospíšilík, 2023), a system of continuously adjustable side walls of length $L = 2.50$ m was made (Fig. 2) and inserted into the channel to provide the required approach channel width. The wall system consisted of two water-resistant plywood sheets with a smooth surface with a thickness of 0.018 m, which were connected above the surface of the water by pairs of threaded rods and fastened with nuts, which were used to set the required width of the approach channel B . A pair of walls abutted the weir plate through the seal, which ensured water tightness. The wall system was loaded by concrete blocks to prevent the components from floating out of place. The same measurement equipment was used (except the DN32 flow meter) as described by Pospíšilík (Pospíšilík and Zachoval, 2023). More information about this research is presented in Gabriel (2023).

A summary of the ranges of all of the data used from the aforementioned experimental research is given in Table 1. The division is made gradually according to the notch angle and then according to the date of the research.

EVALUATION AND RESULTS

The C_B function (7) was determined by the ratio of the measured discharge for partially contracted weir Q to the computed discharge for fully contracted weir $Q_{C,0}$, i.e.

$$C_B = \frac{Q}{Q_{C,0}}, \quad (9)$$

where $Q_{C,0}$ was determined using the equations (Pospíšilík and Zachoval, 2023)

$$k_h = 0.0194 \cdot \alpha^{-0.186} - 0.0088, \quad (10)$$

$$h_l = 0.3 \cdot \alpha^{-0.8} - 0.03, \quad (11)$$

$$C_{e,0} = 0.05 \cdot \exp(-0.08 \cdot \alpha) + 0.59. \quad (12)$$

The dependence of C_B on b/B was determined by an approximation curve over all values

$$C_B = 1 + 0.2 \cdot \left(\frac{b}{B}\right)^{2.2} \quad (13)$$

with the coefficient of determination $R^2 = 0.89$. The approximation



Fig. 2. TPW and adjustable sidewalls (left), flow straightener (right).

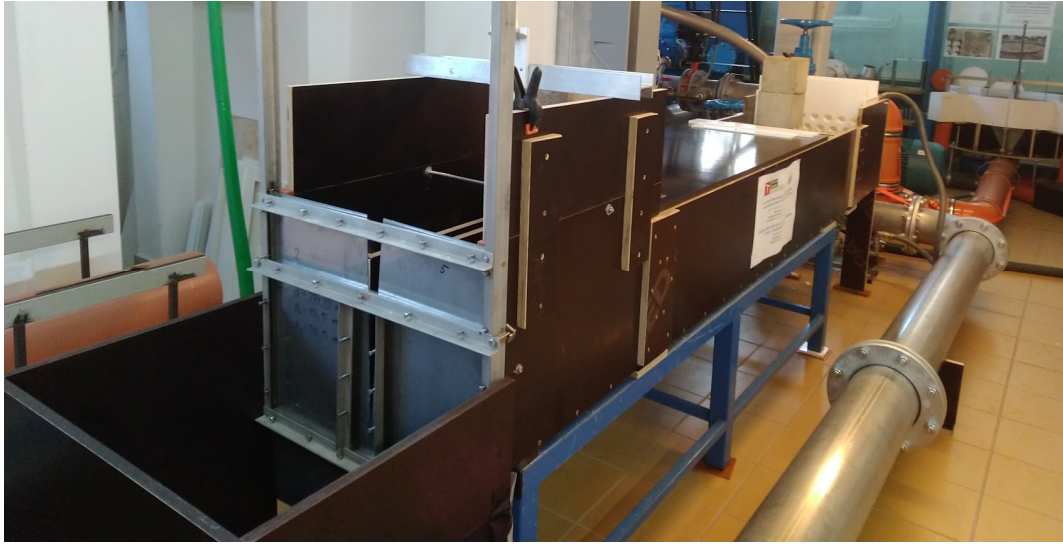


Fig. 3. Experimental equipment.

Table 1. Ranges of measured quantities.

Author	α (°)	B (m)	L (m)	h (m)	T (°C)	$Q \cdot 10^{-3}$ (m ³ ·s ⁻¹)
Pospíšilík (2023)	5.25	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.080–0.183	21.9–22.3	0.138–1.048
Gabriel (2023)	5.25	0.017, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.039–0.183	16.6–20.0	0.222–1.177
Pospíšilík (2023)	10.18	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.060–0.185	22.0–23.3	0.129–2.033
Gabriel (2023)	10.18	0.032, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.040–0.179	16.3–16.9	0.046–2.226
Pospíšilík (2023)	15.19	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.041–0.193	22.4–24.9	0.071–3.297
Gabriel (2023)	15.19	0.048, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.039–0.180	16.0–16.5	0.062–3.305
Pospíšilík (2023)	20.23	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.040–0.182	22.5–25.5	0.089–3.723
Gabriel (2023)	20.23	0.064, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.035–0.190	15.9–16.7	0.065–4.322
Pospíšilík (2023)	25.21	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.040–0.167	22.2–24.9	0.108–3.726
Gabriel (2023)	25.21	0.080, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.036–0.178	15.8–16.4	0.085–5.306
Pospíšilík (2023)	30.53	0.05, 0.10, 0.20, 0.30, 0.40, 0.50	0.82	0.050–0.155	21.3–24.8	0.226–3.739
Gabriel (2023)	30.53	0.098, 0.10, 0.20, 0.30, 0.40, 0.50	2.50	0.039–0.179	17.5–17.9	0.123–6.248
Pospíšilík (2023)	60.67	0.50	0.82	0.020–0.115	15.7–43.4	0.047–3.656
Gabriel (2023)	60.67	0.211, 0.30, 0.40, 0.50	2.50	0.040–0.175	17.7–18.0	0.258–10.720
Numachi and Hutizawa (1941b)	90	0.3, 0.44, 0.54, 0.64	4.60	0.059–0.237	7.4–10.2	0.124–41.071
Pospíšilík (2023)	91.17	0.50	0.82	0.020–0.092	16.5–42.6	0.076–3.660
Gabriel (2023)	91.17	0.367, 0.40, 0.50	2.50	0.038–0.139	18.0–18.2	0.415–10.711

curve is shown together with the values determined from the measurements in Fig. 4. The values determined by the measurements are color-coded according to the notch angle. The validity of the approximation curve is given by the range of b/B values from 0.02 to 1. With regard to the fact that the coefficient C_B becomes close to 1 as b/B decreases and the weir in the range $b/B \leq 0.2$ creates fully developed contractions, it is possible to assume that $C_B = 1$ for $b/B < 0.02$ and extend the validity of Equation (13) to the whole possible range of b/B , i.e., from 0 to 1.

The graph in Fig. 4 shows that the b/B ratio determines an area which fulfils the definition of fully contracted weirs in terms of the effect of contractions on the discharge (within the scope of validity of the above equations). In the case of a required relative error of up to +1%, the fulfilment is valid up to $b/B = 0.25$, while in the case of a required relative error of up to +2%, the fulfilment is valid up to $b/B = 0.35$ (Fig. 4).

DISCUSSION

To evaluate the error of the discharge computed using the above Equations (8), (10), (12), (13) and (4) with the constraint condition (11) Q_C , the relative error of discharge was calculated for each value

$$\delta Q_C = \frac{Q_C - Q}{Q}. \quad (14)$$

The evaluation of the relative error of discharge δQ_C depending on the relative width of the approach channel expressed by the b/B ratio is depicted graphically in Fig. 5. In the graph, the values for each notch angle listed in the legend are color-coded. The graph is supplemented by red dashed lines delineating bands of relative error of $\pm 1\%$ and $\pm 2\%$. It can be seen from the graph that in the b/B interval from 0.02 to 0.60 most of the values are

with a relative error of up to $\pm 1\%$, and almost all of the values are in the range up to $\pm 2\%$. In the interval b/B from 0.06 to 1.00, the dispersion of relative error values is greater (up to about $\pm 3\%$). Greater dispersion may be caused by the position of the profile for measuring the water surface level (the relative distance of the profile from the weir plate) and by the roughness of the side walls and bottom surfaces, as there are already relatively high velocities in the approach channel, causing friction losses. The increase in dispersion may also be due to a relative error in determining the width of the approach channel (Table 1).

To provide a summary, the relative error of discharge δQ_C is described by a frequency histogram showing the normal distribution of $N(-0.108, 1.07)$. In the frequency histogram, the values for each notch angle listed in the legend are color-coded. It can be seen from the frequency histogram (Fig. 6) that 76.0% of the δQ_C values fall within the $\pm 1\%$ range, 93.5% of the values fall within the $\pm 2\%$ interval and 99.7% of the values fall within the $\pm 3\%$ range.

The evaluation of the influence of the approach channel length is carried out graphically in Fig. 7. Fig. 7 shows the dependence of the relative error of discharge δQ_C on the h/L ratio.

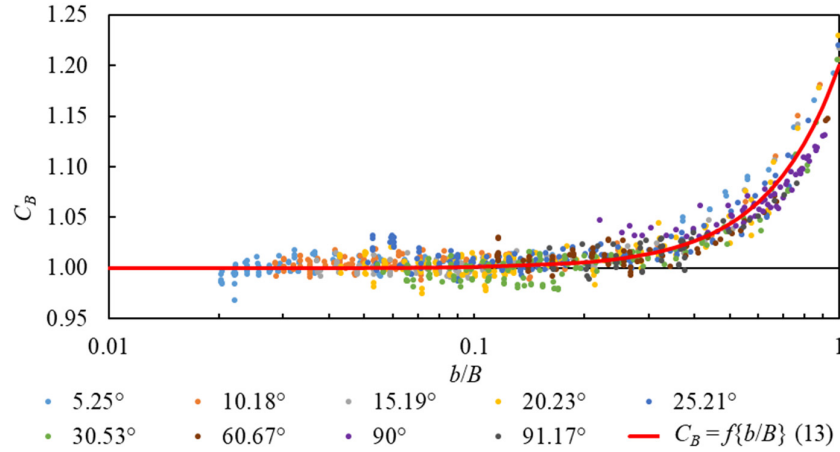


Fig. 4. Dependence of the C_B on the b/B for α in the legend.

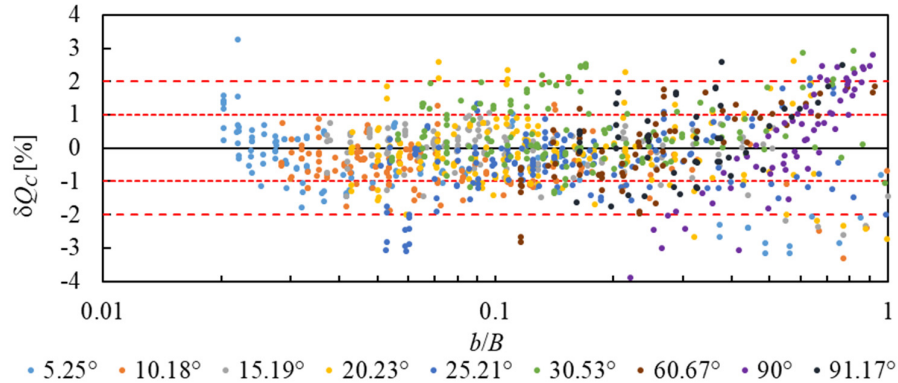


Fig. 5. Evaluation of the dependence of the δQ_C on the b/B for α in the legend.

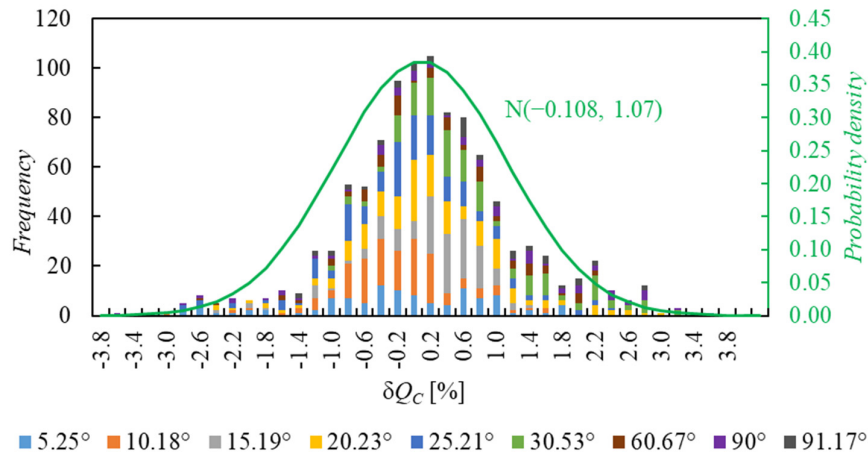


Fig. 6. Frequency histogram for δQ_C .

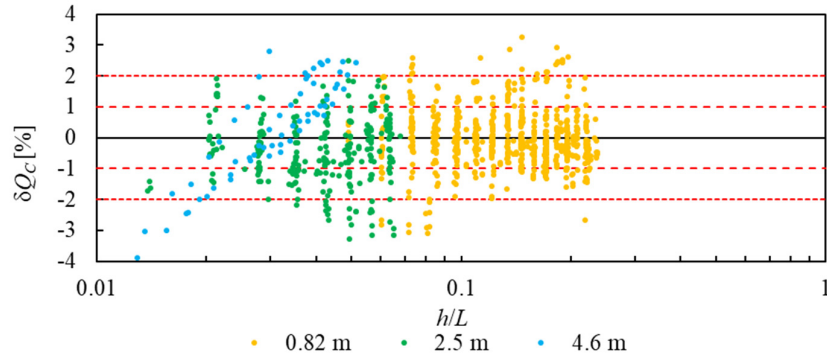


Fig. 7. Evaluation of the dependence of the δQ_C on the h/L for L in the legend.

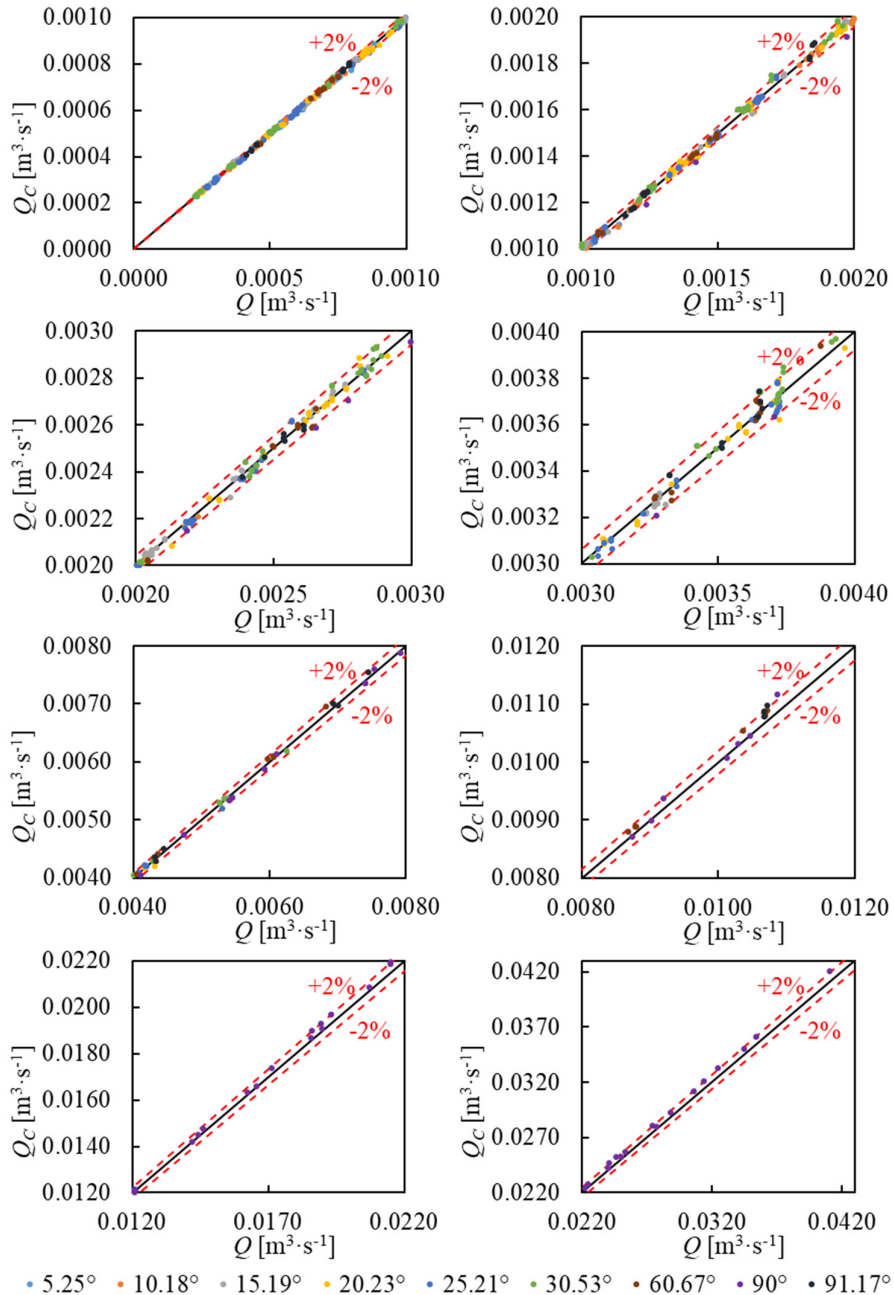


Fig. 8. Comparison of Q_C and Q for α in the legend.

The individual lengths of the approach channel listed in the legend are distinguished by colour. The graph is again supplemented by red dashed lines delineating the bands of relative error $\pm 1\%$ and $\pm 2\%$. It is clear from the graph that the measurements by Pospíšilík (0.82 m) and Gabriel (2.5 m) show that the h/L ratio does not affect the determination of the discharge, i.e. the lengths of the approach channels were sufficient in both cases and a fully developed velocity profile was created. To some extent, this is also confirmed by the Numachi and Hutizawa measurements (4.6 m), but with a larger dispersion of values, but this is due to the greater relative error of discharge δQ_C expressed in relation to b/B (Fig. 5, 90°).

The evaluation of the influence of the discharge values is carried out by comparing computed discharge Q_C and the experimentally measured discharge Q in Fig. 8. Fig. 8 contains graphs with progressively increasing intervals of discharge for legibility. The graphs are supplemented with lines of agreement and relative error intervals $\pm 2\%$ from the line of agreement. The graphs show that the values of relative errors are practically unchanged (up to $\pm 2\%$) throughout the measured discharge. This shows that the equations used to calculate the discharge in the range of measured discharges (from $0.0002 \text{ m}^3 \cdot \text{s}^{-1}$ to $0.042 \text{ m}^3 \cdot \text{s}^{-1}$) are independent of the discharge value. Due to the direct link between Q_C and h , the above independence also applies to the head h in the range from 0.039 m to 0.237 m.

Verification of Q_C determination by the above equations could not be carried out due to the insufficient quantity of suitable experimental data from other authors available for α , b/B and P .

CONCLUSION

The article describes the influence of the relative width of an approach channel of rectangular cross-section on the discharge of clean water through a TPW with a triangular notch and zero weir height. The evaluation is performed for the modified form of the Kindsvater and Shen equation. To quantify the influence, the basic effective discharge coefficient $C_{e,0}$ and the coefficient of the relative width of the approach channel C_B were introduced. The basic effective discharge coefficient was taken from the publication by Pospíšilík and Zachoval (2023). The coefficient of the relative width of the approach channel was described for all notch angles by a single ratio b/B , where b is the width of the triangular notch at the measured level of the water surface, which can be directly calculated from the head.

To determine the equation describing the dependence of the influence coefficient C_B on the b/B ratio, data from three extensive experimental investigations carried out by different authors using different devices changing the width of the approach channel were analysed. The total range of the measurement data was in the range of notch angles α from 5.25° to 91.17° and relative widths b/B from 0.02 to 1.00.

The coefficient of the relative width of the approach channel was described by a power function with a large coefficient of determination. The validity of the relation extends across the entire real range of the b/B ratio, i.e. from 0 to 1. The relative error depending on the b/B ratio was evaluated, which practically confirmed that high accuracy was achieved across the entire range of measured data. To provide a summary, a frequency histogram with the definition of normal data distribution was evaluated.

The influence of the relative length of an approach channel of rectangular cross-section on discharge flowing over TPWs with a triangular notch was also analysed. The analysis confirmed that for the range of relative approach channel lengths used in the experiments, relative length had no effect on discharge.

The evaluation of the relative error was performed by comparing the calculated discharge with the measured discharge over the entire range of discharges. The evaluation showed that the interval of the relative error of discharge is in most cases up to $\pm 2\%$ over the entire discharge range.

Acknowledgements. The contribution was created with the support of project FAST-J-22-7822 “Influence of the width of the approach channel on the discharge coefficient of a thin-plated weir with a triangular notch”.

NOMENCLATURE

b	the width of the triangular notch at the measured level of the water surface (m)
B	approach channel width (m)
C_e	effective discharge coefficient
$C_{e,0}$	basic effective discharge coefficient (for a fully contracted weir)
C_B	coefficient of the relative width of the approach channel
g	standard acceleration due to gravity (m/s^2)
h	head (m)
H	total head (m)
h_e	effective head (m)
h_l	limit head (m)
h_{max}	maximum head (m)
k_h	height representing the combined effects of viscosity and surface tension (m)
L	channel length (m)
P	weir height (m)
Q	discharge, measured discharge (m^3/s)
Q_C	computed discharge (m^3/s)
$Q_{C,K-S}$	computed discharge by the Kindsvater–Shen equation (m^3/s)
$Q_{C,0}$	computed discharge for fully contracted weirs (m^3/s)
Q_I	discharge of ideal liquid without surface tension (m^3/s)
R^2	coefficient of determination
T	temperature ($^\circ\text{C}$)
Z_b	bottom level (m)
Z_h	water surface level upstream in front of the weir at a distance from $2h_{max}$ to $4h_{max}$ (m)
Z_P	crest level (m)
α	notch angle ($^\circ$)
δQ_C	relative error of discharge

REFERENCES

- Ali, A.A.M., Ibrahim, M., Diwedat, A.I., 2015. The discharge coefficient for a compound sharp crested V-notch weir. *Asian J. Eng. Technol.*, 3, 5, 494–501.
- Allerton, R.W., 1932. Flow of water over triangular weirs. Thesis. Princeton University, Princeton.
- Barr, J., 1910. Experiments upon the flow of water over triangular notches. *Engineering*, 89, 435–437, 470–473.
- Barrett, F.B., 1931. The flow of water over triangular weirs. Thesis. Princeton University, Princeton.
- Bautista-Capetillo, C., Robles, O., JÚnez-Ferreira, H., Playán, E., 2013. Discharge coefficient analysis for triangular sharp-crested weirs using low-speed photographic technique. *J. Irrig. Drain. Eng. - ASCE*, 140, 3, 06013005.
- Bazin, H., 1890. Recent experiments on the flow of water over weirs. *Proceedings of the Engineers' Club of Philadelphia*, 7, 260–310.
- Bos, M.G., 1989. Discharge measurement structures. 3. ILRI.
- Carter, R.W., 1956. A comprehensive discharge equation

- for rectangular-notch weirs. Thesis. Georgia Institute of Technology, Atlanta.
- Cone, V.M., 1916. Flow through weir notches with thin edges and full contractions. *J. Agric. Res.*, 5, 23, 1051–1113.
- Gabriel, P., 2023. Influence of approach channel width on flow over thin-plate weir with triangular notch. BS. Thesis. Brno University of Technology, Brno. (In Czech.) <https://dspace.vutbr.cz/bitstream/handle/11012/211464/final-thesis.pdf?sequence=1>
- Greve, F.W., 1930. Calibration of 16 triangular weirs at Prude. *Eng. News-Rec.*, 105, 5, 166–167.
- Greve, F.W., 1932. Flow of water through circular, parabolic, and triangular vertical notch-weirs. *Eng. Bulletin Purdue University*, 40, 2–84.
- Greve, F.W., 1945. Flow of liquids through vertical circular orifices and triangular weirs. *Eng. Bull. Purdue University*, 29, 3, 1–68.
- Hager, W.H., 1990. The V-notch weir. *Wasser Energie Luft = Eau énergie air = Acqua energia aria*, 82, 1–2, 9–14. (In German.) <http://doi.org/10.5169/seals-939771>
- Hattab, M.H., Mijic, A., Vernon, D., 2019. Optimised triangular weir design for assessing the full-scale performance of green infrastructure. *Water*, 4, 11, 773–790. <https://doi.org/10.3390/w11040773>
- Chanson, H., Wang, H., 2013. Unsteady discharge calibration of a large V-notch weir. *Flow Meas. Instrum.*, 29, 19–24. <https://doi.org/10.1016/j.flowmeasinst.2012.10.010>
- ISO 1438, 2017. Hydrometry – Open channel flow measurement using thin-plate weirs. 3. ISO, Geneva.
- Kindsvater, C., Carter, R., 1957. Discharge characteristics of the rectangular thin-plate weirs. *J. Hydraul. Div. - ASCE*, 83, 1–36. <https://doi.org/10.1061/TACEAT.0007696>
- King, H.W., 1916. Flow of water over right-angled V-notch weir. *The Michigan Technic*, 29, 3, 189–195.
- Lenz, A.T., 1942. Viscosity and surface tension effects on V-notch weir coefficients. *Trans. Am. Soc. Civ. Eng.*, 351–374.
- Martínez, J., Reca, J., Morillas, M.T., López, J.G., 2005. Design and calibration of a compound sharp-crested weir. *J. Hydraul. Eng. - ASCE*, 131, 2, 112–116. [https://doi.org/10.1061/\(ASCE\)0733-9429\(2005\)131:2\(112\)](https://doi.org/10.1061/(ASCE)0733-9429(2005)131:2(112))
- Milburn, P., Burney, J., 1988. V-notch weir boxes for measurement of subsurface drainage system discharges. *Can. Agr. Eng.*, 30, 2, 209–212.
- Novak, P., Čábelka, J., 1981. Models in Hydraulic Engineering: Physical Principles and Design Applications. Pitman, London.
- Numachi, F., Hutizawa, S., 1941a. On the overflow coefficient of a right-angled triangular weir (2. Notice). *J. Soc. Mech. Eng.*, 44, 286, 286. (In Japanese.) https://doi.org/10.1299/jsmemag.44.286_5_1
- Numachi, F., Hutizawa, S., 1941b. On the overflow coefficient of a right-angled triangular weir (2. Notice). *Trans. Jpn. Soc. Mech. Eng.*, 7, 27–3, 5–9. (In Japanese.) https://doi.org/10.1299/kikai1938.7.27-3_5
- Numachi, F., Hutizawa, S., 1942a. On the overflow coefficient of a right-angled triangular weir (3. Notice). *J. Soc. Mech. Eng.*, 45, 308, 725. (In Japanese.) https://doi.org/10.1299/jsmemag.45.308_725_2
- Numachi, F., Hutizawa, S., 1942b. On the overflow coefficient of a right-angled triangular weir (3. Notice). *Trans. Jpn. Soc. Mech. Eng.*, 8, 33–3, 37–40. (In Japanese.) https://doi.org/10.1299/kikai1938.8.33-3_37
- Numachi, F., Kurokawa, T., Hutizawa, S., 1940a. On the overflow coefficient of a right-angled triangular weir. *J. Soc. Mech. Eng.*, 43, 275, 45. (In Japanese.) https://doi.org/10.1299/jsmemag.43.275_45_2
- Numachi, F., Kurokawa, T., Hutizawa, S., 1940b. On the overflow coefficient of a right-angled triangular weir. *Trans. Jpn. Soc. Mech. Eng.*, 6, 22–3, 10–14. (In Japanese.) https://doi.org/10.1299/kikai1938.6.22-3_10
- Numachi, F., Saito, I., 1948. On allowable shortest length of channel for triangular notch. *J. Soc. Mech. Eng.*, 51, 357, 229–230. (In Japanese.) https://doi.org/10.1299/jsmemag.51.357_229_2
- Numachi, F., Saito, I., 1951. On allowable shortest length of channel for triangular notch. *Trans. Jpn. Soc. Mech. Eng.*, 17, 56, 1–3. (In Japanese.) <https://doi.org/10.1299/kikai1938.17.1>
- Piratheepan, M., Winston, N.E.F., Pathirana, K.P.P., 2007. Discharge measurements in open channels using compound sharp-crested weirs. *J. Inst. Eng.*, 40, 3, 31–38. <https://doi.org/10.4038/engineer.v40i3.7144>
- Pospíšilík, Š., 2020. 2.67° triangular-notch thin-plate weir. In: *Juniorstav 2020*. ECON Publishing, Brno, pp. 598–603. (In Czech.)
- Pospíšilík, Š., 2023. Influence of the relative approach channel width on the flow over thin-plate weir with triangular notch of the angle 30.53°. In: *Juniorstav 2023*. ECON Publishing, Brno, pp. 588–594.
- Pospíšilík, Š., Zachoval, Z., 2023. Discharge coefficient, effective head and limit head in the Kindsvater-Shen formula for small discharges measured by thin-plate weirs with a triangular notch. *J. Hydrol. Hydromech.*, 71, 1, 35–48. <https://doi.org/10.2478/johh-2022-0040>
- Schoder, E.W., Turner, K.B., 1929. Precise weir measurements. *Trans. Am. Soc. Civ. Eng.*, 1929, 93, 999–1190.
- Shen, J., 1981. Discharge characteristics of triangular-notch thin plate weirs: Studies of flow of water over weirs and dams. Geological survey water-supply paper, 1617-B. U. S. Government Printing Office, Washington.
- Strickland, T.P., 1910. Mr. James Barr's experiments upon the flow of water over triangular notches. *Engineering*, 90, 598.
- Thomson, J., 1858. On experiments on the measurement of water by triangular notches in weir boards. In: *Proc. Twenty-eight Meeting of the British Association for the Advancement of Science*. John Murray, London, pp. 181–185.
- Thomson, J., 1861. On experiments on the gauging of water by triangular notches. In: *Proc. Thirty-first Meeting of the British Association for the Advancement of Science*. John Murray, London, pp. 151–158.
- Yarnall, D.R., 1912. The V-notch weir method of measurement. *J. Am. Soc. Mech. Eng.*, 34, 2, 1479–1494.
- Yarnall, D.R., 1927. Accuracy of the V-notch-weir method of measurement. *Trans. Am. Soc. Mech. Eng.*, 48, 939–964.
- Zmítka, J., 2020. Full-width thin-plate rectangular weir influenced by channel width. MS. Thesis. Brno University of Technology, Brno. (In Czech.) <https://dspace.vutbr.cz/handle/11012/184201>

Received 27 November 2023

Accepted 12 March 2024