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Effect of water surface slope and friction slope on the value of the estimated Manning's roughness coefficient in gravel-bed streams

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Abstract: The aim of the study was to assess the possibility of using the empirical formulas to determine the roughness coefficient in gravel-bed streams, the bed slopes of which range from 0.006 to 0.047. Another aim was to determine the impact of taking into account the conditions of non-uniform flow on the application of these formulas and to develop the correlation relationships between the roughness coefficient and water surface slope and also between the roughness coefficient and friction slope in order to estimate the roughness coefficient n in gravel-bed streams.

The studies were conducted in eight measuring sections of streams located in the Kraków-Częstochowa Upland, southern Poland. The roughness coefficient n_0 for these sections was calculated from the transformed Bernoulli equation based on the results of surveys and hydrometric measurements. The values of n_0 were compared with the calculation results obtained from fourteen empirical formulas presenting the roughness coefficient as a function of slope.

The Lacey, Riggs, Bray and Sauer formulas were found to provide an approximate estimate of the n value, while the best roughness coefficient estimation results were obtained using the Riggs formula. It was also found that taking into account the non-uniform flow and using the friction slope in the formulas instead of the bed slope or water surface slope did not improve the estimated values of the roughness coefficient using the tested formulas. It was shown that the lack of differences in the RMSE and MAE error values calculated for the developed correlation equations between the roughness coefficient and the friction slope or with the water surface slope also indicate no influence of the assumed friction slope or water surface slope on the value of the estimated roughness coefficient.

Keywords: Roughness coefficient; Bed slope; Uniform flow; Non-uniform flow.

1 INTRODUCTION

Accurate estimation of the Manning's coefficient of roughness n is crucial in determining the water flow rate in open channels. If the estimated roughness is too low, this can result in an overestimated flow in the calculations, an underestimation of the flood flow surface levels and vice versa (Ladson et al., 2002). The roughness coefficient n was introduced by Manning into the Chézy equation:

$$n = (R^{2/3} S_f^{1/2}) / V \quad (1)$$

where R is a hydraulic radius, S_f is the friction slope, and V is the mean flow velocity.

The Manning equation is most often used to estimate the hydraulic parameters of water flow in open channels and applied in computational models (Bjerklie et al., 2005; Strupczewski and Szymkiewicz 1996; Zhu et al., 2020). The main limitation of the Manning equation is the assumption of a constant value of the friction coefficient regardless of the depth and velocity of water flow (Bellos et al., 2018). On the other hand, Ferguson (2010) considers this method of determining the hydraulic parameters of water flow to be non-optimal. In the engineering approach, the Manning equation calculates the water flow velocity for a given bed slope and assumed value of roughness coefficient. When the flow is uniform then the friction slope S_f can be replaced by the bed slope S (Ferguson, 2007). This means the parallelism of the bed slope and water surface slope S_w. The roughness coefficient is taken from the tables for a given type of mineral material in the watercourse bed. The roughness coefficient can also be calculated for the determined friction factor f presented in the Darcy-Weisbach equation:

$$f = (8gRS_f) / V^2 \quad (2)$$

The Equations (1) and (2) show the relationship between the friction factor and the roughness coefficient in the form (Yen, 2002):

$$R^{1/6} / n = (8g / f)^{1/2} \quad (3)$$

However, in engineering practice it is not a commonly used procedure. This coefficient can also be estimated using empirical formulas in which it is presented as a function of the bed grain size, slope, hydraulic radius and other parameters. The basic formulas describing the relationship between the roughness coefficient and sediment grain size were developed on the basis of the equation proposed by Strickler in 1923 (Lane, 2014):

$$n = a\, d_{50}^{1/6} \quad (4)$$

The formula developed by Strickler allows to determine the roughness coefficient based on the grain median, i.e. d_{50}, and on the determined value of the coefficient a. The formulas developed by various authors, following the example of Strickler's (4), may also adopt a grain size diameter other than d_{50}. Raudkivi (1976) used d_{65}, while Keulegan (1938), Meyer-Peter and Mueller (1948) and Bray (1982) used d_{90}. The development of empirical formulas was based not only on the size, shape and distribution of the grains of the bed material, but also on the wetted perimeter. These types of formulas have been developed by, among others, Limerinos (1970), Griffiths (1981), Bray (1982), Phillips and Ingersoll (1997). In these formulas, the

ratio of hydraulic radius to a characteristic roughness element size, such as grain size or a bed-form characteristic, defined as the relative submergence, was intended to help quantify the variability of flow resistance due to different flow depth. The relationships between the roughness coefficient and the bed slope, water surface slope or friction slope were developed (Table 1) apart from the formulas for calculating the roughness coefficient value described above. In the equations (5) of Lacey (1946), (7) of Bray (1979), of Jarrett (1984) and of Sauer (1990), the values are given in imperial units of measurement.

Table 1. Roughness coefficient in the function of slope and others parameters.

No	Author / Authors	Equation	Description
(5)	Lacey (1946)	$n = 0.0928\ S^{1/6}$	Low gradient stream, sediment of gravel size or smaller
(6)	Riggs (1976)	$n = 0.21\ A^{-0.33} R^{0.667}\ S_w^{0.095}$	Uniform cross-sectional area, nearly full natural channel, Water-surface slopes:0.0003–0.018, hydraulic radius up to 19 ft.
(7)	Bray (1979)	$n = 0.104\ S_w^{0.177}$	Gravel-bed channel, $h/d_{90} > 5$ and water-surface slope: 0.00022 – 0.015.
(8)	Bray (1982)	$n = 1/80\ R^{0.067}\ S_w^{0.21}$	Inappropriate for high-gradient streams and narrow channels with dense streambank vegetation, high within-bank flow: $h/d_{90} > 3$
(9)	Brownlie (1983)	$n = 1.694(R/d_{50})^{0.1374}\ S^{0.1112}\ \delta^{0.1605}\ 0.034 d_{50}^{0.167}$	Sand bed: dune and ripple in bed - lower regime flow
(10)	Brownlie (1983)	$n = 1.0213(R/d_{50})^{0.0662}\ S^{0.0395}\ \delta^{0.1282}\ 0.034 d_{50}^{0.167}$	Sand bed: flat bed and antidune - upper regime flow
(11)	Jarrett (1984)	$n = 0.39\ S_f^{0.38}\ R^{-0.16}$	Stable bed and bank materials (gravels, cobbles and boulders) energy gradients from 0.002 to 0.039 and hydraulic radius from 0.5 to 7 ft.
(12)	Bruschin (1985)	$n = (d_{50}^{1/6}/12.38)\ ((R/d_{50})\ S_f)^{1/7.2}$	Sand bed, upper regime flow, bed slope less than 0.037
(13)	Gessler (1990)	$n = R^{1/3} / ((2.21 + 2.03 \log(0.0251/S_f))\ (8g)^{0.5})$	Armored canal beds at the discharge that formed the armor coat
(14)	Sauer (1990)	$n = 0.11\ S_w^{0.18}\ R^{0.08}$	Calibrated to data from Barnes (1967): bottom of fine sand, silt, sand, gravel, boulders. Water-surface slopes: 0.0003–0.018, hydraulic radius up to 19 ft.
(15)	Rickenmann (1994)	$n = 0.01786\ (S^{0.33}\ d_{90}^{0.45}) / (g^{0.44}\ Q^{0.11})$	Wide range of flow conditions, mountain streams and torrents, bed-slope range: 0.008–0.63
(16)	Dingman and Sharma (1997)	$n = 0.217\ A^{-0.173}\ R^{0.267}\ S_w^{0.156}$	Calibrated to data from Barnes (1967): bottom of fine sand, silt, sand, gravel, boulders, water-slope range: 0.0002–0.052
(17)	Rice et al. (1998)	$n = 0.029\ (d_{50}\ S)^{0.147}$	Channels with rock riprap laid on steep slopes, slope range: 0.025–0.22
(18)	Yochum et al. (2014)	$n = 0.18\ S_w^{0.79}$	Cascade and step-pool streams with beds dominated by gravel, cobbles, or boulders at low flow - water-surface slope: 0.019–0.17.

where: No. – number of equation, S – slope of bed, S_w – slope of water surface, S_f – friction slope or energy gradient, R – hydraulic radius, T – top width of stream, A – cross-sectional area of flow, h – mean depth, Q – water discharge, d_x – the grain diameter for which x% is finer, δ – the geometric standard deviation of the sediment mixture.

The use of Brownlie equations (1983) (Table 1) requires determination of the flow regime. If the slope is greater than 0.006, flow is always of upper regime. Otherwise, the transition is correlated with the grain Froude number (F_g) as follows (Fischenich and Little, 2007): if $F_g \leq F_g$', then lower regime flow is observed, and if $F_g > F_g$', then upper regime flow is observed, where F_g and F_g':

$$F_g = V / ((s-1) g d_{50})^{0.5} \quad (19)$$

$$F_g' = 1.74 / S^{1/3} \quad (20)$$

where: s is specific gravity of sediment particles, S is the bed slope, V is mean velocity, and d_{50} is the grain diameter for which 50% is finer. A parameter δ in formulas (9) and (10) in the table 1 - i.e. the geometric standard deviation of the sediment mixture, is determined from the formula:

$$\delta = 0.5((d_{84}/d_{50}) + (d_{50}/d_{16})) \quad (21)$$

In the calculations of the friction factor f, friction slope S_f should be assumed as the slope value (Ferguson, 2007; Machiels et al., 2009). In the case of significant non-uniformity of the flow in the calculations, the use of the water surface slope S_w or mean bed slope S may result in obtaining a systematic error (Jarrett,

1984; Hicks and Mason, 1991). The flow is generally assessed based on the formulas developed for the friction slope determined experimentally for uniform flow conditions. In simple empirical formulas, Jarrett (1984), Bruschin (1985) and Gessler (1990) introduced a friction slope, also called an energy gradient (Table 1). Under steady flow conditions, the effects of bottom friction are precisely balanced by gravitational forces. Thus, for a uniform flow, the friction slope is assumed to be equal to the channel bed slope (e.g. Brownlie (1983), Lacey (1946), Rickenmann (1994) and Rice et al. (1998) – Table 1). For hydraulic conditions corresponding to a uniform flow, some authors provide the water surface slope in the proposed formulas, e.g. Riggs (1976), Bray (1979 and 1982), Saurer (1990), Dingman and Sharma (1997) as well as Yochum et al. (2014) (Table 1). When using these formulas for flow conditions other than uniform, the bed slope or water surface slope should be replaced by the friction slope. Determining the friction slope poses no problem when data from measurements are available surveying measurements will be made.

In the case of a design channel with non-uniform flow and irregular cross-section, the calculation of the friction slope requires not only calculation of bed and water surface slopes, but also the calculation of the height of the velocity line from the formula $\alpha V2/2g$, it is necessary to adopt the roughness coefficient if the calculations are conducted using the Manning's equation, or to adopt the friction factor when the Darcy-Weisbach equation is used. These coefficients are therefore adopted in order to calculate the height of the velocity line in the calculation cross-sections of the designed reach to calculate the friction slope. And yet, the friction slope itself was intended to be used for the estimation of these coefficients. The solution of this task requires iterative calculations, which is in contradiction with the practical and efficient estimation of the roughness coefficient, or the friction factor values as a function of the friction slope. The aim of this study was 1) to assess the possibility of using the empirical formulas presented to determine the roughness coefficient in streams with gravel bottoms, whose bed slopes range from 0.006 to 0.047, 2) to assess the impact of taking into account the conditions of non-uniform flow on using these formulas, 3) to develop the correlation dependencies between the roughness coefficient and water surface slope or friction slope for the estimation of the n values in gravel-bed streams. The obtained results may prove helpful in the assessment of hydraulic conditions of water flow in streams with similar morphological features. As a result of these studies, it will be possible to indicate an empirical formula or formulas that will allow to estimate the roughness coefficient on the basis of the stream bed slope, water surface slope or friction slope.

2 MATERIAL AND METHODS

Field studies were carried out on the reaches of the Będkówka, Racławka, Bolechówka and Wierzchówka streams selected for the experiment. These streams are tributaries of the Rudawa river, which is a tributary of the Vistula river. The catchments of these streams are located in the Olkusz Upland, which is a mesoregion being a part of the Kraków-Częstochowa Upland macroregion. They are located within the Kraków poviat, in the Lesser Poland voivodeship (Fig. 1). The basic parameters of the analyzed streams are presented in Table 2.

Measurement reaches with a length of 3 to 5 cross-section widths were selected for the tests. In every reach, the following cross-sections were determined: upper (initial) and lower (final), as well as three sections between the upper and lower section. The bottom ordinates were estimated in each of the eight cross-sections, in order to determine the bed slope as well as the water surface slope at a given flow. Hydrometric measurements were also conducted in the upper and lower section of each measurement reach. In these cross-sections, hydrometric risers were determined at intervals of 10 cm and hydrometric measurements were conducted in each riser at different heights above the bottom. Hydrometric measurements were conducted with a Valeport Model 801 induction flowmeter. Based on the results of hydrometric measurements, the water flow rate was calculated using the Harlacher method (Michalec and Zwolenik, 2020). Then, the hydraulic parameters for this reach were calculated according to the methodology provided by Nitsche et. al. (2012). A representative channel cross-section was interpolated from the vertical mean of multiple measured cross-sections. Using this reach-averaged cross-section, all hydraulic

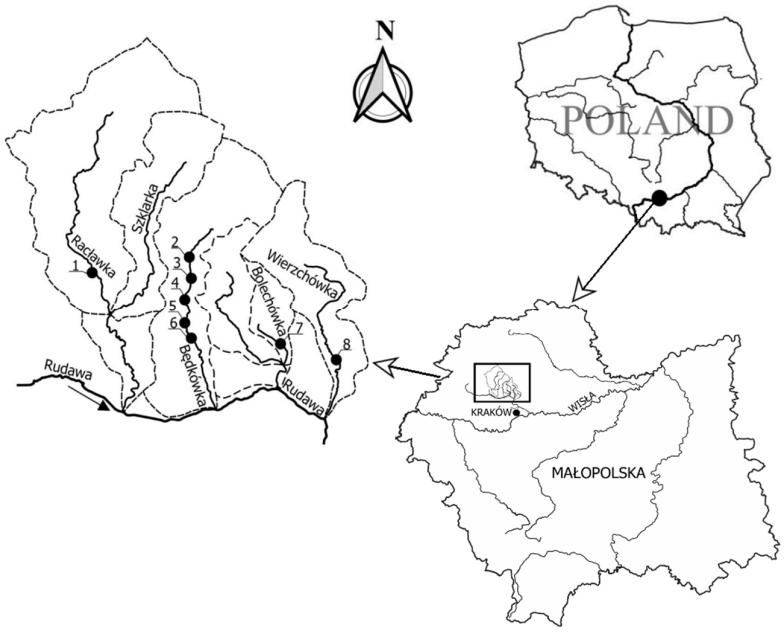


Fig. 1. Streams: Będkówka, Racławka, Bolechówka and Wierzchówka together with the location of measurement sections (from 1 to 8).

Table. 2. Basic parameters of the studied streams.

Stream	Lenght of stream (km)	Cachment area (km^2)	Mean bed slope of stream (–)	Mean bed slope of reach (–)
Racławka	16.52	39.78	0.009	0.006
Będkówka	12.00	24.90	0.018	0.006–0.023
Bolechówka	4.83	18.40	0.008	0.030
Wierzchówka	9.00	17.29	0.009	0.047

parameters were solved (i.e. mean flow depth h, hydraulic radius R and cross sectional area A) corresponding to reach-averaged discharge and reach-averaged velocity (Nitsche et al., 2012). Surveys and hydrometric measurements were conducted in three series on each of the eight reaches (Fig. 1). The results of each measurement series were obtained from the measurements conducted within 2 days under constant hydrological and meteorological conditions in the catchment area of the studied streams.

Based on the surveys and hydrometric measurements, the water flow rate Q, was calculated, followed by the calculation of the Manning's mean roughness coefficient n_0 by using the formula developed by Barnes (1967). This formula (22), derived from the Bernoulli equation, enables the calculation of the average Manning's roughness coefficient for sections of the watercourse with many cross-sections.

$$n_0 = \frac{1}{Q}\sqrt{\frac{(h_1 + h_{v1}) - (h_m + h_{vm}) - \sum_{i=2}^{m}(k_{i-1,i}\,\Delta h_{v_i\text{-}1,i})}{\sum_{i=2}^{m}\frac{L_{i-1,i}}{Z_{i-1}Z_i}}} \qquad (22)$$

where: h – water depth, h_v – velocity head, $k\Delta h_v$ – energy loss due to contraction of velocity or deceleration of velocity in contracting or expanding reaches, where k is a coefficient adopted as zero for contracting reaches and 0.5 for expanding reaches (Lee et al., 2017), $Z = AR^{2/3}$, where A is the cross-section area, R is the hydraulic radius and m is number of cross-sections.

The uncertainty of the mean rate flow Q, which is the arithmetic mean of the rate flow Q, calculated from the measurements in five sections, was determined on the basis of the standard deviation of the mean. Due to the small number of hydrometric measurements (n = 5), in order to avoid underestimating the specified uncertainty, the standard deviation of the mean was multiplied by the Student's t-test distribution coefficient ($t_{n,\alpha}$). This coefficient was adopted for the confidence level α = 95% and degrees of freedom (n – 1 = 4). The relative uncertainty (RU) was calculated as the quotient of the standard uncertainty, i.e. the standard deviation of the mean (SD) and the mean rate flow Q. The standard error (SE) was also calculated.

In order to determine the granulometric composition of the bed-material, sediment samples were taken from the bed of the streams between the upper and lower cross-sections. On the basis of the developed curves and particle size distribution, the characteristic diameters of the sediment were determined. Sieve analyzes were performed for three samples collected in each measuring reach. The mean percentage composition of the separated sediment fractions was determined. Fractions smaller than 1 mm were analyzed hydrometrically. On the basis of the developed curves and particle size distribution, the characteristic diameters of the sediment were determined. The standard deviation, i.e. the uncertainty of the arithmetic mean (Kolwalczyk et al., 2012), was calculated to describe the accuracy of determining the true value of the measured diameter d_{50}. The standard deviation of the mean (SD) was calculated for the d_{50} determined from the three specific grain size composition. Due to the small number of collected sediment samples (n = 3), in order to avoid underestimating the specified uncertainty, the standard deviation of the mean was multiplied by the Student's t-test distribution coefficient ($t_{n,\alpha}$). This coefficient was adopted for the confidence level α = 95% and degrees of freedom (n – 1 = 2). The relative uncertainty (RU) was calculated as the quotient of the standard uncertainty, i.e. the standard deviation of the mean (SD) and the mean d_{50}. The standard error (SE) was also calculated.

The hydraulic parameters calculated for the studied stream reaches allowed to calculate the roughness coefficient value from the Equation (22), which was compared with the n values calculated using the formulas presented in Table (1). In the first stage of calculations, the slope was used in these equations in accordance with the formulas themselves, using the bed slope, water surface slope or friction slope, respectively. The results of these calculations were compared with the values of n_0 from the measurements for each of the measurement reaches made in each measurement series. In the second stage of the calculations, due to the non-uniform flow, the friction slope was used in each formula (Table 1), regardless of the parameter given by its author, replacing the bed slope and water surface slope with friction slope, respectively. The results of all calculations were finally compared with each other in order to find the impact of changing the type of slope in the examined formulas on the results of the calculations compared to the actual values obtained from the measurements.

3 RESULTS

Hydrometric measurements in cross-sections were conducted at depths h ranging from 0.08 to 0.19 m in the first measurement series, from 0.10 to 0.21 m in the second measurement series and from 0.13 to 0.30 m in the third measurement series (Table 3). The bed slope and water surface slope were determined based on the measurements. The friction slope was calculated by taking into account the average flow velocity calculated from the measurements in the upper and lower cross-sections of the measurement reaches. The values of water surface slopes and friction slopes are presented in Table 3. This table also includes the roughness coefficient values calculated by the formula (22) for each of the measurement sections in a given measurement series.

The uncertainty of the mean rate flow Q of each reach, expressed by the standard deviation, is 0.0015–0.0039 for measurements series S1, 0.0014-0.0034 for measurements series S2, and 0.0024–0.0042 for measurements series S3. Taking into account the distribution coefficient ($t_{n,\alpha}$) of 2.7764, the corrected value of the standard deviation of the mean ($SDt_{n,\alpha}$) is 0.0043–0.0108 for measurements series S1, 0.0039–0.0095 for measurements series S2, and 0.0067–0.0117 for measurements series S3. Relative uncertainty (RU) and standard error (SE) for mean rate flow Q calculated in all reaches in measurement series are respectively: RU from 89.96 to 110.04 and SE from 0.0007 to 0.0017 for S1, RU from 92.44 to 107.56 and SE from 0.0006 to 0.0015 for S2 and RU from 94.30 to 105.70 and SE from 0.0011 to 0.0019 for S3.

Based on the specific grain size composition of the sediment collected from the bed (Table 4), the relative submergence h/d_{50} was calculated (Table 3) and the roughness criterion was determined according to Bathurst et al. (1981). The h/d_{50} values ranging from 2 to 7.5 indicate average roughness for nineteen measurement data and small roughness ($h/d_{50} > 7.5$) for five measurement data, i.e. on reaches IV and VI in measurement series S3, i.e. at the highest flows and on the reach VIII for all three measurement series. The uncertainty of the mean diameter d50, expressed by the standard deviation of the mean for each reach, is presented in Table 4. Taking into account the distribution coefficient ($t_{n,\alpha}$) of 4.303, the corrected value of the standard deviation of the mean ($SDt_{n,\alpha}$), relative uncertainty (RU) and standard error (SE) was calculated. The results are presented in Table 4.

3.1 Verification of empirical formulas

Table 3 presents the values of the roughness coefficients calculated using the formula (22). The values of these coefficients were compared with those calculated according to the formulas presented in Table 1. It was found that the roughness coefficients calculated by the formula (5) by Lacey (1946) are the most similar to those obtained as a result of the measurements – most of the points in the graph in Figure 2.a) are located on the proportionality line with a few cases of the value underestimation.

The Lacey's formula (5) was developed for low gradient streams. According to Yochum et al. (2012, 2014) the limit value is the dimensionless bed slope of 0.02–0.03, distinguishing

Table 3. Basic hydraulic parameters on the reach.

No.	h (m]	Q ($m^3 \cdot s^{-1}$)	R (m)	h/d_{50} (–)	S_w (–)	S_f (–)	n_0 ($m^{-1/3} \cdot s$)
Measurement series S1							
I	0.08	0.158	0.073	2.3	0.0171	0.0158	0.055
II	0.13	0.078	0.130	2.0	0.0205	0.0226	0.063
III	0.16	0.084	0.126	2.5	0.0142	0.0134	0.060
IV	0.11	0.087	0.105	4.9	0.0117	0.0086	0.054
V	0.08	0.099	0.081	2.1	0.0156	0.0157	0.061
VI	0.19	0.112	0.161	5.6	0.0082	0.0078	0.057
VII	0.11	0.051	0.088	2.6	0.0092	0.0077	0.056
VIII	0.15	0.115	0.140	7.0	0.0177	0.0197	0.062
Measurement series S2							
I	0.10	0.198	0.097	2.9	0.0080	0.0065	0.048
II	0.15	0.084	0.140	2.3	0.0141	0.0155	0.059
III	0.17	0.093	0.138	2.8	0.0103	0.0095	0.055
IV	0.14	0.105	0.136	6.4	0.0057	0.0083	0.052
V	0.12	0.115	0.114	2.9	0.0050	0.0053	0.051
VI	0.21	0.129	0.178	6.3	0.0059	0.0070	0.052
VII	0.12	0.059	0.103	2.8	0.0058	0.0031	0.054
VIII	0.17	0.168	0.157	8.0	0.0177	0.0194	0.060
Measurement series S3							
I	0.13	0.261	0.141	3.9	0.0022	0.0018	0.043
II	0.19	0.149	0.165	2.8	0.0196	0.0100	0.056
III	0.23	0.171	0.171	3.7	0.0103	0.0007	0.048
IV	0.19	0.209	0.186	8.8	0.0044	0.0021	0.047
V	0.18	0.228	0.169	4.4	0.0035	0.0038	0.046
VI	0.30	0.243	0.239	8.9	0.0047	0.0043	0.049
VII	0.24	0.144	0.204	5.8	0.0049	0.0025	0.051
VIII	0.21	0.336	0.199	10.1	0.0140	0.0190	0.055

where: No. – number of the reach, h – mean depth in the reach, Q – water flow rate, R – hydraulic radius, d_{50} – the grain size diameter for which 50% is finer, S_w – water-surface slope, S_f – friction slope, n_0 – resistance coefficient calculated from the measurement results.

Table 4. Characteristic diameters of bed sediment collected from the bed on the reach and uncertainty analysis for d_{50}.

No. of the reach	Characteristic diameters d_i (mm)												Uncertainty analysis parameters for d_{50}			
	d_5	d_{10}	d_{16}	d_{35}	d_{50}	d_{65}	d_{75}	d_{80}	d_{84}	d_{85}	d_{90}	d_{95}	DS	$SDt_{n,\alpha}$	RU (%)	SE
I	6.1	11	14	28	33	42	49	51	53	54	61	69	1.0	4.3	87.0 – 113.0	0.58
II	6.2	9.5	13	45	65	68	70	72	74	75	77	79	1.0	4.3	94.4 – 106.6	0.58
III	4.8	6.3	9	28	63	68	69	70	71	72	75	78	1.2	5.3	91.6 – 108.4	0.71
IV	3.3	5	6	13	22	33	47	53	62.5	63	67	74	0.6	2.5	88.7 – 111.3	0.33
V	5.5	10	17	20	40	51	63	68	71	72	78	81	0.7	3.0	92.5 – 107.5	0.40
VI	5	6.4	10	26	34	60	47	63	67	68	70.5	72	0.6	2.5	92.7 – 107.3	0.33
VII	2	4.5	6.2	23	41	60	66	69	70	70.5	74	79	0.6	2.5	93.9 – 106.1	0.33
VIII	4.3	5.1	6.4	12.5	21	34	50	59	65	66	70	72	0.6	2.5	88.2 – 111.8	0.33

where: DS – standard deviation, $SDt_{n,\alpha}$ – corrected value of the standard deviation of the mean, RU – relative uncertainty, SE – standard error.

between watercourses with low and high bed slopes. The bed slope of the examined reaches ranged from 0.005 to 0.047, and twelve of the twenty-four reaches had a bed slope smaller than 0.02, which is the lower limit of the Lacey range of slopes. For these reaches, roughness coefficients were obtained, the positions of which in Figure 2.a are at the greatest distance from the n_0 and n proportionality curve, where n_0 is the roughness calculated from the measurements, n – roughness estimated by the formulas from Table 1. This means that for the cross-sections located on the reaches with bed slope of <0.02 (low gradient streams) the worst results were obtained from the Lacey formula (5), which is dedicated to this range of slopes.

Much greater differentiation of n_0 values was obtained using the Riggs' formula (6) (1976) resulting in both underestimation and overestimation of the n value (Fig. 2.b). None of the hydraulic parameters affected the significant dispersion of the calculated roughness coefficient values. It can be assumed that the reason for significant discrepancies in the values of n determined by the Riggs' formula (6) and those obtained from the measurements, is the range of bed slopes. Riggs developed the formula (6) based on the measurement data of Barnes (1967), i.e. for dimensionless water surface slope of 0.0003–0.018, while in the studied streams the water surface slope was outside this range, i.e. it was from 0.0022 to 0.0205, which can be the reason for such significant discrepancies of the results.

The formulas: (7) of Bray (1979), (8) of Bray (1982) and (14) of Sauer (1990) allow for an approximate estimation of the roughness coefficient value, which is, however, somewhat overestimated (Fig 2.c and Fig 2.i). Slight discrepancies between the results of calculations of the roughness coefficient by the Bray's formula (7), compared to the one determined from the measurements may result from a slight exceedance of the maximum value of the water surface slope, for which this formula was developed. The upper limit of the dimensionless water surface slope range, according to Bray, is 0.015, while the measurements provided the values up to 0.0205. The reason for the discrepancy in the results may also be the range of the relative submergence h/d_{90} of the examined reaches. Bray developed the formula for h/d_{90} greater than 5, and the value of h/d_{90} for the examined cross-sections ranged from 1.05 to 4.27. Also the results of calculating the value of n using the formula (8) by Bray (1982) (Fig 2.d) are underestimated, but their position is close to line of perfect agreement. This formula is recommended by Bray (1982) for the channels, whose $h/d_{90} > 3$. Only five sections do not meet this condition. The Sauer's formula (14), like the Riggs' formula (6), was also developed on the basis of measurement data by Barnes (1967). In this case, despite the fact that the measured water surface slope is outside the range given by Sauer (1990), values of the roughness coefficient estimated using this formula indicate the possibility of its application in the case of the studied streams.

The values of n obtained from calculations by the formula (16) by Dingman and Sharma (1997) proved to be significantly overestimated (Fig. 2.j), which indicates that this formula should not be used to determine the roughness coefficient of the studied streams, despite the fact that the range of the water surface slopes corresponded to the range for which the formula (16) was developed. The roughness coefficients calculated by the formulas (9) and (10) of Brownlie (1983) (Fig. 2.e and Fig. 2.f), by the formula (12) of Bruschin (1985) (Fig. 2.g), by the formula (13) of Gessler (1990) (Fig. 2.h), as well as by the formula (17) of Rice et al. (1998) (Fig. 2.k) and by the formula (18) of Yochum et al. (2014) (Fig. 2.l) are also smaller than the one calculated from the measurements. Significant differences in the roughness coefficient values determined on the basis of measurements and according to empirical formulas were obtained using the formula (18) of Yochum et al. (2014) (Fig. 2.l). This formula was developed on the basis of measurement data at low flows, the water surface slope of which ranged from 0.019 to 0.17. The formula (18) of Yochum et al. (2014) was applied for low flows, where the water surface slope was smaller than the range for which this formula was developed, except for two reaches. Despite this, the calculation result also for these two cases is several times smaller than that obtained from the measurements.

Among the verified two Brownlie formulas, a smaller underestimation of the n value was obtained by using the formula (9) (Fig. 2.e) corresponding to the lower regime flow and bed slopes smaller than 0.006. On the other hand, the studied streams are characterized by an average bed slope greater than 0.006 (Table 2). Thus, by using the Brownlie's formula (10) for slopes greater than 0.006, the obtained calculation results should be closer to those obtained on the basis of the measurement results. However, this was not achieved as the results obtained using the formula (10) were worse (Fig 2.f) than those obtained by using the formula (9) (Fig. 2.e).

The results of roughness coefficient calculations by the formulas (11) of Jarrett (1984) and (15) of Rickenmann (1994) were not compared graphically with those calculated from the measurements. The roughness coefficient calculated by the Jarrett's formula (11) (1984) proved to be, on average, several times higher than that determined from the measurements. In some cases, it was even twenty or thirty times overstated. The formula (11) according to Jarrett (1984) is applicable for channels with energy gradients from 0.002 to 0.09 and for the tested reaches it ranges from 0.0007 to 0.0226, whereas for ten examined reaches the energy gradient was within the range given by Jarrett (1984). Also the range of the hydraulic radius calculated from the measurements does not correspond to the range given by Jarrett (1984). The hydraulic radius of the examined cross-sections ranged from 024 to 0.78 feet and only ten of these reaches were characterized with hydraulic radius greater than 0.5 feet. The roughness coefficient values obtained by using the formula (15) of Rickenmann (1994) were, on average, over eighty times smaller than those determined from the measurements. Despite the fact that only nine sections are characterized by a dimensionless bed slope smaller than that given by Rickenmann (1994), the value of which is 0.008 – 0.63, significantly underestimated values of the roughness coefficient were obtained for all sections.

Most of the verified formulas (Table 1) give an estimate of n depending on the bed slope or water surface slope S_w. Only the formulas (11) according to Jarrett, (12) according to Brunschin and (13) according to Gessler present the value of n in the function of friction slope S_f. During the measurements non-uniform flow was observed in the tested reaches of the streams, therefore the roughness coefficients were calculated for the friction slope using each of the formulas presented in Table 1. The values of the roughness coefficient calculated by the Lacey's formula (5) after the introduction of the friction slope (Fig. 3.a) proved to be underestimated compared to the ones obtained from the measurements and also more underestimated compared to the results of measurements with the bed slope S adopted in this formula (Fig. 2.a). The formula (6) by Riggs proved to be insensitive to the use of friction slope (Fig. 3.b) instead of the water surface slope (Fig. 2.b), because the scatter of the results of estimated values of n in relation to the proportionality line is similar regardless of whether S or S_f is used in this formula. The introduction of S_f into the Bray's formula (7) resulted in obtaining overestimated n values (Fig. 3.c) compared to the measurement results, also providing worse results, i.e. all of them were overestimated compared to the results of calculations with the water surface slope (Fig. 2.c). On the other hand, the change of S_f into

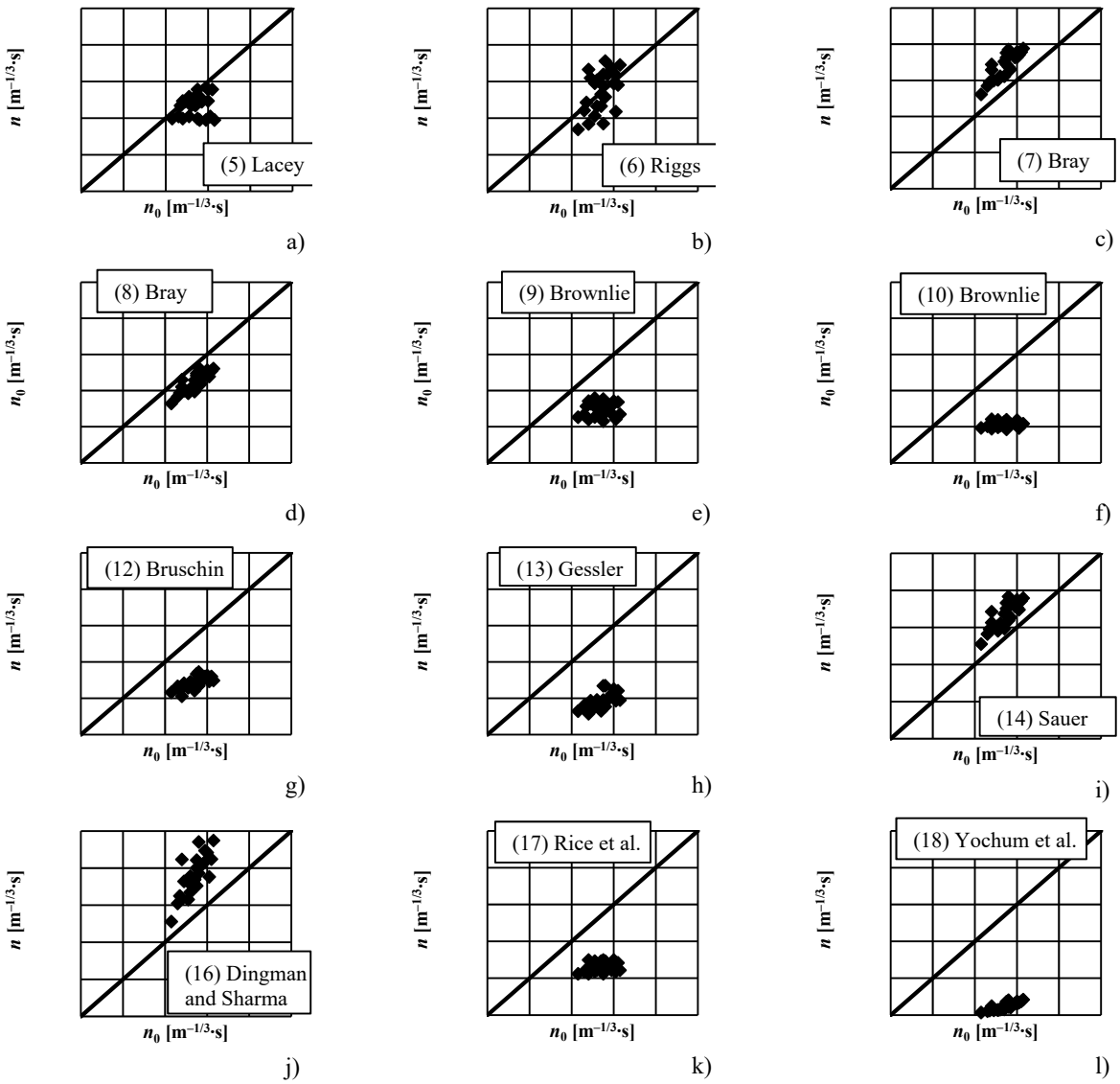


Fig. 2. Comparison of roughness coefficients calculated on the basis of the measurement results (n_0) and using empirical formulas (n) according to: a) (5) Lacey, b) (6) Riggs, c) (7) Bray, d) (8) Bray, e) (9) Brownlie, f) (10) Brownlie, g) (12) Bruschin, h) (13) Gessler, i) (14) Sauer, j) (16) Dingman and Sharma, k) (17) Rice et al., l) (18) Yochum et al. (2014).

S_w in the Bray's formula (8) (Fig. 2.d and Fig. 3.d), and in formulas (9) and (10) by Brownlie, does not cause a significant difference in the results, as can be seen when comparing Figures 2.e) and 3.e), and 2.d) and 3.d), respectively. The friction slope used in the Bray's formula (8) caused that some of the results became distant from the proportionality curve (Fig. 3.d). The use of S_f in the formula (9) by Brownlie causes the points to be moved away from the proportionality curve in Figure 3.e). This also applies to the formula (17) of Rice et al. (Figs. 2.k and 3.k). Using the formulas (14) of Sauer and (16) by Dingman and Sharmy with S_f, slightly better results were obtained (Fig. 3.i and 3.j) compared to the results of calculations using these formulas with S_w – points in Figures 2.i) and 2.j) are located closer to the proportionality curve. The change of S_f to S_w in the formula (18) by Yochum et al. (2014) causes such slight differences in the results that they are imperceptible and therefore they have not been presented in Figure 3. In these analyses, the results obtained from the formulas (11) of Jarrett and (15) of Rickenman also differed significantly from those obtained from the measurements.

3.2 An attempt to develop empirical formulas

The correlation relationships were developed for the measured water surface slope S_w and the calculated friction slope S_f, presented in Table 3, based on the example of the formula (5) proposed by Lacey (1946). These formulas have the form of:

$$n = 0.0837\ S_f^{0.0895} \tag{23}$$

$$n = 0.1038\ S_w^{0.1391} \tag{24}$$

Due to the small amount of data (N = 24), the developed formulas are an initial attempt to develop a relationship for streams with gravel bottom and bed slopes ranging from 0.0022 to 0.0205. The coefficient of determination r^2 for the developed equations amounts to 0.599 for the Equation (23) and 0.716 for the Equation (24). All parameters of independent variables are statistically significant at p = 0.05%.

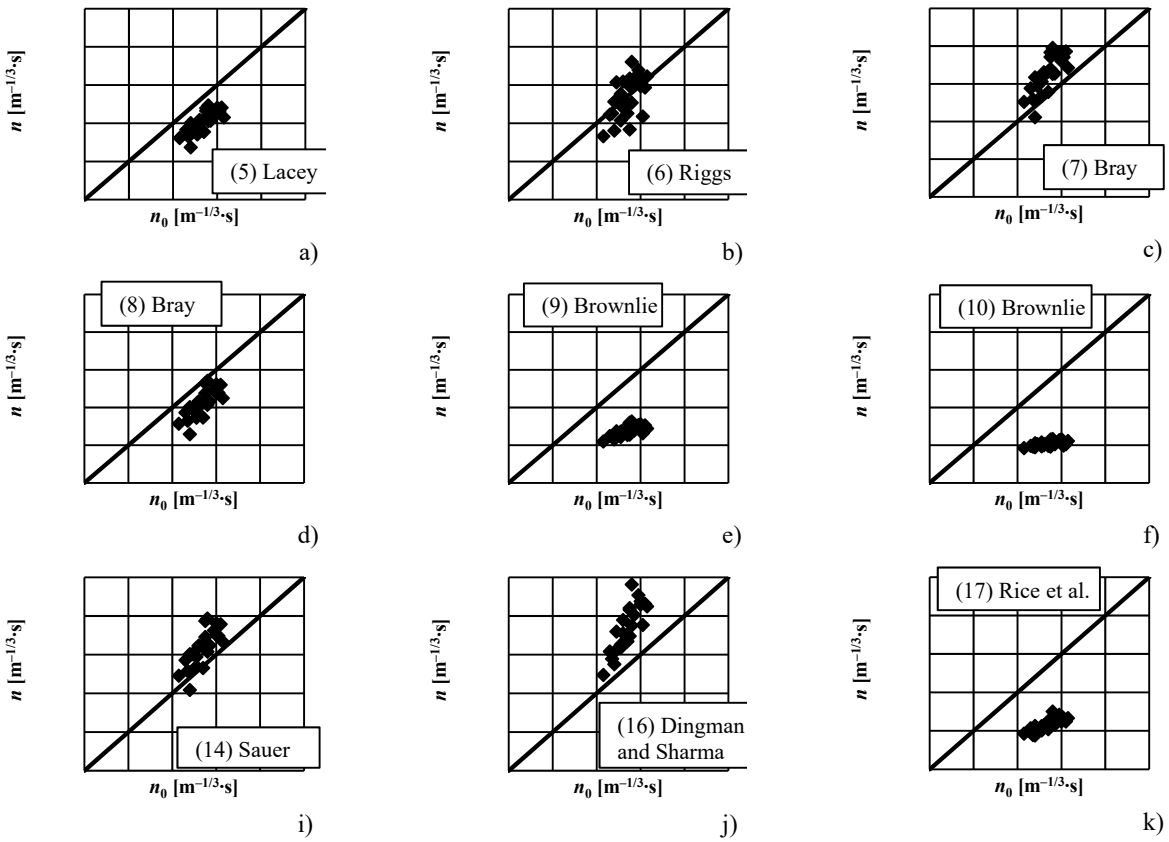


Fig. 3. Comparison of roughness coefficients calculated on the basis of the measurement results (n_0) and using empirical formulas (n), in which a friction slope was introduced in place of a slope, i.e. equations: a) (5) Lacey, b) (6) Riggs, c) (7) Bray, d) (8) Bray, e) (9) Brownlie, f) (10) Brownlie, i) (14) Sauer, j) (16) Dingman and Sharma, k) (17) Rice et al.

The predictive effectiveness of the developed models was statistically assessed using the following measures: root mean square error: RMSE = $(\Sigma(x_p - x_m)2/N)^{1/2}$ and mean absolute error: MAE = $\Sigma|x_p - x_m|/N$, where: x_p – value of predicted roughness coefficient or resistance coefficient, x_m – value of the measured roughness coefficient or resistance coefficient, N is the number of data.

The RMSE and MAE values for the model written in the form of Equation (23), in which the independent variable is the friction slope, are 0.0033 and 0.0029, respectively. For the Equation (24) with the water surface slope, the RMSE and MAE values are 0.0028 and 0.0024, respectively.

Control calculations were also performed to verify the influence of the applied slope in a given formula. Thus, the values of the roughness coefficient were calculated from the formula (23) by introducing the water surface slope calculated for the examined reaches instead of the friction slope. The predictive efficiency of this model for the independent variables S_w were determined by RMSE and MAE, the values of which are 0.0034 and 0.0029, respectively. The calculations with the formula (24) were also performed for the friction slope instead of for the water surface slope. The calculated RMSE and MAE values are 0.0041 and 0.0031 respectively.

4 DISCUSSION

The obtained results, verifying the application of the tested formulas (Table 1), indicate that the estimated values of the roughness coefficient for streams with gravel bottoms located in the Kraków-Częstochowa Upland can be obtained from the Lacey's formula (5), which shows the dependence of this coefficient on the bed slope. The *n* values calculated for the bed slopes of the examined streams, proved to be partially underestimated. Also Nguyen and Fenton (2004) found that the roughness coefficient values determined using the Lacey's formula are underestimated as compared to the values determined from the measurements, but in the case of those studies this observation concerned the entire range of measurements.

A parameter that is less determining while calculating the *n* value is the water surface slope, applied in the formulas (6) of Riggs, (7) and (8) of Bray and (14) of Sauer. On the basis of the obtained results, it was found that by using these formulas, it is possible to obtain the roughness coefficient values similar to the real ones. The formula (6) by Riggs gave both underestimated and overestimated *n* values. When using the Bray's formula (7), the *n* values were overestimated for the entire data range, similarly as in the case of the formula (14) of Sauer. On the other hand, the underestimated values of the roughness coefficient were obtained from the Bray's formula (8). According to Nguyen and Fenton (2004), the Bray's and Sauer's formulas underestimated the values of *n*.

Out of the fourteen verified formulas, only the formula (11) of Jarrett, (12) of Bruschin and (13) of Gessler were developed for the friction slope and only these formulas in their original form can be used in the conducted research due to the non-uniform flow occurring in the studied streams. However, the values of *n* calculated by the Jarrett's Equation (11) were over ten times overestimated, and the values obtained from the formulas (12) of

Bruschin and (13) of Gessler – were significantly underestimated (Fig. 2), which excluded their use. Machiels et al. (2011) state that in simple empirical formulas, which combine the channel roughness, flow variables and bed slope, the bed slope can be replaced with friction slope and thus determine the friction effect for the flow conditions other than uniform flow. Replacing the bed slope or water surface slope with friction slope in the assessment of applicability of the verified empirical formulas, did not improve the estimated n values. In the case of the Lacey's formula (5), worse results were obtained, i.e. the n values were more underestimated (Fig. 3.a). Using the formula (7) of Bray, in which the friction slope was used instead of the water surface slope, still results in obtaining overestimated n values (Fig. 3.c), while the formula (8) of Bray brings some of the results closer to the proportionality line. The roughness coefficients obtained from the Saurer's formula (16) with friction slope proved to be less overestimated (Fig. 3.i) than while using the water surface slope (Fig. 2.i). After applying the friction slope in the formula (6) of Riggs, some of the obtained results were less overestimated (Fig. 3.b) without causing significant changes in the arrangement of points in relation to the proportionality curve (Fig. 2.b). The results of the calculations with the other formulas using the friction slope appeared to be as unsatisfactory as of the calculations conducted for the bed slope or water surface slope.

The formulas (5) of Lacey, (6) of Riggs, (7) and (8) of Bray and (14) of Sauer, regardless of the water flow conditions, i.e. uniform or non-uniform flow, allow only an approximate estimate of the roughness coefficient in the studied streams. It is indicated by the calculated differences in the results Δ, as a percentage deviation of the estimated n value from the values obtained from the n_0 measurements. The values of Δ calculated for the slopes given in these formulas, i.e. without taking into account the non-uniform flow conditions, are from –38 to 3% for the formula (5) of Lacey, –33 to 39% for the formula (6) of Riggs, from 12 to 41% for the formula (7) of Bray, 27 to –4% for the formula (8) of Bray and from 10 to 43% for the formula (14) of Sauer (Fig. 2). When the friction slope was used in these formulas, the difference in the results (Fig. 3) is as follows: for the formula (5) of Lacey from –43 to –12%, (6) of Riggs: from –33 to 29%, (7) of Bray: from –41 to 12%, (8) of Bray: from –46 to –3% and (14) of Sauer: from –13 to 41%. For the remaining formulas, the obtained differences were significantly higher – the highest for the formula (6) of Riggs, amounting to 62%. The calculated differences in the results are significant and it can be concluded that there is no single equation that can consistently and accurately estimate the n values for all channels, as also demonstrated by Coon (1998). The differences in the obtained results are also caused by the fact that empirical formulas are developed for specific conditions that limit their application geographically, resulting from a specific research area or a deliberately selected research site in a channel with perfect characteristics (Coon, 1998). Another issue is the application of a given formula to the hydraulic conditions of the analyzed reach of a watercourse, taking into consideration the scope of conditions, for which the formula was developed. In many publications using empirical formulas, the Authors of these publications do not specify the scope of their application, or this description is not complete. A very extensive description of the formulas and their application is presented by Lang et al. (2004), however this description does not provide the range of slopes, relative submergence or hydraulic radius corresponding to the application of these formulas. Moreover, when applying some of the formulas, the authors of the studies do not pay attention to the source materials which provide statistical parameters of the formulas developed. An example is the formula (7) of Bray, for which the coefficient of determination is only 0.298, and yet it is widely used, e.g. in the studies by Coon (1998), Lang et al. (2004), Ferguson (2007), Ghani et al. (2007), López et al. (2007), Sefick et al. (2015), Zhu et al. (2020), Toufik and Mahmoud (2021), as well as in this study.

5 CONCLUSIONS

Out of the fourteen empirical formulas assessed, only the formulas (5) of Lacey, (6) of Riggs, (7) and (8) of Bray as well as (14) of Sauer allow for an approximate estimation of the roughness coefficient in the studied streams of the Kraków-Częstochowa Upland.

In the studied streams, the water flow was non-uniform and therefore the roughness coefficient was calculated using the tested formulas for the friction slope. Based on the obtained results of calculations for the estimated roughness coefficient values, it is not possible to clearly indicate the formula enabling the estimation of this coefficient. The Riggs equation (6) allows to obtain the roughness coefficient value that differs from the one obtained from the measurements by –33 to 29%. In the case of the remaining formulas, the differences in the results reached even 41% for the formula (7) of Bray and (14) of Sauer, and 43% for the formula (5) of Lacey and (8) of Bray.

Taking into account the occurring non-uniform flow and using the friction slope in the formulas instead of the bed slope did not improve the values of the roughness coefficient estimated using the examined formulas. The RMSE and MAE values calculated for the developed correlation Equations (23) and (24) showed that the adopted friction slope or water surface slope had no effect on the value of the estimated roughness coefficient. RMSE and MAE values for the formula (23), in which the friction slope was replaced by the water surface slope, amount to 0.0034 and 0.0029, respectively. These values do not differ from those obtained for this very equation, in which the friction slope was used according to its original form, because then the RMSE and MAE values are 0.0033 and 0.0029, respectively. When replacing the water surface slope with the friction slope in the Equation (24), the RMSE and MAE values for the estimated n value were 0.0041 and 0.0031, respectively. On the other hand, the values of RMSE and MAE for the calculations performed with the formula (24) with the water surface slope, i.e. in accordance with the provisions for this formula, are 0.0028 and 0.0024, respectively. In this case, after changing the type of slope in the formula, higher error values were obtained, i.e. by 46 and 29%, respectively. These values do not disqualify the change of slope in the formula (24), because the RMSE and MAE values mean that the actual values of the roughness coefficient obtained with this equation will deviate from the predicted ones by the absolute n values equal to only 0.41–0.31%. The obtained results allow to state that there are no significant differences in the obtained results, meaning that not taking into account the water flow conditions, i.e. uniform or non-uniform flow, does not affect the correct determination of the resistance impact acting in the examined streams. The roughness coefficient in streams with gravel beds, the morphological characteristics of which are similar to those presented in this paper, can be calculated using the Equations (23) and (24).

REFERENCES

Barnes, H.H., 1967. Roughness characteristics of natural channels. Water-Supply Paper 1849. U.S. Geological Survey.

Bathurst, J.C., Li, R.M., Simons, D.B., 1981. Resistance equation for large-scale roughness. J. Hydraul. Div. ASCE, 107,

12, 1593–1613. https://doi.org/10.1061/JYCEAJ.0005780
Bellos, V., Nalbantis, I., Tsakiris, G., 2018. Friction modeling of flood flow simulations. Journal of Hydraulic Engineering, 144, 12. DOI: 10.1061/(ASCE)HY.1943-7900.0001540
Bjerklie, D.M., Dingman, S.L., Bolster, C.H., 2005. Comparison of constitutive flow resistance equations based on the Manning and Chezy equations applied to natural rivers. Water Resour. Res., 41, 11, W11502. DOI: 10.1029/2004WR003776
Bray, D.I., 1982. Flow resistance in gravel-bed rivers. In: Hey, R.D., Bathurst, J.C., Thorne, C.R. (Eds.): Gravel-Bed Rivers. John Wiley and Sons, Chichester, UK, pp. 109–137.
Bray, D.I., 1979. Estimating average velocity in gravel-bed rivers. J. Hydraul. Div. ASCE, 105, 9, 1103–1122.
Brownlie, W.R., 1983. Flow depth in sand-bed channels. Journal of Hydraulic Engineering, 109, 7, 959–991.
Bruschin, J., 1985. Discussion of "Brownlie (1983): Flow depth in sand-bed channels". Journal of Hydraulic Engineering, 111, 736–739.
Coon, W.F., 1998. Estimates of Roughness Coefficients for Natural Stream Channels with Vegetated Banks. Report, US Geological Survey Water-Supply, Paper No 2441.
Dingman, S.L., Sharma, K.P., 1997. Statistical development and validation of discharge equations for natural channels. Journal of Hydrology, 199, 13–35.
Ferguson, R., 2007. Flow resistance equations for gravel- and boulder-bed streams. Water Resour. Res., 43, W05427. https://doi.org/10.1029/2006WR005422
Ferguson, R., 2010. Time to abandon the Manning equation? Earth Surf. Process. Landf., 35, 1873–1876. https://doi.org/10.1002/esp.2091
Fischenich, C., Little, C., 2007. Sediment sampling and analysis for stream restoration projects. U.S. Army Engineer Research and Development Center, Technical Note – Ecosystem Management and Restoration Research Program. http://hdl.handle.net/11681/3972
Gessler, J., 1990. Friction factor of armoured river beds. Journal of Hydraulic Engineering, 116, 531–543.
Ghani, A.A.B, Zakaria, N.R., Kiat, Ch.Ch., Ariffin, J., Hasan, Z.A., Ghaffar, A.B.A., 2007. Revised equations for Manning's coefficient for sand-bed rivers. International Journal of River Basin Management, 5, 4, 329–346. DOI: 10.1080/15715124.2007.9635331
Griffiths, G.A., 1981. Flow resistance in coarse gravel bed rivers. J. Hydraul. Div. ASCE, 107, 7, 899–918.
Hicks, D.M., Mason, P.D., 1991. Roughness Characteristics of New Zealand Rivers. DSIR Water Resources Survey, Wellington, 329 p.
Jarrett, R.D., 1984. Hydraulics of high-gradient rivers. J. Hydraul. Eng., 110, 11, 1519–1539.
Keulegan, G.H., 1938. Laws of turbulent flow in open channels. Journal of the National Bureau of Standards, Research Paper 1151, 21, 707–741.
Kowalczyk, A., Szlachta, A., Hanus, R., 2012. Standard uncertainty determination of the mean for correlated data using conditional averaging. Metrology and Measurement Systems, Rzeszów, Poland, 787–796.
Lacey, G., 1946. A general theory of flow in alluvium. Journal of the Institution of Civil Engineers, Paper No 5518, 27, 1, 16–47.
Ladson, A., Anderson, B., Rutherfurd, I., van de Meene, S., 2002. An Australian handbook of stream roughness coefficients: How hydrographers can help. In: Proceeding of 11th Australian Hydrographic conference, Sydney, 3–6 July, 2002.
Lane, S.N., 2014. Acting, predicting and intervening in a socio-hydrological world. Hydrol. Earth Syst. Sci., 18, 927–952.
Lang, S., Ladson, T., Anderson, B., 2004. A review of empirical equations for estimating stream roughness and their application to four streams in Victoria. Australasian Journal of Water Resources, 8, 1, 69–82, DOI: 10.1080/13241583.2004.11465245
Lee, K., Firoozfar, A.R., Muste, M., 2017. Technical Note: Monitoring of unsteady open channel flows using the continuous slope-area method. Hydrol. Earth Syst. Sci., 21, 1863–1874. https://doi.org/10.5194/hess-21-1863-2017
Limerinos, J.T., 1970. Determination of the Manning coefficient from measured bed roughness in natural channels. U.S. Geological Survey Journal of Research, 4, 3, 285–291.
López, R., Barragán, J., Colomer, M.A., 2007. Flow resistance equations without explicit estimation of the resistance coefficient for coarse-grained rivers. J. Hydrol., 338, 1–2, 113–121. http://dx.doi.org/10.1016/j.jhydrol.2007.02.027.
Machiels, O., Erpicum, S., Archambeau, P., Dewals, B., Pirotton, M., 2011. Theoretical and numerical analysis of the influence of the bottom friction formulation in free surface flow modelling. Water SA, 37, 2, 221–228.
Machiels, O., Erpicum, S., Archambeau, P., Dewals, B. and Pirotton, M., 2009. Bottom friction formulations for free surface flow modeling. In: 8th NCTAM Congress, Belgium. http://hdl.handle.net/2268/28208
Meyer-Peter, E., Muller, R., 1948. Formulas for Bed-load Transport. In: Proc. Third Meeting of IAHR, Stockholm, Sweden, pp. 39–64.
Michalec, B., Zwolenik, M., 2020. Preliminary verification of empirical formulas to define the roughness coefficient. Acta Sci. Pol. Formatio Circumiectus, 19, 4, 21–32. https://doi.org/10.15576/ASP.FC/2020.19.4.21
Nitsche, M., Rickenmann, D., Kirchner, J.W., Turowski, J.M., Badoux, A., 2012. Macroroughness and variations in reach-averaged flow resistance in steep mountain streams. Water Resour. Res., 48, W12518. DOI: 10.1029/2012WR012091.
Nguyen, H.T., Fenton, J.D., 2004. Using two-point velocity measurements to estimate roughness in streams. In: Rutherfurd, I.D., Wiszniewski, I., Askey-Doran, M., Glazik, R. (Eds.): Proc. 4th Austral. Stream Management Conf., Launceston, Tasmania, 19-22 Oct 2004. Dept. of Primary Industries, Water and Environment, Hobart, pp. 445–450.
Phillips, J.V., Ingersoll, T.L., 1997. Comparison of verified roughness coefficients for gravel-bed streams in central Arizona with other areas of the western United States. In: Floodplain Management in a Multi-Faceted World. In: Proceedings of the 21st Annual Conference of the Association of State Floodplain Managers; Little Rock, Arkansas, April 27-May 2 1997. Natural Hazards Research Centre, University of Colorado, pp. 158–158.
Raudkivi, A.J., 1976. Loose Boundary Hydraulics. Pergamon Press.
Rice, C.E., Kadavy, K.C., Robinson, K.M., 1998. Roughness of loose rock riprap on steep slopes. J. Hydraul. Eng., 124, 2, 179–185.
Rickenmann, D., 1994. An alternative equation for the mean velocity in gravel-bed rivers and mountain torrents. In: Cotroneo, G.V., Rumer, R.R. (Eds.): Proceedings of the Conference Hydraulic Engineering '94, vol. 1. ASCE, New York.
Riggs, H C., 1976. A simplified slope-area method for estimating flood discharges in natural channels. U.S. Geological Survey Journal of Research, 4, 3, 285–291.
Sauer, V.B., 1990. US Geological Survey, Written communication to W.F. Coon. Estimation of roughness coefficients for natural stream channels with vegetated banks, 1998. US Geological.
Sefick, S.A., Kalin, L., Kosnicki, E., Schneid, B.P., Jarrell, M.S., Anderson, C.J., Paller, M.H., and Feminella, J.W., 2015. Empirical estimation of stream discharge using channel geometry

in low-gradient, sand-bed streams of the Southeastern Plains. Journal of the American Water Resources Association (JAWRA), 1–12. DOI: 10.1111/jawr.12278

Strupczewski, W.G., Szymkiewicz, R., 1996. Analysis of paradoxes arising from the Chezy formula with constant roughness: I. Depth-discharge curve. Hydrological Sciences Journal, 41, 5, 659–673.

Toufik, H., Mahmoud, H., 2021. Friction coefficient equation in a gravel bed under bedload regime using the dimensional analysis. KSCE Journal of Civil Engineering, 25, 4252–4260 https://doi.org/10.1007/s12205-021-1576-6

Yen, B.C., 2002. Open channel flow resistance. Journal of Hydraulic Engineering, 128, 1, 20–39. http://dx.doi.org/10.1061/(ASCE)0733-9429(2002)128:1(20)

Yochum, S.E., Bledsoe, B.P., David, G.C.L., Wohl, E., 2012. Velocity prediction in high-gradient channels. Journal of Hydrology, 424–425, 84–98.

Yochum, S.E., Comiti, F., Wohl, E., David, G.C.L., Mao, L., 2014. Photographic Guidance for Selecting Flow Resistance Coefficients in High-Gradient Channels. Gen. Tech. Rep. RMRS-GTR-323. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Research Station. 91 p.

Zhu, X., Liu, B., Liu, Y., 2020. New method for estimating roughness coefficient for debris flows. Water, 12, 9, 2341. https://doi.org/10.3390/w12092341

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