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UDK: 336.77:061.1EU
DOI: 10.2478/jcbtp-2021-0030

Journal of Central Banking Theory and Practice, 2021, 3, pp. 169-192
Received: 01 March 2020; accepted: 27 May 2020

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Measuring Sovereign Credit Risk of the EU countries[1]

Abstract: European countries have increased significantly their public debt since the Global Financial Crisis. The increasing trend and the high concentration of public debt in portfolios of financial institutions can lead to a financial turmoil we witnessed during the European Sovereign Debt Crisis. Financial stability authorities therefore look for models to measure the sovereign credit risk and develop"what-if"scenarios to assess a potential repercussion of a financial institution rescue or of an economic contraction on sovereign credit risk. The presented article introduces adjustments to the sovereign contingent claims analysis that is based on the Merton´s Credit Risk Model and the Black-Scholes option pricing techniques. The article proposes adjustments by introducing a new view on a stylised liability side of a central government balance sheet, seniority of its items, and a new proxy for risk measure of junior claims. We show reliable results using derived risk sensitivities for 20 EU countries with decent forward looking ability and propose potential stress-testing framework with an application for the Czech Republic.

Keywords: sovereign credit risk, contingent claims analysis, stress-testing.

JEL Classification: G32, H63, G01.

[1] Vojtěch Siuda acknowledges the support by the Czech Science Foundation Grant 18-05244S Innovation Approaches to Credit Risk Management and support from IGA F1/46/2019. Milan Szabo acknowledges the support by IGA F1/18/2017.
All errors and omissions are ours. The views expressed in this paper are those of the authors and may differ from those of the Czech National Bank.

1. Introduction

The government bond market is the largest in terms of issue volume, and also the most liquid one. There are undoubtedly good economic and practical reasons for this. The state has high credibility as an issuer thanks to its power to collect taxes. The intertemporal aspect of sovereign debt service helps maintain the value of government bonds. For these reasons, sovereign credit risk is also often regarded as the lowest risk in the economy (the floor). Therefore, sovereign securities have been widely represented in portfolios of banks and other financial institutions. The specific position and concentration of these securities in banks' balance sheets are also connected with central bank collateral policy and, in particular, with the preferential treatment given to government bonds in banking regulations on credit and liquidity risk (Bank for International Settlements, 2013).

However, too high concentration of sovereign securities in the balance sheets forces national prudential regulators to carry out a more detailed analysis of sovereign exposures and, due to maintaining financial stability, appropriate stress test of sovereign risk as a source of systemic risk. The systemic risk is based on both asset quality deterioration and the downward liquidity spiral that might arise if a close relationship between the financial and sovereign sectors was to decouple suddenly and values of sovereign securities rapidly drop. The prudential view regarding the sovereign credit risk is further reinforced owing to the recent experience from the sovereign debt crisis which disrupted the zero sovereign credit risk paradigm.

Authorities and rating agencies around the world are estimating the level of sovereign government risk mostly with a mixture of macroeconomic, financial and market indicators.[2] This traditional approach is usually robust. However, there are some disadvantages associated with it. Firstly, employed econometric models usually do not account for nonlinear relationship, which is very natural to default processes. Secondly, they often possess a high dependence on historical data meaning they can only recognize documented sovereign distresses. Additionally, the distress periods take often binary values for modelling purposes. This introduces possible drawbacks hidden in the definition and thresholds used to identify distress periods, (Gramlich, Miller, Oet, & Ong, 2013). Thirdly, the usual models require a wide list of inputs. The data demanding nature makes the usual models

[2] For example see methodology of (Asanović, 2017) S&P (S&P Global Ratings, 2017) or European Commission (Berti, Salto, & Lequien, 2012). The importance of sovereign debt credit rating is also elaborated in (Tahmoorespour, Ariff, & Alireza, 2019).

cumbersome for forecasting and stress-testing that could otherwise provide additional insights into a sensitivity of credit risk.

Our study provides an alternative approach to the stress-testing of sovereign credit risk as well as forward-looking measure of sovereign credit risk by applying option pricing techniques to simplified government balance sheet and expected fiscal expenditures. We adjusted the model based on Merton's Credit Risk model for sovereign entities that shows reliable values for both distance to distress and risk-neutral credit spread. The advantage of our approach is an inclusion of non-linear relationships in estimating credit risk instead of linearly combining stocks and flows of a various macroeconomic indicators. Furthermore, we pioneered the method with focus on the stability of risk-adjusted economic balance sheet of a sole government and not the joint balance sheet of government and central bank, which opens the possibility of a sovereign default on a domestic currency debt. We believe that this kind of indicator can cover some of the effects which the standard linear indicators cannot, thus offering a complementary framework in the set of current models.

The paper proceeds as follows. After a short literature review, we introduce the sovereign contingent claims analysis of (Gapen, Gray, Xiao, & Lim, 2005), further abbreviated as GGXL. Then we propose our adjustments to their model and show our results for EU countries. We follow with a brief framework for a stress-testing that is applied for the Czech Republic.

2. Overview of sovereign contingent claims analysis

Contingent claims analysis (CCA) is widely used for analysing and measuring the risk in private sector. It was pioneered by (Black & Scholes, 1973) and (Merton, 1974), who have generalized this technique for pricing options and corporate debt. Option pricing methodology has been adapted for many contingent claims, including the so-called Merton's model, which is a common name for CCA application on credit risk.

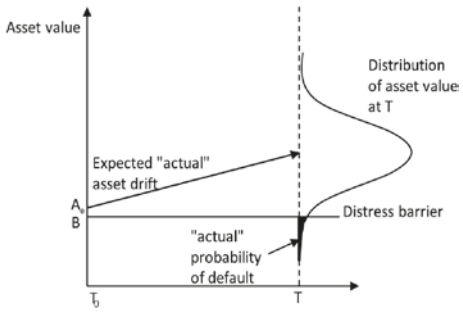
First models connecting sovereign characteristics with contingent claims were introduced in 1980s. Contingent service models introduced by (Grossman & van Huyck, 1988) treated sovereign debt service as a contingent on a realization of income. Sovereign can service its debt fully only in times of realization of sufficiently high income, otherwise it defaults. In their view, default could be excusable when associated with the above described contingencies, or unjustifiable. The current standard of measuring sovereign credit risk with contingent claims were intro-

duced between 2005 and 2008 by GGXL. This approach has been widely applied to measure sovereign credit risk in emerging countries (Keller, Kunzel, & Marcos, 2007), (Brière, Ferrarini, & Ramayandi, 2016), (Cevik, Dibooglu, & Kutan, 2013).

The original Merton model is based on three major principles: a) liabilities have different priority; b) assets follow a stochastic process; and c) value of liabilities is derived from assets. GGXL meet points a) and b) directly, point c) is met indirectly. The value of sovereign assets is driven by a stochastic process with a positive drift (risk free rate) and a diffusion component. Sovereign risk can be then captured by the decline of a market value of sovereign assets to a level of a distress barrier or as an increase in volatility of sovereign assets (see figure 1). The liabilities are divided by an assumed priority to junior and senior claims.

As of c), while the market value of central government's liabilities is quite easy to estimate, the market value of sovereign assets cannot be directly observed. Hence we need to derive the value of assets from liabilities. With knowledge about seniority of sovereign liabilities, we can use the CCA and option price techniques to estimate the market value and volatility of sovereign assets.

Figure 1: Sovereign risk in CCA



Source: Adapted from (Gray & Malone, 2008)

For the construction of a balance sheet, GGXL consolidates the government and monetary authorities. Assets of consolidated sovereign balance sheet are represented by international reserves (held by central bank and denominated in foreign currency) and other public sector assets minus implicit guarantees to financial (or other) sector. Liabilities are described as foreign currency debt (senior claims) and domestic currency liabilities: domestic currency debt and base money (junior claims).

According to seniority, GGXL claim that the seniority is defined by behaviour of policymakers during stress periods. Subject to limited capability to acquire foreign currency, the government strives to stay solvent on its foreign currency debt, which makes this kind of debt senior. On the other side, the government has much more options to control debt in a domestic currency. Government may directly order the mandatory rollover or restructuring of the domestic currency debt, lengthen maturities of the debt, start to control capital flows and convert-

ibility restriction, or they can raise the monetary base and let the inflation to decrease the real value of the domestic debt. The risk measure of junior claims is defined as a volatility of foreign exchange rate in their framework and distress barrier is set to a short term plus one half of a long term central government debt.

3. Adjustments in our approach

The presented method of GGXL is, in our opinion, very inappropriate for EU countries with a low level of foreign currency debts and independent central banks. Another source of motivation for the adjustments stemmed from a different view of sovereign credit risk and seniority of liabilities, as it will be discussed further.

3.1. Balance sheet adjustments

Firstly, we do not consolidate the balance sheet of central government and central bank. The risk of government bonds is clearly defined as a risk of government (and its policies) itself. Moreover, for EU countries, the central bank is independent (regarding constitutional and European law) and government has no power to force the central bank to monetize its debt or running inflation to discount its debt.

Secondly, leaving the consolidation we need to bring a different stylized balance sheet. We still include foreign and domestic debt in liabilities per se. Moreover, government also has liabilities which arise from its declared fiscal expenditures. Fiscal expenditures are a different kind of liabilities.[3] However, they are representing the cash value of government liabilities and need to be counted when thinking about sovereign risk.

What we cannot pinpoint explicitly are contingent liabilities, which can be perceived as a potential future government debt and includes e.g. guarantees to financial sector. Due to the uncertainty of estimates in contingent liabilities we include their materialisation only in stress-testing exercise where they become one of the key drivers.[4]

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[3] The biggest difference is in stock vs. flow character of debt and fiscal expenditures. On the other hand, the rolling of debt and repetitive character of fiscal expenditures are reducing the true difference between stock and flow characteristics.

[4] Contingent liabilities are strongly non-linear which corresponds to default processes in the financial sector. Estimated value of these liabilities is not included in the balance sheet; however, their materialisation is included in the fiscal risk measure (page 5) from a historical point of view.

Turning to the asset side of the stylized balance sheet, assets, in our view, are defined indirectly. Generally, asset is a resource that provides us with some kind of benefit in the future. We are looking for a market value of whole sovereign asset, which generates the income (mainly tax) of government. With this income (and implied asset value) the government should be able to stay current on its liabilities - both debt and fiscal.[5]

3.2. Seniority of liabilities in our approach

Historically, sovereign states often preferred to cut budget expenditures and made drastic reforms of public finance rather than defaulting on a domestic or foreign debt. Recent examples can be Estonia in 2009, which cut about 9% of GDP, respectively near 30% of total fiscal expenditures (Bank of Estonia, 2010), or Greece, which cut about 30% of fiscal expenditures between years 2010-2016 (Keravitis, 2018). The budget expenditures can be hence considered as a junior in comparison with debt. Therefore, 30% of budget expenditures are chosen to represent junior claims in our approach. This reducible part has also an equity-like character for government; it can be cut (as a kind of imaginary buffer) without immediate strong impact on amount of money collected from taxes.

The senior claims are, on the other hand, constructed from the local and foreign currency debts and from the rest of mandatory expenditures. This is in line with recent researches indicating that the domestic currency debt is similarly risky as foreign currency debt (Amstad, Packer, & Shek, 2018). Furthermore, the argument of inflating away the domestic debt was criticised in (Engel & Park, 2017) or (Summers, 2019). The inclusion of the local currency debt into the senior claims is also highly motivated by the high exposure of banks mainly to local sovereign debt (home bias). This results in the well-known doom-loop effect of which fiscal authorities are aware, (Farhi & Tirole, 2017). Finally, it enables us to reflect on the relation between total debt and financial instability as discussed in (Dumičić, 2019).

[5] One can think about a more precise definition of assets. For example, corporations are running business in a country and paying taxes from profit. The higher tax income is collected by a government, the higher must be the assets (asset is the power to collect taxes). Thus, the calculated market value of assets can be viewed as a set of all factors that determine the sum of collected taxes.

3.3. Risk measure of junior claims in our approach

Having based junior claims on fiscal expenditures, risk measure of junior claims should be driven by a development in fiscal numbers. We construct an artificial budget indicator which monitors the development of fiscal revenues and expenditures and also the structural development of budget expenditures regarding its incumbency. Also, the risk measure considers the development in recent history and cumulating the risk through last 6 quarters. The fiscal risk measure is calculated as follows:

$$\text{fiscal risk measure}_t = \max(0.01, (1 + s_t) * \sum_{i=0}^{5} (ex_{t-i} - re_{t-i})) \quad (1)$$

where *s* represents the change in ratio of mandatory expenditures to all expenditures. *Ex* stands for the growth of fiscal expenditures and *re* is the growth of revenues. Fiscal expenditures and revenues are adjusted for transactions with EU budget. The indicator takes similar values as a volatility of equity markets[6]. In times of imprudent behaviour and large increase in fiscal expenditures in relative to fiscal revenues, the indicator starts to rise quickly. When contingent liabilities materialise[7], fiscal risk measure tends to spike over one and even more depending on extent of a public help. Volatility over 100% may be seen as unrealistic; however, there are many examples of traded companies which stocks' volatility exceeded 100%[8]. Giving a huge bailout to maintain a stability of a financial sector will cause large imbalance in the budget but its impact on fiscal risk measure will be partially limited by an "improvement" in structural characteristics.

To obtain the final risk measure of junior liabilities we have used two options: (1) The junior risk driver is represented solely by the presented fiscal risk measure (eq. 1), and (2) The junior risk driver is an average of fiscal risk measure and sovereign CISS indicator of (Garcia-de-Andoain & Kremer, 2018).[9]

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[6] Volatility is presented by standard deviation of market returns and indicates a variability of observed returns in relation to mean return. Fiscal risk measure has a similar distribution as a yearly stock market volatility with a slightly fatter tail. According to economic interpretation, fiscal risk measure can be viewed as a deviation of a hypothehical budget neutrality – balanced budget.

[7] For example, it can be a bailout to financial sector, or to some significant and large non-financial corporation.

[8] E.g. financial stocks during 2008-2009.

[9] Countries without sovereign CISS indicator, the Country level index of financial stress were used (Slovakia, Slovenia and Bulgaria).

4. Estimation of risk characteristics

Assuming that the unknown value of sovereign assets A and junior claims J follow lognormal diffusion process with a constant risk measure and risk-free rate r, the value of junior claims at time T can be computed using Black and Scholes formulae (Black & Scholes, 1973):

$$J = A \cdot N(d_1) - Be^{-rT} \cdot N(d_2) \tag{2}$$

$$d_1 = \frac{\ln \frac{A}{B} + (r + \frac{\sigma_A^2}{2})\, T}{\sigma_A \cdot \sqrt{T}}$$

$$d_2 = d_1 - \sigma_A \cdot \sqrt{T}$$

Where σ_A is an unknown volatility of sovereign assets, B stands for the distress barrier from figure 1, and N(.) is the cumulative standard normal distribution. We follow GGXL to derive the level of distress barrier as short-term and one half of long-term central government debt.

To solve the unknown level and the volatility of assets while assuming the Black and Scholes assumptions, we can use the following relationship that results from the Ito´s lemma and links the level and the risk of junior claims:

$$J = A \cdot N(d_1) \cdot \frac{\sigma_A}{\sigma_J} \tag{3}$$

with σ_J as a risk measure of junior claims which is approximated by fiscal risk measure from formula (1) or the average of fiscal risk measure and sovereign CISS indicator.

The risk neutral probability of default is then obtained by a cumulative normalized distribution function of $-d_2$:

$$PD = N(-d_2) \tag{4}$$

Another metrics that measures risk sensitivity is risk neutral credit spread. This can be calculated as yield-to-maturity of the risky debt (the central government debt) minus the risk free rate. The exact formula for the risk neutral credit spread using the Black & Scholes is as follows:

$$RNCS = y - r_f = \frac{1}{T} \ln \left(N(d_2) - \frac{A_t}{Be^{-rT}} N(-d_1) \right) \tag{5}$$

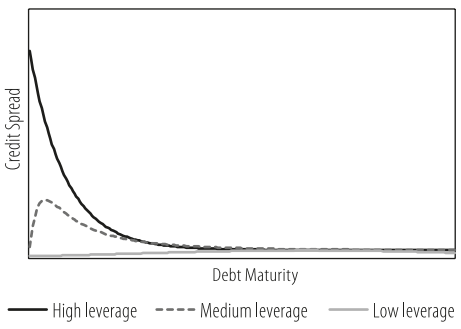
This risk-neutral credit spread is obtained without using any market information about actual CDS values or bond spreads. Only market information employed is the risk-free rate, the rest are derived from the balance sheet relations.

The credit spread is function of asset's volatility and leverage (here represented indirectly as the assets to debt ratio). While the debt does not evolve very quickly in case of the sovereign, the more important risk driver is the volatility of assets. Increase in the assets to debt ratio and decrease in volatility of assets decreases the credit spread. Converse situation leads to increase in the credit spread.

There are three possible shapes of term structure of implied credit spreads under the Merton model (see Figure 2): (1) An entity with low leverage (thus lower debt to assets ratio) has spread for short maturities close or equal to zero, as the assets to cover short term turbulences are sufficient. Spread increase (albeit very slowly) with longer maturities; (2) Medium leverage entity is hump-shaped, with low spreads for short maturities. Spread rises for mid-term maturities before it starts to gradually decrease for longer maturities; (3) High leverage entities (mainly those with urgent fiscal problems and stress) have decreasing term structure of spreads. For short maturities the spread is enormous and represents high probability that in the short term the value of assets jumps under the value of distress barrier. The spread is decreasing with time as assets steadily rise and cover liabilities.

For more detailed description of Merton model and derivation of risk characteristics see Annex 1.

Figure 2: Shapes of implied credit spread



Source: Adapted from (Wang, 2009), modified by authors

5. Data and results

Our dataset is collected from various sources (see Table 1 below). Data has quarterly frequency and covers the period from December 2005 to December 2017 (for CDS from March 2008 to December 2017 only). For the risk free rate we use the interest rate swap curve rate from Refinitiv. All balance sheet data were normalized by a nominal GDP. The risk characteristics were estimated for 20

EU countries[10] with two risk measures of junior claims. Firstly as a purely fiscal based, and secondly as a combination of fiscal and market based driver (sovereign CISS). The risk neutral probability of default and risk neutral credit spread were calculated for 1-, 2-, 3-, 5-, and 10-year maturities.

Table 1: Data

Input	Source	Frequency
Debt data, GDP	Eurostat	quarterly
IRS, CDS	Refinitiv	quarterly
Budget data	The Economist Intelligence Unit	quarterly
Sovereign CISS	ECB Statistical Data Warehouse	quarterly
Budget structure	Eurostat	yearly

Source: Authors

We have to report the frequent issue with Merton model, that mainly during calm periods the implied spread is near to 0. These issues are often mentioned with respect to the contingent claim analysis; see for similar issues in application on corporates (Eom, Helwege, & Huan, 2004), and for a sovereign application (Duyvesteyn & Martens, 2015). However, since the goal is to develop forward-looking indicator/approach and a base for stress-testing framework, we do not mind.

5.1. Results with fiscal risk as a sole driver of junior risk

Decreasing term structures of CDS rates were estimated for three of the countries during the turbulent period of the European debt crisis (Ireland, Greece and Spain), while humble-shaped term structure of CDS rates were estimated for four countries (PT and EU emerging countries). Standard slightly increasing curves were found for the rest of the countries. In reality, the decreasing CDS term structure during the peak of European debt crisis were observed for Ireland, Greece and Portugal, on the other hand, humble-shaped for Spain and Italy. This proves that the model identified almost fully the countries with concerns about their ability to repay debts. The highest probability of default were recorded for Ireland from 4Q 2010 to 4Q 2011, followed by Greece.

[10] The countries included in analysis were: Austria (AT), Belgium (BE), Bulgaria (BG), the Czech Republic (CZ), Denmark (DK), France (FR), Greece (GR), Hungary (HU), Ireland (IE), Italy (IT), Lithuania (LT), the Netherlands (NL), Poland (PL), Portugal (PT), Slovakia (SK), Slovenia (SI), Spain (ES), Sweden (ES) and the United Kingdom (UK).

To use this indicator for macroprudential purposes, some kind of prediction or forward-looking ability is required. Figure 3 reports correlations between the model implied RNCS and CDS market data. Pearson´s correlation of variables, where none is lagged or led, does not report appealing results. The situation changes when we lag the model implied risk neutral credit spread. The best results are obtained when the lag is set for three periods. Generally, higher correlation was observed for developed, more indebted countries, lower for EU emerging countries. The best results were observed for countries with the most serious problems during the European debt crisis, the southern euro area countries[11], and Ireland. Average correlation for those countries was between 0.58 for 1Y maturity and 0.73 for 10Y maturity.

Figure 3: Correlation matrix

	Correlation of model implied RNCS and CDS market data, without lagged or lead variable						Correlation of model implied RNCS and CDS market data, with RNCS lagged by 3 periods				
	1Y tenor	2Y tenor	3Y tenor	5Y tenor	10Y tenor		1Y tenor	2Y tenor	3Y tenor	5Y tenor	10Y tenor
AT	0.068		0.086	0.138	0.253	AT	0.425		0.580	0.672	0.762
BE	0.323	0.340	0.364	0.407	0.427	BE	0.613	0.713	0.721	0.738	0.769
BG	0.060	0.120	0.141	0.151	0.152	BG	0.133	0.161	0.163	0.163	0.138
CZ	0.086	0.156	0.189	0.236	0.307	CZ	0.097	0.197	0.264	0.356	0.519
DK	-0.058	-0.052	-0.019	0.075	0.120	DK	0.450	0.459	0.542	0.616	0.678
ES	0.283	0.259	0.258	0.231	0.139	ES	0.505	0.595	0.649	0.675	0.648
FR	0.122	0.167	0.230	0.344	0.370	FR	0.813	0.812	0.819	0.810	0.808
GR	0.972	0.952	0.927	0.897	0.858	GR	0.943	0.973	0.962	0.944	0.930
HU	0.359	0.292	0.269	0.286	0.314	HU	0.556	0.526	0.500	0.416	0.328
IE	0.895	0.918	0.914	0.895	0.835	IE	0.422	0.491	0.539	0.629	0.717
IT	-0.007	-0.002	0.000	-0.016	-0.119	IT	0.423	0.403	0.414	0.433	0.432
LT	0.121	0.159	0.190	0.230	0.335	LT	0.024	0.075	0.115	0.165	0.228
NL		0.727	0.719	0.463	0.553	NL		0.082	0.179	0.805	0.765
PL	0.084	0.148	0.192	0.235	0.274	PL	0.304	0.382	0.418	0.494	0.594
PT	0.235	0.312	0.406	0.444	0.471	PT	0.849	0.943	0.947	0.935	0.892
SE	-0.065	-0.067	-0.074	-0.033	-0.067	SE	0.095	0.114	0.125	0.153	0.186
SI	-0.201	-0.239	-0.229	-0.208	-0.185	SI	-0.154	-0.068	0.013	0.102	0.170
SK	0.493	0.556	0.580	0.569	0.530	SK	0.667	0.694	0.713	0.753	0.761
UK	0.099	0.169	0.206	0.282	0.228	UK	0.185	0.176	0.181	0.310	0.344
Mean	0.215	0.273	0.281	0.296	0.305	Mean	0.408	0.429	0.466	0.535	0.561

Source: Refinitiv, Authors' calculation.

Note: For blank values no CDS data were found. The reported correlation was measured from 4Q 2007 to 4Q 2017, except for Greece, where the data provided were from 4Q 2005 to 3Q 2010. Correlation was computed using the standard Paerson's method.

[11] Greece, Italy, Spain and Portugal.

5.2. Results with fiscal risk as a combination of fiscal risk and sovereign CISS

The model implied RNCS curve reported the decreasing structure for a period of European debt crisis for the same three countries as in the previous approach. An additional country with a humble-shaped CDS curve identified was Italy. The highest probability of default, ranging between 23% for 1Y PD to 30% for 10Y PD, were recorded again for Ireland from 4Q 2010 to 4Q 2011. Ireland was followed by Greece and Spain in 2Q and 3Q 2010.

Figure 4: Correlation matrix

	Correlation of model implied RNCS and CDS market data, without lagged or lead variable						Correlation of model implied RNCS and CDS market data, with RNCS lagged by 3 periods				
	1Y tenor	2Y tenor	3Y tenor	5Y tenor	10Y tenor		1Y tenor	2Y tenor	3Y tenor	5Y tenor	10Y tenor
AT	0.483		0.416	0.519	0.737	AT	-0.108		0.064	0.243	0.399
BE	0.825	0.824	0.808	0.837	0.855	BE	0.174	0.416	0.507	0.598	0.599
BG	0.051	0.141	0.181	0.225	0.294	BG	0.168	0.221	0.237	0.252	0.251
CZ	0.434	0.589	0.656	0.713	0.709	CZ	-0.015	0.137	0.247	0.367	0.468
DK	0.500	0.530	0.540	0.621	0.737	DK	0.126	0.131	0.110	0.097	0.083
ES	0.488	0.609	0.677	0.715	0.689	ES	0.578	0.725	0.805	0.857	0.825
FR	0.296	0.438	0.540	0.636	0.648	FR	-0.021	0.097	0.168	0.260	0.310
GR	0.958	0.888	0.848	0.804	0.753	GR	0.676	0.805	0.851	0.850	0.822
HU	0.362	0.332	0.370	0.443	0.586	HU	0.274	0.407	0.432	0.381	0.346
IE	0.932	0.967	0.964	0.945	0.887	IE	0.485	0.577	0.614	0.677	0.735
IT	0.662	0.822	0.870	0.886	0.763	IT	0.126	0.298	0.427	0.559	0.565
LT	0.188	0.281	0.356	0.440	0.539	LT	0.023	0.118	0.187	0.251	0.298
NL	0.004	0.232	0.287	0.617	0.664	NL	-0.196	-0.294	-0.366	0.693	0.690
PL	0.111	0.217	0.307	0.385	0.452	PL	0.217	0.277	0.336	0.434	0.521
PT	0.679	0.718	0.768	0.781	0.765	PT	0.895	0.868	0.850	0.842	0.803
SE	0.183	0.355	0.454	0.566	0.676	SE	0.087	0.268	0.366	0.433	0.476
SI	-0.131	-0.149	-0.128	-0.121	-0.148	SI	-0.127	-0.076	0.026	0.125	0.156
SK	0.389	0.461	0.485	0.476	0.440	SK	0.677	0.724	0.740	0.765	0.763
UK	0.095	0.214	0.332	0.498	0.473	UK	0.198	0.347	0.483	0.694	0.683
Mean	0.395	0.471	0.512	0.578	0.606	Mean	0.223	0.336	0.373	0.494	0.515

Source: Refinitiv, Authors' calculation

Note: For blank values no CDS data were found. The reported correlation was measured from 4Q 2007 to 4Q 2017, except from Greece, where the data provided were from 4Q 2005 to 3Q 2010. Correlation was computed using standard Paerson's method.

Figure 4 shows correlation between the model implied RNCS with a combination of junior risk measure and market CDS. In comparison with the previous model, the correlation was much higher when no variable was either lagged or led. The combined risk measure of junior liabilities estimates the CDS spreads more

accurately in real time, which could lead us to a conclusion that this model specification is better. However, when we lag the implied RNCS, the correlation gets significantly lower. This suggests that the forward-looking ability of this indicator is quite limited with the specified risk measure of junior claims. The correlations across different countries were similar to the previous model specification. For numerical comparison of estimated RNCS and real CDS values please see the Figures in Annex 2.

Our results in both specifications report weaker correlation and less robust estimates for post-Soviet European emerging countries than for European developed countries. There are two arguments for this result: (a) the higher implicit risk of these countries; and (b) generally lower part of fiscal expenditures on GDP. Higher implicit risk can be addressed to standard risk perception of market participants between developed and emerging economies. It could be explained by lower institutional quality and legal system, greater corruption and lower general wealth of the country. Lower ratio of fiscal expenditures to GDP is decreasing their intended buffer characteristics in our model.

6. Stress-testing framework

Stress testing can provide an additional insight into the credit risk assessment. Employing an adverse scenario of economic and financial development negatively affects the financial sector soundness and its stability. Stress-testing of sovereign entities should have an implication for macroprudential policy makers. According to the high concentration in financial sector balance sheets and the non-zero risk, the government bonds are without a doubt source of systemic risk (Dumičić, 2019) and also source of possible contagion effect in the form of downward liquidity spiral, (Alogoskoufis & Langfield , 2018).

Estimating sovereign risk with the contingent claims analysis opens up the possibility for utilization of stress-testing for sovereign entities and quality of government bonds in balance sheets of financial sector. According to the results of stress-test, the microprudential or macroprudential policy may actively react on the risks arising from sovereign risk. Basically, a regulator has two options: (a) setting a non-zero capital requirement for exposures to sovereign balance sheets, which enhances the resilience of a financial sector against unexpected losses;[12] or (b) setting a limit on sovereign exposures of an affected country, which should

[12] According to our approach, the level of capital requirement for sovereign exposures will be intuitively defined as: CR (%) = PD (%) *LGD (%). LGD will be subject to estimation.

reduce concentration risk and narrow the channel for contagion. However, policy makers should also count with undesirable effects of their actions. Action (a) can raise the pressure on financial sector, and deepen credit contraction and economic recession. Policy tool (b) may start or accelerate fire sales of government bonds that can negatively affect market liquidity and cause additional losses by repricing bonds balance sheets.

Under our approach, the sovereign exposures could be stress-tested in 2 different ways. (1) Defining a severe development of economy with asset prices turmoil, a financial and an economic crisis, etc. with future fiscal revenues, expenditures and debt projected in line with country's macroeconomic development. In this kind of stress test exercise, the materialisation of contingent liabilities in a form of bail-out for part of the financial sector can appear. (2) Using a Monte Carlo simulation method generating numerous paths of future development of individual variables and calculating the maximal values of risk characteristics on defined confidence level over a time horizon (Value at Risk method). However, the disadvantage connected with this approach is that the whole story about economical background of the stress remains hidden to analysts.

6.1. Stress scenario: Case of the Czech Republic

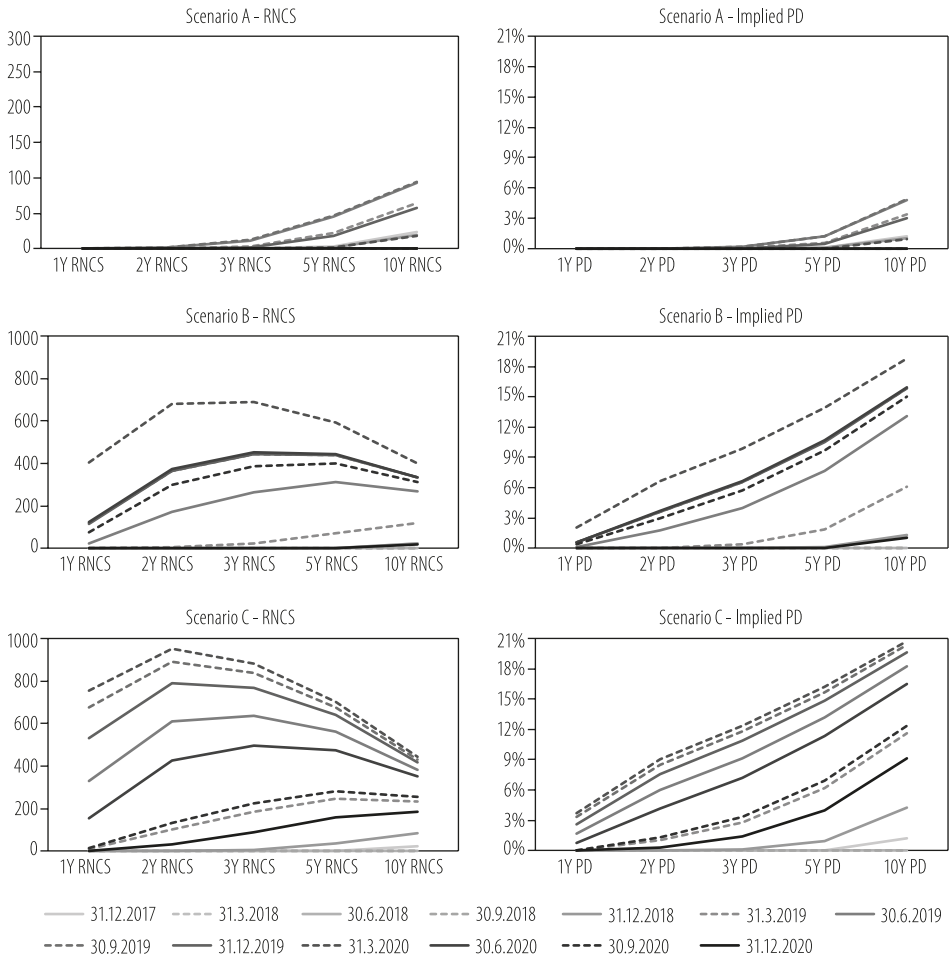
We prepared an example of how an adverse scenario and contingent liabilities materialisation can be implemented into sovereign contingent claims following our adjustments. Let us assume three imaginary scenarios: scenario (A) counts with mild recession[13] but there is no need for government aid to financial sector; scenario (B) considers mild recession with a bailout of 7% GDP; and (C) projects mild recession with a bailout of 14% GDP.[14] In our example, the government makes countercyclical policy and supporting economy through higher fiscal activity. In line with this, the dynamics of revenues and expenditures causes higher fiscal deficits and leads to a rise in distress barrier and a partial rise in a risk measure of junior claims. Alongside, in scenarios (B) and (C), the realisation of systemic risk has forced government to save the financial institutions, which will strongly increase the pressure on distress barrier and junior claims risk measure.

[13] This is very similar to the last observed recession in Czech economy in 2012–2013.

[14] These values were chosen arbitrarily and should represent values which are slightly below and above the average amount of bailout. The World Bank had estimated the average fiscal costs of bailout to 12.8% of GDP (Honohan & Klingebiel, 2000). (Amaglobeli, End, Jarmuszek, & Palomba, 2015) have identified, that the median direct fiscal costs of banking crises was 6% of GDP and the median increase in public debt was 14% of GDP. Comprehensive review of fiscal aid to financial sector during financial crisis in Europe can be found in (Grossman & Woll, 2014).

Figure 5 shows estimated values of risk characteristics for the Czech Republic. The situation is quite calm in scenario (A), where there are no fiscal costs of bail-out. Implied PD and RNCS increases somewhat in 2019 as a result of expansive fiscal policy, but quickly falls to zero values in 2020. Also the term structure of RNCS is standard increasing for the whole period. This suggests that the sovereign entity's balance sheet is strong and healthy enough to carry out a counter-cyclical policy.

Figure 5: Estimated risk characteristics in stress test scenarios



Source: Authors' calculation

Tensions for sovereign entity's bondholders arise, when contingent liabilities materialise. In scenario (B), the projected government capital injection was totally 350 bln CZK. As a result, there is a dramatic jump up in risk characteristics, which is driven by a rise in the distress barrier and mainly rise in the fiscal risk measure. The peak of sovereign distress is also followed by a switch in the shape of the implied RNCS term structure. Humble shaped structure is observed from 2Q 2019 to 3Q 2020, which indicates real concerns and higher uncertainty of debt sustainability mainly in a mid-term horizon. In scenario (C), we observe similar development as in (B); however, the shock is stronger and has longer duration.

7. Conclusion

This paper introduces adjusted methodology for estimating sovereign risk and its utilization on the case of EU sovereign countries. This approach applies contingent claims analysis and option pricing techniques developed by (Black & Scholes, 1973), (Merton, 1974), and initially modified for sovereign entities by (Gapen, Gray, Xiao, & Lim, 2005). We contribute to the discussion by introducing a significantly adjusted approach to contingent claims analysis for sovereign countries. We have widened the risk sources and their basic understanding. The analysis shows promising results. Especially for countries that experienced real concerns about their capability to stay current on their debts during the European debt crisis.

The adjustments we make are based on different understanding of sovereign risk driver and balance sheet structure. The main changes are in (a) balance sheet items, (b) seniority of liabilities, and (c) risk measure of junior liabilities. For the balance sheet we omit consolidation of central government and central bank and include fiscal expenditures. In line with recent researches, we do not differ in denomination of central government debt as they are similarly risky. Thus whole debt with irreducible part of fiscal expenditures is defined as senior liabilities. In the authors` opinion, making both domestic and foreign currency debt as senior to fiscal expenditures better captures relationship between these factors of sovereign risk. In line with the different structure of junior liabilities we also introduce the different risk measure of junior liabilities, which is derived from dynamics of fiscal revenues and expenditures and the dynamics of the structure in fiscal expenditures. We also employ the alternative risk measure compounded not only from the fiscal indicator, but also from the sovereign composite indicator of systemic stress (sov. CISS).

The main results of our model are estimates of risk characteristics such as probability of default and risk neutral credit spread for different maturities. The correlation with observed CDS spreads shows a decent forward-looking ability. Our model indicated spikes during the European debt crisis and a decreasing or humble shaped term structure of credit spreads. Denmark was the only country where the spike in CDS during 2011–2012 was almost not recognised by any of our two indicators (see Figure 5).

In the authors` view, an increase in the probability of risk materialisation shown separately by both our indicator and standard sovereign risk indicators (which are based on macroeconomic and financial variables) can be seen as a supportive signal that calls for the tightening of macroprudential policy in a form of additional capital requirements or lowering a concentration of sovereign exposures.

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Annex 1: The Merton model

Suppose at time t that company's asset A_t are financed by equity J_t and the value of debt $Be^{-r(T-t)}$, where D_t stands for market value of debt with face value B, r for risk free rate and T is time to maturity. Together, these create a balance sheet with identity:

$$A_t = J_t + D_t \tag{A1_1}$$

At maturity time T, when $A_T > D_T$, then the debtholder obtains whole B and shareholder still gets $A_T - B_T$, otherwise the entity defaults on its debt and debtholder gets A_T and shareholder nothing. This makes equity value in time T equal to:

$$J_T = \max(A_T - B_T, 0) \tag{A1_2}$$

which is a payoff of the European call option. Assuming that assets follow the geometric Brownian motion with risk neutral dynamics, we can rewrite it in stochastic differential equation:

$$dA_t = rA_t dt + \sigma_A A_t dW_t \tag{A1_3}$$

Where W_t is a Brownian motion realization under risk-neutral measure. Applying the Black-Scholes formula for equity value gives us:

$$J_t = A_t N(d_1) - Be^{-r(T-t)} N(d_2) \tag{A1_4}$$

Where $N(.)$ is the cumulative standard normal distribution and d_1 and d_2 are quantities given by:

$$d_1 = \frac{\ln \frac{A_t}{B} + (r + \frac{\sigma_A^2}{2})\,(T-t)}{\sigma_A \cdot \sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_A \cdot \sqrt{T-t}$$

The probability that the option will expire out of money equals the probability that at time T the $A_T < B$, which is the trigger for default:

$$PD = N(-d_2) \tag{A1_5}$$

Standard way of observing market value of assets A_t and volatility of assets σ_A is connected with auxiliary equation: Starting with geometric brownian motion for equity J_t:

$$dJ_t = \mu J_t dt + J_t \sigma_J dW_t \tag{A1_6}$$

After applying Ito's lemma and option delta (derivation of the value of equity with respect to value of the assets) we get:

$$\begin{aligned} dJ_t &= \left(\frac{\partial J}{\partial t}\mu_t A_t \frac{\partial J}{\partial A} + \frac{1}{2}\sigma_A^2 A_t^2 \frac{\partial J}{\partial A^2}\right) dt + \sigma_A A_t \frac{\partial J}{\partial A} = \\ &= \left(\frac{\partial J}{\partial t}\mu_t A_t \frac{\partial J}{\partial A} + \frac{1}{2}\sigma_A^2 A_t^2 \frac{\partial J}{\partial A^2}\right) dt + \sigma_A A_t N(d_1) \end{aligned} \tag{A1_7}$$

After few steps we obtain the equation A1_8:

$$J_t \sigma_E = A_t \sigma_A N(d_1) \tag{A1_8}$$

A1_4 and A1_6 can be solved simultaneously for asset value and volatility of assets. With knowledge of both of these we can derive a model implied risk-neutral credit spread. The debtholder can hedge its risky position by buying a put option on assets with strike B. His payoff will be $B - A_T$ when $A_T < B$ at time T, otherwise 0. Combining debt and put option guarantee a risk free payoff of B at time T:

$$P_t + D_t = Be^{-r(T-t)} \tag{A1_9}$$

Applying Black-Scholes formula for European put option gives us:

$$P_t = Be^{-r(T-t)}N(d_2) - A_t N(-d_1) \tag{A1_10}$$

The debt is risky and should carry a risk premium. Denoting the credit spread with s, the value of risky debt follows:

$$D_t = Be^{-(r+s)(T-t)} \tag{A1_11}$$

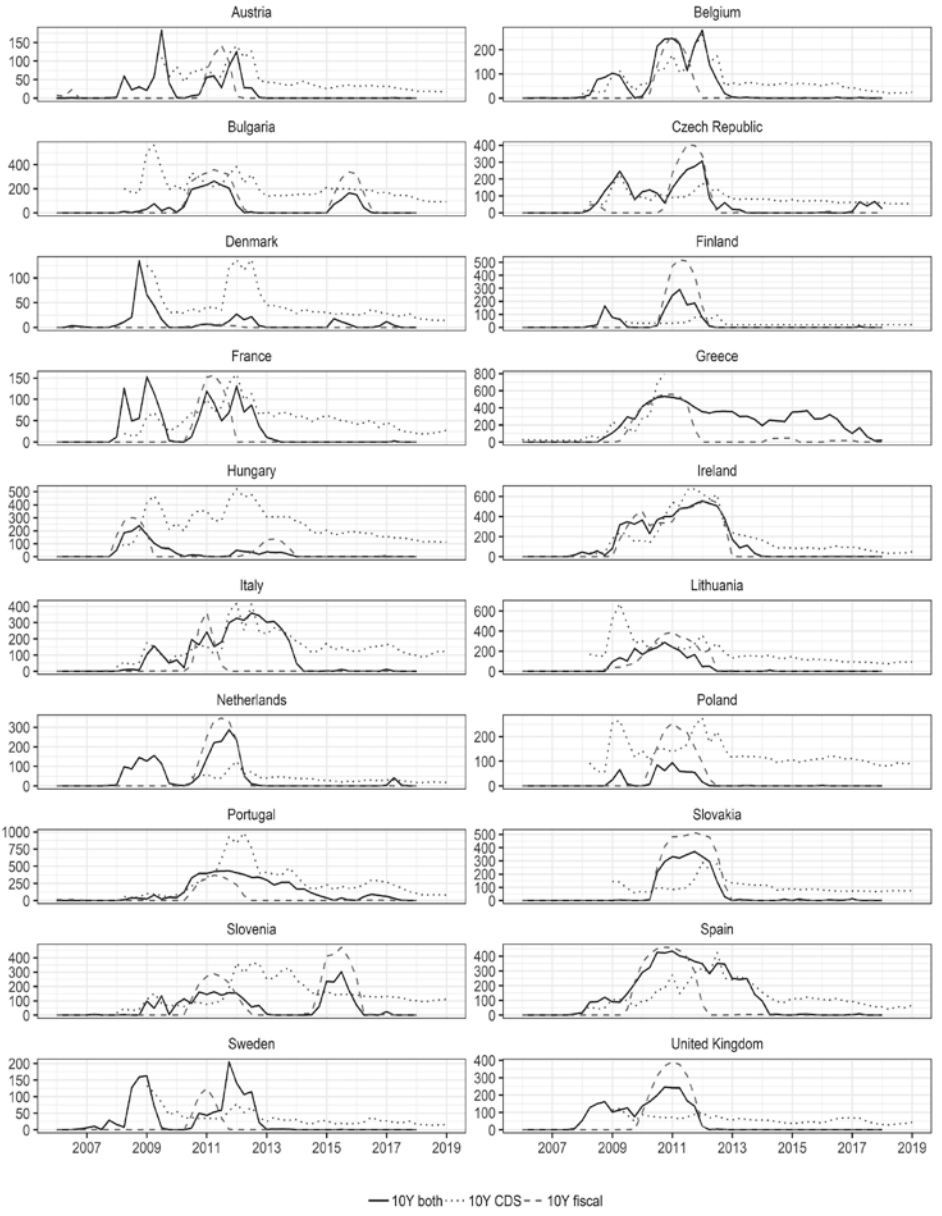
Using A1_9, A1_10 and A1_11 we can analytically formulate the risk neutral credit spread as:

$$RNCS = y - r_f = \frac{1}{T-t}\ln\left(N(d_2) - \frac{A_t}{B}e^{r(T-t)}N(-d_1)\right) \tag{A1_12}$$

For t equals to zero we obtain instantaneous risk characteristics.

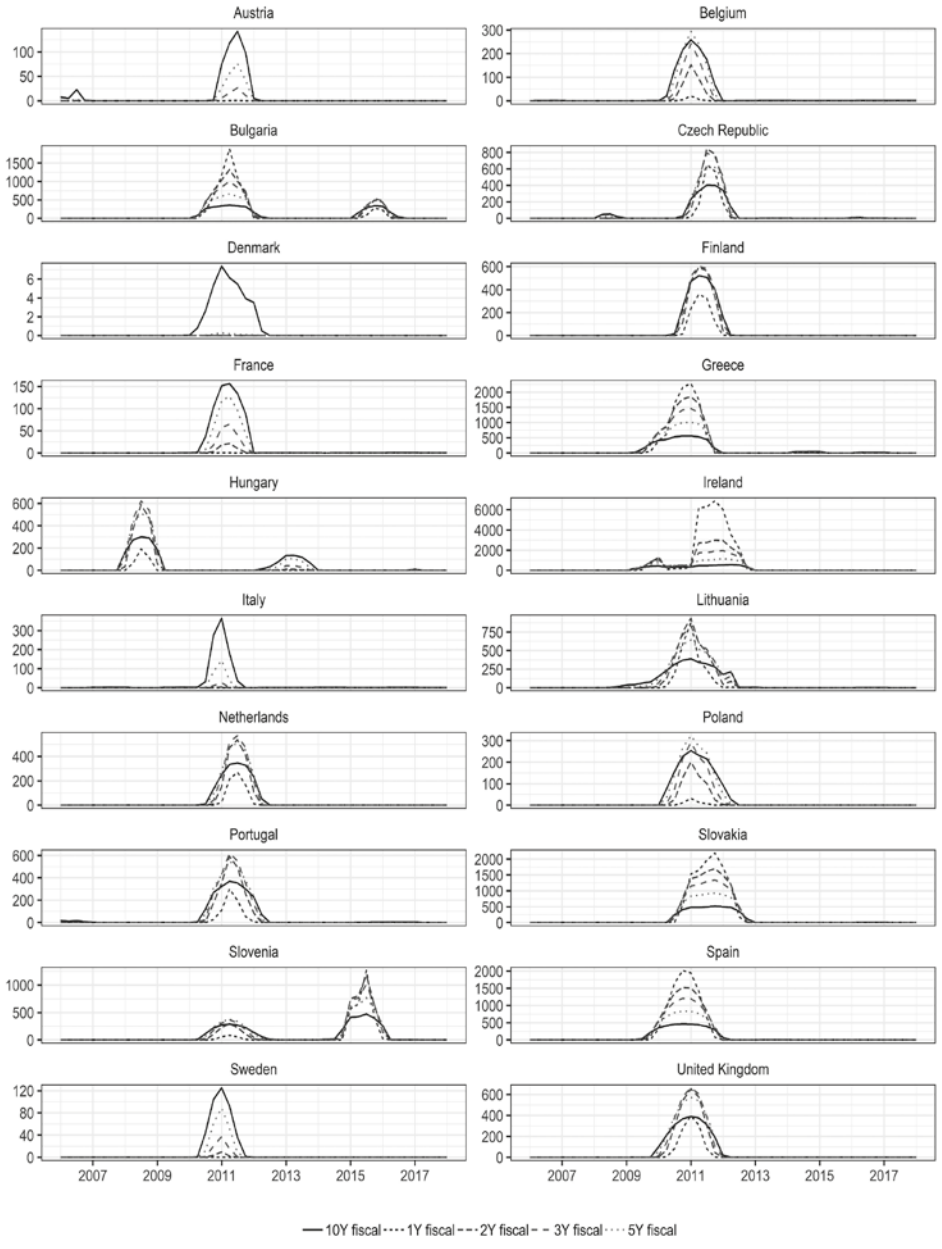
Annex 2: Implied Risk Characteristics

Figure A1: Implied risk neutral credit spreads and observed CDSs



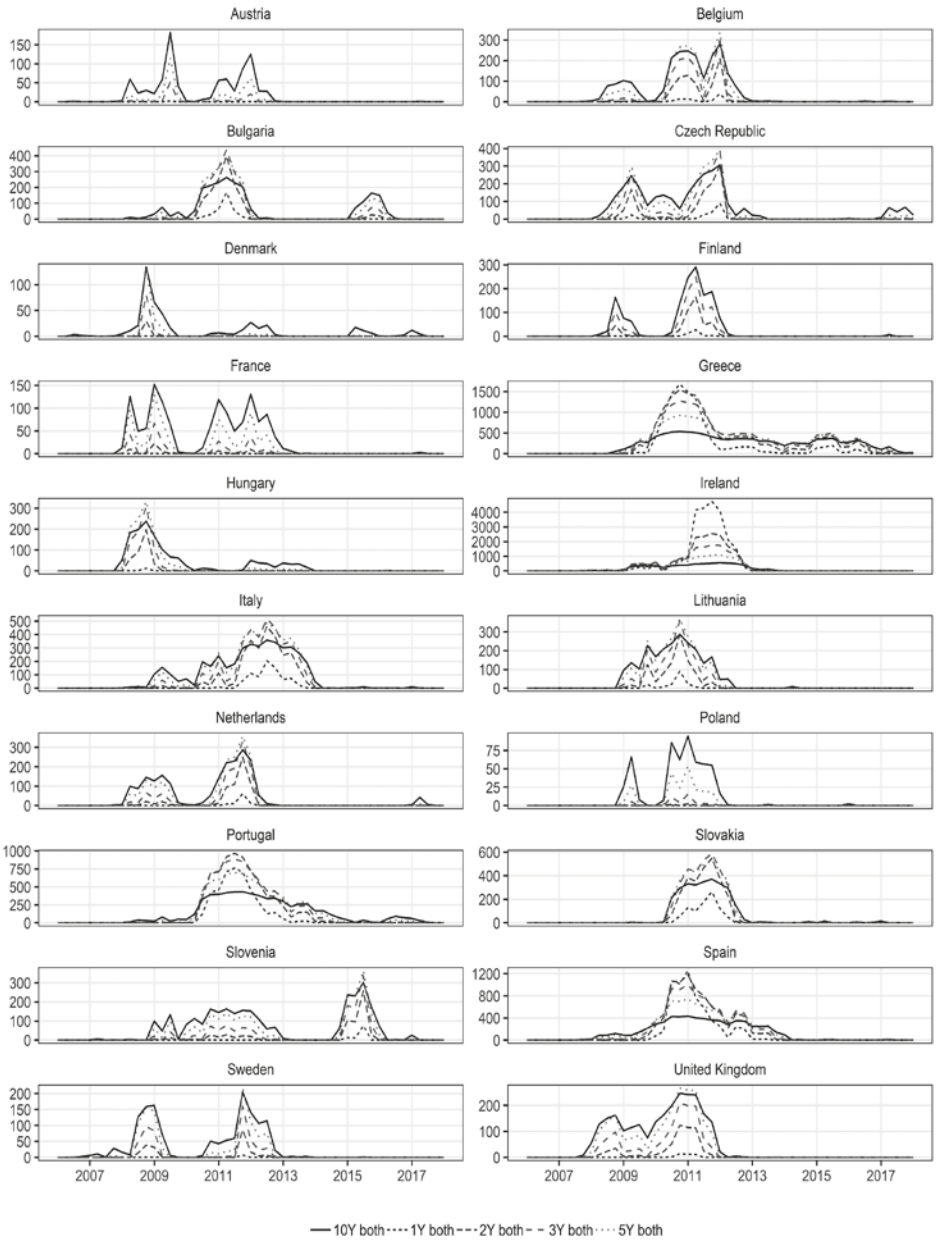
Source: Refinitiv, Authors` calculation

Figure A2: Term structure of estimated RNCS with fiscal risk measure



Source: Authors' calculation

Figure A3: Term structure of implied risk-neutral credit spreads with both risk measures



Source: Authors' calculation