A new generalization of Samade distribution with properties and its application to lung cancer data[1]

P. PANDIYAN AND R. JOTHIKA

Abstract

In this study, a new generalization of Samade distribution has been proposed. The weighted Samade distribution (WSD) is a new distribution designed for modeling real-life data. The survival function, hazard function, moments, moment-generating function, order statistics, and entropies of this distribution have all been determined. The parameters of the suggested model are estimated using the maximum likelihood estimation technique. Finally, the helpfulness and application of the distribution have been demonstrated by lung cancer data.

Mathematics Subject Classification 2000: 60E05, 62E15
Key Words and Phrases: Weighted Samade Distribution; Reliability Analysis; maximum likelihood estimation technique; Order Statistics; Entropies.

1. INTRODUCTION

The concept of weighted distributions provides a group approach to the issues of fact interpretation and model specification. When samples are collected from developed distributions and unique distributions, weighted distributions provide a way to become the fashions to the burden features. In order to simulate the ascertainment bias, Fisher (1934) created a novel concept of distributions called the weighted distribution. Rao (1965) subsequently developed this approach in a unified way while modeling the statistical data, since the standard distributions were not sufficient to appropriately record these observations with equal probabilities. Thus, weighted models were created under these conditions to represent the observations using a weighted function. The concept of length biased sampling was first introduced by Zelen (1974) and Cox (1969). Saghir and associates (2017) covered the weighted distributions characterizations and the short angle.

[1] This research was supported by the Funding Agency

10.2478/jamsi-2024-0002
©P. Pandiyan, R. Jothika
This is an open access article licensed under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0).

sciendo

The two weighted distributions produced by the exponential distribution were added by Mahdavi (2015). The weighted exponential distribution and its homes with the utility were proposed by Dey and Perk (2015). The Size-Biased Distribution Theory was separately established by Van Deusen (1986) and applied to the distributions of breast height diameter (DBH). Lappi and Bailey (1987) then used weighted distributions to examine the HPS diameter increment data. Taillie et al. (1995) employed weights to model fish population stocks in fisheries. The sampling mechanism typically chooses the units based on a probability that is proportionate to a unit size measure. This distribution is referred to as size skewed. Weighted distributions are explained in detail in a number of reliable sources. Several writers have studied and researched the numerous weighted probability models, and they have included illustrations of their use in different fields.

Many studies in the fields of biology, branching processes, and reliability use weighted distributions. Compact paperwork contains the statistical properties of the version, including joint pdf, joint cdf, joint survival feature, product moments, and marginal conditional density. With the help of Zubair et al., the weighted exponential families of distributions are presented (2019). The weighted Nwike distribution is a two-parameter continuous distribution that Maryam et al. (2022) suggested. The Shambhu distribution is a one-parameter continuous distribution that Shanker et al. (2021) suggested. Aderoju (2021) has introduced the Samade distribution, a two-parameter lifespan distribution.

The weighted method known as weighted Samade distribution has been used in this study to examine a new extension of the Samade distribution. In comparison to the other distributions, we give a new distribution that has better features and applications for modelling real-life data sets.

2. WEIGHTED SAMADE DISTRIBUTION (WSD)

The Samade distribution's probability density function (pdf) is provided by

$$f(x;\alpha,\theta) = \frac{\theta^4}{\theta^4+6\alpha}(\theta + \alpha x^3)e^{-\theta x} \; ; \qquad x,\theta,\alpha > 0 \tag{1}$$

The Samade distribution's cumulative distribution function (cdf) is provided by

$$F(x;\alpha,\theta) = 1 - \left(\frac{\theta^4+\alpha\left(6+x\theta\left(6+x\theta(3+x\theta)\right)\right)}{6\alpha+\theta^4}\right)e^{-\theta x};\ x,\theta,\alpha > 0 \qquad (2)$$

Let X be a non-negative random variable that has a probability density function $f(x)$. Assume that $w(x)$ is the non-negative weight function. and then the probability density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))};x > 0$$

Where, $w(x)$ is a non-negative weight function and

$$E(w(x)) = \int w(x)f(x) < \infty$$

$$f_w(x) = \frac{x^c f(x)}{E(x^c)} \qquad (3)$$

Where,

$$E(x^c) = \int_0^\infty x^c f(x)dx$$

For different weighted models, we can have different choices of the weight function $w(x)$. In this research paper, we introduce weighted Model of Samade distribution, in weights we use $w(x) = x^c$ in order to get the weighted Samade distribution and the probability density function (pdf) of weighted version is given as

$$f_w(x) = \frac{w(x)f(x)}{E\left(w(x)\right)} \ ; \qquad x > 0$$

$$f_w(x) = \frac{x^c f(x)}{E(x^c)} \ ; \qquad c > 0 \qquad (4)$$

Where,

$$E(x^c) = \int_0^\infty x^c f\,(x;\alpha,\theta)dx$$

$$E(x^c) = \int_0^\infty x^c \frac{\theta^4}{\theta^4+6\alpha}(\theta + \alpha x^3)e^{-\theta x}dx$$

$$E(x^c) = \frac{\theta^4}{\theta^4+6\alpha}\int_0^\infty x^c\,(\theta + \alpha x^3)e^{-\theta x}dx$$

$$E(x^c) = \frac{\theta^4}{\theta^4+6\alpha}\int_0^\infty x^c\,\theta e^{-\theta x}dx + \alpha\int_0^\infty x^{c+3}\,e^{-\theta x}dx$$

$$E(x^c) = \frac{\theta^4}{\theta^4+6\alpha}\theta\int_0^\infty x^{(c+1)-1}\,e^{-\theta x}dx + \alpha\int_0^\infty x^{(c+4)-1}\,e^{-\theta x}dx$$

$$E(x^c) = \frac{\theta^4}{\theta^4+6\alpha}\left[\theta\frac{\Gamma c+1}{\theta^{c+1}} + \alpha\frac{\Gamma c+4}{\theta^{c+4}}\right]$$

$$E(x^c) = \frac{\theta^4}{\theta^4+6\alpha}\left[\frac{\theta^4\Gamma c+1+\alpha\Gamma c+4}{\theta^{c+4}}\right]$$

$$E(x^c) = \frac{\theta^4 c!+\alpha(c+3)!}{\theta^c(\theta^4+6\alpha)} \tag{5}$$

The probability density function (pdf) for the weighted Samade Distribution is obtained by substituting equations (1) and (5) into equation (3).

$$f_w(x) = \frac{x^c\frac{\theta^4}{\theta^4+6\alpha}(\theta+\alpha x^3)e^{-\theta x}}{\frac{\theta^4 c!+\alpha(c+3)!}{\theta^c(\theta^4+6\alpha)}}$$

$$f_w(x;\alpha,\theta) = \frac{\theta^{c+4}}{\theta^4 c!+\alpha(c+3)!}\,x^c\,(\theta+\alpha x^3)e^{-\theta x} \tag{6}$$

The weighted Samade distribution's cumulative distribution function (cdf) can be derived as

$$F(x;\alpha,\theta) = \int_0^x f(x;\alpha,\theta)\,dx$$

$$F(x;\alpha,\theta) = \int_0^x \frac{x^c\theta^{c+4}(\theta+\alpha x^3)e^{-\theta x}}{\theta^4 c!+\alpha(c+3)!}dx$$

$$F(x;\alpha,\theta) = \frac{\theta^{c+4}}{\theta^4 c!+\alpha(c+3)!}\int_0^x x^c\,(\theta+\alpha x^3)$$

$$F(x;\alpha,\theta) = \frac{\theta^{c+4}}{\theta^4 c!+\alpha(c+3)!}\theta\int_0^x x^c e^{-\theta x}dx + \alpha\int_0^x x^{c+3}\,e^{-\theta x}dx \tag{7}$$

After simplification, the cumulative distribution function of the weighted Samade distribution was derived.

$$F(x;\alpha,\theta) = \frac{\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!} \tag{8}$$

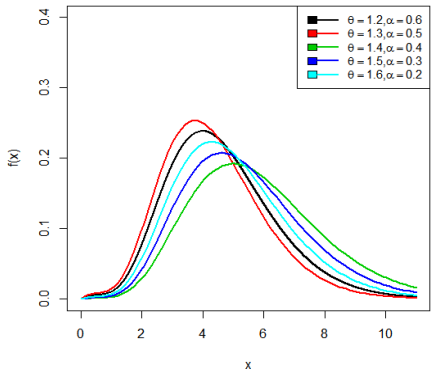


Figure.1: pdf plot of weighted Samade distribution

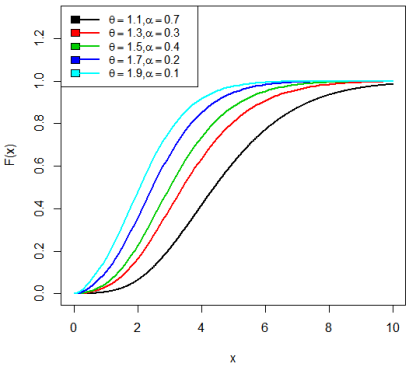


Figure.2 cdf plot of weighted Samade distribution

3. RELIABILITY ANALYSIS

The analysis of the weighted Samade distribution's survival function, also known as the reliability function, hazard function, and reverse hazard function is covered in this section.

3.1 Survival Function

The weighted Samade distribution's survival function or reliability function has the following expression

$$S(x;\alpha,\theta) = 1 - F(x;\alpha,\theta)$$

$$S(x;\alpha,\theta) = 1 - \frac{\theta^{c+4}[\theta^4\gamma(c+1,\theta x) + \alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c! + \alpha(c+3)!} \tag{9}$$

3.2 Hazard Function

The failure rate or hazard function that corresponds to the weighted Samade distribution is written as

$$H(x;\alpha,\theta) = \frac{f(x;\alpha,\theta)}{S(x;\alpha,\theta)}$$

$$H(x;\alpha,\theta) = \frac{\frac{x^c\theta^{c+4}(\theta+\alpha x^3)e^{-\theta x}}{\theta^4 c!+\alpha(c+3)!}}{\frac{\theta^4 c!+\alpha(c+3)!-\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!}}$$

$$H(x;\alpha,\theta) = \frac{x^c\theta^{c+4}(\theta+\alpha x^3)e^{-\theta x}}{\theta^4 c!+\alpha(c+3)!-\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]} \quad (10)$$

3.3 Reverse Hazard Rate

For the weighted Samade distribution, the reverse hazard rate can be calculated as follows.

$$hr(x) = \frac{f(x;\alpha,\theta)}{F(x;\alpha,\theta)}$$

$$hr(x) = \frac{x^c\theta^{c+4}(\theta+\alpha x^3)e^{-\theta x}}{\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]} \quad (11)$$

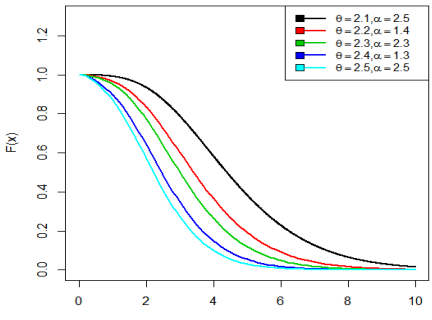


Figure.3 survival function of weighted samade distribution

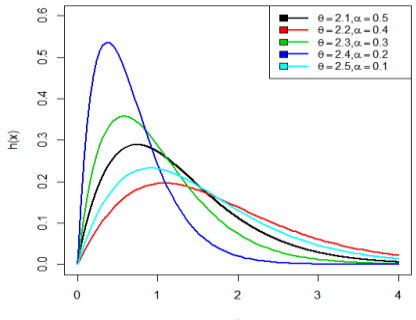


Figure.4 hazard plot of weighted Samade distribution

4. MOMENTS AND ASSOCIATED MEASURES

Assume that X is the random variable that follows a weighted Samade distribution then the r^{th} order moment, that is $E(X^r)$ can be obtained as

$$\mu_r' = E(X^r) = \int_0^\infty x^r f(x;\alpha,\theta)dx$$

$$\mu_r' = \int_0^\infty x^r \left(\frac{x^c\theta^{c+4}(\theta+\alpha x^3)}{\theta^4 c!+\alpha(c+3)!}\right)e^{-\theta x}dx$$

$$\mu_r{}' = \frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!} \int_0^\infty x^r x^c \, (\theta + \alpha x^3) e^{-\theta x} dx$$

$$\mu_r{}' = \frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!} \theta \int_0^\infty x^{r+c} \, e^{-\theta x} dx + \alpha \int_0^\infty x^{r+c+3} \, e^{-\theta x} dx$$

$$\mu_r{}' = \frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!} \theta \int_0^\infty x^{(r+c+1)-1} \, e^{-\theta x} dx + \alpha \int_0^\infty x^{(r+c+4)-1} \, e^{-\theta x} dx$$

$$\mu_r{}' = \frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!} \left[\theta \left(\frac{\Gamma r + c + 1}{\theta^{(r+c+1)}} \right) + \alpha \left(\frac{\Gamma r + c + 4}{\theta^{(r+c+4)}} \right) \right]$$

$$\mu_r{}' = \frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!} \left[\frac{\theta^4 \Gamma r + c + 1 + \alpha \Gamma r + c + 4}{\theta^{(r+c+4)}} \right]$$

$$\mu_r{}' = \frac{\theta^4 \Gamma r + c + 1 + \alpha \Gamma r + c + 4}{\theta^r (\theta^4 c! + \alpha(c+3)!)} \tag{12}$$

The weighted Samade distribution's moment and variance can be obtained by substituting r=1 and r=2.

$$\mu_1{}' = \frac{\theta^4 \Gamma c + 2 + \alpha \Gamma c + 5}{\theta(\theta^4 c! + \alpha(c+3)!)}$$

$$\mu_2{}' = \frac{\theta^4 \Gamma c + 3 + \alpha \Gamma c + 6}{\theta^2(\theta^4 c! + \alpha(c+3)!)}$$

Variance $\mu_2 = \mu_2{}' - (\mu_1{}')^2$

$$\mu_2 = \left[\frac{\theta^4 (c+3) + \alpha(c+6)}{\theta^2(\theta^4 c! + \alpha(c+3)!)} - \left(\frac{\theta^4 (c+2) + \alpha(c+5)}{\theta(\theta^4 c! + \alpha(c+3)!)} \right)^2 \right] \tag{13}$$

5. MOMENT GENERATING FUNCTION AND CHARACTERISTIC FUNCTION

If X has a weighted Samade distribution, its moment generating function (mgf) is as follows:

$M_X(t) = E(e^{tx})$

$M_X(t) = \int_0^\infty e^{tx} f(x; \alpha, \theta) dx$ (14)

Using Taylor's series expansion

$$M_X(t) = \left(1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots\right)$$

$$M_X(t) = \sum_{r=0}^{\infty} \left(\frac{t^r}{r!}\right) \int_0^{\infty} f(x;\alpha,\theta)dx$$

$$M_X(t) = \sum_{r=0}^{\infty} \left(\frac{t^r}{r!}\right) \mu_r{}'$$

$$M_X(t) = \sum_{r=0}^{\infty} \left(\frac{t^r}{r!}\right) \left(\frac{\theta^4 \Gamma r + c + 1 + \alpha \Gamma r + c + 4}{\theta^r(\theta^4 c! + \alpha(c+3)!)}\right) \tag{15}$$

The weighted Samade distribution's characteristic function can be obtained in the same way.

$$\phi_X(it) = E(e^{itx})$$

$$\phi_X(it) = \int_0^{\infty} e^{itx} f(x;\alpha,\theta)dx$$

$$\phi_X(it) = \sum_{r=0}^{\infty} \left(\frac{it^r}{r!}\right) \left(\frac{\theta^4 \Gamma r + c + 1 + \alpha \Gamma r + c + 4}{\theta^r(\theta^4 c! + \alpha(c+3)!)}\right) \tag{16}$$

6. ENTROPIES

Entropy is an essential term in several fields, including probability, statistics, physics, economics, and communication theory. Entropies are applied to quantify a system's diversity, uncertainty, or randomness. An indicator of the uncertainty's variation is the entropy of a random variable X.

6.1 Renyi Entropy

In ecology and statistics, the Renyi entropy is significant as a diversity index. Renyi entropy, which is determined by, is also significant in quantum information because it can be used to quantify entanglement for a given probability distribution.

$$R_\lambda = \frac{1}{1-\lambda} \log \int_0^{\infty} \left(f(x)\right)^{\lambda} dx \qquad \lambda > o, \lambda \neq 1$$

$$R_\lambda = \frac{1}{1-\lambda}\log\int_0^\infty \left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}x^c(\theta+\alpha x^3)e^{-\theta x}\right)^\lambda dx$$

$$R_\lambda = \frac{1}{1-\lambda}\log\left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}\right)^\lambda \int_0^\infty x^{\lambda c}(\theta+\alpha x^3)^\lambda e^{-\theta\lambda x}dx \qquad (17)$$

Using binomial expansion to the above equation (17), we have

$$(\theta+\alpha x^3)^\lambda = \sum_{i=0}^{\lambda}\binom{\lambda}{i}\theta^i(\alpha x^3)^{\lambda-i}$$

Again, using power series expansion

$$\theta^i = \sum_{j=0}^{1}\frac{(\log(\theta))^j i^j}{j!}$$

$$\alpha^{\lambda-i} = \sum_{k=0}^{\lambda-i}\frac{(\log(\alpha))^k(\lambda-i)^k}{k!}$$

$$= \frac{1}{1-\lambda}\log\left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}\right)^\lambda \sum_{i=0}^{\lambda}\sum_{j=0}^{i}\sum_{k=0}^{\lambda-i}\binom{\lambda}{i}\frac{(\log(\theta))^j i^j}{j!}\frac{(\log(\alpha))^k(\lambda-i)^k}{k!}\int_0^\infty x^{(\lambda c+3(\lambda-i)+1)-1}e^{-\theta\lambda x}dx$$

$$= \frac{1}{1-\lambda}\log\left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}\right)^\lambda \sum_{i=0}^{\lambda}\sum_{j=0}^{i}\sum_{k=0}^{\lambda-i}\binom{\lambda}{i}\frac{(\log(\theta))^j i^j(\log(\alpha))^k(\lambda-i)^k}{j!\,k!}\frac{\Gamma(\lambda c+3(\lambda-i)+1)}{(\theta\lambda)^{\lambda c+3(\lambda-i)+1}}$$

6.2 Tsallis entropy

The Boltzmann-Gibbs (B-G) statistical mechanics generalization, which was first introduced by Tsallis, has attracted a lot of interest. First, the Tsallis entropy (Tsallis, 1988) mathematical expression for a continuous random variable—which is defined as follows—was introduced in order to suggest this generalization of (B-G) statistics.

$$T_\lambda = \frac{1}{\lambda-1}\left[1-\int_0^\infty \big(f(x)\big)^\lambda dx\right] \qquad \lambda>0, \lambda\neq 1$$

$$T_\lambda = \frac{1}{\lambda-1}\left[1-\int_0^\infty \left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}x^c(\theta+\alpha x^3)e^{-\theta x}\right)^\lambda dx\right]$$

$$T_\lambda = \frac{1}{1-\lambda}\left[1-\left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}\right)^\lambda \int_0^\infty x^{\lambda c}(\theta+\alpha x^3)^\lambda e^{-\theta\lambda x}dx\right] \quad (18)$$

Using binomial expansion to the above equation (18), we have

$$(\theta+\alpha x^3)^\lambda = \sum_{i=0}^{\lambda}\binom{\lambda}{i}\theta^i(\alpha x^3)^{\lambda-i}$$

After simplification we get

$$T_\lambda = \frac{1}{1-\lambda}\left[1-\left(\frac{\theta^{c+4}}{\theta^4 c!+\alpha(c+3)!}\right)^\lambda \sum_{i=0}^{\lambda}\sum_{j=0}^{i}\sum_{k=0}^{\lambda-i}\binom{\lambda}{i}\frac{(\log(\theta))^j i^j(\log(\alpha))^k(\lambda-i)^k}{j!k!}\ \frac{\Gamma(\lambda c+3(\lambda-i)+1)}{(\theta\lambda)^{\lambda c+3(\lambda-i)+1}}\right]$$

7. ORDER STATISTICS

There are many uses for order statistics in various areas of statistical theory and practice. Let $X_{(1)}, X_{(2)}, X_{(3)}, \ldots X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, X_3, \ldots X_n$ from a continuous population with cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$ and then the pdf of p^{th} order statistics $X_{(p)}$ is obtained as

$$\mathrm{f}_{X(r)}(x) = \frac{n!}{(r-1)(n-r)!}\mathrm{f}_X(x)[\mathrm{F}_X(x)]^{r-1}[1-\mathrm{F}_X(x)]^{n-r}$$

$$\mathrm{f}_{X(r)}(x) = \frac{n!}{(r-1)(n-r)!}\left(\frac{x^c\theta^{c+4}(\theta+\alpha x^3)}{\theta^4 c!+\alpha(c+3)!}\right) \times\left(\frac{\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!}\right)^{r-1} \times\left(1-\frac{\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!}\right)^{n-r}$$

Hence, the probability density function (pdf) of the highest order statistic, denoted as $X_{w(n)}$ can be derived as follows

$$\mathrm{f}_{x(n)}(x) = \left(\frac{n\,x^c\theta^{c+4}(\theta+\alpha x^3)}{\theta^4 c!+\alpha(c+3)!}e^{-\theta x}\right) \times\left(\frac{\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!}\right)^{n-1} \quad (19)$$

Consequently, the probability density function (pdf) of the first-order statistic, denoted as $X_{w(1)}$ can be determined as follows

$$f_{x(1)}(x) = \left(\frac{n\,x^c\theta^{c+4}(\theta+\alpha x^3)}{\theta^4 c!+\alpha(c+3)!}e^{-\theta x}\right) \times \left(\frac{1-\theta^{c+4}[\theta^4\gamma(c+1,\theta x)+\alpha\theta\gamma(c+4,\theta x)]}{\theta^4 c!+\alpha(c+3)!}\right)^{n-1} \quad (20)$$

8. LIKELIHOOD RATIO TEST

Assume that a random sample $x_1, x_2, x_3 \ldots, x_n$ is taken from a weighted Samade distribution to test the hypothesis, we have,

$H_0 : f(x) = f(x,\theta)$ against $H_1 : f(x) = f_w(x,c,\theta)$

The following test statistic is used to determine if the random sample of size n derives from the weighted Samade distribution or the Samade distribution.

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n}\frac{f_w(x_i,c,\theta)}{f(x_i,\theta)}$$

$$= \prod_{i=1}^{n}\left(\frac{\frac{\theta^{c+4}}{\theta^4 c!+\alpha(c+3)!}x^c(\theta+\alpha {x_i}^3)e^{-\theta x_i}}{\frac{\theta^4}{\theta^4+6\alpha}(\theta+\alpha {x_i}^3)e^{-\theta x_i}}\right)$$

$$= \prod_{i=1}^{n}\left(\frac{\theta^c(\theta^4+6\alpha)}{\theta^4 c!+\alpha(c+3)!}{x_i}^c\right)$$

The null hypothesis is rejected if

$$\Delta = \left(\frac{\theta^c(\theta^4+6\alpha)}{\theta^4 c!+\alpha(c+3)!}\right)^n \prod_{i=1}^{n}{x_i}^c > k$$

$$\Delta = \prod_{i=1}^{n}{x_i}^c > k\left(\frac{\theta^c(\theta^4+6\alpha)}{\theta^4 c!+\alpha(c+3)!}\right)^n$$

$$\Delta^* = \prod_{i=1}^{n}{x_i}^c > k^*$$

Where,

$$k^* = k\left(\frac{\theta^4 c! + \alpha(c+3)!}{\theta^c(\theta^4 + 6\alpha)}\right)^4$$

A chi-square variate with one degree of freedom is distributed as 2 log Δ for large sample sizes (n). The null hypothesis is thus rejected when the probability value is provided by

$P(\Delta^* > \alpha^*)$, where $\alpha^* = \prod_{i=1}^{n} {x_i}^c$ is less than level of significance. And $\prod_{i=1}^{n} {x_i}^c$ is the observed value of the statistic Δ^*.

9. MAXIMUM LIKELIHOOD ESTIMATION

This section focuses on the maximum likelihood of the weighted Samade distribution's parameters. Consider $(x_1, x_2, x_3 \ldots\ldots x_n)$ to be a random sample of size n from weighted Samade distribution.

After that, the likelihood function is provided by

$$L(x) = \prod_{i=1}^{n} f(x;\alpha,\theta)$$

$$L(x) = \prod_{i=1}^{n}\left(\frac{{x_i}^c\theta^{c+4}(\theta + \alpha {x_i}^3)e^{-\theta x_i}}{\theta^4 c! + \alpha(c+3)!}\right)$$

$$L(x) = \left(\frac{\theta^{c+4}}{\theta^4 c! + \alpha(c+3)!}\right)^n \prod_{i=1}^{n} {x_i}^c(\theta + \alpha {x_i}^3)\, e^{-\theta x_i} \tag{21}$$

The definition of the log likelihood function is

$$\log L = n(\log\theta^{c+4} - \log\theta^4\, c! - \log\alpha(c+3)!) + \theta\sum_{i=1}^{n} c\log x_i + \alpha\sum_{i=1}^{n}(1 + {x_i}^3) - \theta\sum_{i=1}^{n} x_i$$

$$\log L = n(c+4)(\log\theta - 4\log\theta - \log c! - \alpha\log(c+3)!) + \theta c\sum_{i=1}^{n}\log x_i + \alpha\sum_{i=1}^{n}(1 + {x_i}^3) - \theta\sum_{i=1}^{n} \tag{22}$$

After differentiating equation (22) with respect to θ, α, and c, the maximum likelihood estimates of θ, α, and c can be determined; these estimates have to satisfy the normal equations.

$$\frac{\partial \log L}{\partial \theta} = \frac{n(c+4)}{\theta} - \frac{4n}{\theta} - \sum_{i=1}^{n} x_i = 0 \tag{23}$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{n(c+3)}{\alpha} - \frac{3n}{\alpha} - \sum_{i=1}^{n} x_i = 0 \tag{24}$$

$$\frac{\partial \log L}{\partial c} = n \log \theta - n\big(\theta^4 \Psi(c) + \alpha \Psi(c+3)\big) + \sum_{i=1}^{n} \log x_i = 0 \tag{25}$$

where

$\Psi(.)$ denotes the digamma function.

The likelihood equations (23) (24) and (25) have a complex form, which makes solving the system of nonlinear equations extremely challenging. For this reason, we estimate the necessary parameter of the weighted Samade distribution using R and Wolfram Mathematica.

10. DATA ANALYSIS

The data set includes 64 lung cancer patients' survival periods (measured in months), as reported by Pena (2002). Below are the data sets:

0.99, 1.28, 1.77, 1.97, 2.17, 2.63, 2.66, 2.76, 2.79, 2.86, 2.99, 3.06, 3.15, 3.45, 3.71, 3.75, 3.81, 4.11, 4.27, 4.34, 4.40, 4.63, 4.73, 4.93, 4.93, 5.03, 5.16, 5.17, 5.49, 5.68, 5.72, 5.85, 5.98, 8.15, 8.62, 8.48, 8.61, 9.46, 9.53, 10.05, 10.15, 10.94, 10.94, 11.24, 11.63, 12.26, 12.65, 12.78, 13.18, 13.47, 13.96, 14.88, 15.05, 15.31, 16.13, 16.46, 17.45, 17.61, 18.20, 18.37, 19.06, 20.70, 22.54, 23.36.

The weighted Samade distribution is compared to other distributions using metrics such as -2 logL, Akaike Information Criteria Corrected (AICC), Akaike Information Criteria (AIC), and Bayesian Information Criteria (BIC).

A distribution with smaller values of AIC, BIC, AICC, and -2 logL is considered preferable. The following formula can be used to evaluate AIC, BIC, AICC, and -2logL:

$$AIC = 2k - 2\log L, \qquad BIC = k\log n - 2\log L, \qquad AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

where the variables are displayed in Table 1 and denote the number of parameters (k), sample size (n), and maximal value (-2logL) of the log likelihood function.

Table 1: Performance of the Distributions

Distributions	MLE	-2logL	AIC	BIC	AICC
Weighted Samade distribution	α=0.53779456 (0.00000000) $\hat{\theta}$ =0.82615740 (0.05728325)	290.3415	295.3415	302.3212	295.5377
Samade distribution	$\hat{\theta}$ =0.21805246 (0.02102116)	302.1632	304.1632	307.3427	304.2277

AIC, BIC, AICC, and -2logL values are lower in the weighted Samade distribution than in the Samade distribution, according to Table 1. Our conclusion is that the weighted Samade distribution provides a better match than the standard Samade distribution.

11. CONCLUSION

A weighted Samade distribution has been proposed in this research. By applying the weighted approach to the baseline distribution, the suggested distribution is produced. Along with reliability metrics, a few statistical features have been covered. Through the application of maximum likelihood technique, the parameters of the new distribution have been determined. A goodness of fit analysis was conducted to show how the new distribution applied to a real-life data collection. In comparison to alternative distributions, the proposed distribution seems to offer the best fit to the data, according to the AIC, BIC, and AICC criteria.

ACKNOWLEDGMENTS

We thank my co-research scholars G. Prabhavathi and M. Sakthivel who have helped to format this article in TEX format.

REFERENCES

1. Aderoju, S. (2021). *Samade probability distribution its Properties and Application to real lifetime data*. Asian Journal of Probability and Statistics, 14(1), 1-11.
2. Cox D. R. (1969*). Some Sampling Problems in Technology, In New Development in Survey Sampling*, Johnson, N. L and Smith, H., Jr. (eds.) New York Wiley-Inter science, 506-527.
3. Dey S., Ali, S. & Perk, C. (2015), *On Weighted Exponential Distribution*. Journal of Statistical Computational and Simulation.
4. Fisher, R. A. (1934). *The effects of methods of ascertainment upon the estimation of frequencies*. Annals of Eugenics. 6(3), 13-25.
5. J. Lappi and R.L Bailey (1987). *Estimation of diameter increment function or other tree relations using angle-count samples*, Forest science 33: 725-739.
6. Mahdavi, A. (2015). *Two weighted distributions generated by exponential distribution*. Journal of Mathematical Extension. (9), 1-12.
7. Maryam Mohiuddin, Shabir A. Dar, Arshad A. Khan, M. Ahajeeth and Hilal Al Bayatti (2022). On *Weighted Nwikpe Distribution: Properties and Applications*. International journal of information science letters. 11(1), 85-95.
8. P. Vanitha, P. Pandiyan, "*A Stochastic Model For Lung Cancer.*", Advances in Mathematics Scientific journal, Volume 11, Number 10, Oct 2022, pp. 943-954. https: // doi.org /10.37418 / amsj.11.10.10.
9. Pandiyan. P, "*Calculating human immune virus infected persons survival time through stochastic model* ", Bio-Science Research bulletin, Volume 29, Number 1, 2013, pp. 1-4. 2013.
10. Rama Shanker (2016). *Shambhu Distribution and Its Applications*, International Journal of Probability and Statistics. 5(2): 48-63 DOI: 10.5923/j.ijps.20160502.03
11. Rao, C. R. (1965). *On discrete distributions arising out of methods of ascertainment, in Classical and Contagious Discrete Distribution.* Patil, G. P. ed. Pergamon Press and Statistical Publishing Society, Calcutta. 320-332.
12. Saghir, A., Hamedani, G. G., Tazeem, S. & Khadim, A. (2017). *Weighted Distribution: a brief review, perspective and characterization*. IJSP. 6(3), 109-131.
13. Tsallis, C. (1988). *Possible generalization of boltzmann-gibbs Statistics*. Journal of Statistical Physics. (52), 479-487.
14. T. Elizabeth Sangeetha, P. Pandiyan, (2021) "*A Stochastic Model for Chronic Kidney Disease*.", Advances in Mathematics Scientific journal, Volume 10, Number 10, Oct 2021, pp. 3363-3372.
15. Van Deusen P.C., (1986). *Fitting assumed distributions to horizontal point sample diameters*. Forest Science 32(1): 146-148.
16. Zelen, M. (1934). *Problems in Cell Kinetic and the early Detection of Disease, in Reliability and Biometry,* F. Proschan & R. J. Sering, eds, SIAM, Philadelphia, 701-706.
17. Zubair, A., Elgarhy, M. & Hamedani, G. (2019). *Weighted Exponentiated Family of Distributions: Properties, Applications and Characterizations*. Journal of Iranian Statistical Society. 19(1), 209-228.

P. Pandiyan, Professor,
Department of Statistics,
Annamalai University,
Annamalai Nagar, Tamil Nadu, India-608002.

R. Jothika, Research Scholar,
Department of Statistics,
Annamalai University,
Annamalai Nagar, Tamil Nadu, India-608002.

Received November 2023