# Performance study of the $M/M(a,b)/1/MWV$ queuing system with types of breakdowns

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**Abstract**

This paper analyses the $M/M(a,b)/1/MWV$ queuing model with various types of breakdowns. During peak times, the system has breakdown due to server unavailability. The model considers system with two types of breakdowns. The system may breakdown in two different ways during busy and working vacation stages. Customer arrives to the system with parameter $\lambda_v$ which follows Poisson distribution and server provides service in regular busy period with parameter $\mu_{rb}$ and under the multiple working vacations it provides service with parameter $\mu_{wv}$ with exponential distribution. In this model batches of customers are served under General Bulk Service Rule. The system may breakdown at any time. The breakdown that occurs during working vacation is denoted as $\beta_{v1}$ and the breakdown during busy state is denoted as $\beta_{v2}$. The steady-state equation, the performance of measures for the system and particular cases of described model are derived. Finally, a real life example demonstrated the validity of the model in a proper way.

**General Terms:** Breakdown, Vacation, Batch Service, queue length.
**Mathematics Subject Classification 2000:** 60K25, 60M20, 90B22
**Keywords:** Busy period, Multiple Working vacations (MWV), Working state.

## 1. INTRODUCTION

The Danish engineer Agner Krarup Erlang first introduced mathematical models to predict the behavior of telephone networks. Erlang's work laid the foundation for what would later become known as queuing theory. Mathematical models are used in queuing theory to represent different types of queuing systems. These models provide a way to analyze and predict the behavior of queues and evaluate performance measures.

In queuing theory, the batch arrival process refers to a type of arrival pattern where multiple entities arrive simultaneously or in groups rather than individually. Instead of arrivals occurring one at a time, they occur in batches. The batch queue concept was analysed by [Bailey 1954], he analysed specific-batch service and used the embedded-Markov chain technique for determining the average distribution of queue size. Countless authors generalized his procedure and developed many models. [Claeys et al. 2013] analyzed the batch service queuing model. [Niranjan Indhira 2016] established the detailed review of the batch service model.

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[Medhi 1975] discussed the Poisson queue with GBSR. The General Bulk Service Rule was initially described by Neuts. He developed the matrix algorithmic solution for the model. The GBSR concept was analysed by many researchers, some of them are, [Curry Feldman 1985] introduced the M/M/1 model with GBSR. [Mary Begum 2009] proposed the M/M(a,b)/1/WV and obtain closed form analytical solution of GBS queuing model. [Haridass Arumuganathan 2011] studied the GBSR system with threshold policy for secondary jobs. [Sasikala Kandaiyan 2016] explored the detailed survey of bulk service model.

The vacation queuing concept emerged significantly during past few years. Instead of the server being stopped in its service during the vacation period, server serves at a different rate during MWV. The service rate varies depending on the arrival state. [Servi Finn 2002] introduced the working vacation queue model. [Tian Zhang 2006] established the queuing models' theory & applications. [Baba 2012] derived the $M^X/M/1$ model. [Ibe Isijola 2014] analysed the M/M/1/MV system with differentiated vacations.

[Gray et al. 2000] proposed the procedure VQM with server breakdowns. [Jain Jain 2002] explained the types of breakdown in VQM. $M^X/M/1/MWV/BD$ studied by [Remona et al. 2016]. [Rajadurai et al. 2018] calculated some of the random breakdown in the MWV queuing model. [Rajan et al. 2020] studied the batch arrival queuing model with breakdown and repair. Multiple vacation two server queue with breakdown was derived by [Yang et al. 2020]. [Agrawal et al. 2021] presented the single server working vacation queue with two types of breakdowns.

This paper analyses the $M/M(a,b)/1/MWV$ with various types of breakdowns. The steady-state equation, the performance of measures for the system and the particular cases of described model are derived. Also analysed the numerical example, by using tables and graphs.

## 2. METHODOLOGY

In this paper, analysed the $M/M(a,b)/1/MWV$ queuing model with types breakdowns. Instead of the server being fully idle during the vacation period, server serves at a different rate during multiple working vacations. The service rate varies depending on the arrival state. Customers arrive to the system with parameter $\lambda_v$ which follows Poisson distribution, server provides service in regular busy period with parameter $\mu_{rb}$ and during the busy period and under the multiple working vacations server provides service with parameter $\mu_{wv}$ with exponential distribution. In this model batches of

customers are served under General Bulk Service Rule. Thus each batch of service contains minimum $'a'$ units and maximum $'b'$ units. Suppose the customers waiting in the queue for a time period that is less than $'a'$ and server begins a vacation random variable V with parameter $\xi$. Also the system may breakdown at any time. Here the breakdown during working vacation is denoted as $\beta_{v1}$ and the breakdown during busy state is denoted as $\beta_{v2}$.

Consider the customers in the queue at time t denoted as $N_c(t)$, and L(t)=0,1 or 2 according to whether the server is idle, busy or regular busy on vacation state respectively. Let

$$\begin{aligned} R_n^I(t) &= Pr\{N_c(t)=n, L(t)=0\} \qquad 0 \le n \le a-1 \\ Q_n^V(t) &= Pr\{N_c(t)=n, L(t)=1\} \qquad n \ge 0 \\ P_n^B(t) &= Pr\{N_c(t)=n, L(t)=2\} \qquad n \ge 0 \end{aligned}$$

When the server is idle: $L(t)=0$, the size of the queue and system are the same.
When the server is busy or working during the vacation: $L(t)=1$ *or* $2$, , the total number of customers in the system is the sum of the number of customers in queue and the size of the service batch that contains particular $a \le x \le b$ customers.
Probabilities of the steady state are,

$$Q_n^V = \lim_{t\to\infty} Q_n^V(t); \qquad R_n^I = \lim_{t\to\infty} R_n^I(t); \qquad P_n^B = \lim_{t\to\infty} P_n^B(t);$$

exist and the Chapman-Kolmogrov equations satisfied by them in the steady state are given by,

$$\lambda_v R_0^I = \mu_{rb} P_0^B + \mu_{wv} Q_0^V \tag{1}$$

$$\lambda_v R_n^I = \lambda_v R_{n-1}^I + \mu_{rb} P_n^B + \mu_{wv} Q_n^V \qquad 1 \le n \le a-1 \tag{2}$$

$$(\lambda_v + \xi + \mu_{wv} + \beta_{v1}) Q_0^V = \lambda_v R_{a-1}^I + \mu_{wv} \sum_{n=a}^{b} Q_n^V \tag{3}$$

$$(\lambda_v + \xi + \mu_{wv} + \beta_{v1}) Q_n^V = (\lambda_v + \beta_{v1}) Q_{n-1}^V + \mu_{wv} Q_{n+b}^V \qquad n \ge 1 \tag{4}$$

$$(\lambda_v + \mu_{rb} + \beta_{v2}) P_0^B = \mu_{rb} \sum_{n=a}^{b} P_n^B + \xi Q_0^V \tag{5}$$

$$(\lambda_v + \mu_{rb} + \beta_{v2}) P_n^B = (\lambda_v + \beta_{v2}) P_{n-1}^B + \mu_{rb} P_{n+b}^B + \xi Q_n^V \qquad n \ge 1 \tag{6}$$

## 3. STEADY STATE SOLUTION

To solve the steady state equation, the forward shifting operators $E$ on $P_n^B$ and $Q_n^V$ are introduced then,

$$E(P_n^B) = P_{n+1}^B; \qquad E(Q_n^V) = Q_{n+1}^V \qquad for\ n \geq 0$$

Thus the equation (4) gives homogeneous difference equation as,

$$[\lambda_v + \beta_{v1} + \mu_{wv}E^{b+1} - (\lambda_v + \beta_{v1} + \xi + \mu_{wv})E]Q_n^V = 0 \tag{7}$$

The characteristics equation (7) is obtained,

$$z(u) = \lambda_v + \beta_{v1} + \mu_{wv}u^{b+1} - (\lambda_v + \beta_{v1} + \xi + \mu_{wv})u = 0$$

by taking $x(u) = (\lambda_v + \beta_{v1} + \xi + \mu_{wv})u$ and $y(u) = \lambda_v + \beta_{v1} + \mu_{wv}u^{b+1}$, it is found that $|y(u)| < |x(u)|$ on $|u| = 1$. By Rouche's theorem $z(u)$ has unique root $r_v$ inside the contour $|u| = 1$. Thus equation (7) has a homogeneous solution as,

$$Q_n^V = r_v^n Q_0^V \tag{8}$$

From equation (6) yields,

$$[\lambda_v + \beta_{v2} + \mu_{rb}E^{b+1} - (\lambda_v + \beta_{v2} + \mu_{rb})E]P_n^B = -\xi r_v^{n+1}Q_0^V \tag{9}$$

Now by applying Rouche's theorem to equation (9), we get

$$[\lambda_v + \beta_{v2} + \mu_{rb}E^{b+1} - (\lambda_v + \beta_{v2} + \mu_{rb})E]P_n^B = 0$$

The above equation has unique root r with $|r| < 1$. Also equation (9) gives a non-homogeneous solution as,

$$P_n^B = \left[Zr^n - \frac{\xi r_v^{n+1}}{[\lambda_v + \beta_{v2} + \mu_{rb}r_v^{b+1} - (\lambda_v + \beta_{v2} + \mu_{rb})r_v]}\right]Q_0^V \tag{10}$$

$$P_n^B = (Zr^n + Z^* r_v^n)Q_0^V \tag{11}$$

where

$$Z^* = \frac{\xi r_v}{[\lambda_v(r_v - 1) + \beta_{v2}(r_v - 1) + \mu_{rb}r_v(1 - r_v^b)]} \tag{12}$$

The expression for $R_n^I$ is obtained by adding equation (1) and (2) and by substituting $P_n^B$ and $Q_n^V$ value we get,

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_{wv}}{\lambda_v}\frac{(1-r_v^{n+1})}{(1-r_v)}\right]Q_0^V \tag{13}$$

Now to calculate Z, substitute $P_n^B$ and $Q_n^V$ values in equation (5) and we get,

$$Z\left[(\lambda_v+\mu_{rb}+\beta_{v2}) - \frac{\mu_{rb}(r^a - r^{b+1})}{(1-r)}\right] = \left[\xi - Z^*\left((\lambda_v+\mu_{rb}+\beta_{v2}) - \frac{\mu_{rb}(r_v^a - r_v^{b+1})}{(1-r_v)}\right)\right] \tag{14}$$

the above expression can be simplified as

$$\frac{Z\mu_{rb}(1-r^a)}{(1-r)} = \frac{\xi}{(1-r_v)} - \frac{Z^*\mu_{rb}(1-r_v^a)}{(1-r_v)} \tag{15}$$

Hence the probability queue size of the steady-state equation in terms of $Q_0^V$ is obtained,

$$Q_n^V = (r_v^n)Q_0^V \qquad n \geq 0 \tag{16}$$

$$P_n^B = (Zr^n + Z^* r_v^n)Q_0^V \qquad n \geq 0 \tag{17}$$

where

$$Z = \frac{(1-r)}{\mu_{rb}(1-r^a)}\left[\frac{\xi}{(1-r_v)} - \frac{Z^*\mu_{rb}(1-r_v^a)}{(1-r_v)}\right] \tag{18}$$

$$Z^* = \frac{\xi r_v}{[\lambda_v(r_v-1) + \beta_{v2}(r_v-1) + \mu_{rb}r_v(1-r_v^b)]} \tag{19}$$

and

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_{wv}}{\lambda_v}\frac{(1-r_v^{n+1})}{(1-r_v)}\right]Q_0^V \tag{20}$$

by using normalizing condition and the calculated value of $Q_0^V$

$$\sum_{n=0}^{\infty} Q_n^V + \sum_{n=0}^{\infty} P_n^B + \sum_{n=0}^{a-1} R_n^I = 1$$

By substituting $P_n^B$, $Q_n^V$ and $R_n^I$ we observe that,

$$\sum_{n=0}^{\infty} r_v^n Q_0^V + \sum_{n=0}^{\infty}(Zr^n + Z^* r_v^n)Q_0^V + \\ + \sum_{n=0}^{a-1}\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_{wv}}{\lambda_v}\frac{(1-r_v^{n+1})}{(1-r_v)}\right]Q_0^V = 1$$

Then we get,

$$(Q_0^V)^{-1} = \omega(r_v, \mu_{wv}) + Z\omega(r, \mu_{rb}) + Z^*\omega(r_v, \mu_{rb}) \tag{21}$$

where

$$\omega(x,y) = \frac{1}{(1-x)}\left(1 + \frac{y}{\lambda_v}(c - \frac{x(1-x^a)}{(1-x)})\right)$$

## 4. PERFORMANCE MEASURES

### 4.1. Mean queue length

The expected queue length is given by,

$$L_q = \sum_{n=1}^{\infty} n(Q_n^V + P_n^B) + \sum_{n=1}^{a-1} nR_n^I$$

By substituting $P_n^B$, $Q_n^V$ and $R_n^I$ we observe that,

$$L_q = \sum_{n=0}^{\infty} n(r_v^n Q_0^V) + \sum_{n=0}^{\infty} n(Zr^n + Z^* r_v^n)Q_0^V + \sum_{n=0}^{a-1} n\left[\frac{\mu_{rb}}{\lambda_v}\left(\frac{Z(1-r^{n+1})}{(1-r)} + \frac{Z^*(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_{wv}}{\lambda_v}\frac{(1-r_v^{n+1})}{(1-r_v)}\right]Q_0^V$$

$$L_q = Z\omega^*(r, \mu_{rb}) + Z^*\omega^*(r_v, \mu_{rb}) + \omega^*(r_v, \mu_{wv}) \tag{22}$$

where

$$\omega^*(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)}\left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right]$$

and $Z$ & $Z^*$ are given by equations (18) & (19).

If $Pr_{(wv)}$, $Pr_{(busy)}$ *and* $Pr_{(idle)}$ denote the probability that the server in idle, regular busy and busy vacation period then

$$Pr_{(idle)} = \sum_{n=0}^{a-1} R_n^I \tag{23}$$

where the $R_n^I$ is given by equation (20).

$$Pr_{(busy)} = \sum_{n=0}^{\infty} P_n^B = \left(\frac{Z}{(1-r)} + \frac{Z^*}{(1-r_v)}\right)Q_0^V \tag{24}$$

$$Pr_{(wv)} = \sum_{n=0}^{\infty} Q_n^V = \frac{Q_0^V}{(1-r_v)} \tag{25}$$

## 5. PARTICULAR CASES

### 5.1. case 1: $M/M/1$ model

By letting a=b=1 and $\beta_{v1} = \beta_{v2} = 0$ in equations (16) to (20) becomes,

$$\begin{aligned} Q_n^V &= (r_v^n)Q_0^V & n \geq 0 \\ P_n^B &= \frac{Z^*}{r_v}(r_v^{n+1} - r^{n+1})Q_0^V & n \geq 0 \\ \textit{and} \qquad R_0^I &= \frac{Q_0^V}{r_v} \end{aligned}$$

where

$$r = \frac{\lambda_v}{\mu_{rb}} = \rho_v, \qquad Z = -\frac{Z^*\rho_v}{r_v} \qquad \textit{and} \qquad Z^* = \frac{\xi r_v}{\mu_{rb}(1-r_v)(r_v-\rho_v)}$$

The above equations coincide with the $M/M/1/WV$ queuing model analyzed by [Xu et al. 2009].

### 5.2. case 2: $M/M(a,b)/1/MWV$ model

Using $\beta_{v1} = \beta_{v2} = 0$ in equations (16) to (21) results in

$$Q_n^V = (r_v^n)Q_0^V \qquad n \geq 0$$

$$P_n^B = (Zr^n + Z^* r_v^n)Q_0^V \qquad n \geq 0$$

$$R_n^I = \left[\frac{\mu_{rb}}{\lambda_v}(Zg_n(r) + Z^* g_n(r_v) + g_n(r_v)\right]Q_0^V \qquad 0 \leq n \leq a-1$$

where

$$Z = \frac{(1-r)}{\mu_{rb}(1-r^a)}\left[\frac{\xi}{(1-r_v)} - \frac{Z^*\mu_{rb}(1-r_v^a)}{(1-r_v)}\right]$$

$$Z^* = \frac{\xi r_v}{\lambda_v(r_v-1) + \mu_{rb}r_v(1-r_v^b)}$$

Further

$$L_q = Z\omega^*(r,\mu_{rb}) + Z^*\omega^*(r_v,\mu_{rb}) + \omega^*(r_v,\mu_{wv})$$

where

$$\omega^*(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda_v(1-x)}\left[\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right]$$

Thus, it can be observed that our specified model coincides with the $M/M(a,b)/1/MWV$ queuing model analysed by [Mary Begum 2009].

## 6. NUMERICAL ANALYSIS

Consider the nuts and bolts manufacturing process, in this case machine is taken as a single server. Machine can process multiple nuts and bolts at once in batches, each batch has minimum $'a'$ and maximum $'b'$ units, after a batch is completed the machine processes the next batch. In busy period system it operates at its full capacity, consider surface treatments and shaping process are done in busy period. In working vacation period system operates in low service rate, cleaning and coating are done in working vacation state. Electricity outage or maintenance causes the machines breakdown. Consider various parameter like, regular service rate $(\mu_{rb})$, the service rate during multiple working vacation $(\mu_{wv})$, the arrival rate $(\lambda_v)$, vacation parameter $(\xi)$ and the breakdowns $(\beta_{v1})$ and $(\beta_{v2})$. With the aid of the above parameters numerical analysis is carried out to analyse the performance measures of the specified model. For that purpose consider the equations (7) and (9), where $r_v$ and $r$ are the roots of the following characteristics equations,

$$[\lambda_v + \beta_{v1} + \mu_{wv}u^{b+1} - (\lambda_v + \beta_{v1} + \xi + \mu_{wv})u] = 0$$

$$[\lambda_v + \beta_{v2} + \mu_{rb}u^{b+1} - (\lambda_v + \beta_{v2} + \mu_{rb})u] = 0$$

respectively and that lie in the interval (0,1).

Let as assume the arrival rate $\lambda_v = 50$, service during busy period $\mu_{rb} = 0.9$, the breakdown rates $\beta_{v1} = 0.001$ & $\beta_{v2} = 0.002$ and by varying service rate during working vacation period $\mu_{wv}$ and vacation parameter $\xi$. By using these parameters various performance measures are calculated and tabulated below.

Table I: Various Performance Measures with respect to $\mu_{wv}$ & $\xi$

| $\mu_{wv}$ | $r_v$ | $\xi$ | $L_q$ | $L_s$ | $W_q$ | $W_s$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.9969 | 0.1 | 308.885 | 309.2553 | 6.1777 | 6.1851 |
| | 0.9959 | 0.09 | 225.4425 | 225.8129 | 4.50885 | 4.5163 |
| | 0.9945 | 0.08 | 160.5996 | 160.97 | 3.21199 | 3.2194 |
| | 0.9934 | 0.07 | 127.368 | 127.7384 | 2.54736 | 2.5548 |
| | 0.9929 | 0.06 | 113.5859 | 113.9563 | 2.27172 | 2.2791 |
| 0.10 | 0.9955 | 0.1 | 208.2967 | 208.6671 | 4.16593 | 4.1733 |
| | 0.9946 | 0.09 | 166.8809 | 167.2513 | 3.33762 | 3.345 |
| | 0.9934 | 0.08 | 130.0864 | 130.4567 | 2.60173 | 2.6091 |
| | 0.9926 | 0.07 | 110.5429 | 110.9133 | 2.21086 | 2.2183 |
| | 0.9914 | 0.06 | 88.92154 | 89.29191 | 1.77843 | 1.7858 |
| 0.15 | 0.9947 | 0.1 | 173.6351 | 174.0054 | 3.4727 | 3.4801 |
| | 0.9934 | 0.09 | 133.0982 | 133.4686 | 2.66196 | 2.6694 |
| | 0.9926 | 0.08 | 113.2807 | 113.6511 | 2.26561 | 2.273 |
| | 0.9918 | 0.07 | 97.03434 | 97.40471 | 1.94069 | 1.9481 |
| | 0.9902 | 0.06 | 74.20205 | 74.57242 | 1.48404 | 1.4914 |
| 0.20 | 0.9938 | 0.1 | 145.6551 | 146.0255 | 2.9131 | 2.9205 |
| | 0.9925 | 0.09 | 114.6294 | 114.9998 | 2.29259 | 2.3000 |
| | 0.9906 | 0.08 | 84.82537 | 85.19574 | 1.69651 | 1.7039 |
| | 0.9895 | 0.07 | 69.94547 | 70.31584 | 1.39891 | 1.4063 |
| | 0.989 | 0.06 | 62.25426 | 62.62463 | 1.24509 | 1.2525 |

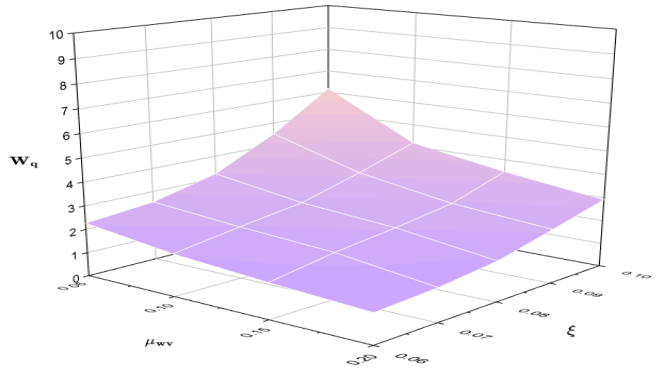



Fig. 1: Expected Waiting time of the Queue with respect to $\mu_{wv}$ & $\xi$

Table I and Figure 1 show the effect of vacation parameter $\xi$ and service rate during working vacation $\mu_{wv}$. We notice that as the vacation parameter increases the expected waiting time of the queue will be increasing. Also the average length of the queue, average length of the system and average waiting time of the system increase. Also we find that as the vacation parameter increases waiting time of the queue will be increased.

Consider service during busy period $\mu_{rb} = 0.9$, vacation parameter $\xi = 0.01$, the breakdown rates $\beta_{v1} = 0.001$ & $\beta_{v2} = 0.002$ and by varying service rate during working vacation period $\mu_{wv}$ and the arrival rate $\lambda_v$. Now by using these parameters various performance measures are calculated and tabulated below.

Table II: Various Performance Measures with respect to $\mu_{wv}$ & $\lambda_v$

| $\mu_{wv}$ | $r_v$ | $\lambda_v$ | $L_q$ | $L_s$ | $W_q$ | $W_s$ |
|---|---|---|---|---|---|---|
| 0.05 | 0.9975 | 45 | 384.25732 | 384.59065 | 8.53905 | 8.54646 |
| | 0.9971 | 44 | 330.306 | 330.63192 | 7.50695 | 7.51436 |
| | 0.9968 | 43 | 298.69274 | 299.01126 | 6.94634 | 6.95375 |
| | 0.9965 | 42 | 272.50969 | 272.8208 | 6.48833 | 6.49573 |
| | 0.9959 | 41 | 231.74075 | 232.04446 | 5.65221 | 5.65962 |
| 0.10 | 0.9973 | 45 | 351.71183 | 352.04516 | 7.81582 | 7.82323 |
| | 0.9969 | 44 | 305.09059 | 305.41652 | 6.93388 | 6.94128 |
| | 0.9965 | 43 | 269.17628 | 269.4948 | 6.25991 | 6.26732 |
| | 0.9961 | 42 | 240.67254 | 240.98365 | 5.7303 | 5.73771 |
| | 0.9958 | 41 | 222.84763 | 223.15134 | 5.43531 | 5.44272 |
| 0.15 | 0.9967 | 45 | 283.00305 | 283.33638 | 6.28896 | 6.29636 |
| | 0.9964 | 44 | 258.44421 | 258.77014 | 5.87373 | 5.88114 |
| | 0.9959 | 43 | 225.69482 | 226.01333 | 5.24872 | 5.25612 |
| | 0.9956 | 42 | 209.58934 | 209.90045 | 4.99022 | 4.99763 |
| | 0.9952 | 41 | 191.34759 | 191.6513 | 4.66701 | 4.67442 |
| 0.2 | 0.9964 | 45 | 255.6197 | 255.95303 | 5.68044 | 5.68785 |
| | 0.996 | 44 | 228.82935 | 229.15528 | 5.20067 | 5.20807 |
| | 0.9957 | 43 | 212.00055 | 212.31907 | 4.93025 | 4.93765 |
| | 0.9952 | 42 | 188.85777 | 189.16889 | 4.49661 | 4.50402 |
| | 0.9949 | 41 | 177.11345 | 177.41715 | 4.31984 | 4.32725 |

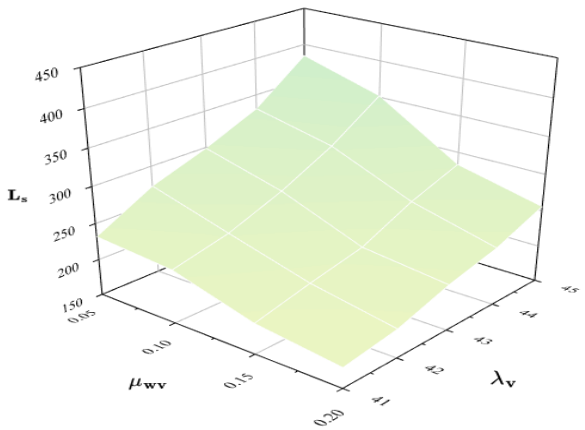



Fig. 2: Expected Length of the System with respect to $\mu_{wv}$ & $\lambda_v$

The effect of arrival rate $\lambda_v$ & service rate during the working vacation $\mu_{wv}$ is shown in Table II and Figure 2. As the arrival rate $\lambda_v$ decreases the average length of the system will be decreasing. Other performance measures also decrease. As a result arrival process is low; length of the system is also low.

Let arrival rate $\lambda_v = 50$, vacation parameter $\xi = 0.01$, the breakdown rates $\beta_{v1} = 0.001$ & $\beta_{v2} = 0.002$ and by varying service rate during working vacation period $\mu_{wv}$ and during busy period $\mu_{rb}$, various performance measures are calculated and tabulated below.

Table III: Various Performance Measures with respect to $\mu_{rb}$ & $\mu_{wv}$

| $\mu_{wv}$ | $r_v$ | $\mu_{rb}$ | $L_q$ | $L_s$ | $W_q$ | $W_s$ |
|---|---|---|---|---|---|---|
| | 0.9991 | 0.5 | 1059.792 | 1060.459 | 21.1958 | 21.209 |
| | 0.9984 | 0.6 | 595.4574 | 596.0129 | 11.9091 | 11.92 |
| 0.05 | 0.9975 | 0.7 | 380.6135 | 381.0897 | 7.61227 | 7.6218 |
| | 0.9968 | 0.8 | 297.5945 | 298.0112 | 5.95189 | 5.9602 |
| | 0.9959 | 0.9 | 232.026 | 232.3963 | 4.64052 | 4.6479 |
| | 0.9988 | 0.5 | 785.6978 | 786.3645 | 15.714 | 15.727 |
| | 0.998 | 0.6 | 470.5315 | 471.0871 | 9.41063 | 9.4217 |
| 0.10 | 0.9973 | 0.7 | 348.6511 | 349.1272 | 6.97302 | 6.9825 |
| | 0.9965 | 0.8 | 268.5762 | 268.9928 | 5.37152 | 5.3799 |
| | 0.9958 | 0.9 | 223.6878 | 224.0582 | 4.47376 | 4.4812 |
| | 0.9985 | 0.5 | 620.4676 | 621.1342 | 12.4094 | 12.423 |
| | 0.9974 | 0.6 | 355.403 | 355.9586 | 7.10806 | 7.1192 |
| 0.15 | 0.9967 | 0.7 | 280.3949 | 280.8711 | 5.6079 | 5.6174 |
| | 0.9959 | 0.8 | 225.4125 | 225.8292 | 4.50825 | 4.5166 |
| | 0.9952 | 0.9 | 192.4832 | 192.8536 | 3.84966 | 3.8571 |
| | 0.9981 | 0.5 | 481.1505 | 481.8172 | 9.62301 | 9.6363 |
| | 0.9973 | 0.6 | 338.8475 | 339.403 | 6.77695 | 6.7881 |
| 0.20 | 0.9964 | 0.7 | 253.3908 | 253.867 | 5.06782 | 5.0773 |
| | 0.9957 | 0.8 | 212.0812 | 212.4979 | 4.24162 | 4.25 |
| | 0.9949 | 0.9 | 178.5609 | 178.9312 | 3.57122 | 3.5786 |

Table III and Figure 3 describe that service rate during busy period decreases as the average waiting time of the system will increase. As a result, server provides service in higher rate and waiting time of the system will be decreasing.

Assume arrival rate $\lambda_v = 50$, service during busy period $\mu_{rb} = 0.9$, vacation parameter $\xi = 0.01$, and by varying service rate during working vacation period $\mu_{wv}$ and the breakdown rate$\beta_{v1}$ & $\beta_{v2}$, various performance measures are calculated and tabulated below.

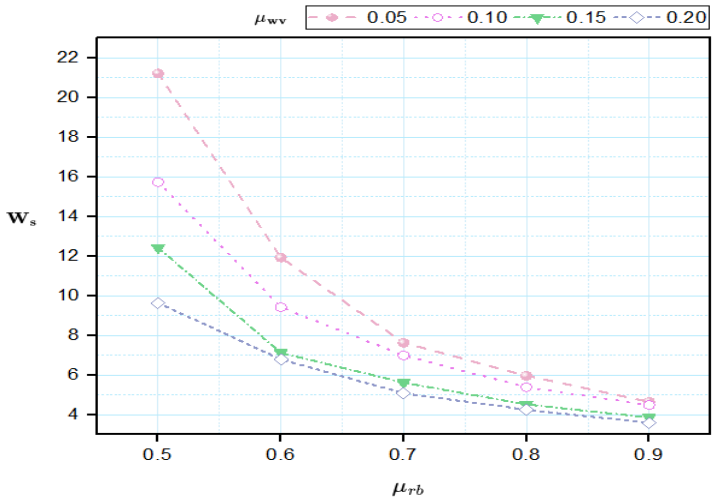



Fig. 3: Expected Waiting time of the System with respect to $\mu_{wv}$ & $\mu_{rb}$

Table IV: Various Performance Measures with respect to $\mu_{wv}$, $\beta_{v1}$ & $\beta_{v2}$

| $\mu_{wv}$ | $r_v$ | $\beta_{v2}$ | $L_q$ | $L_s$ | $W_q$ | $W_s$ |
|---|---|---|---|---|---|---|
| | 0.9987 | 0.009 | 746.14511 | 746.51548 | 14.9229 | 14.9303 |
| | 0.9984 | 0.008 | 604.80058 | 605.17095 | 12.096 | 12.1034 |
| 0.05 | 0.9979 | 0.007 | 459.10184 | 459.47221 | 9.18204 | 9.18944 |
| | 0.9975 | 0.006 | 384.56776 | 384.93813 | 7.69136 | 7.69876 |
| | 0.9971 | 0.005 | 330.62678 | 330.99715 | 6.61254 | 6.61994 |
| | 0.9985 | 0.009 | 641.94116 | 642.31153 | 12.8388 | 12.8462 |
| | 0.998 | 0.008 | 478.89213 | 479.2625 | 9.57784 | 9.58525 |
| 0.10 | 0.9977 | 0.007 | 415.18922 | 415.55959 | 8.30378 | 8.31119 |
| | 0.9973 | 0.006 | 352.35123 | 352.72161 | 7.04702 | 7.05443 |
| | 0.9969 | 0.005 | 305.79585 | 306.16622 | 6.11592 | 6.12332 |
| | 0.9979 | 0.009 | 452.21397 | 452.58434 | 9.04428 | 9.05169 |
| | 0.9974 | 0.008 | 363.02654 | 363.39691 | 7.26053 | 7.26794 |
| 0.15 | 0.997 | 0.007 | 313.21312 | 313.58349 | 6.26426 | 6.27167 |
| | 0.9967 | 0.006 | 283.83941 | 284.20979 | 5.67679 | 5.6842 |
| | 0.9964 | 0.005 | 259.40502 | 259.77539 | 5.1881 | 5.19551 |
| | 0.9977 | 0.009 | 408.58367 | 408.95404 | 8.17167 | 8.17908 |
| | 0.9973 | 0.008 | 346.07455 | 346.44492 | 6.92149 | 6.9289 |
| 0.20 | 0.9968 | 0.007 | 290.11726 | 290.48763 | 5.80235 | 5.80975 |
| | 0.9964 | 0.006 | 256.67178 | 257.04215 | 5.13344 | 5.14084 |
| | 0.996 | 0.005 | 230.00746 | 230.37783 | 4.60015 | 4.60756 |

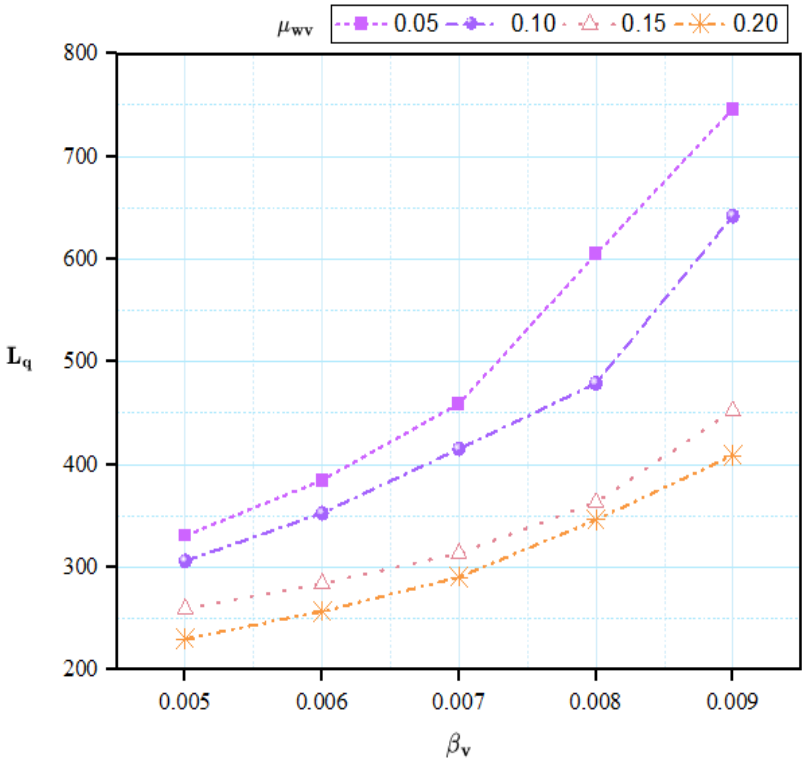



Fig. 4: Expected Length of the Queue with Respected to $\mu_{wv}$, $\beta_{v1}$ & $\beta_{v2}$

Table IV and Figure 4 illustrate the effect of breakdown rate $\beta_{v1}$ & $\beta_{v2}$ and service during working vacation $\mu_{wv}$, we observe that when breakdown rate $\beta_{v1}$ & $\beta_{v2}$ decreases the length of the queue will decrease sufficiently. The breakdown of machine leads to several impacts on the systems operation. Due to breakdowns, manufacturing process may be delayed, and then the queue length increases. This can affect the system performances. Understanding the queuing system breakdown situations may help us to understand the system performances and complete the process on time.

The probabilities for various states $Pr(idle)$, $Pr(wv)$ and $Pr(busy)$ are analysed and the characteristic values are calculated and tabulated (Table V) by using the equations (22) to (24). The chosen parameters are $\lambda_v = 50$, $\mu_{rb} = 0.9$, $\beta_{v1} = 0.001$, $\beta_{v2} = 0.002$ $a = 100$ & $b = 150$, notice that $Pr(busy)$, $Pr(idle)$ increase and $Pr(wv)$ decreases as $\xi$ increases for particular value for $\mu_{wv}$.

Table V: $Pr_{(busy)}$, $Pr_{(wv)}$ & $Pr_{(idle)}$ with respect to $\mu_{wv}$ and $\xi$

| $\mu_{wv}$ | $r_v$ | $\xi$ | $Pr_{(busy)}$ | $Pr_{(wv)}$ | $Pr_{(idle)}$ |
|---|---|---|---|---|---|
| 0.05 | 0.9994 | 0.01 | 0.1692127 | 0.80681 | 0.01055 |
| | 0.9991 | 0.02 | 0.2141959 | 0.7413 | 0.01841 |
| | 0.9987 | 0.03 | 0.2225349 | 0.71055 | 0.02653 |
| | 0.9984 | 0.04 | 0.2366487 | 0.67583 | 0.03345 |
| | 0.9979 | 0.05 | 0.2304775 | 0.65632 | 0.04141 |
| 0.10 | 0.9991 | 0.01 | 0.1223198 | 0.84666 | 0.01601 |
| | 0.9988 | 0.02 | 0.173048 | 0.77323 | 0.02485 |
| | 0.9985 | 0.03 | 0.2006115 | 0.72368 | 0.03294 |
| | 0.998 | 0.04 | 0.2028617 | 0.6948 | 0.04246 |
| | 0.9977 | 0.05 | 0.2158951 | 0.65976 | 0.04942 |
| 0.15 | 0.9989 | 0.01 | 0.1036358 | 0.85808 | 0.02238 |
| | 0.9985 | 0.02 | 0.1459109 | 0.78953 | 0.03327 |
| | 0.9979 | 0.03 | 0.1574779 | 0.7474 | 0.04572 |
| | 0.9974 | 0.04 | 0.1688346 | 0.7073 | 0.05589 |
| | 0.997 | 0.05 | 0.1802061 | 0.66956 | 0.06406 |
| 0.20 | 0.9985 | 0.01 | 0.0802587 | 0.86857 | 0.03366 |
| | 0.9981 | 0.02 | 0.121529 | 0.79902 | 0.04522 |
| | 0.9977 | 0.03 | 0.1465647 | 0.74649 | 0.05566 |
| | 0.9973 | 0.04 | 0.1631712 | 0.70277 | 0.06514 |
| | 0.9968 | 0.05 | 0.171152 | 0.66499 | 0.07505 |

## CONCLUSION

In this paper, we analyse the $M/M(a,b)/1/MWV$ queuing system with different types of breakdown. The system breaks down in two different ways during busy and working vacation stages. The steady-state solution, the performance of measures for the system and particular cases are calculated. Finally, in the form of tables and graphs, numerical results have been analysed. Further, a real life example illustrates the validity of the model. In future, the model may be extended to multiple working vacations for heterogeneous arrival of queuing system also.

## CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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## REFERENCES

AGRAWAL, P. K., JAIN, A., AND JAIN, M. 2021. *M/M/1 queueing model with working vacation and two type of server breakdown.* In Journal of Physics: Conference Series. Vol. 1849, 1, 012021. IOP Publishing.

BABA, Y. 2012 *The M X/M/1 queue with multiple working vacation.* In American Journal of Operations Research. Vol.2 No.2 (2012), Paper ID 19941. Scientific Research Publishing. doi: 10.4236/ajor.2012.22025.

BAILEY, N. T. 1954. *On queueing processes with bulk service.* In Journal of the Royal Statistical Society: Series B (Methodological) 16, 1, 80–87.

CLAEYS, D., STEYAERT, B., WALRAEVENS, J., LAEVENS, K., AND BRUNEEL, H. 2013. *Analysis of a versatile batch-service queueing model with correlation in the arrival process.* Performance Evaluation 70, 4, 300–316.

CURRY, G. L. AND FELDMAN, R. M. 1985. *An M/M/1 queue with a general bulk service rule.* Naval research logistics quarterly 32, 4, 595–603.

GRAY, W. J., WANG, P. P., AND SCOTT, M. 2000. *A vacation queueing model with service breakdowns.* Applied Mathematical Modelling 24, 5-6, 391–400.

HARIDASS, M. AND ARUMUGANATHAN, R. 2011. *Analysis of a batch arrival general bulk service queueing system with variant threshold policy for secondary jobs.* International Journal of Mathematics in Operational Research 3, 1, 56–77.

IBE, O. C. AND ISIJOLA, O. A. 2014. *M/M/1 multiple vacation queueing systems with differentiated vacations.* Modelling and Simulation in Engineering 2014, 18–18.

JAIN, M. AND JAIN, A. 2010. *Working vacations queueing model with multiple types of server breakdowns.* Applied Mathematical Modelling 34, 1, 1–13.

MARY, K. J. R. AND BEGUM, A. 2009. *Closed Form Analytical Solution of the General Bulk Service Queuing Model* $M/M(a,b)/1/MWV$. International conference on mathematical and computational model, PSG college of Technology.

MEDHI, J. 1975. *Waiting time distribution in a Poisson queue with a general bulk service rule.* Management Science 21, 7, 777–782.

NIRANJAN, S. AND INDHIRA, K. 2016. *A review on classical bulk arrival and batch service queueing models.* International journal of pure and applied Mathematics 106, 8, 45–51.

RAJADURAI, P., SARAVANARAJAN, M., AND CHANDRASEKARAN, V. 2018. *Cost optimisation analysis of retrial queue with K optional phases of service under multiple working vacations and random breakdowns.* International Journal of Industrial and Systems Engineering 29, 2, 193–222.

RAJAN, B. S., GANESAN, V., AND RITA, S. 2020. *Batch arrival Poisson queue with breakdown and repairs.* International Journal of Mathematics in Operational Research 17, 3, 424–435.

REMONA, J. M., VINNARASI, S., AND MARY, K. J. R. 2016. *Operating Characteristics of Mx/M/1/MWV/BD Queuing Systems.*

SASIKALA, S. AND KANDAIYAN, I. 2016. *Bulk service queueing models - a survey.* In International Journal of Pure and Applied Mathematics 106, 43–56. doi: 10.12732/ijpam.v106i6.6.

SEENIVASAN, M. AND CHANDIRALEKA, S. 2022. *Single Server Queueing Model with Multiple*

*Working Vacation and with Breakdown.* 2022 Second International Conference on Advances in Electrical, Computing, Communication and Sustainable Technologies (ICAECT), 1-5.

SERVI, L. D. AND FINN, S. G. 2002. *M/M/1 queues with working vacations (m/m/1/wv).* Performance Evaluation 50, 1, 41–52.

TIAN, N. AND ZHANG, Z. G. 2006. *Vacation queueing models: theory and applications.* Vol. 93. Springer Science & Business Media.

XU, X.-L., LIU, M.-X., AND ZHAO, X.-H. 2009. *The bulk input M [X]/M/1 queue with working vacations.* Journal of Systems Science and Systems Engineering 18, 3, 358–368.

YANG, D.-Y., CHEN, Y.-H., AND WU, C.-H. 2020. *Modelling and optimisation of a two-server queue with multiple vacations and working breakdowns.* In International Journal of Production Research 58, 10, 3036–3048.

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