
A new generalized transmuted distribution

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Abstract

We introduced Transmuted another Two-Parameter Sujatha Distribution by using Quadratic Rank Transmutation Map technique. Various necessary statistical properties of Transmuted another Two-Parameter Sujatha Distribution are obtained. The reliability measures of proposed model are also derived and model parameters are estimated by using maximum likelihood estimation method. The significance of transmuted parameter has been tested by using likelihood ratio statistic. Finally, an application to real data sets is presented to examine the significance of newly introduced model by computing Kolmogorov statistic, p-value, AIC, BIC, AICC, HQIC.

General Terms: Method Comparison, Simulation, Bayesian Inference.

Mathematics Subject Classification: 60E05, 62F10

Keywords: Quadratic Rank Transmutation Map technique, Simulation study, Transmuted parameter, Statistical properties and Maximum likelihood estimation.

1. INTRODUCTION

Data analysts and researchers fit appropriate model to various real life data sets depending on the nature of the real life data among other things. A model with large number of parameters brings more flexibility and covers more variation from the data. There are many techniques for obtaining generalized probability models. One of the techniques for adding extra parameter to the existing models is Quadratic Rank Transmutation Map technique (QRTM). Data analysts make use of Quadratic Rank Transmutation Map technique for analyzing very complex data. Shaw and Buckley (2007) introduced Quadratic Rank Transmutation Map technique for generalization of classical probability models [1]. Aryal and Tsokos (2009) worked on transmuted generalized extreme value distribution and studied its applications and properties [2]. Hassan, Wani & Para (2018) introduced three parameter Quasi Lindley distribution by using weighting technique and studied various properties of that model [3]. Hassan, Wani, Shafi and Sheikh (2020) introduced Lindley-Quasi Xgamma Distribution (LQXD) and studied its applications along with properties [4]. Merovci (2013) developed transmuted Rayleigh distribution and studied its

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necessary properties and applications [5]. Para and Jan (2018) formulated transmuted inverse log-logistic distribution and obtained its various characteristic properties [6]. Haq (2016) studied transmuted exponentiated inverse Rayleigh distribution and obtained its various properties [7]. Hassan, Wani, Shafi (2020) introduced Poisson Pranav distribution and obtained its various mathematical properties along with obtaining applications of the proposed model [8].

Here we have incorporated an extra parameter known as transmuted parameter to Another Two-Parameter Sujatha distribution [9].

A continuous random variable X is said to follow another Two-Parameter Sujatha Distribution (ATPSD) if its probability density function is of the form

$$g_b(x) = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} x > 0, \theta > 0, \alpha \geq 0 \quad (1.1)$$

And is denoted by $X \sim \text{ATPSD}(\alpha, \theta)$

The cumulative distribution function of Another Two-Parameter Sujatha Distribution is given by

$$G_b(x) = 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \quad x > 0, \theta > 0, \alpha \geq 0 \quad (1.2)$$

By using Quadratic Rank Transmutation Map technique. The c.d.f of transmuted model $F_\tau(x)$ is of the below form

$$F_\tau(x) = (1 + \lambda)G_b(x) - \lambda[G_b(x)]^2, -1 \leq \lambda \leq 1,$$

Which on differentiation yields the p.d.f $f_\tau(x)$ of transmuted model as

$$f_\tau(x) = g_b(x)[1 + \lambda - 2\lambda G_b(x)]$$

Where $g_b(x)$ and $G_b(x)$ are p.d.f and c.d.f of base model respectively.

2. TRANSMUTED ANOTHER TWO-PARAMETER SUJATHA DISTRIBUTION (TATPSD)

A non-negative random variable X is said to have Transmuted another Two-Parameter Sujatha Distribution (TATPSD) if its cumulative distribution function $F_T(x)$ is obtained as

$$F_T(x, \alpha, \theta, \lambda) = (1 + \lambda)G_b(x) - \lambda[G_b(x)]^2, -1 \leq \lambda \leq 1,$$

$$F_T(x, \alpha, \theta, \lambda) = G_b(x)\{1 + \lambda(1 - G_b(x))\} \quad (2.1)$$

Putting the value of $G_b(x)$ from equation (1.2) in equation (2.1) we get

$$F_T(x, \alpha, \theta, \lambda) = \left\{1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)}\right] e^{-\theta x}\right\} \left\{(1 + \lambda) - \lambda\left\{1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)}\right] e^{-\theta x}\right\}\right\} \quad (2.2)$$

The graphs of c.d.f of Transmuted Another Two-Parameter Sujatha Distribution are given below:

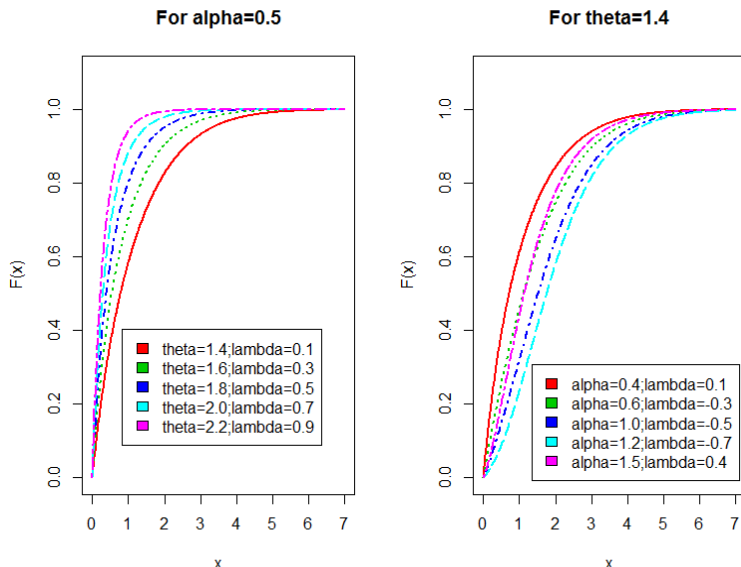


Figure 1(a) Graph of distribution function

Figure 1(b) Graph of distribution function

Where $G_b(x)$ is the c.d.f of base distribution (Another Two-Parameter Sujatha Distribution). For $\lambda = 0$ in (2.1) we get the base distribution.

As can be seen from Figures 1(a) and 1(b) c.d.f starts at zero and ends with one for different parameter values which shows that c.d.f is accurate.

The corresponding probability density function of Transmuted Another Two-Parameter Sujatha Distribution $f_{\tau}(x, \alpha, \theta, \lambda)$ is obtained by differentiating (2.1) as

$$f_{\tau}(x, \alpha, \theta, \lambda) = g_b(x)[1 + \lambda - 2\lambda G_b(x)] \quad (2.3)$$

Putting the value of $g_b(x)$ and $G_b(x)$ in equation (2.3)

$$f_{\tau}(x, \alpha, \theta, \lambda) = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \quad (2.4)$$

$$x > 0, \theta > 0 \text{ and } \alpha > 0, -1 \leq \lambda \leq 1$$

Where α, θ & λ are shape, scale and transmuted parameters respectively.

Which is the p.d.f of Transmuted another Two-Parameter Sujatha distribution.

Where $g_b(x)$ and $G_b(x)$ are p.d.f and c.d.f of base model respectively.

As can be seen from figure 2(a) and 2(b) the Transmuted another Two-Parameter Sujatha Distribution is positively skewed as is clear from graphs given below:

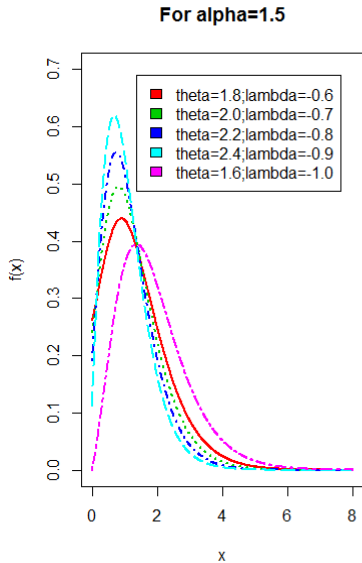


Figure 2(a) Graph of Probability Density Function

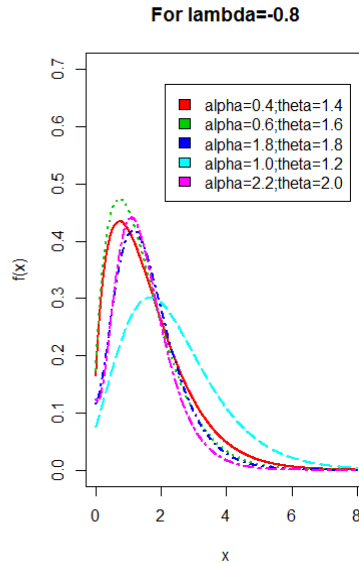


Figure 2(b) Graph of Probability Density Function

3. SPECIAL CASES

CASE 1: If we put $\lambda = 0$, then Transmuted another Two-Parameter Sujatha distribution (2.4) reduces to Another Two-Parameter Sujatha distribution with shape parameter α & scale parameter θ and having probability density function as:

$$f(x) = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \quad x > 0, \theta > 0, \alpha \geq 0$$

CASE 2: If we put $\alpha = 0, \lambda = 0$, then Transmuted another Two-Parameter Sujatha distribution (2.4) reduces to Exponential distribution having probability density function as:

$$f(x) = \theta e^{-\theta x} \quad x > 0, \theta > 0$$

4. RELIABILITY ANALYSIS

We explored survival function, hazard rate and reverse hazard rate of the proposed transmuted another Two-Parameter Sujatha distribution in this segment of paper.

4.1 Reliability function $R(x)$

The reliability function or survival function $R(x, \alpha, \theta, \lambda)$ is the measure of chance that a system survives beyond a specified time(t).

Mathematically

$$R(x, \alpha, \theta, \lambda) = P(X > t)$$

It can be obtained as complement of the cumulative distribution function of the model. The reliability function or the survival function of Transmuted another Two-Parameter Sujatha distribution is calculated as:

$$R(x, \alpha, \theta, \lambda) = 1 - \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\}$$

The graphs of reliability function are given below:

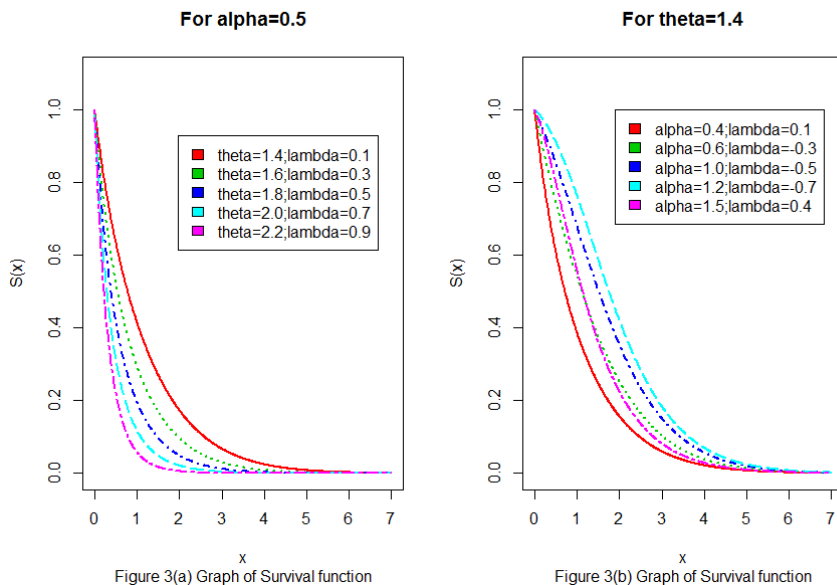


Figure 3(a) Graph of Survival function

Figure 3(b) Graph of Survival function

As can be seen from Figures 3(a) and 3(b) survival function starts at one and ends with zero for different parameter values which shows that survival function is accurate.

4.2 Hazard function

The hazard function of TATPSD is obtained as

$$H.R = h(x; \alpha, \theta, \lambda) = \frac{f(x, \alpha, \theta, \lambda)}{R(x, \alpha, \theta, \lambda)}$$

$$= \frac{\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left\{ 1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right\} e^{-\theta x} \right\}}{1 - \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\}}$$

The graph of hazard rate of Transmuted another Two-Parameter Sujatha distribution is given below

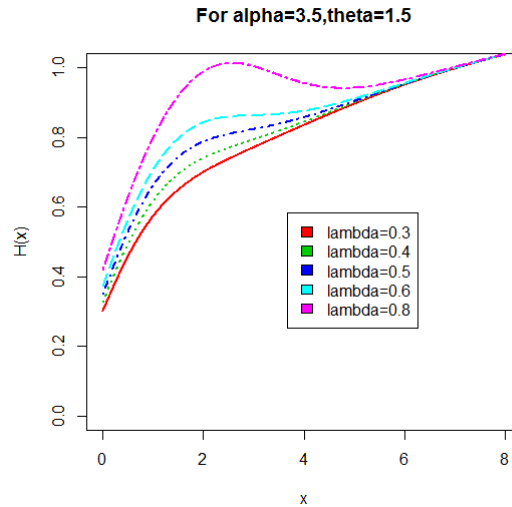


Figure 4 Graph of Hazard Function

The above graph of hazard rate of TATPSD reveals increasing, decreasing and constant hazard rate for different values of transmuted parameter. The shape of hazard rate graph is bathtub showing the flexibility of proposed model and greater applicability in real life.

4.3 Reverse hazard rate

The reverse hazard rate of the TATPSD is given as:

$$R.H.R = h(x, \alpha, \theta, \lambda) = \frac{f(x, \alpha, \theta, \lambda)}{F(x, \alpha, \theta, \lambda)}$$

$$= \frac{\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\}}{\left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\}}$$

5. STATISTICAL PROPERTIES

In this section we have obtained the different structural and statistical properties of the proposed Transmuted another Two-Parameter Sujatha model. These include moments, moment generating function and characteristic function

5.1 Moments

Suppose X is a random variable following Transmuted another Two-Parameter Sujatha distribution with parameters α, θ and λ . Then the r^{th} moment about origin for TATPSD is given by

$$\begin{aligned}\mu_r' &= E(X^r) = \int_0^\infty x^r f(x, \alpha, \theta, \lambda) dx \quad r=1, 2, 3 \dots n \\ &= \int_0^\infty x^r \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} dx \\ \mu_r' &= \left(\frac{r!}{\theta^r(\theta^2 + \alpha\theta + 2\alpha)} \right. \\ &\quad \left. \left\{ (1 - \lambda)(\theta^2 + (r+1)\alpha\theta + \alpha(r+1)(r+2)) + \right. \right. \\ &\quad \left. \left. 2\lambda \left\{ \frac{1}{2^{r+3}}(4\theta^2 + 2\alpha\theta(r+1) + \alpha(r+1)(r+2)) + \frac{1}{(\theta^2 + \alpha\theta + 2\alpha)} \right. \right. \right. \right. \\ &\quad \left. \left. \left. \frac{\alpha(r+1)(r+2)}{2^{r+5}}(4\theta^2 + 2\alpha\theta(r+3) + \alpha(r+3)(r+4)) + \frac{\alpha(\alpha\theta + 2)(r+1)}{2^{r+4}} \right\} \right\} \right) \quad (5.1.1)\end{aligned}$$

5.2 Moment generating function and characteristic function of TATPSD

We will derive moment generating function and characteristic function of TATPSD in this section of paper.

THEOREM 5.2.1: If X has the TATPSD(θ, α, λ), then the moment generating function $M_X(t)$ and characteristic generating function $\varphi_X(t)$ are

$$\begin{aligned}M_X(t) &= \\ &\left(\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \right. \\ &\quad \left\{ (1 - \lambda) \left(\frac{1}{\theta - t} + \frac{\alpha}{(\theta - t)^2} + \frac{2\alpha}{(\theta - t)^3} \right) + 2\lambda \right. \\ &\quad \left. \left\{ \left(\frac{1}{(2\theta - t)} + \frac{\alpha}{(2\theta - t)^2} + \frac{2\alpha}{(2\theta - t)^3} \right) + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} \right. \right. \\ &\quad \left. \left. \left\{ \theta \left(\frac{2}{(2\theta - t)^3} + \frac{6\alpha}{(2\theta - t)^4} + \frac{24\alpha}{(2\theta - t)^5} \right) + (\alpha\theta + 2) \left(\frac{1}{(2\theta - t)^2} + \frac{2\alpha}{(2\theta - t)^3} + \frac{6\alpha}{(2\theta - t)^4} \right) \right\} \right\} \right) \end{aligned}$$

and

$$\varphi_X(t) =$$

$$\left(\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left\{ (1-\lambda) \left(\frac{1}{\theta - it} + \frac{\alpha}{(\theta - it)^2} + \frac{2\alpha}{(\theta - it)^3} \right) + 2\lambda \left\{ \left(\frac{1}{(2\theta - it)} + \frac{\alpha}{(2\theta - it)^2} + \frac{2\alpha}{(2\theta - it)^3} \right) + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} \right\} \right\} \right)$$

respectively.

And hence show that Another Two-Parameter Sujatha distribution and Exponential distribution are particular cases of Transmuted another Two-Parameter Sujatha distribution.

PROOF: We begin with the well-known definition of the moment generating function given by

$$\begin{aligned} M_X(t) &= E(e^{tx}) = \int_0^\infty e^{tx} f(x; \alpha, \theta, \lambda) dx \\ &= \int_0^\infty e^{tx} \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1-\lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} dx \\ &= \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \int_0^\infty e^{tx} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1-\lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} dx \\ M_X(t) &= \left(\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left\{ (1-\lambda) \left(\frac{1}{\theta - t} + \frac{\alpha}{(\theta - t)^2} + \frac{2\alpha}{(\theta - t)^3} \right) + 2\lambda \left\{ \left(\frac{1}{(2\theta - t)} + \frac{\alpha}{(2\theta - t)^2} + \frac{2\alpha}{(2\theta - t)^3} \right) + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} \right\} \right\} \right) \\ &\quad \left(\left\{ \theta \left(\frac{2}{(2\theta - t)^3} + \frac{6\alpha}{(2\theta - t)^4} + \frac{24\alpha}{(2\theta - t)^5} \right) + (\alpha\theta + 2) \left(\frac{1}{(2\theta - t)^2} + \frac{2\alpha}{(2\theta - t)^3} + \frac{6\alpha}{(2\theta - t)^4} \right) \right\} \right) \end{aligned} \quad (5.2.1)$$

Which is the m.g.f of Transmuted another Two-Parameter Sujatha distribution.

For $\lambda = 0$ in equation (5.3.1) we get

$$M_X(t) = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left(\frac{1}{\theta - t} + \frac{\alpha}{(\theta - t)^2} + \frac{2\alpha}{(\theta - t)^3} \right)$$

Which is m.g.f of *Another Two-Parameter Sujatha distribution* with parameters α, θ

For $\lambda = 0, \alpha = 0$ in equation (5.3.1) we get

$$M_X(t) = \frac{\theta}{(\theta - t)}$$

Which is the m.g.f of Exponential distribution

Also we know that $\varphi_X(t) = M_X(it)$

Therefore,

$$\varphi_X(t) = \left(\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left((1 - \lambda) \left(\frac{1}{\theta - it} + \frac{\alpha}{(\theta - it)^2} + \frac{2\alpha}{(\theta - it)^3} \right) + 2\lambda \left(\frac{1}{(2\theta - it)} + \frac{\alpha}{(2\theta - it)^2} + \frac{2\alpha}{(2\theta - it)^3} \right) + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} \right) \right. \\ \left. \left(\left\{ \theta \left(\frac{2}{(2\theta - it)^3} + \frac{6\alpha}{(2\theta - it)^4} + \frac{24\alpha}{(2\theta - it)^5} \right) + (\alpha\theta + 2) \left(\frac{1}{(2\theta - it)^2} + \frac{2\alpha}{(2\theta - it)^3} + \frac{6\alpha}{(2\theta - it)^4} \right) \right\} \right) \right) \quad (5.2.2)$$

Which is the characteristic function of Transmuted another Two-Parameter Sujatha distribution.

5.3 Mean deviation about mean and median of TATPSD

We have derived the expressions for mean deviation about mean and median of TATPSD in this section.

THEOREM 5.3.1: If X has the TATPSD(θ, α, λ), then the mean deviation about mean ($\delta_1(X)$) and mean deviation about median ($\delta_2(X)$) are given as:

$$\delta_1(X) = \left[2\mu \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right. \\ \left. - 2 \left\{ \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A + 2\lambda \left\{ B + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C + (\alpha\theta + 2)D) \right\} \right] \right\} \right]$$

Where

$$A = \frac{1}{\theta^4} \{ \theta^2 + 2\theta\alpha + 6\alpha - e^{-\theta\mu} (\theta^2 + \mu\theta^3 + 2\alpha\theta + 2\mu\alpha\theta^2 + \mu^2\theta^3\alpha + 6\alpha\theta\mu + 6\alpha + 3\mu^2\theta^2\alpha + \mu^3\theta^3\alpha) \}$$

$$B = \left(\frac{1}{16\theta^4} \{ 4\theta^2 + 4\alpha\theta + 6\alpha - e^{-2\theta\mu} (4\theta^2 + 8\mu\theta^3 + 4\alpha\theta + 8\mu\alpha\theta^2 + 8\mu^2\theta^3\alpha + 12\alpha\theta\mu + 6\alpha + 12\mu^2\theta^2\alpha + 8\mu^3\theta^3\alpha) \} \right)$$

$$C = \frac{1}{64\theta^5} \left\{ \begin{aligned} &24\theta^2 + 48\alpha\theta + 120\alpha - e^{-2\theta\mu} \\ &(24\theta^2 + 48\mu\theta^3 + 48\alpha\theta + 48\mu^2\theta^4 + 32\mu^3\theta^5 + 96\mu\theta^2\alpha + 96\mu^2\theta^3\alpha + 64\mu^3\theta^4\alpha \\ &+ 32\mu^4\theta^5\alpha + 32\mu^5\theta^5\alpha + 240\mu\theta\alpha + 120\alpha + 240\theta^2\mu^2\alpha + 160\mu^3\theta^3\alpha + 80\mu^4\theta^4\alpha) \end{aligned} \right\}$$

$$D = \frac{1}{32\theta^5} \left\{ \begin{aligned} &8\theta^2 + 12\alpha\theta + 24\alpha - e^{-2\theta\mu} \\ &(8\theta^2 + 16\mu\theta^3 + 12\alpha\theta + 16\mu^2\theta^4 + 24\mu\theta^2\alpha + 24\mu^2\theta^3\alpha + 16\mu^3\theta^4\alpha \\ &+ 48\mu\theta\alpha + 24\alpha + 48\theta^2\mu^2\alpha + 32\mu^3\theta^3\alpha + 16\mu^4\theta^4\alpha) \end{aligned} \right\}$$

And

$$\delta_2(X) = \left[\mu - 2 \left\{ \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A' + 2\lambda \left\{ B' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C' + (\alpha\theta + 2)D') \right\} \right] \right\} \right]$$

Where

$$A' = \left(\frac{1}{\theta^4} \{ \theta^2 + 2\theta\alpha + 6\alpha - e^{-\theta M} (\theta^2 + M\theta^3 + 2\alpha\theta + 2M\alpha\theta^2 + M^2\theta^3\alpha + 6\alpha\theta M + 6\alpha + 3M^2\theta^2\alpha + M^3\theta^3\alpha) \} \right)$$

$$B' = \left(\frac{1}{16\theta^4} \{ 4\theta^2 + 4\alpha\theta + 6\alpha - e^{-2\theta M} (4\theta^2 + 8M\theta^3 + 4\alpha\theta + 8M\alpha\theta^2 + 8M^2\theta^3\alpha + 12\alpha\theta M + 6\alpha + 12M^2\theta^2\alpha + 8M^3\theta^3\alpha) \} \right)$$

$$C' = \left(\frac{1}{64\theta^5} \left\{ \begin{aligned} &24\theta^2 + 48\alpha\theta + 120\alpha - e^{-2\theta M} \\ &(24\theta^2 + 48M\theta^3 + 48\alpha\theta + 48M^2\theta^4 + 32M^3\theta^5 + 96M\theta^2\alpha + 96M^2\theta^3\alpha + 64M^3\theta^4\alpha \\ &+ 32M^4\theta^5\alpha + 32M^5\theta^5\alpha + 240M\theta\alpha + 120\alpha + 240\theta^2M^2\alpha + 160M^3\theta^3\alpha + 80M^4\theta^4\alpha) \end{aligned} \right\} \right)$$

$$D' = \frac{1}{32\theta^5} \left\{ \begin{aligned} &8\theta^2 + 12\alpha\theta + 24\alpha - e^{-2\theta M} \\ &(8\theta^2 + 16M\theta^3 + 12\alpha\theta + 16M^2\theta^4 + 24M\theta^2\alpha + 24M^2\theta^3\alpha + 16M^3\theta^4\alpha \\ &+ 48M\theta\alpha + 24\alpha + 48\theta^2M^2\alpha + 32M^3\theta^3\alpha + 16M^4\theta^4\alpha) \end{aligned} \right\}$$

respectively.

Proof: Mean deviation about mean and mean deviation about median are defined as

$$\delta_1(X) = \int_0^\infty |x - \mu| f_\tau(x, \alpha, \theta, \lambda) dx$$

$$\text{And } \delta_2(X) = \int_0^\infty |x - M| f_\tau(x, \alpha, \theta, \lambda) dx$$

respectively.

Where μ and M are mean and median respectively of random variable $X \sim \text{TATPSD}$. The measures $\delta_1(X)$ and $\delta_2(X)$ can be obtained by using the simplified relationships.

$$\begin{aligned} \delta_1(X) &= \int_0^\mu (\mu - x) f_\tau(x, \alpha, \theta, \lambda) dx + \int_\mu^\infty (x - \mu) f_\tau(x, \alpha, \theta, \lambda) dx \\ \delta_1(X) &= 2\mu F_\tau(\mu) - 2 \int_0^\mu x f_\tau(x, \alpha, \theta, \lambda) dx \end{aligned} \quad (5.3.1)$$

And

$$\begin{aligned} \delta_2(X) &= \int_0^M (M - x) f_\tau(x, \alpha, \theta, \lambda) dx + \int_M^\infty (x - M) f_\tau(x, \alpha, \theta, \lambda) dx \\ \delta_2(X) &= \mu - 2 \int_0^M x f_\tau(x, \alpha, \theta, \lambda) dx \end{aligned} \quad (5.3.2)$$

Where

$$f_\tau(x, \alpha, \theta, \lambda) = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\}$$

Now

$$\int_0^\mu x f_\tau(x, \alpha, \theta, \lambda) dx = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A + 2\lambda \left\{ B + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C + (\alpha\theta + 2)D) \right\} \right] \quad (5.3.3)$$

Where

$$A = \frac{1}{\theta^4} \{ \theta^2 + 2\theta\alpha + 6\alpha - e^{-\theta\mu}(\theta^2 + \mu\theta^3 + 2\alpha\theta + 2\mu\alpha\theta^2 + \mu^2\theta^3\alpha + 6\alpha\theta\mu + 6\alpha + 3\mu^2\theta^2\alpha + \mu^3\theta^3\alpha) \}$$

$$B = \frac{1}{16\theta^4} \{ (4\theta^2 + 4\alpha\theta + 6\alpha - e^{-2\theta\mu} (4\theta^2 + 8\mu\theta^3 + 4\alpha\theta + 8\mu\alpha\theta^2 + 8\mu^2\theta^3\alpha + 12\alpha\theta\mu + 6\alpha + 12\mu^2\theta^2\alpha + 8\mu^3\theta^3\alpha)) \}$$

$$C = \left| \frac{1}{64\theta^5} \left\{ \begin{aligned} &24\theta^2 + 48\alpha\theta + 120\alpha - e^{-2\theta\mu} \\ &(24\theta^2 + 48\mu\theta^3 + 48\alpha\theta + 48\mu^2\theta^4 + 32\mu^3\theta^5 + 96\mu\theta^2\alpha + 96\mu^2\theta^3\alpha + 64\mu^3\theta^4\alpha \\ &+ 32\mu^4\theta^5\alpha + 32\mu^5\theta^5\alpha + 240\mu\theta\alpha + 120\alpha + 240\theta^2\mu^2\alpha + 160\mu^3\theta^3\alpha + 80\mu^4\theta^4\alpha) \end{aligned} \right\} \right|$$

$$D = \frac{1}{32\theta^5} \left\{ \begin{aligned} &8\theta^2 + 12\alpha\theta + 24\alpha - e^{-2\theta\mu} \\ &(8\theta^2 + 16\mu\theta^3 + 12\alpha\theta + 16\mu^2\theta^4 + 24\mu\theta^2\alpha + 24\mu^2\theta^3\alpha + 16\mu^3\theta^4\alpha \\ &+ 48\mu\theta\alpha + 24\alpha + 48\theta^2\mu^2\alpha + 32\mu^3\theta^3\alpha + 16\mu^4\theta^4\alpha) \end{aligned} \right\}$$

And

$$\int_0^M x f_\tau(x, \alpha, \theta, \lambda) dx = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A' + 2\lambda \left\{ B' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C' + (\alpha\theta + 2)D') \right\} \right] \quad (5.3.4)$$

Where

$$A' = \frac{1}{\theta^4} \left\{ \theta^2 + 2\theta\alpha + 6\alpha - e^{-\theta M} \right. \\ \left. (\theta^2 + M\theta^3 + 2\alpha\theta + 2M\alpha\theta^2 + M^2\theta^3\alpha + 6\alpha\theta M + 6\alpha + 3M^2\theta^2\alpha + M^3\theta^3\alpha) \right\}$$

$$B' = \left(\frac{1}{16\theta^4} \left\{ \theta^2 + 4\alpha\theta + 6\alpha - e^{-2\theta M} \right. \right. \\ \left. \left. (4\theta^2 + 8M\theta^3 + 4\alpha\theta + 8M\alpha\theta^2 + 8M^2\theta^3\alpha + 12\alpha\theta M + 6\alpha + 12M^2\theta^2\alpha + 8M^3\theta^3\alpha) \right\} \right)$$

$$C' = \frac{1}{64\theta^5} \left\{ \begin{aligned} &24\theta^2 + 48\alpha\theta + 120\alpha - e^{-2\theta M} \\ &(24\theta^2 + 48M\theta^3 + 48\alpha\theta + 48M^2\theta^4 + 32M^3\theta^5 + 96M\theta^2\alpha + 96M^2\theta^3\alpha + 64M^3\theta^4\alpha \\ &+ 32M^4\theta^5\alpha + 32M^5\theta^5\alpha + 240M\theta\alpha + 120\alpha + 240\theta^2M^2\alpha + 160M^3\theta^3\alpha + 80M^4\theta^4\alpha) \end{aligned} \right\}$$

$$D' = \frac{1}{32\theta^5} \left\{ \begin{aligned} &8\theta^2 + 12\alpha\theta + 24\alpha - e^{-2\theta M} \\ &(8\theta^2 + 16M\theta^3 + 12\alpha\theta + 16M^2\theta^4 + 24M\theta^2\alpha + 24M^2\theta^3\alpha + 16M^3\theta^4\alpha \\ &+ 48M\theta\alpha + 24\alpha + 48\theta^2M^2\alpha + 32M^3\theta^3\alpha + 16M^4\theta^4\alpha) \end{aligned} \right\}$$

Using expressions (5.3.1), (5.3.2), (5.3.3) and (5.3.4) and expression for c.d.f (2.2) we obtain mean deviation about mean ($\delta_1(X)$) and mean deviation about median ($\delta_2(X)$)

$$\delta_1(X) = \left[\begin{aligned} &2\mu \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \\ &- 2 \left\{ \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A + 2\lambda \left\{ B + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C + (\alpha\theta + 2)D) \right\} \right] \right\} \end{aligned} \right]$$

&

$$\delta_2(X) = \left[\mu - 2 \left\{ \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A' + 2\lambda \left\{ B' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C' + (\alpha\theta + 2)D') \right\} \right] \right\} \right]$$

6. BONFERRONI AND LORENZ CURVES AND INDICES OF TATPSD

The Bonferroni curve($B(p)$), Lorenz curve($L(p)$), Bonferroni index (B) and Gini index (G) have find applicability in fields of economics, demography, reliability, life testing and medical sciences. The Bonferroni and Lorenz curves are defined as

$$B(p) = \frac{1}{p\mu} \int_0^q x f_{\tau}(x, \alpha, \theta, \lambda) dx \quad (6.1)$$

$$L(p) = \frac{1}{\mu} \int_0^q x f_{\tau}(x, \alpha, \theta, \lambda) dx \quad (6.2)$$

Where $\mu = E(x)$ is the mean of TATPSD and $q = F^{-1}(p)$.

The Bonferroni and Gini indices are defined as

$$B = 1 - \int_0^1 B(p) dp \quad (6.3)$$

$$G = 1 - 2 \int_0^1 L(p) dp \quad (6.4)$$

Using the p.d.f (2.4) of TATPSD we get

$$\int_0^q x f_{\tau}(x, \alpha, \theta, \lambda) dx = \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A'' + 2\lambda \left\{ B'' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C'' + (\alpha\theta + 2)D'') \right\} \right] \quad (6.5)$$

Where

$$A'' = \frac{1}{\theta^4} \left\{ \theta^2 + 2\theta\alpha + 6\alpha - e^{-\theta q} \right. \\ \left. (\theta^2 + q\theta^3 + 2\alpha\theta + 2q\alpha\theta^2 + q^2\theta^3\alpha + 6\alpha\theta q + 6\alpha + 3q^2\theta^2\alpha + q^3\theta^3\alpha) \right\}$$

$$B'' = \frac{1}{16\theta^4} \left\{ 4\theta^2 + 4\alpha\theta + 6\alpha - e^{-2\theta q} \right. \\ \left. (4\theta^2 + 8q\theta^3 + 4\alpha\theta + 8q\alpha\theta^2 + 8q^2\theta^3\alpha + 12\alpha\theta q + 6\alpha + 12q^2\theta^2\alpha + 8q^3\theta^3\alpha) \right\}$$

$$C'' = \frac{1}{64\theta^5} \left\{ \begin{aligned} &24\theta^2 + 48\alpha\theta + 120\alpha - e^{-2\theta q} \\ &(24\theta^2 + 48q\theta^3 + 48\alpha\theta + 48q^2\theta^4 + 32q^3\theta^5 + 96q\theta^2\alpha + 96q^2\theta^3\alpha + 64q^3\theta^4\alpha \\ &+ 32q^4\theta^5\alpha + 32q^5\theta^5\alpha + 240q\theta\alpha + 120\alpha + 240\theta^2q^2\alpha + 160q^3\theta^3\alpha + 80q^4\theta^4\alpha) \end{aligned} \right\}$$

$$D'' = \frac{1}{32\theta^5} \left\{ \begin{aligned} &8\theta^2 + 12\alpha\theta + 24\alpha - e^{-2\theta q} \\ &(8\theta^2 + 16q\theta^3 + 12\alpha\theta + 16q^2\theta^4 + 24q\theta^2\alpha + 24q^2\theta^3\alpha + 16q^3\theta^4\alpha \\ &+ 48q\theta\alpha + 24\alpha + 48\theta^2q^2\alpha + 32q^3\theta^3\alpha + 16q^4\theta^4\alpha) \end{aligned} \right\}$$

Using equation (6.5) in (6.1) & (6.2) we get

$$B(p) = \frac{1}{p\mu} \left[\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A'' + 2\lambda \left\{ B'' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C'' + (\alpha\theta + 2)D'') \right\} \right] \right] \quad (6.6)$$

And

$$L(p) = \frac{1}{\mu} \left[\left[\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A'' + 2\lambda \left\{ B'' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C'' + (\alpha\theta + 2)D'') \right\} \right] \right] \right] \quad (6.7)$$

Using (6.6) & (6.7) in (6.3) & (6.4) we get

$$B = 1 - \frac{1}{\mu} \left[\left[\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A'' + 2\lambda \left\{ B'' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C'' + (\alpha\theta + 2)D'') \right\} \right] \right] \right] \quad (6.8)$$

$$L = 1 - \frac{2}{\mu} \left[\left[\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} \left[(1 - \lambda)A'' + 2\lambda \left\{ B'' + \frac{\alpha\theta}{(\theta^2 + \alpha\theta + 2\alpha)} (C'' + (\alpha\theta + 2)D'') \right\} \right] \right] \right] \quad (6.9)$$

7. ORDER STATISTICS OF TATPSD

Assuming $X_{(1)}, X_{(2)}, X_{(3)}, \dots, X_{(n)}$ to be the ordered statistics of the random sample $x_1, x_2, x_3, \dots, x_n$ obtained from the Transmuted another Two-Parameter Sujatha distribution with cumulative distribution function $F_T(x, \alpha, \theta, \lambda)$ and probability density function $f_T(x, \alpha, \theta, \lambda)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by:

$$f_{(r)}(x, \alpha, \theta, \lambda) = \frac{n!}{(r-1)!(n-r)!} f(x, \alpha, \theta, \lambda) [F(x, \alpha, \theta, \lambda)]^{r-1} [1 - F(x, \alpha, \theta, \lambda)]^{n-r}.$$

$$r=1, 2, 3, \dots, n$$

Using the equations (2.2) and (2.4), the probability density function of r^{th} order statistics of transmuted another Two-Parameter Sujatha distribution is given by:

$$f_{(r)}(x, \alpha, \theta, \lambda) =$$

$$\left[\frac{n!}{(r-1)!(n-r)!} \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right] \left[\left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right]^{r-1} \left[\left\{ 1 - \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right\} \right]^{n-r}.$$

Then, the p.d.f of first order statistic $X_{(1)}$ of Transmuted another Two-Parameter Sujatha distribution is given by:

$$f_{(1)}(x, \alpha, \theta, \lambda) =$$

$$\left[n \frac{\theta^2}{(\alpha\theta + 2)^2} \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right] \left[\left\{ 1 - \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right\} \right]^{n-1}.$$

and the pdf of n^{th} order statistic $X_{(n)}$ of Transmuted another Two-Parameter Sujatha distribution is given as:

$$f_{(n)}(x, \alpha, \theta, \lambda) =$$

$$\left[n \frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right] \left[\left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \left\{ (1 + \lambda) - \lambda \left\{ 1 - \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right]^{n-1}.$$

8. ESTIMATION OF PARAMETERS OF TATPSD

Assuming $x_1, x_2, x_3, \dots, x_n$ to be the random sample of size n drawn from Transmuted another Two-Parameter Sujatha distribution having density function given by (2.4), then the likelihood function of Transmuted another Two-Parameter Sujatha distribution is given as:

$$L(x|\alpha, \theta, \lambda) = \prod_{i=1}^n \left[\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)} (1 + \alpha x + \alpha x^2) e^{-\theta x} \left\{ (1 - \lambda) + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\} \right]$$

The log likelihood function becomes:

$$\log L = \left(\begin{aligned} & \left\{ 3n \log \theta - n \log(\theta^2 + \alpha\theta + 2\alpha) - \theta \sum_{i=1}^n \{1 + \alpha x_i + \alpha x_i^2\} x_i + \sum_{i=1}^n \log \{ \} \right\} \\ & + \sum_{i=1}^n \log \left\{ (1 - \lambda) + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x_i} \right\} \right\} \end{aligned} \right) \quad (8.1)$$

Differentiating the log-likelihood function with respect to α , θ and λ . This is done by partially differentiate (8.1) with respect to θ, α and λ equating the result to zero, we obtain the following normal equations,

$$\frac{\partial \log L}{\partial \theta} = \left[\begin{aligned} & \left[\frac{3n}{\theta} - \frac{n(2\theta + \alpha)}{(\theta^2 + \alpha\theta + 2\alpha)} - \sum_{i=1}^n x_i \right. \\ & \left. + \sum_{i=1}^n \left\{ \frac{2\lambda e^{-\theta x_i} \left\{ \frac{(\theta^2 + \alpha\theta + 2\alpha)(2\alpha\theta x_i^2 + 2\theta\alpha^2 x_i + 2\alpha x_i) - (\alpha\theta x_i(\theta x_i + \alpha\theta + 2)(2\theta + \alpha))}{(\theta^2 + \alpha\theta + 2\alpha)^2} \right\} - x_i \left(1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right) \right\}}{(1 - \lambda) + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x_i} \right\}} \right] \right] = 0 \quad (8.2) \end{aligned}$$

$$\frac{\partial \log L}{\partial \alpha} = \left(\begin{aligned} & \left[-\frac{n(\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} + \sum_{i=1}^n \left(\frac{x_i + x_i^2}{1 + \alpha x_i + \alpha x_i^2} \right) \right. \\ & \left. + \sum_{i=1}^n \left\{ \frac{2\lambda e^{-\theta x_i} \{ (\theta^2 x_i^2 + 2\alpha\theta^2 x_i + 2\theta x_i)(\theta^2 + \alpha\theta + 2\alpha) - \alpha\theta x_i(\theta x_i + \alpha\theta + 2)(\theta + 2) \}}{(\theta^2 + \alpha\theta + 2\alpha)^2 \left\{ (1 - \lambda) + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x_i} \right\} \right\}} \right\} \right] \right) = 0 \quad (8.3) \end{aligned}$$

$$\frac{\partial \log L}{\partial \lambda} = \sum_{i=1}^n \left[\frac{\left\{ 2 \left[1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x_i} - 1 \right\}}{\left\{ (1 - \lambda) + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x_i(\theta x_i + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x_i} \right\} \right\}} \right] = 0 \quad (8.4)$$

MLEs of α, θ, λ cannot be obtained by solving above complex equations as these equations are not in closed form. So we solve above equations by using iteration method through R software.

9. MOTIVATION BEHIND PROPOSED MODEL

Addition of transmuted parameter has increased flexibility in terms of moments of distribution. So it was the flexibility of proposed model which is increased by transmuted parameter which motivated me to work on this model. Also proposed model has increasing as well as decreasing hazard rate as can be seen from graph 4, which is an interesting feature and finds greater applicability in real life.

10. APPLICATIONS OF TATPSD

We have analyzed two real life data sets to show that the transmuted another two-parameter Sujatha distribution can be a better model than another two-parameter Sujatha distribution, exponential distribution and Lindley Pareto distribution. We also tested the significance of transmuted parameter.

DATA SET 1: The data set given in table 1 is related to the burning velocity (cm/sec) of different chemical materials. The source of the data set related to the burning velocity of different chemical materials for the year 2005 is available on the website and has been used by Sajid and Riyaz (2014) [12].

TABLE 1: Burning velocity (cm/sec) of different chemical materials

68	61	64	55	51	68	44	82
60	89	61	54	166	66	50	87
48	42	58	46	67	46	46	44
48	56	47	54	47	80	38	108
46	40	44	312	41	31	40	41
40	56	45	43	46	46	46	46
52	58	82	71	48	39	41	

DATA SET 2: The data set given in table 2 represents the survival times (in months). This data is non-censored data for lung cancer patients obtained from Pena (2002). This data set was recently used by L. S. Diab & E. S. El-Atfy (2017) in

paper “A moment inequality for overall decreasing life class of life distributions with hypothesis testing applications” [13].

TABLE 2: Survival times (in months) for lung cancer patients.

0.99	1.28	1.77	1.97	2.17	2.63	2.66	2.76	2.79	2.86
2.99	3.06	3.15	3.45	3.71	3.75	3.81	4.11	4.27	4.34
4.40	4.63	4.73	4.93	4.93	5.03	5.16	5.17	5.49	5.68
5.72	5.85	5.98	8.15	8.62	8.48	8.61	9.46	9.53	10.05
10.15	10.94	10.94	11.24	11.63	12.26	12.65	12.78	13.18	13.47
13.96	14.88	15.05	15.31	16.13	16.46	17.45	17.61	18.20	18.37
19.06	20.70	22.54	23.36						

These data sets are used here only for illustrative purposes. The required numerical evaluations are carried out using R software version R 3.3.5. We have fitted another two-parameter Sujatha distribution, exponential distribution, Lindley Pareto distribution and transmuted another two-parameter Sujatha distribution to these data sets. The summary statistics of the data set 1 & 2 is displayed in table 3, MLEs of the parameters, model functions are displayed in table 4 for both the data sets and the corresponding log-likelihood values, LR statistic, AIC, AICC, BIC, HQIC, Kolmogorov statistic and p value are displayed in Table 5 & 6 for data sets 1 & 2 respectively.

TABLE 3: Summary statistic of data sets 1 & 2.

Data Set	No. of observations	Min.	First quartile	median	mean	Third quartile	Max.
Data Set 1	55	31.0	44.5	48.0	61.0	62.5	312.0
Data Set 2	64	0.990	3.795	5.915	8.710	12.880	23.360

TABLE 4: ML Estimates, Standard Error of Estimates in parenthesis, model function of proposed model and related models for data sets 1 & 2.

Data set	Distribution	ML estimates (standard errors)	Model Function
1	Transmuted Another Two-Parameter Sujatha Distribution (TATPSD)	$\hat{\theta} = 0.06406471$ $\hat{\alpha} = 0.29756831$ (0.63519694) $\hat{\lambda} = -1.00000000$	$\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)}(1 + \alpha x + \alpha x^2)e^{-\theta x}$ $\left\{ (1 - \lambda) \right.$ $\left. + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\}$
	Another Two-Parameter Sujatha Distribution (TATPSD)	$\hat{\theta} = 0.04879202$ (0.00379731) $\hat{\alpha} = 7.88106111$ (75.65217391)	$\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)}(1 + \alpha x + \alpha x^2)e^{-\theta x}$
	Exponential Distribution (ED)	$\hat{\theta} = 61.000000$ (8.225236)	$\frac{1}{\theta} e^{-\frac{x}{\theta}}$
	Lindley Pareto Distribution (LPD)	$\hat{\theta} = 0.007626414$ (0.003924553) $\hat{\alpha} = 0.916469935$ (0.470474649) $\hat{k} = 1.313641828$ (0.107580641)	$\frac{k\theta^2 e^{\theta} x^{2k-1}}{(\theta + 1)\alpha^{2k}} e^{-\theta\left(\frac{x}{\alpha}\right)^k}$
2	Transmuted Another Two-Parameter Sujatha Distribution (TATPSD)	$\hat{\theta} = 0.28884058$ (0.01689813) $\hat{\alpha} = 0.02223941$ $\hat{\lambda} = -1.00000000$	$\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)}(1 + \alpha x + \alpha x^2)e^{-\theta x}$ $\left\{ (1 - \lambda) \right.$ $\left. + 2\lambda \left\{ \left[1 + \frac{\alpha\theta x(\theta x + \alpha\theta + 2)}{(\theta^2 + \alpha\theta + 2\alpha)} \right] e^{-\theta x} \right\} \right\}$
	Another Two-Parameter Sujatha Distribution (ATPSD)	$\hat{\theta} = 0.3254563$ $\hat{\alpha} = 3.2962191$	$\frac{\theta^3}{(\theta^2 + \alpha\theta + 2\alpha)}(1 + \alpha x + \alpha x^2)e^{-\theta x}$

	Exponential Distribution (ED)	$\hat{\theta} = 8.71000$ (1.08875)	$\frac{1}{\theta} e^{-\frac{x}{\theta}}$
	Lindley Pareto Distribution (LPD)	$\hat{\theta} = 0.02029546$ (0.02490351) $\hat{\alpha} = 0.08779542$ (0.11279414) $\hat{k} = 0.99870938$ (0.09371982)	$\frac{k\theta^2 e^{\theta} x^{2k-1}}{(\theta+1)\alpha^{2k}} e^{-\theta(\frac{x}{\alpha})^k}$

TABLE 5: Model comparison and Likelihood ratio statistic of related models and proposed model for data set 1.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	K-S (D)	P Value	Likelihood Ratio
TATPSD	251.695	509.390	515.41	509.861	511.719	0.1888	0.0395	14.48
ATPSD	258.935	521.871	525.88	522.102	523.424	0.2246	0.0077	
ED	281.098	564.196	566.20	564.271	564.972	0.4454	6.628e-10	
LPD	260.174	526.348	532.37	526.819	528.677	0.2523	0.00181	

TABLE 6: Model comparison and Likelihood ratio statistic of related models and proposed model for data set 2.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	K-S (D)	P Value	Likelihood Ratio
TATPSD	193.137	392.274	398.75	392.674	394.826	0.1156	0.358	5.936
ATPSD	196.105	396.210	400.52	396.407	397.911	0.2542	0.0005	
ED	202.526	407.052	409.21	407.116	407.902	0.1825	0.0281	
LPD	194.481	394.963	401.43	395.363	397.514	0.1162	0.352	

For testing the significance of transmuted parameter λ of our proposed model and for checking superiority of Transmuted Another Two-Parameter Sujatha Distribution over Another Two-Parameter Sujatha Distribution, Lindley Pareto distribution and Exponential Distribution for given data sets 1 & 2 we computed likelihood ratio (LR) statistic. For testing $H_0: \lambda = 0$ versus $H_1: \lambda \neq 0$ the LR statistic for testing H_0 is $\omega_1 = 2\{L(\hat{\theta}) - L(\hat{\theta}_0)\} = 14.48$ for data set 1 & $\omega_2 = 2\{L(\hat{\theta}) - L(\hat{\theta}_0)\} = 5.936$ for data set 2 where $\hat{\theta}$ and $\hat{\theta}_0$ are MLEs under H_1 and H_0 . LR statistic $\omega \sim (\chi_{(1)}^2(\alpha = 0.05) = 3.841)$ as $n \rightarrow \infty$, where 1 = degrees of freedom is the difference in dimensionality. From table 5, 6 $\omega_1 = 14.48 > 3.841$ & $\omega_2 = 5.936 > 3.841$ at 5% level of significance for both the data sets, so we reject H_0 and conclude that transmuted parameter λ plays statistically a significant role.

Further in order to compare the proposed model with related models, we consider the criteria like AIC (Akaike information criterion), AICC (corrected Akaike information criterion), BIC (Bayesian information criterion) and HQIC. The better distribution corresponds to lesser AIC, AICC, BIC and HQIC values.

$$\begin{aligned} \text{AIC} &= 2k - 2\log L & \text{AICC} &= \text{AIC} + \frac{2k(k+1)}{n-k-1} \\ \text{BIC} &= k \log n - 2\log L & \text{HQIC} &= 2k \log(\log(n)) + 2 \log L \end{aligned}$$

where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of the log-likelihood function under the considered model. From Table 5 & 6, it has been observed that the Transmuted Another Two-Parameter Sujatha distribution possesses the lesser AIC, AICC BIC and HQIC values as compared to Another Two-Parameter Sujatha distribution, Lindley Pareto distribution & Exponential distribution for data sets 1 & 2. Hence we can conclude that the Transmuted Another Two-Parameter Sujatha distribution leads to a better fit than Another Two-Parameter Sujatha distribution and exponential distribution for data sets 1 & 2

For testing the goodness of fit of our proposed model Transmuted Another Two Parameter Sujatha distribution over its related models Another Two-Parameter Sujatha distribution, Lindley Pareto distribution & Exponential distribution to the

two data sets we computed Kolmogorov statistic and p value. The better model possesses lesser Kolmogorov statistic value and higher p value. It can be seen from tables 5 & 6 that Transmuted Another Two-Parameter Sujatha distribution possesses lesser Kolmogorov statistic value and higher p value for both the data sets as compared to Another Two-Parameter Sujatha distribution, Lindley Pareto distribution & Exponential distribution.

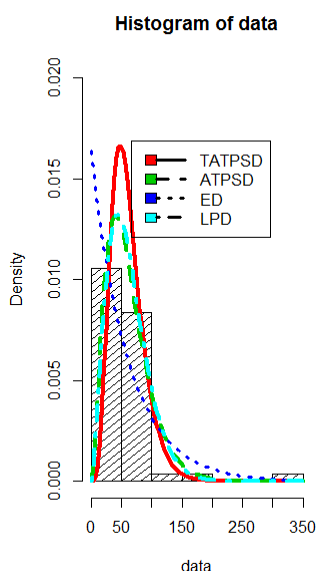


Figure 5 Curve fitting of data set 1

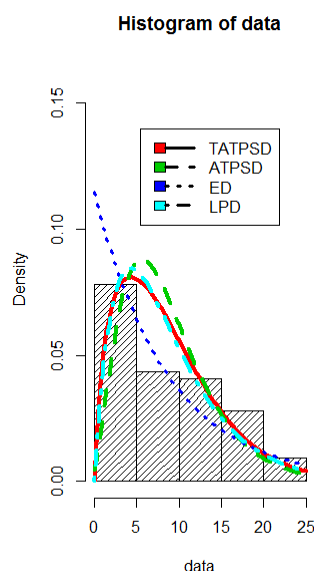


Figure 6 curve fitting of data set 2

11. CONCLUSION

We developed Transmuted another Two-Parameter Sujatha Distribution by using Quadratic Rank Transmutation Map technique. The various important properties of this distribution have been obtained. From the p.d.f plots (graphs 2(a) & 2(b)) it has been observed that our proposed model is positively skewed. Various reliability measures have been obtained for the proposed model. We fitted our model and its related models to two real life data sets and found that our proposed model gives better results for both the data sets than its related models as Kolmogorov statistic value, AIC, BIC, AICC, HQIC are lesser for proposed model on the both data sets than base model and exponential distribution. The significance of

transmuted parameter has been tested and it has been concluded that transmuted parameter plays a significant role. Researchers in future can generalize the proposed model and proposed model finds applicability in life testing.

12. BENEFITS OF THE PROPOSED MODEL

The proposed model can be used in variety of real life situations. As can be seen from application part of the paper, proposed model can be used for modeling of survival time's data. Also statistical significance of transmuted parameter shows the importance of proposed model.

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