Improved strategy for computation of population mean under double stratified sampling framework

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Abstract

The article contains a new technique to estimate the mean of the variate of the interest of the finite population with the help of two auxiliary variates. The technique complies well with the stratified population in which each strata proportion is predicted by taking an initial sample called the first phase sample. When the first phase sample is taken, a second sample is taken from the first sample which is called the second phase sample which is used to estimate the mean of the variate of the interest. In our study, we have considered the population which has two correlated auxiliary variates that pass almost through the origin. In such a situation ratio estimation technique and product estimation technique that provides improved estimates of the mean of the variate of the interest. Our technique considers a ratio-product type exponential estimator of which we have established efficiency theoretically as well as empirically.

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Keywords: Auxiliary variate, Study variate, Double stratified sampling scheme, Bias, Mean square error.

1. INTRODUCTION

In sample survey, surveyors use the auxiliary variates in order to increase the accuracy in the estimation of population parameters when the associated variates are readily and inexpensively available to the entire population. Many times the associated variates are not readily available, in which case, a larger sample is taken provided the collection of information on auxiliary variate is cheap, easy and economical. In many surveys, gathering information on variate of the interest is quite costly and gaining information on auxiliary variates are cheap, easy and economical or auxiliary variate are easily available. If the variate of the interest and auxiliary variates are highly correlated then the efficiency of the estimators may be increased by using auxiliary variate. But this will benefit only if the attainment in precision that is achieved by using the sample auxiliary variate weigh up the loss in precision that has been incurred because of the reduction in the size of the sample for computing mean of the variate of the interest. Surveyors also use auxiliary variates to estimate the population heterogeneity when there is absence of information about

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heterogeneity. Thus, in order to estimate the population mean of different levels of study, our population study is divided into different levels and sample observations have been prepared from each level using the double sampling method.

This article has drawn inspiration from many of the articles like Chand, 1975 and its chain type estimator for the double sampling, Bahl and Tuteja, 1991 product and ratio type exponential estimators, Upadhyay and Singh, 1999 noteworthily contributed in this area and motivated Kadilar and Cingi, 2003, Kadilar and Cingi, 2005, Shabbir and Gupta, 2005. Singh and Vishwakarma, 2007 evolved the method further. Samiuddin and Hanif, 2007, Singh et. al., 2011, Sanaulla et. al., 2014 and Vishwakarma and Singh, 2015 have used two auxiliary variates for double stratified sampling method. All in all, inspired by their work, a need was felt to propose a new methodology in this area, and the results are discussed in a subsequent section.

2. DOUBLE STRATIFIED RANDOM SAMPLING

Let there be a finite population, U_j ($j = 1, 2, 3, \ldots, N$), of size N that can be divided into strata. Let the variates that can be measured with each unit be a target variate y and x and z be the associated variates that information is either fully available or if not then it can be measured easily and economically. Let L be the number of strata that can be formed from the population. Let N_h ($h = 1, 2, 3, \ldots, L$) be size of h^{th}- stratum such that $\sum_a^b N_h = N$ and $W_h = N_h / N$. Let $\left(\bar{y}_h = \sum_{i=1}^{n_h} y_{hi}/n_h,\ \bar{x}_h = \sum_{i=1}^{n_h} x_{hi}/n_h,\ \bar{z}_h = \sum_{i=1}^{n_h} z_{hi}/n_h\right)$ be the unbiased estimators of the variate of the interest and the associated auxiliary variates of the population mean $\left(\bar{Y}_h = \sum_{i=1}^{N_h} y_{hi}/N_h,\ \bar{X}_h = \sum_{i=1}^{N_h} x_{hi}/N_h,\ \bar{Z}_h = \sum_{i=1}^{N_h} z_{hi}/N_h\right)$, based on n_h observations. When information on $\bar{X}_h$ is unknown Double Stratified Sampling method comes to rescue by using first phase sample to note down value on auxiliary variates. The procedures for double stratified sampling method is to select a sample of size n_h' from the h^{th} - stratum using simple random sampling without replacement (*SRSWOR*) where $\sum_{h=1}^{L} n_h' = n'$ and note down the value on the observed auxiliary variate(s) for these units. This will be called a stratified first phase sample. Now, for noting down the

information on variate of the interest, another sample of size $n_h'(n_h < n_h')$ is drawn from the each stratum using SRSWOR such that $\sum_{h=1}^{L} n_h = n$. This will be called second phase sample.

Now, for the given conditions below:

$$\left.\begin{aligned}
&\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h, \bar{y}_h = \bar{Y}_h(1+e_{0h}), e_0 = \sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}/\bar{Y}, \\
&\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h, \bar{x}_h' = \bar{X}_h(1+e_{1h}'), e_1 = \sum_{h=1}^{L} W_h \bar{X}_h e_{1h}'/\bar{X}, \\
&\bar{x}_h = \bar{X}_h(1+e_{1h}), e_1 = \sum_{h=1}^{L} W_h \bar{X}_h e_{1h}/\bar{X}, \\
&\bar{Z} = \sum_{h=1}^{L} W_h \bar{Z}_h, \bar{z}_h' = \bar{Z}_h(1+e_{2h}'), e_2 = \sum_{h=1}^{L} W_h \bar{Z}_h e_{2h}'/\bar{Z}, \\
&\bar{z}_h = \bar{Z}_h(1+e_{2h}), e_2 = \sum_{h=1}^{L} W_h \bar{Z}_h e_{2h}/\bar{Z},
\end{aligned}\right\} \quad (1)$$

The expectations are defined as,

$$\left.\begin{aligned}
&E(e_0) = E(e_1) = E(e_1') = E(e_2) = E(e_2') = 0 \\
&E(e_0^2) = \frac{1}{\bar{Y}^2} \sum_{h=1}^{L} W_h^2 f_{1h} S_{yh}^2 = V_{200} \\
&E(e_1^2) = \frac{1}{\bar{X}^2} \sum_{h=1}^{L} W_h^2 f_{1h} S_{xh}^2 = V_{020} \\
&E(e_1'^2) = \frac{1}{\bar{X}^2} \sum_{h=1}^{L} W_h^2 f_{2h} S_{xh}^2 = V_{020}' \\
&E(e_2^2) = \frac{1}{\bar{Z}^2} \sum_{h=1}^{L} W_h^2 f_{1h} S_{zh}^2 = V_{002} \\
&E(e_2'^2) = \frac{1}{\bar{Z}^2} \sum_{h=1}^{L} W_h^2 f_{2h} S_{zh}^2 = V_{002}'
\end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned}
&E(e_0e_1') = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2 f_{2h} S_{yxh} = V_{110}',\ E(e_0e_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2 f_{1h} S_{yxh} = V_{110},\\
&E(e_0e_2') = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{2h} S_{yzh} = V_{101}',\ E(e_0e_2) = \frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{1h} S_{yzh} = V_{101},\\
&E(e_1'e_1) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 f_{2h} S_{xh}^2 = V_{020}',\ E(e_1'e_2') = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{2h} S_{zxh} = V_{011}',\\
&E(e_1'e_2) = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{2h} S_{zxh} = V_{011}',\ E(e_1e_2') = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{2h} S_{zxh} = V_{011}',\\
&E(e_1e_2) = \frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L} W_h^2 f_{1h} S_{zxh} = V_{011},\ E(e_2'e_2) = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L} W_h^2 f_{2h} S_{zh}^2 = V_{002}'
\end{aligned}\right\} \tag{3}$$

where $f_{1h} = \left(\frac{1}{n_h} - \frac{1}{n_h'}\right),\ f_{2h} = \left(\frac{1}{n_h'} - \frac{1}{N_h}\right)\ f_{3h} = (f_{1h} - f_{2h})$

$$S_{hy}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2,\ S_{hx}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N}(x_{hi} - \bar{X}_h)^2,$$

$$S_{hz}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(z_{hi} - \bar{Z}_h)^2, S_{hyx} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h),$$

$$S_{hyz} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)(z_{hi} - \bar{Z}_h),\ S_{hxz} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)(z_{hi} - \bar{Z}_h).$$

3. SOME AVAILABLE ESTIMATOR

Some of the popular estimators that are available in the literature are reproduced here. Usual unbiased estimator for population mean $\bar{Y}$ in case of double stratified random sampling, is given by

$$\bar{y}_{(st)} = \sum_{h=1}^{L} W_h \bar{y}_h, \tag{4}$$

where

$$W_h = \frac{N_h}{N}$$

The variance of the unbiased estimator is,

$$Var(\bar{y}_{(st)}) = \bar{Y}^2\left(\frac{\sum_{h=1}^{L} W_h^2 f_{1h} S_{yh}^2}{\bar{Y}^2}\right) = \bar{Y}^2 V_{200} \tag{5}$$

The ratio and product estimators for double stratified random sampling is,

$$\hat{\bar{Y}}_{Rd} = \bar{y}_{st}\frac{\bar{x}_{st}^{'}}{\bar{x}_{st}} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\sum_{h=1}^{L} W_h \bar{x}_h^{'}}{\sum_{h=1}^{L} W_h \bar{x}_h} \right) \tag{6}$$

and

$$\hat{\bar{Y}}_{Pd} = \bar{y}_{st}\frac{\bar{x}_{st}}{\bar{x}_{st}^{'}} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\sum_{h=1}^{L} W_h \bar{x}_h}{\sum_{h=1}^{L} W_h \bar{x}_h^{'}} \right) \tag{7}$$

The mean square error of above estimators, for the first order approximation are,

$$MSE(\hat{\bar{Y}}_{Rd}) = \bar{Y}^2 \left(V_{200} + V_{020} - V_{020}^{'} - 2(V_{110} - V_{110}^{'}) \right) \tag{8}$$

and

$$MSE(\hat{\bar{Y}}_{Pd}) = \bar{Y}^2 \left(V_{200} + V_{020} - V_{020}^{'} + 2(V_{110} - V_{110}^{'}) \right) \tag{9}$$

Mohanty (1967) regression-cum-ratio and regression-cum-product type estimator for Y in two-phase stratified sampling, is given by

$$y_{reg}^{R} = \left[\bar{y}_{st} + \beta_1 \left(\bar{x}_{st}^{'} - \bar{x}_{st} \right) \right] \frac{\bar{Z}}{\bar{z}_{st}^{'}} \tag{10}$$

and

$$y_{reg}^{P} = \left[\bar{y}_{st} + \beta_2 \left(\bar{x}_{st}^{'} - \bar{x}_{st} \right) \right] \frac{\bar{z}_{st}^{'}}{\bar{Z}} \tag{11}$$

where β_1 and β_2 are constants.

The MSE of y_{reg}^{R} and y_{reg}^{P} with β_1 and β_2 i.e. $\beta_1 = \beta_2 = \dfrac{\bar{Y}\left(\bar{V}_{110} - \bar{V}_{110}^{'}\right)}{\bar{X}\left(V_{020} - V_{020}^{'}\right)}$ is given by

$$MSE(y_{reg}^{R}) = \bar{Y}^2 \left[V_{200} + V_{020}^{'} - 2V_{101}^{'} - \frac{(V_{110}^{'} - V_{110})^2}{V_{020} - V_{020}^{'}} \right] \tag{12}$$

and

$$MSE(y_{reg}^{R}) = \bar{Y}^2 \left[V_{200} + V_{020}^{'} + 2V_{101}^{'} - \frac{(V_{110}^{'} - V_{110})^2}{V_{020} - V_{020}^{'}} \right] \tag{13}$$

Sanauallah et. al. (2014) generalized exponential chain ratio type estimators for the stratified two-phase sampling method is,

$$\hat{\bar{Y}}_S = \lambda \sum_{h=1}^{L} W_h \bar{y}_h \left[\alpha_h exp\left\{ \alpha \sum_{h=1}^{L} W_h \left(\bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} - \bar{x}_h \right) \Big/ \sum_{h=1}^{L} W_h \left(\bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} + (a-1)\bar{x}_h \right) \right\} \right] \tag{14}$$

where λ, α and a are contants and minimum MSE of the above estimator is

$$MSE\left(\hat{\bar{Y}}_S\right)_{min} = \bar{Y}^2 \left[1 + \left\{ 1 + V_{200} - \frac{(V_{110}^{'} - V_{110} - V_{101}^{'})^2}{V_{020} - V_{020}^{'} + V_{002}^{'}} \right\}^{-1} \right] \tag{15}$$

4. PROPOSED ESTIMATOR

Motivated by Sanaulla et. al. (2014), the following class of exponential estimator for estimation of population mean under double stratified random sampling has been proposed,

$$\hat{\bar{Y}}_{RP}^{s} = \left[\sum_{h=1}^{L} W_h \bar{y}_h + k\left\{ W_h \left(\bar{x}_h^{'} - \bar{x}_h \right) \right\} \right] \left[\alpha H_1 + (1-\alpha) H_2 \right] \tag{16}$$

where

$$H_1 = exp\left[\left\{ \sum_{h=1}^{L} W_h \left(\bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} - \bar{x}_h \right) \right\} \Big/ \left\{ \sum_{h=1}^{L} W_h \left(\bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} + \bar{x}_h \right) \right\} \right]$$

and

$$H_2 = exp\left[\left\{ \sum_{h=1}^{L} W_h \left(\bar{x}_h - \bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} \right) \right\} \Big/ \left\{ \sum_{h=1}^{L} W_h \left(\bar{x}_h + \bar{x}_h^{'} \frac{\bar{z}}{\sum_{h=1}^{L} W_h \bar{z}_h^{'}} \right) \right\} \right]$$

Further simplifying using (1), (2) and (3) , we have

$$\left(\hat{\bar{Y}}_{RP}^{s}-Y\right)^{2}=\bar{Y}^{2}\left[\alpha^{2}\left(e_1^{'}-e_1-e_2^{'}\right)^{2}+\frac{1}{4}\left(e_1^{'}-e_1-e_2^{'}\right)^{2}+e_0^{2}-\alpha\left(e_1^{'}-e_1-e_2^{'}\right)^{2}-e_0\left(e_1^{'}-e_1-e_2^{'}\right)\right.$$
$$\left.+2\alpha e_0\left(e_1^{'}-e_1-e_2^{'}\right)+k^{2}\bar{X}^{2}\left(e_1^{'}-e_1\right)^{2}+(2\alpha-1)k\bar{X}\bar{Y}\left(e_1^{'}-e_1\right)\left(e_1^{'}-e_1-e_2^{'}\right)+2k\bar{X}\bar{Y}e_0\left(e_1^{'}-e_1\right)\right] \tag{17}$$

Taking expectation, we have

$$MSE\left(\hat{\bar{Y}}_{RP}^{s}\right)=\bar{Y}^{2}\left[\alpha^{2}\left(V_{020}-V_{020}^{'}+V_{002}^{'}\right)+\frac{1}{4}\left(V_{020}-V_{020}^{'}+V_{002}^{'}\right)+V_{200}-\alpha\left(V_{020}-V_{020}^{'}+V_{002}^{'}\right)-\left(V_{110}^{'}-V_{110}+V_{101}^{'}\right)\right.$$
$$\left.+2\alpha\left(V_{110}^{'}-V_{110}+V_{101}^{'}\right)\right]+k^{2}\bar{X}^{2}\left(V_{020}-V_{020}^{'}\right)+(2\alpha-1)k\bar{X}\bar{Y}\left(V_{020}-V_{020}^{'}\right)+2k\bar{X}\bar{Y}e_0\left(V_{110}^{'}-V_{110}\right)\Big] \tag{18}$$

Finding optimum value of k and α from (11) we get

$$k_{(opt)}=\frac{\bar{Y}}{\bar{X}}\left[\frac{V_{101}^{'}}{V_{002}^{'}}-\frac{\left(V_{110}^{'}-V_{110}\right)}{\left(V_{020}-V_{020}^{'}\right)}\right] \tag{19}$$

$$\alpha_{(opt)}=\frac{1}{2}+\frac{V_{101}^{'}}{V_{200}^{'}} \tag{20}$$

By substituting the optimum value of k_{opt} and α_{opt} the minimum MSE is,

$$MSE(\hat{\bar{Y}}_{RP}^{s})_{min}=\bar{Y}^{2}\left[V_{200}-\frac{V_{101}^{'2}}{V_{002}^{'}}-\frac{(V_{110}^{'}-V_{110})^{2}}{V_{020}-V_{020}^{'}}\right] \tag{21}$$

Putting $k=0$ and finding optimum value of α, we get

$$\alpha_{opt}=\frac{1}{2}-\left[\frac{V_{110}^{'}-V_{110}-V_{101}^{'}}{V_{020}-V_{020}^{'}+V_{002}^{'}}\right] \tag{22}$$

and using (19) in (11), we have

$$MSE(\hat{\bar{Y}}_{RP}^{s})_{min}=\bar{Y}^{2}\left[V_{200}-\frac{(V_{110}^{'}-V_{110}-V_{101}^{'})^{2}}{V_{020}-V_{020}^{'}+V_{002}^{'}}\right]$$

5. EFFICIENCY COMPARISONS

For carrying out efficiency comparison, the proposed estimator is compared with the above discussed estimators and modified form of Hansen-Hurwitz unbiased estimator.

$MSE(\hat{\bar{Y}}_{RP}^{s})<Var(\bar{y}_{st})$ if

$$\frac{V_{101}^{'2}}{V_{002}^{'}}+\frac{(V_{110}^{'}-V_{110})^2}{V_{020}-V_{020}^{'}}\geq 0 \tag{23}$$

$MSE(\hat{\bar{Y}}_{RP}^{s})<MSE(\bar{y}_{Rd})$ if

$$\frac{V_{101}^{'2}}{V_{002}^{'}}+\frac{(V_{020}-V_{020}^{'}+V_{110}^{'}-V_{110})^2}{V_{020}-V_{020}^{'}} \tag{24}$$

$MSE(\hat{\bar{Y}}_{RP}^{s})<MSE(\bar{y}_{Pd})$ if

$$\frac{V_{101}^{'2}}{V_{002}^{'}}+\frac{(V_{020}-V_{020}^{'}-V_{110}^{'}+V_{110})^2}{V_{020}-V_{020}^{'}} \tag{25}$$

$MSE(\hat{\bar{Y}}_{RP}^{s})<MSE(y_{reg}^{R})$ if

$$\frac{V_{101}^{'2}}{V_{002}^{'}}-2V_{101}^{'}+V_{002}^{'}\geq 0 \tag{26}$$

$MSE(\hat{\bar{Y}}_{RP}^{s})<MSE(y_{reg}^{P})$ if

$$\frac{V_{101}^{'2}}{V_{002}^{'}}+2V_{101}^{'}+V_{002}^{'}\geq 0 \tag{27}$$

$MSE(\hat{\bar{Y}}_{RP}^{s})<MSE\left(\hat{\bar{Y}}_{S}\right)_{min}$ if

$$\frac{\left[V_{110}^{'}\left(V_{020}-V_{020}^{'}\right)+V_{020}^{'}\left(V_{110}^{'}-V_{110}\right)\right]^2}{V_{002}^{'}\left(V_{020}-V_{020}^{'}\right)\left(V_{110}^{'}-V_{110}\right)}\geq 0 \tag{28}$$

6. EMPIRICAL STUDY

To carry out the numerical illustrations some industrial datasets are used to check the usefulness of the proposed estimators of $\bar{Y}$,

Dataset-I: "Source: (Murthy, 1967) **Y**: Production (Output), **X**: No. of workers, **Z**: Fixed Capital"

Dataset-II: "Source: (Sardanal et al., 1992) **Y**: 1983 Workers (in millions), **X**: 1980 Workers (in millions), **Z**: 1982 Gross National Product (in tens of millions of U.S. dollars)"

Dataset-III: "Source: (Gujarati, 2003) **Y**: Average miles per gallon, **X**: Engine horsepower, **Z**: Top speed, miles per hour"

Table 1: Dataset-I

Strata	N_h	n_h	n_h'	S_{yh}	S_{xh}	S_{zh}	$\bar{Y}_h$	$\bar{X}_h$	$\bar{Z}_h$	ρ_{xyh}	ρ_{xzh}	ρ_{yzh}
1	19	5	11	757.08	11.18	109.45	65.16	2669.25	349.68	0.81	0.90	0.93
2	32	8	17	669.11	44.35	109.22	140.00	4657.62	706.60	0.88	0.84	0.92
3	14	3	8	417.00	81.11	277.18	403.21	6537.21	1539.60	0.92	0.93	0.98
4	15	4	9	645.69	171.44	370.96	763.20	7843.67	2620.53	0.97	0.94	0.96

Table 2: Dataset-II

Strata	N_h	n_h	n_h'	S_{yh}	S_{xh}	S_{zh}	$\bar{Y}_h$	$\bar{X}_h$	$\bar{Z}_h$	ρ_{xyh}	ρ_{xzh}	ρ_{yzh}
1	38	16	27	16.46	14.91	1915.17	11.88	13.03	1029.16	0.99	0.74	0.74
2	14	6	10	60.23	58.48	78097.92	26.18	27.35	25671.57	0.99	0.97	0.96
3	11	4	8	34.89	32.66	7588.80	21.88	23.13	5028.82	0.99	0.97	0.97
4	33	14	23	209.08	200.07	20672.75	75.24	79.65	7533.94	0.99	0.30	0.29
5	24	10	17	18.80	18.69	19782.83	20.09	20.28	16314.42	0.99	0.90	0.90

Table 3: Dataset-III

Strata	N_h	n_h	n_h'	S_{yh}	S_{xh}	S_{zh}	$\bar{Y}_h$	$\bar{X}_h$	$\bar{Z}_h$	ρ_{xyh}	ρ_{xzh}	ρ_{yzh}
1	21	06	15	12.14	76.71	19.48	37.55	116.57	114.14	-0.7914	0.9894	-0.7781
2	34	04	17	8.34	31.94	07.10	37.25	93.00	106.50	-0.8339	0.8820	-0.6651
3	26	02	08	5.47	49.55	13.21	26.39	26.39	118.88	-0.7696	0.9669	-0.5935

In Table 5 the Percentage Relative Efficiencies (PREs) of different suggested estimators of $\bar{Y}$ with respect to usual estimator $\bar{y}_{st}$ using the formula given below are tabulated.

$$PRE\left(*, \bar{y}_{st}\right) = \frac{V\left(\bar{y}_{st}\right)}{V\left(*\right)} \tag{30}$$

where $*$ stands for $MSE(\bar{y}_{Rd})$, $MSE(\bar{y}_{Pd})$, $MSE(y_{reg}^{R})$, $MSE(y_{reg}^{P})$, $MSE\left(\hat{\bar{Y}}_S\right)$ and $MSE\left(\hat{\bar{Y}}_{RP}^{s}\right)$.

Table 5: PREs of the estimators of $\bar{Y}$

Estimators	*PREs*		
	Dataset-I	***Dataset-II***	***Dataset-III***
$\bar{y}_{st}$	100.00	100.00	100.00
$\bar{y}_{Rd}$	13.00	101.00	*
$\bar{y}_{Pd}$	*	*	29.00
y_{reg}^{R}	18.73	151.97	*
y_{reg}^{P}	*	*	52.60
$\hat{\bar{Y}}_{S}$	136.05	152.32	193.58
$\hat{\bar{Y}}_{RP}^{s}$	143.00	203.90	208.30

* Data not applicable for this estimator

7. CONCLUSION

In Table-4, it can be noticed with the help of percent relative efficiencies (PREs) that the proposed class of estimators – clearly - outdo the usual unbiased estimator in double stratified sampling $\bar{y}_{st}$, ratio and product estimators $\bar{y}_{Rd}$ and $\bar{y}_{Pd}$ respectively. Also it outperform Mohanty (1967) regression-cum-ratio and regression-cum-product estimators y_{reg}^{R} and y_{reg}^{P} respectively. It is also superior to Sanaullah et al. (2014) estimators $\hat{\bar{Y}}_{S}$ for the population mean $\bar{Y}$. So, in practice the use of the suggested class of estimators could be preferred instead of existing methods to computing statistical inference for industrial data and others for higher accuracy in the estimates.

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