CORRECTION*

Exponentiated quasi power Lindley power series distribution with applications in medical science

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Abstract

The present paper introduces an advanced five parameter lifetime model which is obtained by compounding exponentiated quasi power Lindley distribution with power series family of distributions. The EQPLPS family of distributions contains several lifetime sub-classes such as quasi power Lindley power series, power Lindley power series, quasi Lindley power series and Lindley power series. The proposed distribution exhibits decreasing, increasing and bathtub shaped hazard rate functions depending on its parameters. It is more flexible as it can generate new lifetime distributions as well as some existing distributions. Various statistical properties including closed form expressions for density function, cumulative function, limiting behaviour, moment generating function and moments of order statistics are brought forefront. The capability of the quantile measures in terms of Lambert W function is also discussed. Ultimately, the potentiality and the flexibility of the new class of distributions has been demonstrated by taking three real life data sets by comparing its sub-models.

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1. INTRODUCTION

The modeling of lifetime data has received prominent attention from researchers for the last decade. To predict the ambiguous behaviour of random events as death, appearance of some disease and system failure is a major concern for statisticians. There are diverse lifetime models available for researchers to predict this uncertain behaviour but at times due to complex pattern of data sets, these models do not provide a suitable fit. In order to prevail from this difficulty, researchers have focussed their

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attention on compounding mechanism which is a sound way to develop an appropriate and flexible models to fit the lifetime data of different types.

Keeping this in mind, Adamidis and Loukas (1998), Kus (2007), Tahmasbi (2008) constructed several lifetime distributions through this mechanism that proved to be operative in modeling of lifetime data having different features. Researchers developed many lifetime distributions by this technique which are very flexible and can accommodate different types of data sets. For instance, Chahkandi and Ganjali (2009) obtained a compound class of exponential power series distributions. As Weibull distribution contains the exponential distribution as a special case, Morais and Baretto-Souza (2011) substituted the exponential distribution with a Weibull distribution in this mechanism and obtained a compound class of Weibull power series distributions which contains EPS distribution as a special case. Adil and Jan (2016) introduced a new family of lifetime distributions by compounding a Lindley distribution with power series distribution that contains Lindley Geometric as special case due to Zakerzadeh and Mahmoudi (2012). Moreover, many authors discussed some special cases of the LPS family that are very flexible in terms of density and hazard rate functions. Adil and Jan (2018a, 2018b) obtained a lifetime distribution for series system and generalized version of complementary Lindley power series family of compound lifetime distributions related to parallel system which generalizes most of the lifetime distributions and have versatile properties. Arsalan et al. (2019) introduced the exponential Burr XII power series.

2. EXPONENTIATED QUASI POWER LINDLEY DISTRIBUTION

Manuela Ghica et al. (2017) introduced an Exponentiated Quasi Power Lindley Distribution (EQPLD) defined by its pdf as

$$g(y;\alpha,\theta,\beta,b,)=\frac{\beta\theta^2 b}{\alpha\theta+1}x^{\beta-1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b};x>0,\alpha,\theta,\beta,b>0$$

This new distribution reduces to the quasi Lindley distribution, the exponential distribution and gamma distribution. In terms of reliability, the various shapes of the EQPL distribution give it a benefit, being more flexible to model many real systems

which generally exhibit bath-tub shaped failure rate. The corresponding cdf of the above equation becomes

$$G(y;\alpha,\theta,\beta,b)=\left[1-\left[1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right]e^{-\theta x^{\beta}}\right]^{b};x>0,\alpha,\theta,\beta,b>0$$

2.1. Special cases

Case 1: At $b=1$, EQPLD reduces to Quasi Power Lindley distribution.

Case 2: At $\beta=1$, EQPLD reduces to the Power Lindley distribution introduced by ME Ghitany (2013).

Case 3: At $b=1,\beta=1$, EQPLD reduces to the Quasi Lindley distribution introduced by Shanker and Mishra (2013).

Case 4: At $b=1,\beta=1,\alpha=1$, EQPLD reduces to the Lindley distribution.

3. THE EQPLPS FAMILY

In this section, we derive the family of EQPLPS distributions by compounding the EQPL class of distributions with the power series distributions.

Let N be a discrete random variable following the power series distribution (truncated at zero) with probability mass function given by

$$P(N=n)=\frac{a_n\gamma^n}{C(\gamma)},n=1,2,...$$

Where $a_n\geq 0$ be reliant on n, $C(\gamma)=\sum_{n=1}^{\infty}a_n\gamma^n$ and $\gamma\in(0,s)$ is chosen in such a way that $C(\gamma)$ is finite. The power series family of distributions, contains Poisson, logarithmic, geometric and binomial distributions as special cases. Valuable extents of above distributions truncated at zero are given in table 1.

Table1: Useful Extents Of Zero Truncated Power Series Distribution

Distribution	a_n	$C(\gamma)$	$C'(\gamma)$	$C''(\gamma)$	$C^{-1}(\gamma)$	γ
Poisson	$n!^{-1}$	$e^{\gamma}-1$	e^{γ}	e^{γ}	$\log(\gamma+1)$	$\gamma \in (0,\infty)$
Logarithmic	n^{-1}	$-\log(1-\gamma)$	$(1-\gamma)^{-1}$	$(1-\gamma)^{-2}$	$1-e^{-\gamma}$	$\varphi \in (0,1)$
Geometric	1	$\gamma(1-\gamma)^{-1}$	$(1-\gamma)^{-2}$	$2(1-\gamma^{-3})$	$\gamma(\gamma+1)^{-1}$	$\gamma \in (0,1)$
Binomial	$\binom{m}{n}$	$(\gamma+1)^{m}-1$	$m(\gamma+1)^{m-1}$	$\frac{m(m-1)}{(\gamma-1)^{2-m}}$	$(\gamma-1)^{\frac{1}{m}}-1$	$\gamma \in (0,1)$

Given N, let $X=\max(X_1,X_2,...X_N)$, where $X_i, i=1,2,...N$ are independent and identically distributed (iid) random variables with cdf ().Then the cdf of $X \mid N=n$ is given by

$$F(X \mid N=n)=[G(x;\alpha,\theta,\beta,b)]^n=\left\{\left[1-\left[1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right]e^{-\theta x^{\beta}}\right]^b\right\}^n,$$

$$x>0,\alpha,\theta,\beta,b>0,n\geq 1$$

The EQPLPS is then defined by the marginal cdf of X, which is given by

$$F(x,\alpha,\theta,\beta,b,\gamma)=\sum_{n=1}^{\infty}\frac{a_n\gamma^n}{C(\gamma)}[G(x;\alpha,\theta,\beta,b,\gamma)]^n$$

$$F(x)=\frac{C\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^b\right]}{C(\gamma)},x>0,\alpha>0,\theta>0,\beta>0,b>0,\gamma>0 \qquad (1)$$

Here, a random variable X following Exponentiated Quasi Power Lindley power series distribution with parameters $\alpha,\theta,\beta,b,\gamma$ will be denoted by $X\sim EQPLPS(\alpha,\theta,\beta,b,\gamma)$. This new class of distributions contains several lifetime distributions as special cases which will be discussed in section (9).

4. DENSITY, SURVIVAL AND HAZARD RATE FUNCTIONS

The pdf, survival and hazard functions are respectively given by

$$f(x)=\frac{\beta\theta^2 bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)\gamma e^{-\theta x^{\beta}}\frac{C'\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]}{C(\gamma)},x>0 \qquad (2)$$

$$S(x)=1-\frac{C\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]}{C(\gamma)},x>0$$

And the hazard function is given as

$$h(x)=\frac{\beta\theta^2 bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)\gamma e^{-\theta x^{\beta}}\frac{C'\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]}{C(\gamma)-C\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]},x>0$$

THEOREM 4.1. The Exponentiated Quasi Power Lindley distribution is a limiting case of EQPLPS distribution when $\gamma\to 0^{+}$.

PROOF. From the cdf of EQPLPS distribution , we have

$$\lim_{\gamma\to 0^{+}}F(x)=\lim_{\gamma\to 0^{+}}\frac{C'\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]}{C(\gamma)}$$

We know that

$$C(\gamma)=\sum_{n=1}^{\infty}a_n\gamma^n$$

$$\lim_{\gamma\to 0^{+}}F(x)=\lim_{\gamma\to 0^{+}}\frac{\sum_{n=1}^{\infty}a_n\left[\gamma\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]^{n}}{\sum_{n=1}^{\infty}a_n\gamma^n}$$

Using L' Hospital's rule, it follows that

$$\lim_{\gamma\to 0^+} F(x) = \frac{a_1\left\{\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right\}+\sum_{n=1}^{\infty} a_n\gamma^{n-1}\left[\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^n}{a_1+\sum_{n=1}^{\infty} a_n n\gamma^{n-1}}$$

$$=\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b$$

Which is the cdf of exponentiated quasi power Lindley distribution.

THEOREM 4.2. The densities of the EQPLPS distribution can be expressed as an infinite linear combination of densities of n[th] order statistics of the exponentiated quasi power Lindley distribution

$$f(x)=\sum_{n=1}^{\infty} P(N=n)g_n(x,n) \tag{3}$$

Where $g_n(x,n)=\max(X_1,X_2,...X_n)$ is the n[th] order statistics of exponentiated quasi power Lindley distribution.

PROOF. As we know that

$$C(\gamma)=\sum_{n=1}^{\infty} na_n\gamma^{n-1}$$

Therefore, the pdf of EQPLPS distribution reduces to the expression after using the above argument as follows

$$f(x)=\frac{\beta\theta^2 bx^{\beta-1}}{(\alpha\theta+1)}(\alpha+x^\beta)e^{-\theta x^\beta}\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^{b-1}\sum_{n=1}^{\infty}\frac{na_n\gamma^n}{C(\gamma)}\left[\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{n-1}$$

$$f(x)=\sum_{n=1}^{\infty} P(N=n)g_n(x,n)$$

Where

$$g_n(x,n)=\frac{n\beta\theta^2 bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\left[\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]^{n-1}$$

is the n[th] order statistics of exponentiated quasi power Lindley distribution.

Therefore the densities of EQPLPS distribution can be expressed as an infinite linear combination of the n[th] order statistics of exponentiated quasi power Lindley distribution.

5. MOMENT GENERATING FUNCTION

The moment generating function of EQPLPS distribution can be obtained from (3)

$$M_X(t)=\sum_{n=1}^{\infty}P(N=n)M_{X_{(n)}}(t)$$

Where $M_{X_{(n)}}(t)$ is the moment generating function of n[th] order statistics of exponentiated quasi power Lindley distribution.

$$M_{X_{(n)}}(t)=\frac{n\beta\theta^2 b}{\alpha\theta+1}\int_0^{\infty}e^{tx}x^{\beta-1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\left[\left\{1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right\}^{b}\right]^{n-1}dx$$

$$=\frac{n\beta\theta^2}{\alpha\theta+1}\sum_{j=0}^{n-1}\binom{n-1}{j}\sum_{k=0}^{\infty}\binom{bj+b-1}{k}(-1)^{j+k}\int_0^{\infty}e^{tx}x^{\beta-1}\left(\alpha+x^{\beta}\right)e^{-(\theta+\theta k)x^{\beta}}\left[1+\left(\frac{\theta x^{\beta}}{\alpha\theta+1}\right)\right]^{k}dx$$

Using $e^{tx}=\sum_{l=0}^{\infty}\frac{t^l x^l}{l!}$

$$=\frac{n\theta^2 b}{(\alpha\theta+1)}\sum_{j=0}^{n-1}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\binom{n-1}{j}\binom{bj+b-1}{k}\binom{k}{i}(-1)^{j+k}\frac{t^l}{l!}\left(\frac{\theta}{(\beta\theta+1)}\right)^{k-i}\times$$

$$\left[\frac{\alpha\Gamma\left(k-i+\frac{l}{\beta}+1\right)+(\theta+\theta k)\Gamma\left(k-i+\frac{l}{\beta}\right)}{(\theta+\theta k)^{k-i+\frac{l}{\beta}+1}}\right]$$

And it follows that

$$M_X(t)=\frac{\theta^2 b}{\alpha\theta+1}\sum_{n=1}^{\infty}\frac{na_n\gamma^n}{C(\gamma)}\sum_{j=0}^{n-1}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\binom{n-1}{j}\binom{bj+b-1}{k}\binom{k}{i}(-1)^{j+k}\frac{t^l}{l!}\left(\frac{\theta}{(\beta\theta+1)}\right)^{k-i}\times \left[\frac{\alpha\Gamma\left(k-i+\frac{l}{\beta}+1\right)+(\theta+\theta k)\Gamma\left(k-i+\frac{l}{\beta}\right)}{(\theta+\theta k)^{k-i+\frac{l}{\beta}+1}}\right]$$

The r[th] moment of the EQPLPS distribution about origin is

$$E\left(X^r\right)=\sum_{n=1}^{\infty}P(N=n)\int_0^{\infty}x^r\,g_n(x)dx$$

$$E\left(X^r\right)=\frac{\theta^2 b}{\alpha\theta+1}\sum_{n=1}^{\infty}\frac{na_n\gamma^n}{C(\gamma)}\sum_{j=0}^{n-1}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\binom{n-1}{j}\binom{bj+b-1}{k}\binom{k}{i}(-1)^{j+k}\left(\frac{\theta}{(\beta\theta+1)}\right)^{k-i}\times \left[\frac{\alpha\Gamma\left(k-i+\frac{r}{\beta}+1\right)+(\theta+\theta k)\Gamma\left(k-i+\frac{r}{\beta}+2\right)}{(\theta+\theta k)^{k-i+\frac{r}{\beta}+2}}\right] \quad (4)$$

6. QUANTILE FUNCTION

THEOREM 6.1. If $X\sim EQPLPS(\alpha,\theta,\beta,b,\gamma)$, then the quantile function of X is

$$Q(p)=\left[-\alpha-\frac{1}{\theta}-\frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1-\left[\frac{C^{-1}(vC(\gamma))}{\gamma}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

Where $v\in(0,1)$ and W(.) denotes the Lambert W function (see Corless et al.(1996))

PROOF. The quantile function denoted by Q(p) is the root of the equation

$$\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}=1-\left[\frac{C^{-1}[vC(\gamma)]}{\gamma}\right]^{\frac{1}{b}}:0<v<1 \quad (5)$$

Setting $Z(v) = -\left(1 + \alpha\theta + \theta[Q(v)]^{\beta}\right)$, we may rewrite (5) as

$$Z(v)e^{z(v)} = -(1+\alpha\theta)\left[1 - \left[\frac{C^{-1}[vC(\gamma)]}{\gamma}\right]^{\frac{1}{b}}\right]e^{-(1+\alpha\theta)}$$

So the solution for Z(v) is

$$Z(v) = W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1 - \left[\frac{C^{-1}[vC(\gamma)]}{\gamma}\right]^{\frac{1}{b}}\right)\right]$$

Solving the equation

$$W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1 - \left[\frac{C^{-1}[vC(\gamma)]}{\gamma}\right]^{\frac{1}{b}}\right)\right] = -\left(1 + \alpha\theta + \theta[Q(v)]^{\beta}\right)$$

Which upon solving for Q(v) gives

$$Q(v) = \left[-\alpha - \frac{1}{\theta} - \frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1 - \left[\frac{C^{-1}[vC(\gamma)]}{\gamma}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

Using above equation the quartiles of the EQPLPS distribution can be determined. Median of the exponentiated quasi power Lindley power series distribution is given by

$$Q\left(\frac{1}{2}\right) = \left[-\alpha - \frac{1}{\theta} - \frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1 - \left[\frac{C^{-1}\left[\frac{1}{2}C(\gamma)\right]}{\gamma}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

7. ORDER STATISTICS AND THEIR MOMENTS

Let $X_1, X_2, ...X_n$ be a random sample of size n having EQPLPS distribution. The pdf and cdf of i[th] order statistics say $X_{i:n}$ can be obtained as

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!}[F(x)]^{i-1}[1-F(x)]^{n-i} f(x) \tag{6}$$

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} f(x) \left[\frac{C\left\{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right\}}{C(\gamma)} \right]^{i-1} \left[1 - \frac{C\left\{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right\}}{C(\gamma)} \right]^{n-i}$$

Expression (6) can also be written as

$$f_{i:n}(x) \frac{n!}{(n-i)!(i-1)!} \sum_{k=0}^{n-i} \binom{n-i}{k} (-1)^k f(x)[F(x)]^{k+i-1}$$

The associated cdf of $f_{i:n}(x)$ denoted by $F_{i:n}(x)$ becomes

$$F_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} \sum_{k=0}^{n-i} \frac{\binom{n-i}{k}(-1)^k}{(k+i)} \left[C\left\{ \frac{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{1+\alpha\theta}\right)e^{-\theta x^{\beta}}\right]^{b}}{C(\gamma)} \right\} \right]^{k+i} \tag{7}$$

Expression for r[th] moment of i[th] order statistics with cdf (3.1) can be obtained by using a well- known result given by Barakat et al. (2004) as follows

$$E\left(X_{i:n}^{r}\right) = r \sum_{k=n-i+1}^{n} (-1)^{k-n+i-1} \binom{k-1}{n-i} \binom{n}{k} \int_0^{\infty} x^{r-1} \left[1 - \frac{C\left\{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{1+\alpha\theta}\right)e^{-\theta x^{\beta}}\right]^{b}\right\}}{C(\gamma)} \right]^{k} dx$$

8. PARAMETER ESTIMATION

Let $X_1, X_2, ...X_n$ be a random sample with observed value $x = (x_1, x_2, ...x_n)$ obtained from EQPLPS distribution with parameters $\alpha, \theta, \beta, b \text{ and } \gamma$. Let $\Theta = (\alpha, \theta, \beta, b, \gamma)^T$ be the parameter vector. The log likelihood function is given by

$$l_n = l_n(y, \Theta) = n\log\beta + n\log\gamma + 2n\log\theta + n\log b - n\log(\alpha\theta+1) - n\log C(\gamma) + (\beta-1)\sum_{i=1}^{n}\log x_i - \theta\sum_{i=1}^{n}x_i^{\beta}$$
$$+\sum_{i=1}^{n}\log(\alpha + x_i^{\beta}) + (b-1)\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right] + \sum_{i=1}^{n}\log\left[C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}\right]$$

The corresponding score functions are

$$\frac{\partial l_n}{\partial\theta} = \frac{2n}{\theta} - \frac{n\alpha}{\alpha\theta+1} - \sum_{i=1}^{n}x_i^{\beta} + \frac{\theta(b-1)}{(\alpha\theta+1)^2}\sum_{i=1}^{n}\left[\frac{x_i^{\beta}e^{-\theta x_i^{\beta}}\left(\alpha^2\theta+2\alpha+\alpha\theta x_i^{\beta}+x_i^{\beta}\right)}{\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]}\right]$$
$$+\frac{b\gamma\theta}{(\alpha\theta+1)^2}\sum_{i=1}^{n}\frac{C''\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}{C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^{b-1}x_i^{\beta}e^{-\theta x_i^{\beta}}\left(\alpha^2\theta+2\alpha+\alpha\theta x_i^{\beta}+x_i^{\beta}\right)$$

$$\frac{\partial l_n}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{\beta}\log x_i}{(\alpha+x_i^{\beta})} - \theta\sum_{i=1}^{n}x_i^{\beta}\log x_i + \frac{(b-1)\theta^2}{(\alpha\theta+1)}\sum_{i=1}^{n}\left(x_i^{\beta}\log x_i e^{-\theta x_i^{\beta}}\right)(\alpha+x_i^{\beta}) +$$
$$+\sum_{i=1}^{n}\frac{C''\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}{C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\frac{\left(x_i^{\beta}\log x_i e^{-\theta x_i^{\beta}}\right)(\alpha+x_i^{\beta})}{\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]}$$

$$\frac{\partial l_n}{\partial\alpha} = \frac{n\theta}{\alpha\theta+1} + \sum_{i=1}^{n}\frac{1}{\alpha+x_i^{\beta}} + \frac{\theta^2(b-1)}{(\alpha\theta+1)^2}\sum_{i=1}^{n}\frac{x_i^{\beta}e^{-\theta x_i^{\beta}}}{\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]} + \frac{\gamma b\theta^2}{(\alpha\theta+1)^2}\sum_{i=1}^{n}\frac{C''\left\{\gamma\left[1-\left(1+\frac{\theta y_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}{C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}$$
$$\times\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^{b-1}\left(x_i^{\beta}e^{-\theta x_i^{\beta}}\right)$$

$$\frac{\partial l_n}{\partial b}=\frac{n}{b}+\sum_{i=1}^{n}\log\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]+\gamma\frac{C''\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}{C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b$$

$$\times\log\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b$$

$$\frac{\partial l_n}{\partial \gamma}=\frac{n}{\gamma}-\frac{nC'(\gamma)}{C(\gamma)}+\sum_{i=1}^{n}\frac{C''\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}{C'\left\{\gamma\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b\right\}}\left[1-\left(1+\frac{\theta x_i^{\beta}}{\alpha\theta+1}\right)e^{-\theta x_i^{\beta}}\right]^b$$

MLEs of $\theta,\alpha,\beta,b\,\&\,\gamma$ cannot be obtained by solving above complex equations as these equations are not in closed form. So we solve the above equations by using iteration method through R software.

9. SPECIAL SUB-MODELS OF THE EQPLPS MODEL

9.1. Exponentiated Quasi Power Lindley Poisson Distribution (EQPLPD)

The corresponding cdf, pdf, survival function and hazard function of EQPLPD can be obtained respectively by using $C(\gamma)=e^{\gamma}-1$ and $C'(\gamma)=e^{\gamma}$ in (1) & (2).

$$F(x)=\frac{e^{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}-1}{e^{\gamma}-1},x>0$$

$$f(x)=\frac{\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)\gamma e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\frac{e^{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}}{e^{\gamma}-1},x>0$$

$$S(x)=\frac{e^{\gamma}-e^{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}}{e^{\gamma}-1}$$

$$h(y)=\frac{\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)\gamma e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\frac{e^{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}}{e^{\gamma}-e^{\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]}}$$

For $x,\theta,\alpha,\beta,b>0,\ \ and\ \ \ 0<\gamma<\infty$. The expression for r[th] moment of a random variable following EQPLPS distribution becomes by substituting $a_n=n!^{-1}$ and $C(\gamma)=e^{\gamma}-1$ in (4).

$$E(X^{r})=\frac{n\theta^{2}b}{(\alpha\theta+1)(e^{\theta}-1)}\sum_{n=1}^{\infty}\frac{\gamma^{n}}{n!}\sum_{j=0}^{n-1}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\binom{n-1}{j}\binom{bj+b-1}{k}\left(\frac{\theta}{\alpha\theta+1}\right)^{k-i}(-1)^{j+k}$$
$$\times\left[\frac{\alpha\Gamma\left(k-i+\frac{r}{\beta}+1\right)+(\theta+\theta k)\Gamma\left(k-i+\frac{r}{\beta}+2\right)}{(\theta+\theta k)^{k-i+\frac{r}{\beta}+2}}\right]$$

The pdf and cdf of order statistics of EQPLPD can be obtained by using the cdf and pdf of EQPLPD in (6) and (7).

$$f_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^k\frac{\beta\theta^2 bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^\beta)\gamma e^{-\theta x^\beta}\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^{b-1} e^{\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b} \times\frac{\left[e^{\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b}-1\right]^{k+i-1}}{\left[e^\gamma-1\right]^{k+i}}$$

$$F_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^k}{(k+i)}\left[\frac{e^{\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b}-1}{e^\gamma-1}\right]^{k+i}$$

The quantile function can be obtained by substituting $C(\gamma)=e^\gamma-1$ and $C^{-1}(\gamma)=\log(\gamma+1)$ in (5), we have

$$Q(v)=\left[-\alpha-\frac{1}{\theta}-\frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1-\left[\frac{\log\left[v\left(e^\gamma-1\right)+1\right]}{\gamma}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

9.2. Exponentiated Quasi Power Lindley Logarithmic Distribution (EQPLLD)

The cdf, pdf, survival function and hazard function of EQPLLD is obtained by using $C(\gamma)=-\log(1-\gamma)$ and $C'(\gamma)=(1-\gamma)^{-1}$ in (1) and (2).

$$F(x)=\frac{\log\left[1-\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]}{\log(1-\gamma)},x>0$$

$$f(x)=\frac{\beta\theta^2 bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^\beta)\gamma e^{-\theta x^\beta}\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^{b-1}\frac{\left[1-\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{-1}}{\log(1-\gamma)},x>0$$

$$S(x)=1-\frac{\log\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]}{\log(1-\gamma)},x>0$$

$$h(x)=\frac{\beta\theta^{2}x^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\frac{\left[1-\gamma\left[1-\left(\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]^{-1}}{\log(1-\gamma)-\log\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]\right]}$$

For $x,\alpha,\theta,\beta,b>0,and\ 0<\gamma<1$.The r[th] moment of EQPLLD can be obtained by substituting $a_n=n^{-1}\ \ and\ \ C(\gamma)=-\log(1-\gamma)$ in (4)

The pdf and cdf of EQPLLD can be obtained by substituting its pdf and cdf in (6) and (7).

$$f_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^{k}\frac{\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}$$

$$\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]^{-1}\times\frac{\left[\log\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]\right]^{k+i-1}}{\left[\log(1-\gamma)\right]^{k+i}}$$

$$F_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^{k}}{(k+i)}\left[\frac{\log\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]}{\log(1-\gamma)}\right]^{k+i}$$

By substituting $C(\gamma)=-\log(1-\gamma)$ and $C^{-1}(\gamma)=1-e^{-\gamma}$ in (5), the quantile function of EQPLL distribution is obtained as

$$Q(v)=\left[-\alpha-\frac{1}{\theta}-\frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1-\left[\frac{e^{v\log(1-\gamma)}}{\gamma}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

9.3. Exponentiated Quasi Power Lindley Geometric Distribution (EQPLGD)

The cdf, pdf, survival function and hazard function of EQPLGD can be obtained by using $C(\gamma)=\gamma(1-\gamma)^{-1}$ & $C'(\gamma)=(1-\gamma)^{-2}$ in (1) & (2).

$$F(x)=\frac{(1-\gamma)\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}{1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}},\ x>0$$

$$f(x)=\frac{\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}(1-\gamma)\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]^{-2}$$

$$S(x)=\frac{1-\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}{1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}},\ x>0$$

$$h(x)=\frac{\beta\theta^{2}bx^{\beta-1}}{(\alpha\theta+1)}(1-\gamma)\left(\alpha+x^{\beta}\right)e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\frac{\left[1-\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}\right]^{-1}}{1-\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}}$$

For $x,\alpha,\theta,\beta,b>0$ *and* $0<\gamma<1$. The r[th] moment of EQPLGD can be obtained by substituting $a_n=1$ and $C(\gamma)=\gamma(1-\gamma)^{-1}$ in (4).

The pdf and cdf of order statistics of EQPLGD can be obtained by using the cdf and pdf of EQPLGD in (6) and (7) ,we have

$$f_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^k\frac{\beta\theta^2bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^\beta)e^{-\theta x^\beta}(1-\gamma)^{k+i}\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^{b-1}$$
$$\times\frac{\left[\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{k+i-1}}{\left[1-\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{k+i+1}}$$

$$F_{i:n}(x)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^k}{k+i}\frac{\left[(1-\gamma)\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{k+i}}{\left[1-\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b\right]^{k+i}},x>0$$

By substituting $C(\gamma)=\gamma(1-\gamma)^{-1}$ and $C^{-1}(\gamma)=\gamma(\gamma+1)^{-1}$ in (5), the quantile function of EQPLG distribution is obtained as

$$Q(v)=\left[-\alpha-\frac{1}{\theta}-\frac{1}{\theta}W\left[-(1+\alpha\theta)e^{-(1+\alpha\theta)}\left(1-\left[\frac{v}{\gamma(v-1)+1}\right]^{\frac{1}{b}}\right)\right]\right]^{\frac{1}{\beta}}$$

9.4. Exponentiated Quasi Power Lindley Binomial Distribution (EQPLBD)

The cdf , pdf, survival function and hazard function of EQPLBD can be obtained respectively by taking $C(\gamma)=(\gamma+1)^m-1$ and $C'(\gamma)=m(\gamma+1)^{m-1}$ in (1) and (2).

$$F(x)=\frac{\left[\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b+1\right]^m-1}{(\gamma+1)^m-1},x>0$$

$$f(x)=\frac{m\beta\theta^2bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^\beta)e^{-\theta x^\beta}\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^{b-1}\frac{\left[\gamma\left[1-\left(1+\frac{\theta x^\beta}{\alpha\theta+1}\right)e^{-\theta x^\beta}\right]^b+1\right]^{m-1}}{(\gamma+1)^m-1},x>0$$

$$S(x)=1-\frac{\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m}-1}{(\gamma+1)^{m}-1}$$

$$h(x)=\frac{m\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^{\beta})e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}\frac{\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m-1}}{(\gamma+1)^{m}-\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m}}$$

For $x,\alpha,\theta,\beta,b>0 \; and \; 0<\phi<\infty.$ The r[th] moment of a random variable following EQPLBD becomes by taking $a_n=\binom{m}{n}$ *and* $C(\gamma)=(\gamma+1)^{m}-1$ in (4).

The pdf and cdf of order statistics of EQPLBD can be obtained respectively by using the pdf and cdf of EQPLBD in (6) and (7).

$$f_{i:n}(y)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^{k}\frac{m\beta\theta^{2}bx^{\beta-1}}{\alpha\theta+1}(\alpha+x^{\beta})e^{-\theta x^{\beta}}\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b-1}$$

$$\times\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m-1}\frac{\left[\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m}-1\right]^{k+i-1}}{\left[(\gamma+1)^{m}-1\right]^{k+i}}$$

$$F_{i:n}(y)=\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^{k}}{(k+i)}\frac{\left[\left[\gamma\left[1-\left(1+\frac{\theta x^{\beta}}{\alpha\theta+1}\right)e^{-\theta x^{\beta}}\right]^{b}+1\right]^{m}-1\right]^{k+i}}{\left[(\gamma+1)^{m}-1\right]^{k+i}}$$

10. APPLICATION

To show the superiority of the proposed distribution, we compare its sub–models by taking four real life data sets.

Data set 1. The first data set represents the Lifetime of fatigue of Kevlar 373/epoxy, that are subject to constant pressure at the 90% stress level until all had failed. The data set is

0.0251	0.886	0.0891	0.2501	0.3113	0.3451	0.4763	0.565	0.5671	0.6566	0.6748
0.6751	0.6753	0.7696	0.8375	0.8391	0.8425	0.8645	0.8851	0.9113	0.912	0.9836
1.0483	1.0596	1.0773	1.1733	1.257	1.2766	1.2985	1.3211	1.3503	1.3551	1.4595
1.488	1.5728	1.5733	1.7083	1.7263	1.746	1.763	1.7746	1.8275	1.8375	1.8503
1.8808	1.8878	1.8881	1.9316	1.9558	2.0048	2.0408	2.0903	2.1093	2.133	2.21
2.246	2.2878	2.3203	2.347	2.3513	2.4951	2.526	2.9911	3.0256	3.2678	3.4045
3.4846	3.7433	3.7455	3.9143	4.8073	5.4005	5.4435	5.5295	6.5541	9.096	

Table 10.1: Analysis of model fitting

MODEL	**MLE**	**AIC**	**BIC**
EQPLP	$\hat{\beta}=0.924,\hat{\gamma}=0.000000122\ ,\hat{\theta}=1.110,\hat{b}=1.254,\hat{\alpha}=0.233$	253.13	260.76
EQPLG	$\hat{\beta}=1.060,\hat{\gamma}=0.00000145\ ,\hat{\theta}=0.837,\hat{b}=1.092,\hat{\alpha}=0.525$	253.86	261.49
EQPLL	$\hat{\beta}=1.036,\hat{\gamma}=0.0000000980\ ,\hat{\theta}=0.914,\hat{b}=1.159,\hat{\alpha}=0.483$	253.62	261.25

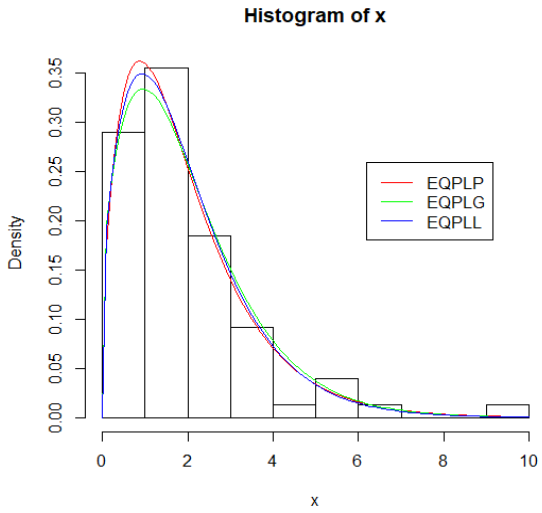


Fig 1: Fitting of EQPLP, EQPLG, EQPLL to the fatigue lifetime data.

Data Set 2. The data set reported by E fron B (1988) and was used by Rama Shanker (2016) represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT). The data set is

6.53	7	10.42	14.48	16.10	22.70	34	41.55	42	45.28	49.40	53.62
63	64	83	84	91	108	112	129	133	133	139	140
140	146	149	154	157	160	160	165	146	149	154	157
160	160	165	173	176	218	225	241	248	273	277	297
405	417	420	440	523	583	594	1101	1146	1417		

Table.10.2: Analysis of model fitting

MODEL	**MLE**	**AIC**	**BIC**
EQPLP	$\hat{\beta} = 0.372, \hat{\gamma} = 2.625, \hat{\theta} = 0.605, \hat{b} = 4.945, \hat{\alpha} = 2.570$	750.37	758.00
EQPLG	$\hat{\beta} = 0.542, \hat{\gamma} = 0.000000839, \hat{\theta} = 0.152, \hat{b} = 1.749, \hat{\alpha} = 0.390$	751.59	759.22
EQPLL	$\hat{\beta} = 0.535, \hat{\gamma} = 0.00000152, \hat{\theta} = 0.160, \hat{b} = 1.789, \hat{\alpha} = 0.384$	751.58	6759.21

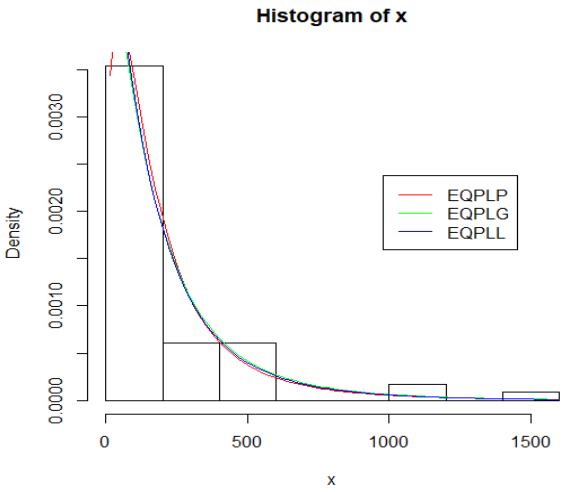


Fig 2: Fitting of EQPLP, EQPLG, EQPLL to the survival data.

Data Set 3. The data set reported by Efron B (1988) and was used by Rama Shanker (2016) represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20	23.56	23.74	25.87	31.98	37	41.35	47.38	55.46	58.36	63.47
68.46	78.26	74.47	81.43	84	92	94	110	112	119	127
130	133	140	146	155	159	173	179	194	195	209
249	281	319	339	432	469	519	633	725	817	1776

Table 10.3: Analysis of model fitting

MODEL	**MLE**	**AIC**	**BIC**
EQPLP	$\hat{\beta}=0.576,\hat{\gamma}=0.00000132,\hat{\theta}=0.125,\hat{b}=1.75,\hat{\alpha}=0.381$	568.43	576.06
EQPLG	$\hat{\beta}=0.578,\hat{\gamma}=0.0000000653,\hat{\theta}=0.126,\hat{b}=1.480,\hat{\alpha}=0.451$	569.03	576.67
EQPLL	$\hat{\beta}=0.597,\hat{\gamma}=0.00000328,\hat{\theta}=0.108,\hat{b}=1.640,\hat{\alpha}=0.411$	568.83	576.47

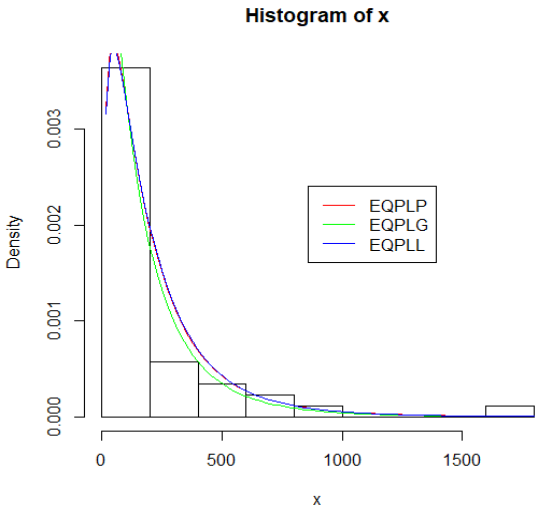


Fig 3: Fitting of EQPLP, EQPLG, EQPLL to the survival data.

Data Set 4. The data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang (2003) and was used by Rama Shanker (2016) in modeling of lifetime data .

Table.10.4: Analysis of model fitting

MODEL	**MLE**	**AIC**	**BIC**
EQPLP	$\hat{\beta} = 0.673, \hat{\gamma} = 0.00000279, \hat{\theta} = 0.552, \hat{b} = 1.415, \hat{\alpha} = 0.406$	827.63	835.26
EQPLG	$\hat{\beta} = 0.727, \hat{\gamma} = 0.00000172, \hat{\theta} = 0.458, \hat{b} = 1.384, \hat{\alpha} = 0.429$	828.34	835.98
EQPLL	$\hat{\beta} = 0.713, \hat{\gamma} = 0.000000556, \hat{\theta} = 0.472, \hat{b} = 1.403, \hat{\alpha} = 0.403$	828.21	835.85

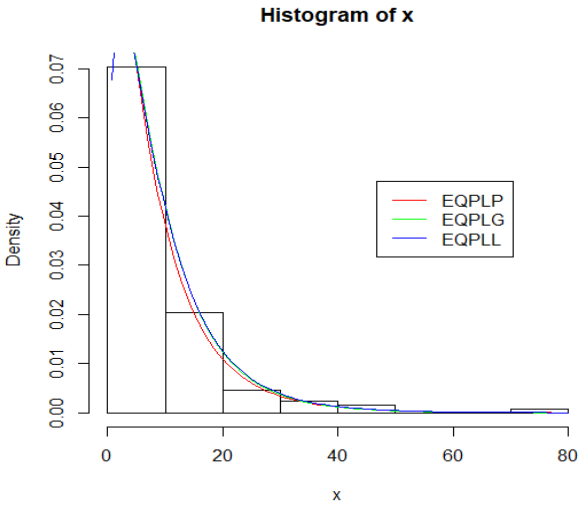


Fig 4: Fitting of EQPLP, EQPLG, EQPLL to the cancer data

All the sub-models fit well but among them EQPLP class of distributions performs excellently well as it possesses the lowest values of AIC and BIC values.

11. CONCLUSION

We have proposed a new five parameter lifetime distribution for parallel system by compounding Exponentiated Quasi Power Lindley distribution with power series distribution. The mathematical properties including density function, moment generating function, order statistics, quantile function have been obtained. The parameters have been estimated by the method of maximum likelihood estimation. The proposed model contains some lifetime sub-classes and has the competency to yield many beneficial and flexible distributions for modelling lifetime data. Ultimately, the sub-models have been compared by applying them to four real life data sets to show the flexibility of the proposed model.

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