

A HYBRIDIZING-ENHANCED QUANTUM-INSPIRED DIFFERENTIAL EVOLUTION ALGORITHM WITH MULTI-STRATEGY FOR COMPLICATED OPTIMIZATION

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Submitted: 26th March 2025; Accepted: 22th August 2025

Abstract

Differential Evolution (DE) has been found to be inefficient and inaccurate for high-dimensional complex problems. Quantum-inspired Differential Evolution (QDE) possesses quantum computational properties, enabling effective handling of high-dimensional problems. However, QDE is plagued by issues of excessive mutation and poor convergence. Therefore, a hybrid enhanced Quantum-inspired Differential Evolution algorithm, termed QAHQDE, is proposed. Within QAHQDE, an improved chaotic strategy is designed. Non-repeating distributed quantum positions are generated, enhancing the diversity of initialized individuals. A quantum-adaptive mutation strategy is adopted to address the over-mutation problem inherent in QDE. The mutation degree is adaptively reduced, and convergence performance is thereby improved. A novel hybrid mutation strategy is constructed. Weighted mutation operators are combined with standard differential evolution. Local and global search capabilities are balanced, and convergence accuracy is enhanced. The performance of QAHQDE was evaluated against 38 algorithms using 48 benchmark functions from CEC2005, CEC2010, and CEC2013, across dimensions $D=100, 500, 1000$, and 3000 . Experimental results demonstrate that QAHQDE outperforms QDE by at least three orders of magnitude. Superior convergence performance, higher convergence accuracy, and excellent stability are exhibited by QAHQDE on most functions.

Keywords: quantum-inspired differential evolution; an improved chaotic strategy; a novel hybrid mutation strategy; a quantum-adaptive mutation strategy; complicated optimization

1 Introduction

Many real-world engineering problems can be formulated as complex optimization tasks [1–4]. In parcel delivery, for instance, a city processing over 1,000 packages per day with 50 delivery vehicles and 200 drop-off points requires optimal routing for each vehicle to minimize time and cost. Over the past two decades, various heuristic algorithms have been proposed and applied to address such problems [5–10]. Methods such as Grey Wolf Optimization (GWO) [11], Sparrow Search Algorithm (SSA) [12], and deep neural networks have demonstrated strong performance in low-dimensional scenarios. However, as engineering systems grow more complex and optimization goals diversify, real-world problems increasingly exhibit large-scale, nonlinear, multi-constrained, and high-dimensional characteristics. In these high-dimensional contexts, traditional optimization methods often experience degraded performance. Consequently, the development of optimization algorithms with higher robustness, improved convergence efficiency, and stronger global search capabilities has become a key area in intelligent optimization research.

Differential Evolution (DE) was introduced by Storn and Price [14] as a stochastic search algorithm for efficient optimization. It is characterized by a simple structure, few control parameters, and strong robustness. DE has been broadly applied in resource scheduling [15–17], artificial neural networks [18–22], vehicle routing [23–26], and numerous other fields [27–39]. Accordingly, many researchers have focused on improving DE performance. For example, Ahmad [40] proposed a DE variant that combines chaotic mapping with opposition-based learning to address poor initial population diversity and local-optima stagnation. Chaotic mapping enhanced population exploration, while opposition-based learning expanded the search range and improved solution quality. Kuma et al. [41] introduced a conservative selection mechanism that retains fitter individuals from

both current and trial populations, improving selection without significantly enhancing exploration. Civicioglu et al. [42] employed elite mutation and Bernstein-polynomial parameter generation to increase mutation directionality and structural control. Yu et al. [43] proposed an uncertainty-based preselection method and a fitness-independent parameter adaptation strategy, and gradually reduced population size to improve exploitation. Liu et al. [44] introduced a distance-constrained adaptive preselection strategy to enhance search efficiency in complex spaces. Despite these improvements, DE remains affected by the “curse of dimensionality,” limiting its efficacy in high-dimensional optimization problems. In addition, other some methods, such as optimization algorithm, machine learning, deep learning, and so on are also proposed in recent years[45–58].

To address this, new methods have been developed for high-dimensional optimization [59]. For example, Xu et al. [60] proposed an importance-guided particle swarm optimization (IGPSO) algorithm, which uses a two-stage neural network trained on positive and negative samples to evaluate feature importance. This guides population initialization and evolution, dynamically adjusting acceleration factors and inertia weights to focus on key feature regions. In another study, Xue et al. [61] developed EAR-FS, which integrates external attention, multi-layer perceptron, and hybrid metaheuristics to evaluate and rank feature importance. A two-stage heuristic search then generates a compact feature subset, significantly reducing dimensionality while maintaining or improving classification performance. To prevent performance collapse in differentiable neural architecture search (DARTS), Xue et al. [62] proposed SWD-NAS, which introduces dual attention modules to dynamically evaluate channel importance and structural weights, along with weight normalization to reduce resource competition and enhance search stability.

To further leverage DE’s original strengths, researchers have explored new perspectives, in-

cluding quantum computing. Quantum Computing (QC) is an advanced computational paradigm renowned for its superior search capabilities. Unlike classical bits, quantum bits (qubits) utilize superposition and entanglement to explore larger state spaces, offering distinct advantages in handling complex, high-dimensional, and constrained problems. Ha et al. [63] were the first to integrate quantum computing concepts into heuristic algorithms, proposing quantum-inspired heuristic methods with notable success in various optimization tasks. Subsequently, Quantum-Inspired Differential Evolution (QDE) has shown promising potential in high-dimensional optimization.

Examples include Dey [64], who presented QIDE, a quantum-inspired DE algorithm for unsupervised clustering that automatically determines optimal cluster numbers. Deng et al. [65] introduced a quantum-adaptive mutation strategy with a PSE framework to monitor and stabilize mutation trends, significantly enhancing convergence and stability. Su et al. [66] developed Adaptive QDE (AQDE), which outperformed other DE variants in global convergence speed and accuracy using real logistics warehouse data. Hu et al. [67] proposed Guided DE (GDE) for optimal quantum gate control, achieving sparse, high-fidelity control pulses under gradient-free conditions. Deng et al. [68] introduced HMCQDE, which combines QDE with a cooperative co-evolution framework to effectively address large-scale, high-dimensional optimization.

Although these QDE variants outperform traditional DE in many high-dimensional tasks, most fail to fully exploit core quantum principles. As a result, issues in initialization balance, convergence accuracy, and stability persist. These shortcomings are largely due to interference from traditional DE mutation strategies, which limit the benefits of quantum computing in high-dimensional optimization.

The significant innovations and contributions of this paper are listed as follow.

- a An improved chaotic strategy is designed for population initialization. Population diversity is enhanced to accelerate the search for optimal solutions.
- b A novel adaptive parameter strategy is introduced. The strength of quantum mutation is ad-

justed dynamically across evolutionary stages to speed up convergence.

- c A hybrid mutation strategy is developed. Weighted mutation operators are integrated with the standard DE mechanism. Different mutation approaches are applied to subpopulations to improve search performance.
- d A new Quantum-Inspired Differential Evolution algorithm, named QAHQDE, is proposed. The algorithm significantly improves initial search efficiency, convergence accuracy, and stability.

The remainder of this paper is structured as follows: Section 2 introduces the fundamental concepts of Quantum-Inspired Differential Evolution (QDE). Section 3 presents the proposed QAHQDE algorithm in detail. Section 4 provides experimental results and comparative analysis. Section 5 concludes the study and discusses future research directions.

2 Quantum differential evolution

QDE is a hybrid algorithm that combines the advantages of quantum evolutionary algorithm (QEA) with the characteristics of DE, aiming to improve the optimization performance in various problems. The difference between a quantum bit and a classical bit allows the quantum bit to represent more states than a classical bit alone. This makes it easier for the quanta to converge to the real value (referring to the actual result of the test function) by interacting with each other during the initialization process and the subsequent quantum revolving door operations. The following are the basic evolutionary operations of the QDE.

2.1 The quantum algorithm

2.1.1 Quantum encoding

The chromosomal code in QDE is no longer real but rather takes the form of a complex number (α, β) , which is typically represented by a Q-bit, as seen in the following equation.

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

where α and β represent the magnitude of 0 and 1 probabilities, respectively. The standardization has

to satisfy the equation $|\alpha|^2 + |\beta|^2 = 1$. ϕ denotes the final state. The following is the definition of a chromosome of length (m):

$$q_i = (|q_{i,1}| |q_{i,1} \wedge |q_{i,m}|) \\ = \begin{bmatrix} |\alpha_{i,1}| & |\alpha_{i,2}| & \dots & |\alpha_{i,m}| \\ |\beta_{i,1}| & |\beta_{i,2}| & \dots & |\beta_{i,m}| \end{bmatrix} \\ = \begin{bmatrix} |\cos(\theta_{i,1})| & |\cos(\theta_{i,2})| & \wedge & |\cos(\theta_{i,m})| \\ |\sin(\theta_{i,1})| & |\sin(\theta_{i,2})| & \wedge & |\sin(\theta_{i,m})| \end{bmatrix} \quad (2)$$

$$\theta_{i,j} = \pi \cdot \text{rand}$$

where i ranges from $\{1, \dots, \text{NP}\}$, and NP is the total population size. j ranges from $[1, D]$ and D is the length of chromosome, $q_{i,j}$ is part of this chromosome. The random variable rand is between (0, 1). $\theta_{i,j}$ denotes the particle of the j -th chromosome in the i -th population.

2.1.2 Feasible solution space transformation

Within the Quantum Evolutionary Algorithm (QEA), the computation value of the objective function necessitates the mapping of quantum chromosomes onto the solution space, which denoted as $\Omega_{QDE} = [a, b]$. The description is describe as follows:

$$q_{i,j} = \frac{(b-a)q_{i,j} + (b+a)}{2} \\ = \left[\begin{array}{c} (b-a)\cos(\theta_{i,j}) + (b+a) \\ (b-a)\sin(\theta_{i,j}) + (b+a) \end{array} \right] / 2 \\ = \left[\begin{array}{c} \text{newcos}(\theta_{i,j}) \\ \text{newsin}(\theta_{i,j}) \end{array} \right] \quad (3)$$

2.1.3 Quantum individual measurement

Quantum observation is the determination of the individual state of the quantum, after which the value of the target can be computed. The operation is described as follows.

$$q_{i,j} = \begin{cases} \text{newcos}(\theta_{i,j}) & \text{newcos}(\theta_{i,j})^2 < \text{rand} \\ \text{newsin}(\theta_{i,j}) & \text{other} \end{cases} \quad (4)$$

2.1.4 Quantum rotation gate

The updating of quantum bits is carried out using a quantum revolving door since the chromo-

somes are in a state of limbo. To achieve population evolution and a better balance between local and global optimization seeking capabilities, new individuals are obtained. As a result, the quantum revolving gate is a crucial part that directly influences the algorithm's performance. The quantum revolving door is shown in Figure 1.

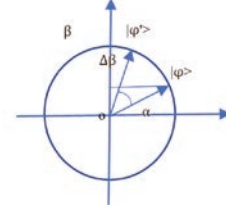


Figure 1. Quantum rotation gate

2.2 Quantum differential evolution

In QDE, in addition to the quantum evolutionary algorithm operation steps described above, three other quantum algorithms combined with differential algorithms are included: the quantum mutation, the quantum crossover, and the quantum selection.

2.2.1 Quantum mutation operation

In each generation evolution of QDE, each quantum individual changes based on the mutation operation, thus promoting the evolution of the population. The mutation strategy for selecting the DE algorithm with Q-bit as the basic vector and mutation operation is described as follows.

$$\Theta_i^{t+1} = \Theta_{r1}^t + F \cdot (\Theta_{r2}^t - \Theta_{r3}^t) \quad (5)$$

where, F is the mutation factor, t represents the current number of iterations, $r1$, $r2$, and $r3$ are random integers within the range of 1 to NP and $r1 \neq r2 \neq r3$.

Range judgments need to be made for mutated individuals to ensure that the specified intervals are not exceeded, and the screening method is described as follows.

$$\Theta_{i,j}^{t+1} = \begin{cases} \Theta_{i,j}^{t+1} & -\pi < \Theta_{i,j}^{t+1} < \pi \\ \pi \cdot \text{rand} & \text{other} \end{cases} \quad (6)$$

2.2.2 Quantum crossover operation

Crossover modifications between quantum chromosomes are carried out to produce new individuals in order to improve the variety of populations. The operation is described as follows.

$$\Theta_{i,j}^{t+1} = \begin{cases} \Theta_{i,j}^{t+1} & \text{rand} < CR \text{ or } j = \text{rand} \\ \Theta_{i,j}^t & \text{other} \end{cases} \quad (7)$$

where, CR is the crossing probability and rand is a random number that is evenly distributed in the range [0, 1].

2.2.3 Quantum selection operation

After determining the fitness value of the target, a greedy strategy is employed to pass on the good individuals to the generation of evolution. The selection procedure is described as follows.

$$\Theta_i^{t+2} = \begin{cases} \Theta_i^{t+1} & f(\Theta_i^{t+1}) < f(\Theta_i^t) \\ \Theta_i^t & \text{other} \end{cases} \quad (8)$$

where, $f(\Theta_i^t)$ is the fitness function.

3 The QAHQDE

Although QDE has better convergence speed and ability to solve high-dimensional problems, it still suffers from low convergence accuracy and insufficient population diversity. Therefore, a new QDE algorithm (QAHQDE) based on chaotic strategy, adaptive parameters and hybrid weighted variation strategy is proposed. First, the population is initialized using chaotic mapping so that the algorithm can find numbers closer to the real value in the initial optimization search. A chaotic strategy is applied to the random numbers in the quantum observation strategy to improve the robustness. Second, the probability of the variance is modified in different search phases, which balances the local and global search abilities, and the adaptive adjustment mechanism of the variance factor is utilized to increase the convergence speed. Finally, the two strategies are dynamically weighted. Meanwhile, the sub-population uses different mutation strategies to generate mutations according to each operation of the quantum vector to increase the diversity

of individuals and avoid falling into local optimality.

3.1 Improved chaotic strategy

In classical QDE algorithms, the initial population is usually generated using random values within a fixed range. However, this often leads to poor diversity and an uneven distribution, causing premature convergence or stagnation in local optima. To alleviate this, chaotic mapping strategies have been widely introduced for initializing the population.

The most commonly used chaotic system is the Logistic map, which exhibits fully chaotic behavior when the control parameter is set to 4. Its standard form is given as follow.

$$Z_{i+1} = \lambda \cdot Z_i \cdot (1 - Z_i) \quad (9)$$

where, $Z_i \in (0, 1)$ is the chaotic variable at iteration i ; λ is the control parameter (set to 4), and Z_1 is the randomly initialized seed. While effective, this mapping often leads to boundary clustering.

To improve the uniformity of the generated population and enhance global search ability, an enhanced chaotic initialization strategy is defined as follow.

$$Z_{i+1} = \lambda \cdot [Z_i(1 - Z_i) + \gamma \cdot (Z_i - 0.5)^3] + \varepsilon_i \quad (10)$$

where, $\lambda = 4$ ensures chaotic behavior; $\gamma \in [0, 1]$ adjusts the shape of nonlinearity; $\varepsilon_i U(\delta, \delta)$ is a small noise term with $\delta = 10^{-4}$; $Z_i \in [0, 1]$ and Z_1 is randomly initialized.

To prevent boundary concentration of values, a cosine-based smoothing transformation is applied.

$$\tilde{Z}_i = \frac{1 - \cos(\pi Z_i)}{2} \quad (11)$$

Finally, normalization is applied to ensure all elements lie within the [0, 1] interval:

$$X_i^{(d)} = \frac{\tilde{Z}_i^{(d)} - \min(\tilde{Z}^{(d)})}{\max(\tilde{Z}^{(d)}) - \min(\tilde{Z}^{(d)})} \quad (12)$$

where, $X_i^{(d)}$ is the normalized value in the d -th dimension for the i -th individual. $\tilde{Z}_i^{(d)}$ denotes the set of transformed values in the d -th dimension.

The final chaotic-initialized population matrix $\mathbf{X} \in [0, 1]^{NP \times D}$ is used as the starting population of the QDE algorithm. This method retains the simplicity of the Logistic map while improving its exploration capability by increasing population diversity and uniformity.

3.2 Parameter adaptive strategy

In the QDE, the mutation factor F plays a pivotal role. As F increases, it boosts the diversity of the population and enhances global search capabilities, yet simultaneously slows down the convergence rate of the algorithm. Conversely, as F decreases, the convergence speed picks up, but at the cost of reduced population diversity, increasing the risk of getting trapped in local optima. To address this trade-off, we employ a parameter adaptive strategy that dynamically adjusts F throughout the algorithm's iterations. Specifically, we maintain a relatively high value of F in the early stages to prioritize population diversity, gradually decreasing it in the later stages to focus more on convergence efficiency. The parameter adaptive strategy is described as follow.

$$F = F_{max} - (F_{max} - F_{min})(t/t_{max}) \quad (13)$$

where, the variance factor has a maximum value and a minimum value, denoted as F_{max} and F_{min} , respectively. $F_{max} = 0.9$ and $F_{min} = 0.2$ are used. t_{max} denotes the maximum number of iterations.

3.3 Hybrid mutation strategy

3.3.1 Weighted dynamic mutation strategy

Solving high-dimensional optimization problems necessitates a careful balance of global and local search capabilities throughout the iterative process. Robust global search enhances population diversity, boosting overall exploration but potentially compromising convergence speed. Conversely, proficient local search accelerates convergence towards optimal solutions, yet it may trap the population in local maxima due to limited diversity. To address this, a hybrid weighted mutation strategy is designed to achieve a harmonious blend of global and local search capabilities, thereby guiding the evolutionary trajectory.

Specifically, Eq. (11) focuses on global search by randomly selecting three individuals for mutation. This approach enhances population diversity but may compromise convergence efficiency. In contrast, Eq.(12) targets the current optimal individual for mutation, emphasizing local search capabilities and enabling faster convergence towards optimal solutions. However, this approach may lead to a reduction in population diversity, trapping the population in local maxima.

To better balance the global and local search capabilities, Eq. (11) and Eq. (12) are combined via a weighting method. A weighting operator μ is introduced, which monotonically decreases with the number of iterations. By adjusting μ , it is possible to ensure that the mutation process is initially more focused on global search, thus maintaining population diversity and improving overall exploration. As the number of iterations increases, μ decreases, shifting the strategy towards local search and accelerating convergence to the optimal solution. This dynamically adjusted weighted mutation strategy achieves an effective balance between global and local search capabilities and greatly improves the efficiency of solving high-dimensional optimization problems. The mutation strategy is described as follows.

QDE/rand/1:

$$\theta_i^{t+1} = \theta_{r1}^t + F \cdot (\theta_{r2}^t - \theta_{r3}^t) \quad (14)$$

QDE/best/1:

$$\theta_i^{t+1} = \theta_{best}^t + F \cdot (\theta_{r1}^t - \theta_{r2}^t) \quad (15)$$

$$\begin{aligned} \theta_i^{t+1} &= \mu [\theta_{r1}^t + F \cdot (\theta_{r2}^t - \theta_{r3}^t)] + \\ &+ (1 - \mu) [\theta_{r1}^t + F \cdot (\text{rand} \cdot \theta_{best}^t - \theta_{r1}^t)] \end{aligned} \quad (16)$$

$$\mu = \exp(-\sqrt{t}) \quad (17)$$

where, $\mu \in [0, 1]$, θ_{best}^t is a best solution for generation t .

3.3.2 Mechanisms of sub-population variation

In quantum differential evolution algorithms, quantum mutation operations play a pivotal role as they directly determine the convergence performance and optimization outcomes of the algorithm. However, traditional mutation operations are limited by a single strategy, which restricts the adjustment of population diversity. This makes balancing

global and local search capabilities challenging and often leads to results deviating from expectations. To address this issue, a mutation mechanism based on subpopulations has been designed. By dividing the population into two subpopulations and applying distinct mutation strategies to each, this mechanism ensures population diversity and significantly boosts algorithm performance.

For the sub-population variation mechanism, two different sub-populations are applied with different variation strategies. One of the subpopulations applies a weighted variation strategy to maximize the balance between global and local optima during the search process.

Another sub-population uses QDE/rand/1 in Eq.(11), which employs random vectors to expand the search scope and allow the algorithm to escape from local optima.

3.4 The model of QAHQDE

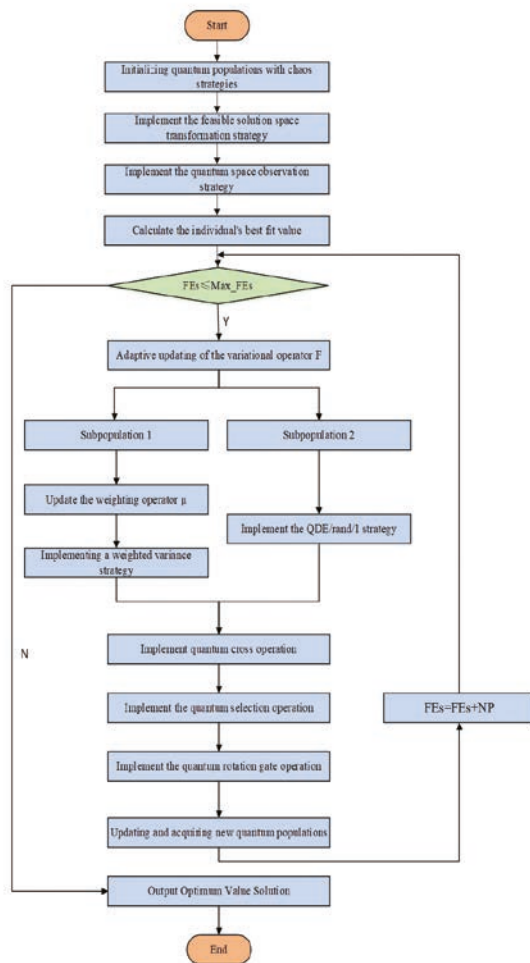


Figure 2. The flow of QAHQDE

The detailed descriptions of QAHQDE are given as follows.

Step 1: Create the starting population by setting the population size NP, dimension D, variation factor F, quantum rotation angle, and other parameters using the chaotic technique.

Step 2: Initialize the populations using the chaos strategy.

Step 3: Implement the workable transformation plan for the solution space.

Step 4: Implement the quantum space observation strategy. Determine the quantum state according to Eq. (4).

Step 5: Use a greedy selection strategy to select the best individual fitness value and update it.

Step 6: Perform step 7 when FEs is less than MaxFEs, otherwise perform step 13.

Step 7: Subdivide the population into two subpopulations with equal individuals.

Step 8: Perform quantum mutation operations.. Subpopulation 1 operates according to Eq.(13) and subpopulation 2 operates according to Eq.(11).

Step 9: A quantum crossover operation is implemented to acquire a fresh population, and the operation is performed on the population according to Eq. (7).

Step 10: Implement a quantum selection operation to perform an operation on the population according to Eq. (8).

Step 11: Implement a quantum rotation gate operation to calculate fitness values and select the new population to perform fitness value calculation and select the new population.

Step 12: Compare the results. If the desired value is reached, perform step 13. Otherwise, return to step 7.

Step 13: Output the scheme with the highest fitness value and terminate the algorithm.

4 Experiment and analysis

4.1 The experiment environment

Emphasizing a testing environment with flexible dimensions ranging from 100 to 3000, the test

functions are required to possess the capability to adjust dimensions to meet experimental demands. Hence, CEC2017, along with a series of other functions that can be extended up to 3000 dimensions, has been chosen. These functions include three main classifications: unimodal, multimodal and hybrid functions. The enhancement or development capabilities inherent in various optimization algorithms can be examined. It also helps to gain insight into the exploration or diversification properties of these algorithms. Their salient features are systematically presented in Tab.1. These algorithms are programmed in MATLAB R2021b and run in an experimental setting using an Intel(R) Core(TM) i7-10700 CPU running at 2.90GHz and Windows. In our experiments, the setting of the parameters, $NP = 90$, $F = 0.5$, $CR = 0.9$, $FES = 5e+5$, was accomplished through some literature and tests. Each function was subjected to 25 independent experiments to avoid chance situations. To provide a fair comparison, the particular variables of the other algorithms are gathered from the relevant references.

In the comparative analysis, the operational efficiency and result metrics of the proposed QAHQDE are juxtaposed against recognized optimization methods such as DE-BBO [69], EO [70], DBO [71], TLBO [72], HMCQDE [68], JADE [73], CoDE [74], L-SHADE [75], CJADE [13], SHADE-ILS [76] and MLSHADE_SPA [77].

5 Experiment results and analysis

5.1 Comparison of sub-strategies

To demonstrate the efficacy of each strategy within the QAHQDE, the strategies in QAHQDE are combined with QDE to form six algorithms QQDE, HQDE, AQDE, AHQDE, QHQDE, and QAQDE, respectively (where Q stands for the sub-population variation strategy, A stands for the parameter adaptive strategy, and H stands for the chaotic strategy). The convergence performance of QAHQDE is shown in Figure 3~Figure 8. The vertical coordinates of Figure 3~Figure 5 are represented by real numbers to observe the performance of the algorithm before convergence. The vertical coordinates of Figure 6~Figure 8 are represented by logarithmic numbers, focusing on observing the performance after convergence.

After an in-depth analysis of the experimental results in Table 3, it can be found that the hybrid mutation strategy effectively frees the algorithm from the constraints of the constraints of a local optimum to a certain extent. The accuracy of the QQDE in solving such functions is significantly improved by orders of magnitude. In contrast, the parameter adaptive strategy and chaos strategy primarily focus on optimizing the convergence rate and initial search values of the algorithm, respectively, which lead to less significant precision enhancements for AQDE and HQDE when compared with QDE. Given the distinct roles played by these three strategies in the algorithm, the integration of any two strategies, such as in QHQDE, enables the algorithm to identify values close to the optimal solution at the initial stage. With the further application of the hybrid mutation strategy, the algorithm shows significant improvement in solving all the selected functions, especially for functions F1, F2, F4, F9, F10 and F11 where the optimal solutions can be successfully found. However, since the two strategies employed in AHQDE do not primarily target enhancing the precision of solving functions, the algorithm's final convergence values for the tested functions are relatively limited. On the other hand, while QAHQDE achieves a certain degree of precision improvement in single-strategy algorithms, its inability to quickly approximate the optimal solution at the initial stage sometimes leads to local optima during the 25 independent runs, thereby affecting the overall average outcome. Conversely, the QAHQDE algorithm, which integrates all three strategies, demonstrates superior performance. It successfully finds the optimal solutions for functions F1, F2, F3, F4, F8, F9, F10, and F11, and while it may not always reach the optimal value for the remaining functions, it still exhibits promising results. Additionally, similar conclusions can be drawn from the analysis of the experimental results presented in Table 3 - Table 4.

After a careful examination of the experimental results in Table 2 - Table 4, we observed that, as the dimension of the test functions increased, QAHQDE maintained its ability to precisely locate the optimal values for functions F1, F2, F3, F4, F9, F10, and F11. While the search accuracy for other functions did decline, the magnitude of this decrease was relatively minor compared to other QDE strategies. Specifically, when $D = 100$, QAHQDE

Table 1. Description of basis functions

Function	Range	$f_{\min}$
$f_1(x) = \sum_{i=1}^n x_i^2$	[-100,100]	0
$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	[-10,10]	0
$f_3(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$	[-100,100]	0
$f_4(x) = \max_i x_i $	[-100,100]	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-30,30]	0
$f_6(x) = \sum_{i=1}^n (x_i + 0.5)^2$	[-100,100]	0
$f_7(x) = \sum_{i=1}^n i x_i^4 + \text{random}[0, 1]$	[-128,128]	0
$f_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	[-500,500]	$-418.9829 \times n$
$f_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	[-5.12,5.12]	0
$f_{10}(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum x_i^2}\right) - \exp\left(\frac{1}{n} \sum \cos(2\pi x_i)\right) + 20 + e$	[-32,32]	0
$f_{11}(x) = \frac{1}{4000} \sum x_i^2 - \prod \cos(x_i/\sqrt{i}) + 1$	[-600,600]	0
$f_{12}(x) = \frac{\pi}{n} \left\{ 10 \sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right\} + \sum u(x_i, 10, 100, 4)$	[-50,50]	0
$f_{13}(x) = 0.1 \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \right\} + \sum u(x_i, 5, 100, 4)$	[-50,50]	0

Table 2. Experimental results of eight algorithms for ablation in 100-dimension

Functions	Index	QDE	AQDE	HQDE	QQDE	AHQDE	QHDE	QAQDE	QAHQDE
F1	Avg.	2.0316E+01	2.6420E-02	1.2853E+02	1.5603E-03	1.6328E-03	0.0000E+00	4.2363E-05	0.0000E+00
	Std.	1.0734E+01	1.3073E-01	6.9523E+01	2.1404E-03	2.6945E-03	0.0000E+00	7.5207E-05	0.0000E+00
F2	Avg.	2.6148E+01	3.1514E+01	1.8219E+00	1.5035E-01	2.8460E+00	0.0000E+00	1.4801E-02	0.0000E+00
	Std.	9.5526E+00	1.2612E+01	2.0890E+00	1.4551E-01	1.2341E+01	0.0000E+00	2.1218E-02	0.0000E+00
F3	Avg.	0.0000E+00	5.0487E-31	0.0000E+00	9.6917E-09	0.0000E+00	1.8201E-09	1.3084E-14	0.0000E+00
	Std.	0.0000E+00	2.5244E-30	0.0000E+00	2.1891E-08	0.0000E+00	8.5324E-09	4.2391E-14	0.0000E+00
F4	Avg.	7.0079E+01	7.0709E+01	7.0531E+01	3.5846E-03	7.0711E+01	0.0000E+00	2.2676E-03	0.0000E+00
	Std.	1.7814E+00	6.5573E-03	6.2497E-01	2.9076E-03	4.3512E-14	0.0000E+00	4.1216E-03	0.0000E+00
F5	Avg.	6.0667E+02	7.6834E+02	5.9357E+03	9.8234E+01	8.1284E+02	9.2574E+01	9.5086E+01	8.0756E+01
	Std.	7.3449E+02	8.4736E+02	2.3148E+03	2.2873E-01	9.4348E+02	4.4515E-01	1.0927E+01	2.7468E+01
F6	Avg.	5.7501E+00	1.5542E-01	1.2301E+02	1.1610E+00	3.9428E-01	5.7978E-01	2.3499E-04	1.3571E-05
	Std.	2.7520E+00	2.5718E-01	7.4003E+01	1.2766E+00	5.0238E-01	3.1086E-01	1.0832E-03	3.7613E-06
F7	Avg.	3.1541E+01	1.1245E+01	7.4713E-02	9.8063E-05	2.1369E-01	3.1207E-05	4.7977E-05	2.4902E-05
	Std.	1.4331E+01	1.2938E+01	1.4067E-02	6.1922E-05	3.3374E-01	1.2462E-05	2.6652E-05	1.8359E-05
F8	Avg.	-4.1536E+04	-4.1898E+04	-4.1383E+04	-4.1729E+04	-4.1898E+04	-4.1721E+04	-4.1898E+04	-4.1898E+04
	Std.	2.7081E+02	1.3477E-04	4.2075E+02	5.0032E+01	1.0028E-04	3.0136E+01	1.6179E-02	1.9792E-02
F9	Avg.	2.2000E+02	1.8781E+02	1.6763E+02	1.4072E-01	2.3700E+02	0.0000E+00	4.6645E-04	0.0000E+00
	Std.	4.1431E+01	4.8721E+01	4.0043E+01	2.0431E-01	5.2256E+01	0.0000E+00	7.7539E-04	0.0000E+00
F10	Avg.	5.2652E+00	1.3961E+01	3.6571E+00	1.1531E-02	2.4645E+00	8.8818E-16	6.5748E-04	8.8818E-16
	Std.	4.4402E+00	2.5053E+00	8.5946E-01	9.1553E-03	5.0829E+00	0.0000E+00	8.0695E-04	0.0000E+00
F11	Avg.	9.3055E-01	7.2911E-02	1.9333E+00	3.5919E-05	3.3560E-02	0.0000E+00	2.4737E-06	0.0000E+00
	Std.	2.1085E-01	9.1094E-02	4.4999E-01	5.0241E-05	3.8759E-02	0.0000E+00	9.5842E-06	0.0000E+00
F12	Avg.	2.7896E-02	8.3653E-03	1.3478E-01	1.8126E-02	1.2791E-02	1.8007E-02	2.2554E-07	2.5887E-07
	Std.	5.9673E-02	1.3948E-02	1.1202E-01	8.2075E-03	1.8103E-02	8.1611E-03	7.3433E-08	9.9212E-08
F13	Avg.	3.5542E-01	5.3249E-02	1.9507E+00	1.6172E+00	3.9601E-03	7.6162E-01	1.7066E-04	1.9474E-04
	Std.	2.8855E-01	1.2736E-01	1.2060E+00	6.6527E-01	5.3833E-03	5.4555E-01	7.9782E-04	9.1879E-04

Table 3. Experimental results of eight algorithms for ablation in 500-dimension

Functions	Index	QDE	AQDE	HQDE	QQDE	AHQDE	QHDE	QAQDE	QAHQDE
F1	Avg.	2.8716E+04	1.5019E+05	4.3828E+04	1.2489E-02	2.1005E+05	0.0000E+00	2.7206E-04	0.0000E+00
	Std.	2.1655E+04	4.9982E+04	3.5707E+04	1.7700E-02	8.6107E+04	0.0000E+00	4.3903E-04	0.0000E+00
F2	Avg.	5.5072E+02	1.3180E+03	7.6688E+02	1.0089E+00	1.1762E+03	0.0000E+00	2.2624E-01	0.0000E+00
	Std.	1.0070E+02	6.2667E+01	1.6625E+02	5.9594E-01	2.2244E+02	0.0000E+00	2.4660E-01	0.0000E+00
F3	Avg.	1.2925E-28	8.1133E+00	0.0000E+00	1.6595E-07	9.3342E-01	7.5266E-08	3.0610E-14	0.0000E+00
	Std.	6.4623E-28	2.9666E+01	0.0000E+00	2.2554E-07	2.6105E+00	1.1050E-07	1.0733E-13	0.0000E+00
F4	Avg.	7.0711E+01	7.0711E+01	7.0711E+01	7.9553E-03	7.0711E+01	5.0886E-15	4.2578E-03	0.0000E+00
	Std.	4.3512E-14	4.3512E-14	4.3512E-14	3.8143E-03	4.3512E-14	5.8668E-15	3.7455E-03	0.0000E+00
F5	Avg.	1.0463E+08	3.4337E+08	9.7773E+07	4.9553E+02	4.0125E+08	4.9138E+02	4.6462E+02	4.2679E+02
	Std.	1.0080E+08	1.3604E+08	1.1630E+08	1.1487E+00	2.1039E+08	2.6006E-01	9.0215E+01	1.1629E+02
F6	Avg.	2.6508E+04	1.5770E+05	4.4270E+04	6.3267E+01	2.4720E+05	2.1134E+01	5.5317E-01	4.3201E-01
	Std.	1.8562E+04	4.1975E+04	3.6888E+04	3.2598E+01	9.7900E+04	4.4504E+00	1.3314E-01	6.3361E-02
F7	Avg.	1.5765E+04	1.8551E+04	5.6185E+03	1.6033E-04	2.4878E+04	3.7595E-05	4.3196E-05	4.0495E-05
	Std.	2.3164E+03	1.4595E+03	1.3554E+03	1.2495E-04	2.9644E+03	2.0285E-05	3.8039E-05	2.2062E-05
F8	Avg.	-2.0807E+05	-2.0487E+05	-2.0702E+05	-2.0792E+05	-2.0450E+05	-2.0797E+05	-2.0889E+05	-2.0890E+05
	Std.	7.4561E+02	1.0898E+03	8.1638E+02	3.5978E+02	1.3149E+03	3.1551E+02	1.1581E+02	1.2750E+02
F9	Avg.	1.7660E+03	2.9757E+03	4.1455E+03	1.1905E+01	4.7397E+03	0.0000E+00	5.5914E-01	0.0000E+00
	Std.	1.7057E+02	3.6902E+02	2.2388E+02	2.2637E+01	2.1136E+02	0.0000E+00	9.7511E-01	0.0000E+00
F10	Avg.	1.8721E+01	1.9661E+01	2.0039E+01	2.2017E-02	1.9181E+01	8.8818E-16	3.9124E-03	8.8818E-16
	Std.	6.4464E-01	1.1768E-01	3.7027E-01	1.5015E-02	2.1287E-01	0.0000E+00	4.4223E-03	0.0000E+00
F11	Avg.	5.6140E+01	9.5934E+02	6.0121E+01	3.1295E-05	9.9697E+02	0.0000E+00	6.6842E-07	0.0000E+00
	Std.	7.3898E+01	2.5736E+02	3.6924E+01	3.7583E-05	5.6103E+02	0.0000E+00	1.5634E-06	0.0000E+00
F12	Avg.	5.5994E+07	1.3517E+09	4.6182E+07	1.6513E-01	1.2656E+09	1.2950E-01	1.0352E-03	9.0918E-04
	Std.	1.0562E+08	5.8137E+08	1.0448E+08	3.5199E-02	8.0247E+08	2.7199E-02	2.0713E-04	1.6491E-04
F13	Avg.	3.3544E+07	1.2961E+09	2.1513E+07	2.6774E+01	1.6037E+09	1.7052E+01	8.7044E-02	1.0862E-01
	Std.	4.8377E+07	5.3278E+08	8.7416E+07	1.0963E+01	9.5337E+08	4.5310E+00	4.5352E-02	8.0645E-02

Table 4. Experimental results of eight algorithms for ablation in 1000-dimension

Functions	Index	QDE	AQDE	HQDE	QQDE	AHQDE	QHQDE	QAQDE	QAHQDE
F1	Avg.	4.7062E+05	1.1149E+06	7.0130E+05	1.5039E-02	1.4308E+06	0.0000E+00	1.1998E-03	0.0000E+00
	Std.	1.2631E+05	1.2050E+05	1.5017E+05	2.1707E-02	1.4336E+05	0.0000E+00	2.4562E-03	0.0000E+00
F2	Avg.	1.0693E+03	1.7464E+03	2.8928E+03	2.7584E+00	3.2551E+03	0.0000E+00	5.3616E-01	0.0000E+00
	Std.	1.2759E+02	4.9259E+01	3.2594E+02	1.9748E+00	7.1234E+01	0.0000E+00	4.4207E-01	0.0000E+00
F3	Avg.	3.6505E-07	2.6952E+03	4.4159E-07	1.7573E-07	1.6731E+01	2.0500E-07	1.4400E-13	0.0000E+00
	Std.	1.7870E-06	1.3445E+04	2.1045E-06	3.3848E-07	3.2887E+01	2.6000E-07	5.8100E-13	0.0000E+00
F4	Avg.	7.0711E+01	7.0711E+01	7.0711E+01	7.1873E-03	7.0711E+01	8.1700E-15	4.0491E-03	0.0000E+00
	Std.	4.3512E-14	4.3512E-14	4.3512E-14	3.9041E-03	4.3512E-14	6.3800E-15	4.4870E-03	0.0000E+00
F5	Avg.	1.5284E+09	3.6303E+09	1.6126E+09	9.9140E+02	4.6890E+09	9.8911E+02	9.9122E+02	8.7405E+02
	Std.	4.4938E+08	6.5904E+08	7.3649E+08	1.1211E+00	7.1319E+08	4.3767E-01	2.0036E+00	2.4877E+02
F6	Avg.	4.4532E+05	1.1489E+06	6.8413E+05	1.5503E+02	1.5188E+06	6.6356E+01	3.7987E+00	3.2937E+00
	Std.	1.6705E+05	1.5666E+05	1.4708E+05	5.2785E+01	1.5917E+05	1.1573E+01	7.0038E-01	3.3666E-01
F7	Avg.	8.9719E+04	1.0086E+05	8.6344E+04	5.5606E-04	1.5608E+05	3.3700E-05	4.4600E-05	5.0609E-05
	Std.	1.1514E+04	4.1863E+03	7.3795E+03	1.3617E-03	4.6764E+03	2.0100E-05	4.4200E-05	3.0521E-05
F8	Avg.	-4.1322E+05	-3.7890E+05	-4.0870E+05	-4.1558E+05	-3.7909E+05	-4.1557E+05	-4.1539E+05	-4.1556E+05
	Std.	2.0893E+03	3.2334E+03	1.5726E+03	5.4352E+02	3.6247E+03	5.9809E+02	9.5782E+02	6.3774E+02
F9	Avg.	5.7254E+03	9.2517E+03	1.0539E+04	1.1984E+01	1.0538E+04	0.0000E+00	1.5261E+00	0.0000E+00
	Std.	4.5390E+02	2.7528E+02	1.3136E+02	1.2415E+01	1.5974E+02	0.0000E+00	3.0234E+00	0.0000E+00
F10	Avg.	1.9638E+01	2.0040E+01	2.0242E+01	1.5519E-02	1.9662E+01	8.8800E-16	4.8770E-03	8.8818E-16
	Std.	3.6297E-01	5.7903E-02	4.1439E-02	8.1702E-03	9.6140E-02	0.0000E+00	5.2053E-03	0.0000E+00
F11	Avg.	1.9239E+03	7.3800E+03	2.8730E+03	3.6973E-05	9.0901E+03	0.0000E+00	2.3400E-06	0.0000E+00
	Std.	7.5985E+02	9.8355E+02	1.1852E+03	3.9349E-05	2.4314E+03	0.0000E+00	4.7700E-06	0.0000E+00
F12	Avg.	3.5720E+09	1.4438E+10	4.5727E+09	2.9581E-01	1.6224E+10	2.3871E-01	2.9473E-03	2.7960E-03
	Std.	2.0580E+09	2.6799E+09	2.7521E+09	5.4221E-02	3.6681E+09	3.3143E-02	5.1300E-04	3.4960E-04
F13	Avg.	4.4390E+09	1.3927E+10	5.9586E+09	5.4795E+01	1.6873E+10	4.6885E+01	3.3077E-01	2.6347E-01
	Std.	1.6892E+09	2.2762E+09	2.6914E+09	2.7674E+01	3.7464E+09	1.0291E+01	3.4923E-01	2.1451E-01

did not demonstrate superior performance in terms of the average values of two functions or the standard deviations of five functions compared to other algorithms. Similarly, for D=500, its performance in terms of the average values of two functions and the standard deviations of four functions fell short of surpassing other algorithms. Furthermore, for D=1000, QAHQDE did not excel in the average values of one function and the standard deviations of two functions. Nevertheless, from an overall trend perspective, as the function dimension increased, the QAHQDE exhibited increasingly robust performance, which indicates better robustness.

After a meticulous examination of Figure 3 - Figure 5, it was observed that, aside from the convergence curves of F2 and F3, the initial convergence performance of AQDE and AHQDE appears slightly inferior to that of QDE. While HQDE exhibits a convergence rate when compared with QDE, the introduction of chaotic strategies enables it to approach the optimal value more closely during the initial phase of the optimization process, thereby demonstrating superior performance in the early stages of convergence. In contrast,

QQDE, QAQDE, QHQDE, and QAHQDE exhibit significant improvements in convergence performance compared to QDE. However, due to the orders-of-magnitude enhancement in search accuracy achieved by these improved algorithms, the current charts are insufficient to comprehensively capture the subtle differences in their relative convergence capabilities. To address this, the convergence patterns using logarithmic-scale graphs are further analysed, and the results are shown in Figure 6 - Figure 8, to provide a more precise comparison of the convergence performance of these algorithms.

After a thorough analysis of Figure 6 - Figure 8, we observed that the QQDE and QHQDE exhibited strikingly similar convergence performance in all figures except for F8 and F10. This similarity can be attributed primarily to the chaos strategy's influence on the initial value search and algorithm robustness. This strategy not only enhances convergence performance but also significantly improves search performance. However, when we compare the logarithmic graphs, the advantages of the chaos strategy become less evident. Notably, the parameter adaptive strategy positively impacts the conver-

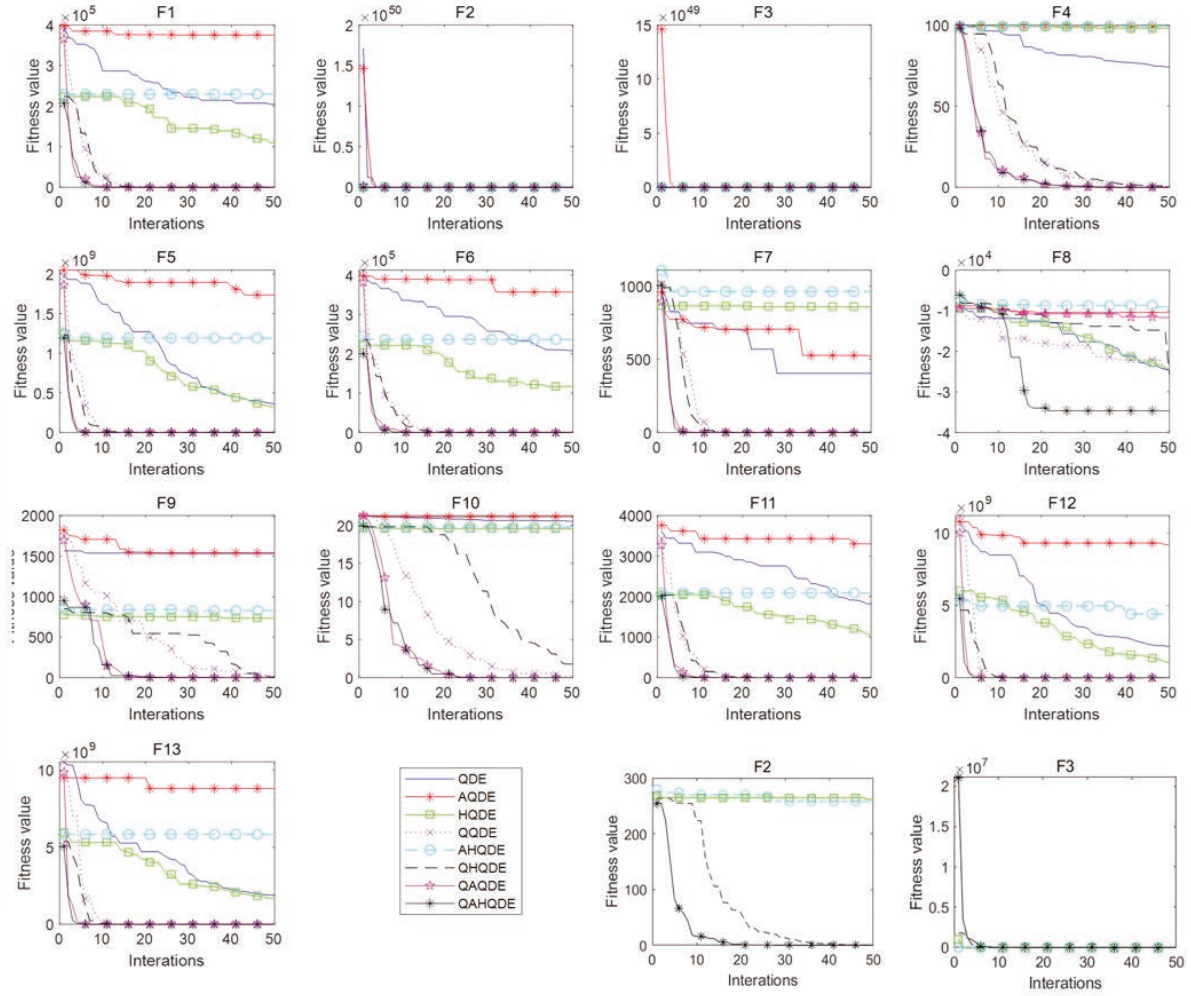


Figure 3. Convergence curves for optimization of ablation test functions (D=100)

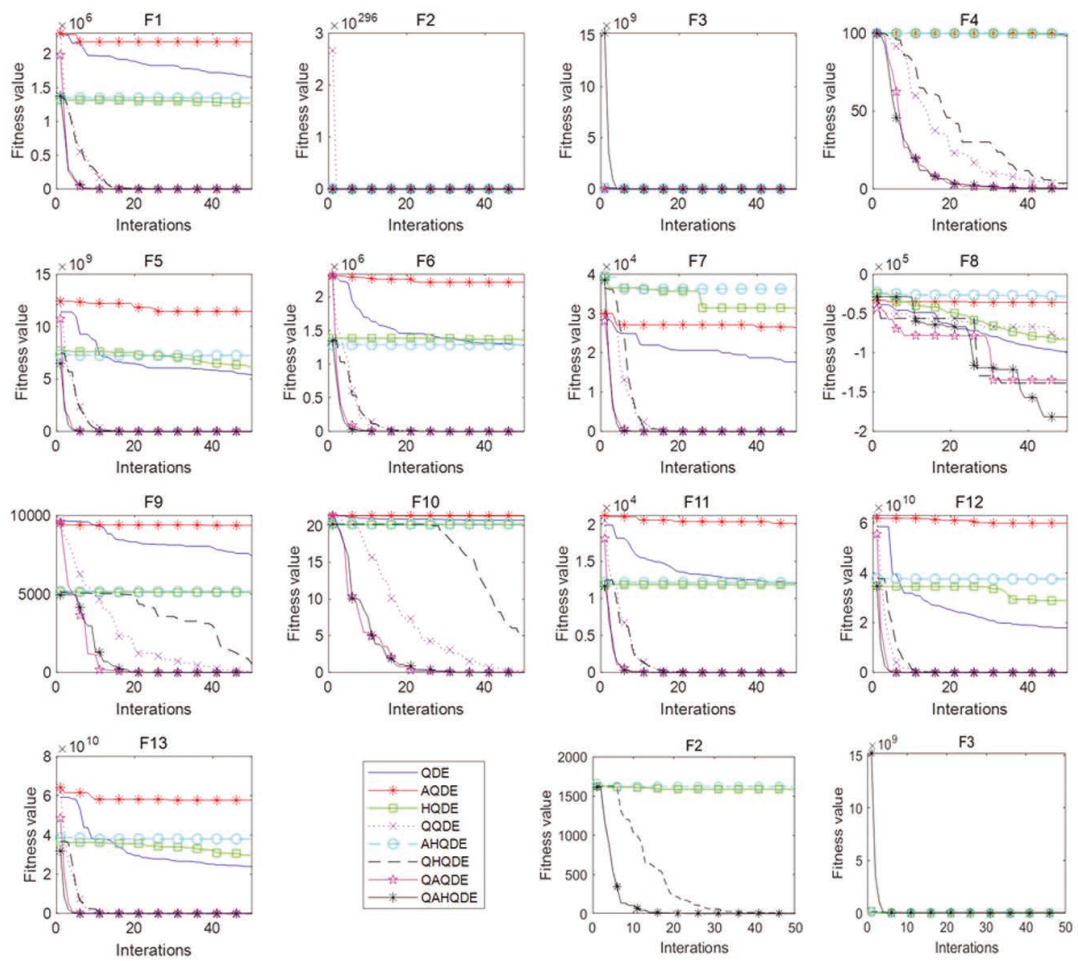


Figure 4. Convergence curves for optimization of ablation test functions (D=500)

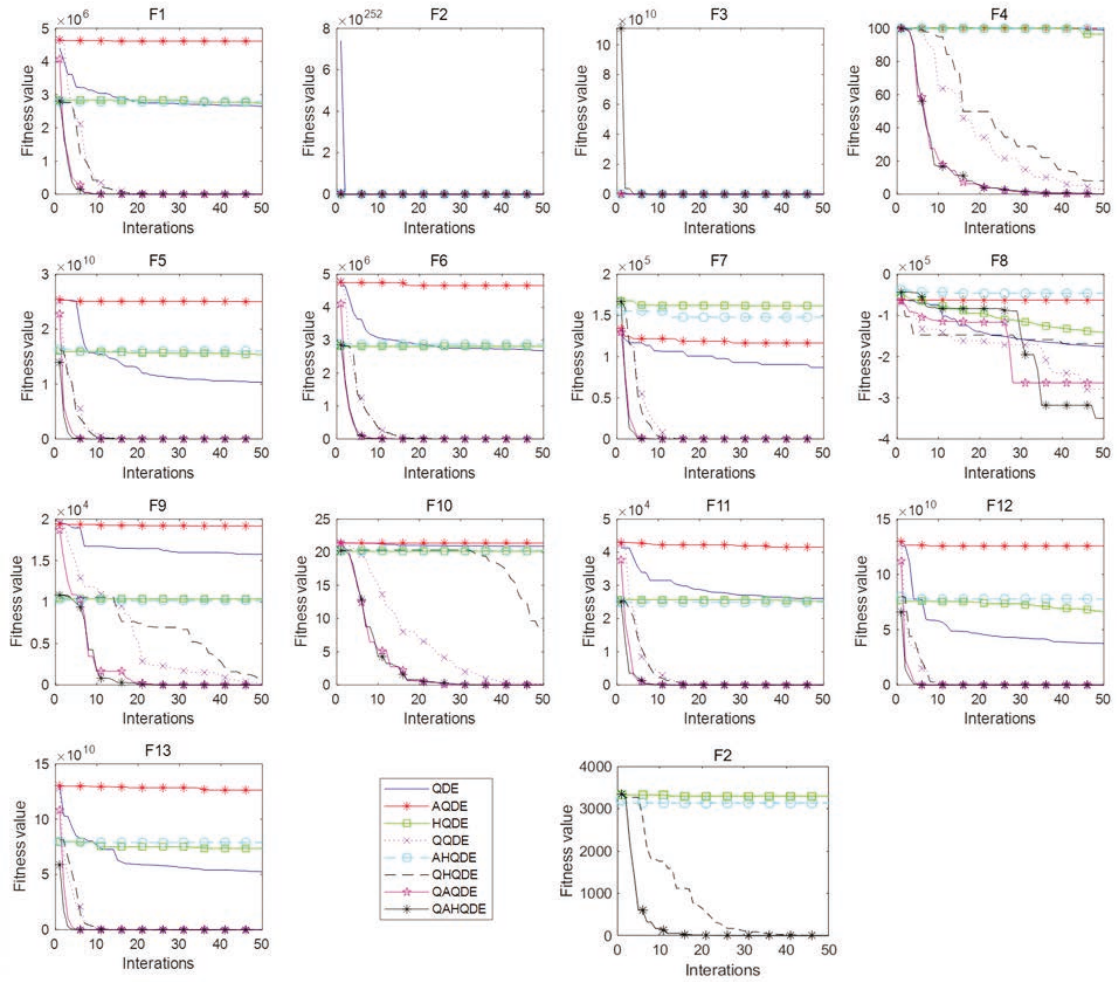


Figure 5. Optimization process logarithmic curves for ablative test functions (D=100)

gence rate of the algorithm, resulting in QAHQDE demonstrating a faster convergence rate compared to QQDE. Moreover, QAHQDE integrates the advantages of all strategies, excelling not only in initial value optimization and convergence rate but also in convergence accuracy. This integration allows each strategy to fully leverage its potential, surpassing the effects of its individual application.

5.2 Comparative evaluation of various algorithms

In order to demonstrate the optimization performance of QAHQDE, the DE-BBO, EO, DBO, TLBO, HMCQDE, JADE and CoDE algorithms are selected for comparison in solving the functions with $D=100$, $D=500$ and $D=1000$. The obtained experiment results are shown in Table 5 - Table 7, with bold type used to indicate the best outcomes. The convergence curves are shown in Figure 9 - Figure 11.

In the Table 5 - Table 7, the QAHQDE shows excellent overall optimization efficacy when dealing with problems of dimensions 100, 500, and 1000, respectively. For functions F1 to F4, F8, F9, and F11, QAHQDE successfully finds optimal solutions in all dimensions. Even when faced with the challenge of some of the functions for which the absolute optimal solution is not found, QAHQDE still shows superior search performance compared to the other algorithms.

Specifically, in the 100-dimensional functions, although the performance of the QAHQDE in finding optimal values for functions F6, F12, and F13 is not the best among all algorithms, it still ensures that the desired range of solutions is achieved. As the dimension is increased to 500, the ability of the QAHQDE to find the optimal values for functions F6, F8, F12, and F13 is clearly superior for the rest of the functions, except for the optimal values for functions F6, F8, F12, and F13, where it is slightly lower than some specific algorithms. When extended to 1000 dimensions, QAHQDE still outperforms other comparative algorithms on most functions.

Based on the visual analysis in Figure 9 - Figure 11, it can be found that QAHQDE is slightly inferior to the other algorithms in the initial convergence speed of function F3 when the dimension

is 100, and QAHQDE is slightly inferior to the HMCQDE algorithm in the early convergence stage of functions F7, F8 and F13. In the case of 500 and 1000 dimensions, although the convergence of solving the function F3 still needs to be improved, and there is a certain gap between the convergence effect and that of HMCQDE when dealing with the functions F6, F8, F12, and F13, QAHQDE shows better convergence performance for all kinds of functions in all other dimensions. It is especially worth mentioning that QAHQDE can achieve significant convergence in just 50 iterations, showing higher convergence efficiency than other algorithms.

In summary, the QAHQDE algorithm shows stronger optimization ability and stability in most function optimization tasks. Even though it is not guaranteed to obtain the best results every time, it is able to quickly approximate and lock in good quality solutions in the comprehension interval. Compared with algorithms optimized specifically for high- or low-dimensional problems, QAHQDE maintains consistently and stably excellent optimization ability on problems spanning different dimensional scales, and its optimization performance does not decline drastically as the problem dimension grows, but rather maintains its consistent efficiency and reliability.

In the Table 8, it can be noted that the running time of the QAHQDE algorithm does not reach the shortest record among all the compared algorithms, however, its execution efficiency shows a more obvious optimization compared to the two algorithms DE-BBO and HMCQDE. Specifically, the runtime of QAHQDE is significantly shorter than that of DE-BBO and HMCQDE. Combined with the in-depth analysis of the convergence performance and results in Table 5~Table 7, it can be seen that although the convergence speed and the optimal solution quality of QAHQDE are slightly inferior to those of HMCQDE for some function problems, at the same time, it has the advantage of overall performance at the cost of shorter runtime. Therefore, considering the two key dimensions of convergence effect and computational resource consumption, it can be concluded that although QAHQDE may lack in individual convergence characteristics, in terms of overall performance, QAHQDE shows a more desirable overall benefit in practical appli-

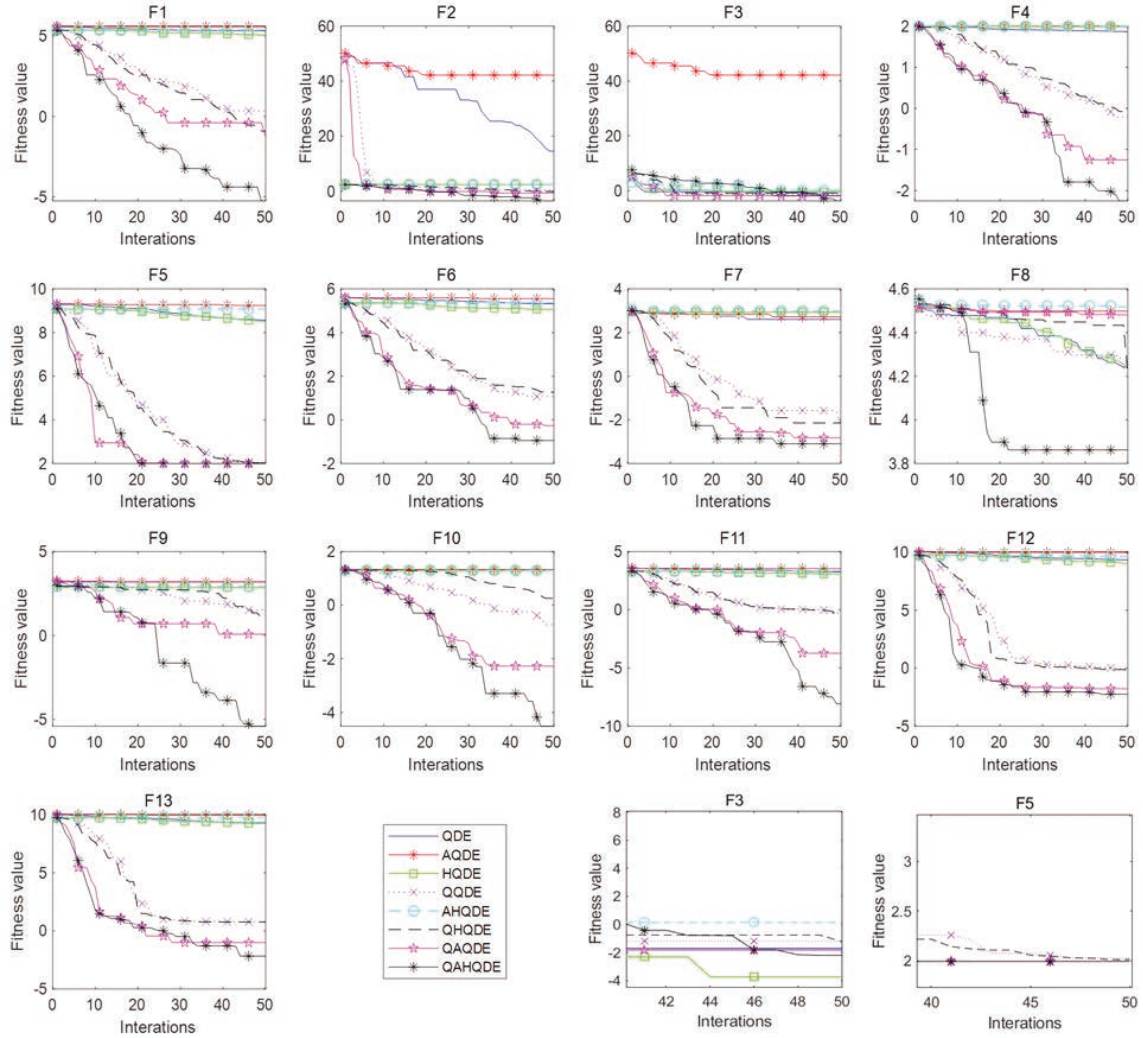


Figure 6. Optimization process logarithmic curves for ablative test functions (D=500)

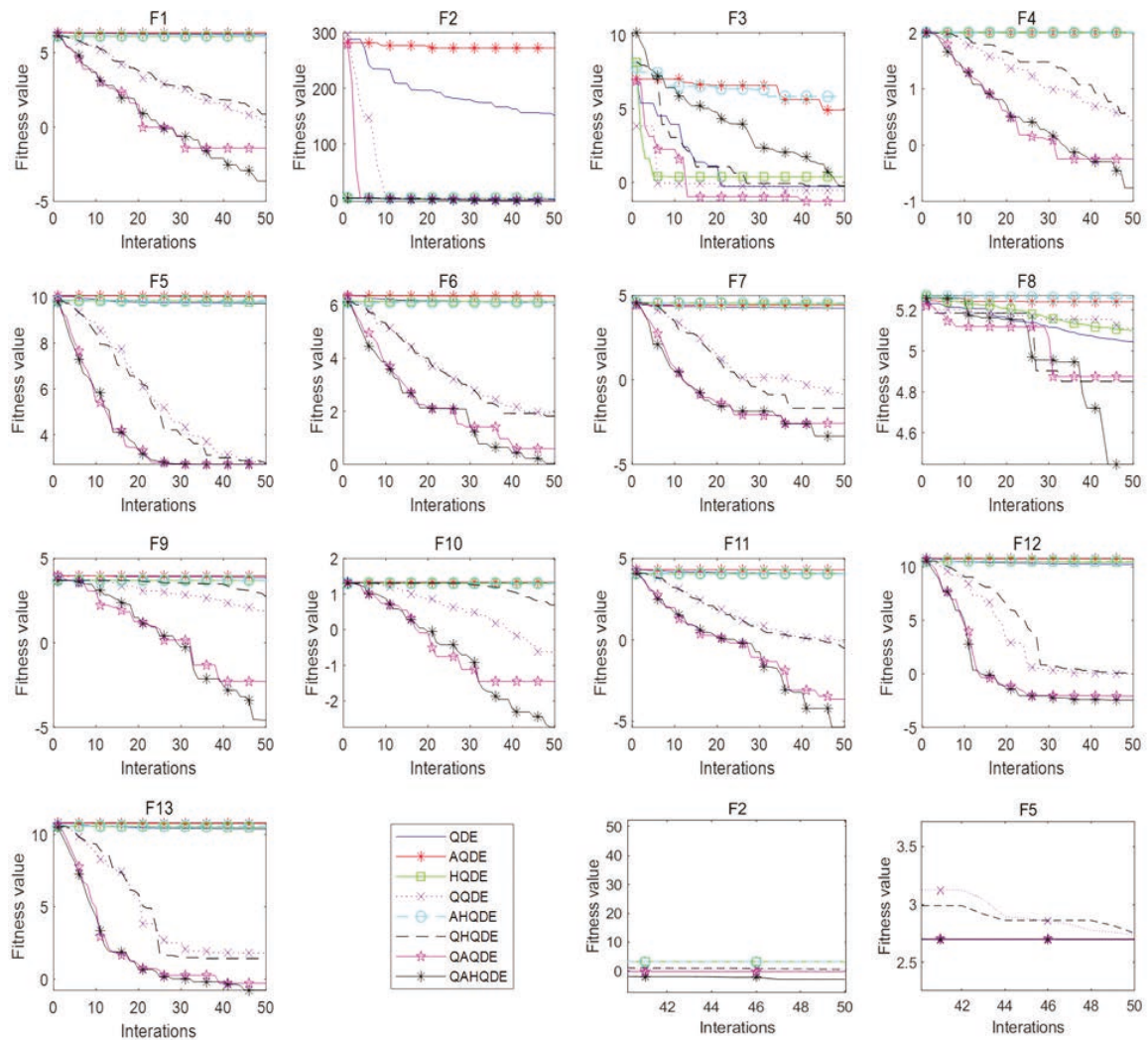


Figure 7. Optimization process logarithmic curves for ablative test functions (D=1000)

Table 5. The experimental results of the eight algorithms in 100 dimensions

Functions	Index	DE-BBO	EO	DBO	TLBO	HMCFQDE	JADE	CoDE	QAHQDE
F1	Avg.	7.6608E+01	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2.0988E-66	7.2460E-26	0.0000E+00
	Std.	8.3025E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	9.0908E-66	7.7354E-26	0.0000E+00
F2	Avg.	5.2173E+00	2.0925E-274	0.0000E+00	0.0000E+00	0.0000E+00	4.7976E-33	1.6508E-13	0.0000E+00
	Std.	3.5183E-01	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.7242E-32	6.3044E-14	0.0000E+00
F3	Avg.	4.2441E-08	1.3754E-62	1.4115E-299	4.6826E-08	2.4261E-07	1.2716E-11	0.0000E+00	0.0000E+00
	Std.	1.4703E-07	6.7153E-62	0.0000E+00	5.8655E-08	5.0046E-07	3.7524E-11	0.0000E+00	0.0000E+00
F4	Avg.	3.1030E+00	1.1035E-80	0.0000E+00	1.0000e-323	9.6358E-12	1.4560E+01	8.6760E-05	0.0000E+00
	Std.	1.4337E-01	5.4615E-80	0.0000E+00	0.0000E+00	2.2031E-11	1.8114E+00	5.6792E-05	0.0000E+00
F5	Avg.	1.2601E+03	8.9557E+01	9.0596E+01	7.4587E+01	1.3140E+02	1.0243E+02	6.8324E+01	8.0756E+01
	Std.	2.4173E+02	5.1304E-01	4.1490E-01	3.6988E+00	5.1239E+01	5.1135E+01	2.3264E+00	2.7468E+01
F6	Avg.	7.2437E+01	6.2596E-18	6.5286E-17	1.8603E-16	0.0000E+00	4.9304E-34	4.9229E-26	1.3571E-05
	Std.	6.5802E+00	2.8564E-17	8.2742E-17	4.3658E-16	0.0000E+00	1.1530E-33	4.0498E-26	3.7613E-06
F7	Avg.	1.2118E-02	8.7196E-05	1.7564E-04	9.6670E-05	2.7052E-02	2.1640E-02	1.0517E-02	2.4902E-05
	Std.	1.6522E-03	4.2083E-05	1.6294E-04	2.2399E-05	8.0408E-03	5.3941E-03	2.9379E-03	1.8359E-05
F8	Avg.	-2.5072E+04	-2.9444E+04	-3.2245E+04	-2.5556E+04	-4.1898E+04	-4.1894E+04	-3.7324E+04	-4.1898E+04
	Std.	1.1756E+03	1.8207E+03	6.2123E+03	2.1271E+03	2.9443E-11	2.3688E+01	5.3104E+02	1.9792E-02
F9	Avg.	1.3735E+02	0.0000E+00	0.0000E+00	1.9103E+00	4.3201E-13	0.0000E+00	2.6176E+02	0.0000E+00
	Std.	1.6091E+01	0.0000E+00	0.0000E+00	9.5516E+00	6.5637E-14	0.0000E+00	2.0691E+01	0.0000E+00
F10	Avg.	2.1511E+00	4.4409E-15	8.8818E-16	7.9936E-15	2.5615E-14	1.6879E+00	3.2863E-14	8.8818E-16
	Std.	8.0940E-02	0.0000E+00	0.0000E+00	0.0000E+00	5.7542E-15	3.2306E-01	9.4554E-15	0.0000E+00
F11	Avg.	1.6712E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.5508E-03	4.2305E-03	0.0000E+00	0.0000E+00
	Std.	6.7212E-02	0.0000E+00	0.0000E+00	0.0000E+00	7.7541E-03	8.3449E-03	0.0000E+00	0.0000E+00
F12	Avg.	7.5960E-02	1.1813E-20	2.3062E-19	1.2440E-03	4.7116E-33	2.9853E-02	4.5421E-28	2.5887E-07
	Std.	7.6354E-03	5.0630E-20	2.8304E-19	6.2201E-03	6.9834E-49	5.8505E-02	3.0212E-28	9.9212E-08
F13	Avg.	2.9829E+00	9.2562E-02	7.1411E-02	1.7754E-01	1.5845E-31	1.5292E+02	8.2687E+01	1.9474E-04
	Std.	2.7665E-01	1.0483E-01	1.0559E-01	1.8213E-01	4.0063E-31	1.5448E+02	3.8315E+01	9.1879E-04

Table 6. The experimental results of the eight algorithms in 500 dimensions

Functions	Index	DE-BBO	EO	DBO	TLBO	HMCFQDE	JADE	CoDE	QAHQDE
F1	Avg.	2.6305E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	2.0142E+00	2.1360E-04	0.0000E+00
	Std.	1.2940E+02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	5.9412E+00	7.6800E-05	0.0000E+00
F2	Avg.	8.0503E+01	2.7058E-231	0.0000E+00	0.0000E+00	0.0000E+00	1.7206E+00	3.0833E-02	0.0000E+00
	Std.	3.3100E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.8062E+00	2.5583E-02	0.0000E+00
F3	Avg.	3.9760E-07	3.6860E-01	3.1366E-35	3.2193E-07	4.4898E-07	4.1304E-10	0.0000E+00	0.0000E+00
	Std.	6.8149E-07	1.8430E+00	1.5683E-34	5.9261E-07	1.1686E-06	1.3747E-09	0.0000E+00	0.0000E+00
F4	Avg.	2.1745E+01	7.9594E+01	4.9000e-324	1.5000E-323	1.4787E-09	3.5364E+01	2.2329E+01	0.0000E+00
	Std.	1.0532E+00	1.8928E+01	0.0000E+00	0.0000E+00	3.0452E-09	1.8708E+00	2.1333E+00	0.0000E+00
F5	Avg.	1.0340E+05	4.8835E+02	4.9221E+02	4.9201E+02	8.4263E+02	3.1875E+03	1.2085E+03	4.2679E+02
	Std.	1.6039E+04	2.2500E+00	5.8981E-01	2.2970E+00	9.9937E+01	1.5842E+03	1.3046E+02	1.1629E+02
F6	Avg.	2.6597E+03	5.7311E+00	8.1370E+00	1.1246E+01	0.0000E+00	6.4111E-01	2.4102E-04	4.3201E-01
	Std.	1.9430E+02	9.1696E-01	9.0951E-01	1.3444E+00	0.0000E+00	7.1676E-01	1.1338E-04	6.3361E-02
F7	Avg.	9.4193E-01	1.6968E-04	1.1831E-04	1.2293E-04	6.8748E-01	4.0870E+00	4.5634E-01	4.0495E-05
	Std.	8.9316E-02	6.3723E-05	1.0055E-04	2.7846E-05	6.4794E-02	8.0402E-01	4.1100E-02	2.2062E-05
F8	Avg.	-9.6521E+04	-1.3961E+05	-1.6663E+05	-1.0914E+05	-2.0949E+05	-1.5253E+05	-8.1976E+04	-2.0890E+05
	Std.	4.1621E+03	4.5146E+03	3.0335E+04	1.3474E+04	6.6685E-11	1.3586E+03	2.7139E+03	1.2750E+02
F9	Avg.	1.6732E+03	0.0000E+00	0.0000E+00	0.0000E+00	1.8954E-11	8.0223E+02	3.2341E+03	0.0000E+00
	Std.	1.1633E+02	0.0000E+00	0.0000E+00	0.0000E+00	1.7938E-12	2.8939E+01	9.6975E+01	0.0000E+00
F10	Avg.	4.4850E+00	6.9988E-15	8.8818E-16	5.1051E-01	1.4115E-13	7.8941E+00	9.4912E-01	8.8818E-16
	Std.	1.1315E-01	1.6281E-15	0.0000E+00	1.2035E+00	1.3377E-14	4.3353E-01	3.7093E-01	0.0000E+00
F11	Avg.	2.4708E+01	0.0000E+00	0.0000E+00	8.4377E-17	5.3291E-16	1.3160E-01	4.6358E-03	0.0000E+00
	Std.	8.8237E-01	0.0000E+00	0.0000E+00	4.8393E-17	1.2820E-16	1.8330E-01	1.2338E-02	0.0000E+00
F12	Avg.	2.0487E+02	8.0770E-03	1.6038E-02	2.3252E-02	9.4233E-34	3.1423E+00	1.3153E-01	9.0918E-04
	Std.	1.8793E+02	1.8058E-03	3.0062E-03	5.1858E-03	3.4917E-49	6.3911E-01	7.4020E-02	1.6491E-04
F13	Avg.	9.2002E+02	4.3750E+01	1.5846E+01	4.7806E+01	4.0004E-31	2.9246E+03	8.3678E+02	1.0862E-01
	Std.	3.1289E+02	1.0988E+00	1.0211E+01	4.6547E+00	6.7255E-31	7.6657E+02	6.7831E+02	8.0645E-02

Table 7. The experimental results of the eight algorithms in 1000 dimensions

Functions	Index	DE-BBO	EO	DBO	TLBO	HMCFQDE	JADE	CoDE	QAHQDE
F1	Avg.	8.1811E+03	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	3.4526E+03	4.9212E+01	0.0000E+00
	Std.	1.7284E+02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	1.1222E+03	3.1169E+01	0.0000E+00
F2	Avg.	1.0655E+02	4.8414E-224	0.0000E+00	0.0000E+00	0.0000E+00	4.9157E+01	3.3103E+01	0.0000E+00
	Std.	1.3970E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	6.6325E+00	6.7993E+00	0.0000E+00
F3	Avg.	6.3620E-07	2.1876E+03	9.8421E-123	9.3176E-07	3.5945E-07	1.2598E-10	0.0000E+00	0.0000E+00
	Std.	9.6490E-07	6.6336E+03	4.9210E-122	1.4907E-06	4.6945E-07	2.7044E-10	0.0000E+00	0.0000E+00
F4	Avg.	3.5893E+01	9.4589E+01	4.9000E-324	1.5000E-323	2.8029E-09	4.1096E+01	2.8100E+01	0.0000E+00
	Std.	9.8812E-01	7.7794E+00	0.0000E+00	0.0000E+00	4.2785E-09	1.8474E+00	2.3575E+00	0.0000E+00
F5	Avg.	4.2289E+05	9.8954E+02	9.8081E+02	9.9134E+02	1.7003E+03	6.0351E+05	8.1057E+03	8.7405E+02
	Std.	5.8360E+04	2.0972E+00	5.2656E+01	6.5413E-01	1.0116E+02	4.3545E+05	8.1637E+02	2.4877E+02
F6	Avg.	8.1575E+03	5.9115E+01	7.3852E+01	7.1327E+01	0.0000E+00	3.5371E+03	4.4094E+01	3.2937E+00
	Std.	2.8579E+02	2.5310E+00	5.0896E+00	2.7778E+00	0.0000E+00	1.1082E+03	1.6784E+01	3.3666E-01
F7	Avg.	6.6651E+00	1.5022E-04	1.6126E-04	1.3282E-04	2.0380E+00	5.3211E+01	2.9992E+00	5.0609E-05
	Std.	9.5516E-01	6.8441E-05	2.1748E-04	2.6972E-05	1.0674E-01	9.7203E+00	4.5162E-01	3.0521E-05
F8	Avg.	-1.7938E+05	-2.5469E+05	-3.2591E+05	-1.7578E+05	-4.1898E+05	-2.1524E+05	-1.1594E+05	-4.1556E+05
	Std.	6.2685E+03	9.5290E+03	7.7706E+04	2.3476E+04	1.1642E-10	2.4172E+03	3.4209E+03	6.3774E+02
F9	Avg.	4.4139E+03	0.0000E+00	0.0000E+00	0.0000E+00	8.1200E-11	3.6740E+03	7.2330E+03	0.0000E+00
	Std.	2.9076E+02	0.0000E+00	0.0000E+00	0.0000E+00	3.8544E-12	7.5133E+01	1.4128E+02	0.0000E+00
F10	Avg.	5.1039E+00	7.5673E-15	8.8818E-16	8.2727E-01	2.8752E-13	1.0985E+01	2.6319E+00	8.8818E-16
	Std.	3.7747E-02	1.1783E-15	0.0000E+00	1.5062E+00	2.4198E-14	3.4778E-01	1.1093E-01	0.0000E+00
F11	Avg.	7.6751E+01	1.3323E-17	0.0000E+00	1.1102E-16	1.7210E-02	3.4105E+01	1.4496E+00	0.0000E+00
	Std.	6.1243E+00	3.6822E-17	0.0000E+00	0.0000E+00	8.6051E-02	1.3338E+01	2.9516E-01	0.0000E+00
F12	Avg.	5.8534E+04	7.7317E-02	1.1502E-01	1.1365E-01	4.7116E-34	1.8880E+05	3.2814E+00	2.7960E-03
	Std.	3.7035E+04	7.9024E-03	3.0619E-02	8.9184E-03	1.7458E-49	2.1006E+05	7.1972E-01	3.4960E-04
F13	Avg.	5.5411E+04	9.7041E+01	4.1058E+01	9.9190E+01	7.3826E-31	1.8365E+05	1.8494E+03	2.6347E-01
	Std.	2.1579E+04	7.1358E-01	2.2670E+01	1.5139E-02	1.3047E-30	1.2356E+05	7.5550E+02	2.1451E-01

Table 8. Times for 13 functions (second)

Algorithm	100	500	1000
DE-BBO	1.20E+03	6.74E+03	1.31E+04
EO	8.53E+02	4.00E+02	7.33E+02
TLBO	3.10E+02	8.21E+02	8.71E+02
HMCFQDE	8.82E+02	9.10E+03	1.98E+04
JADE	6.5 E+01	1.87E+02	3.55E+02
CoDE	2.71E+02	5.50E+02	9.57E+02
QAHQDE	4.60E+02	1.91E+03	3.25E+03

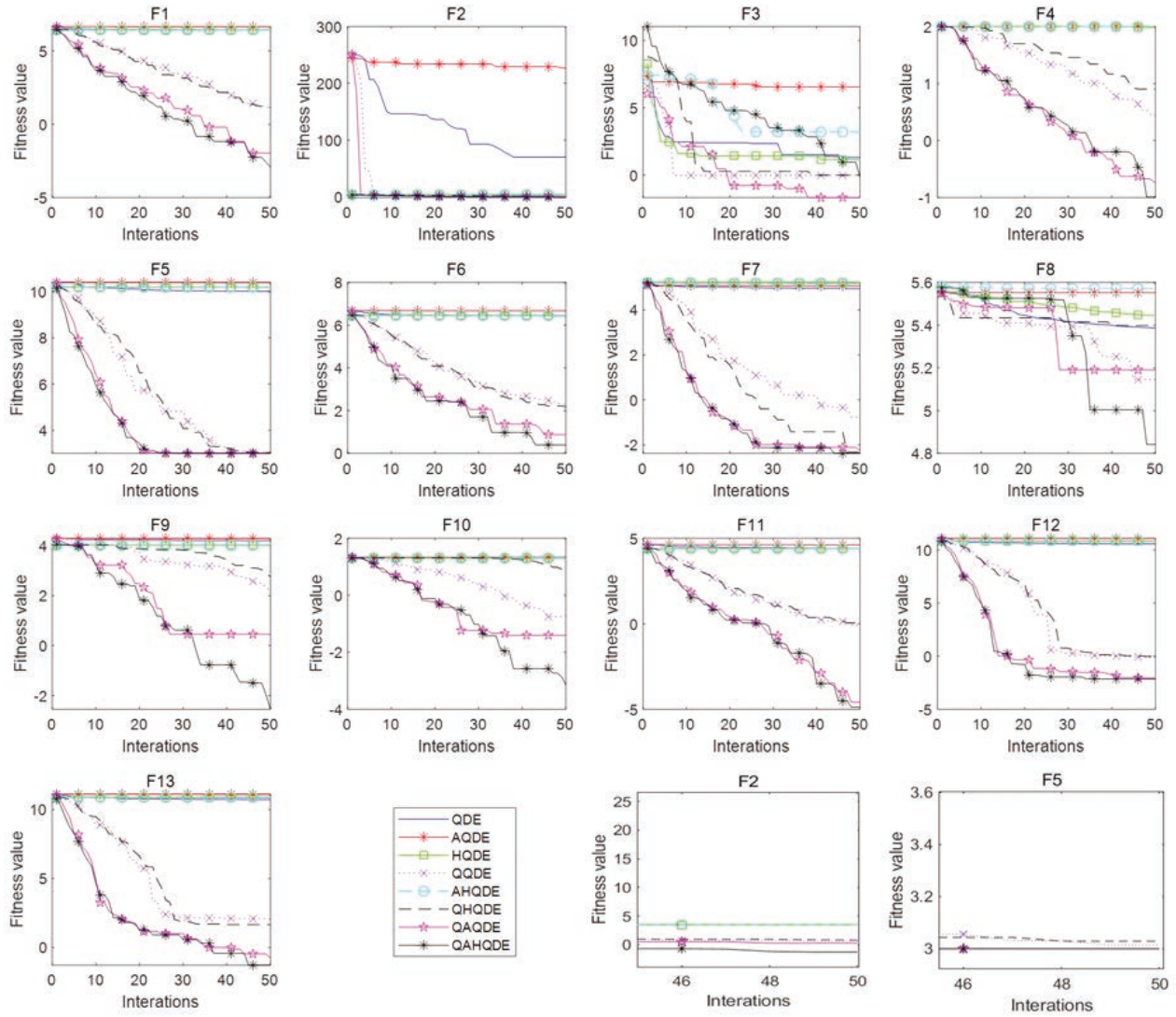


Figure 8. Optimization process curves for different algorithms (D=100)

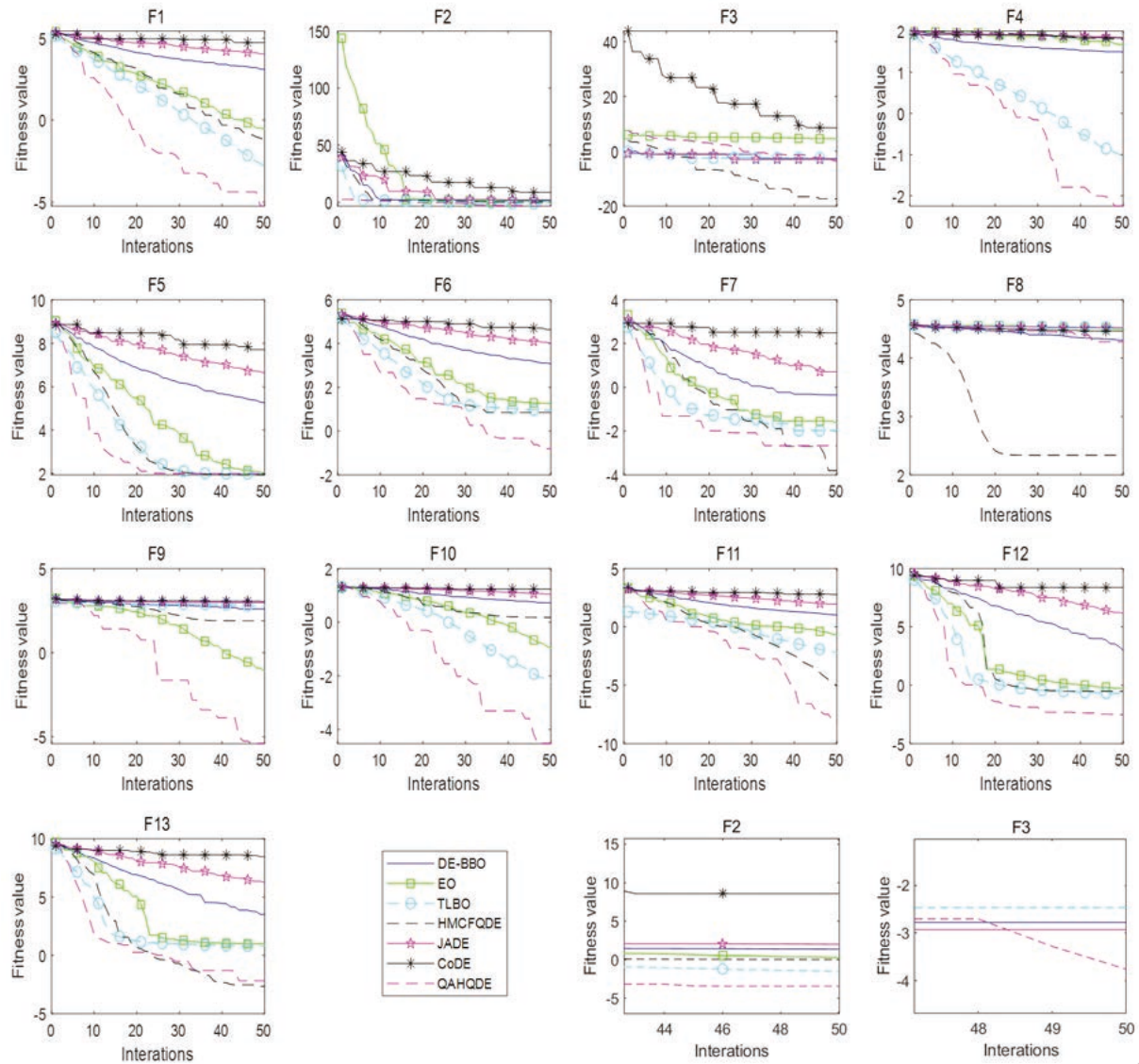


Figure 9. Optimization process curves for different algorithms (D=500)

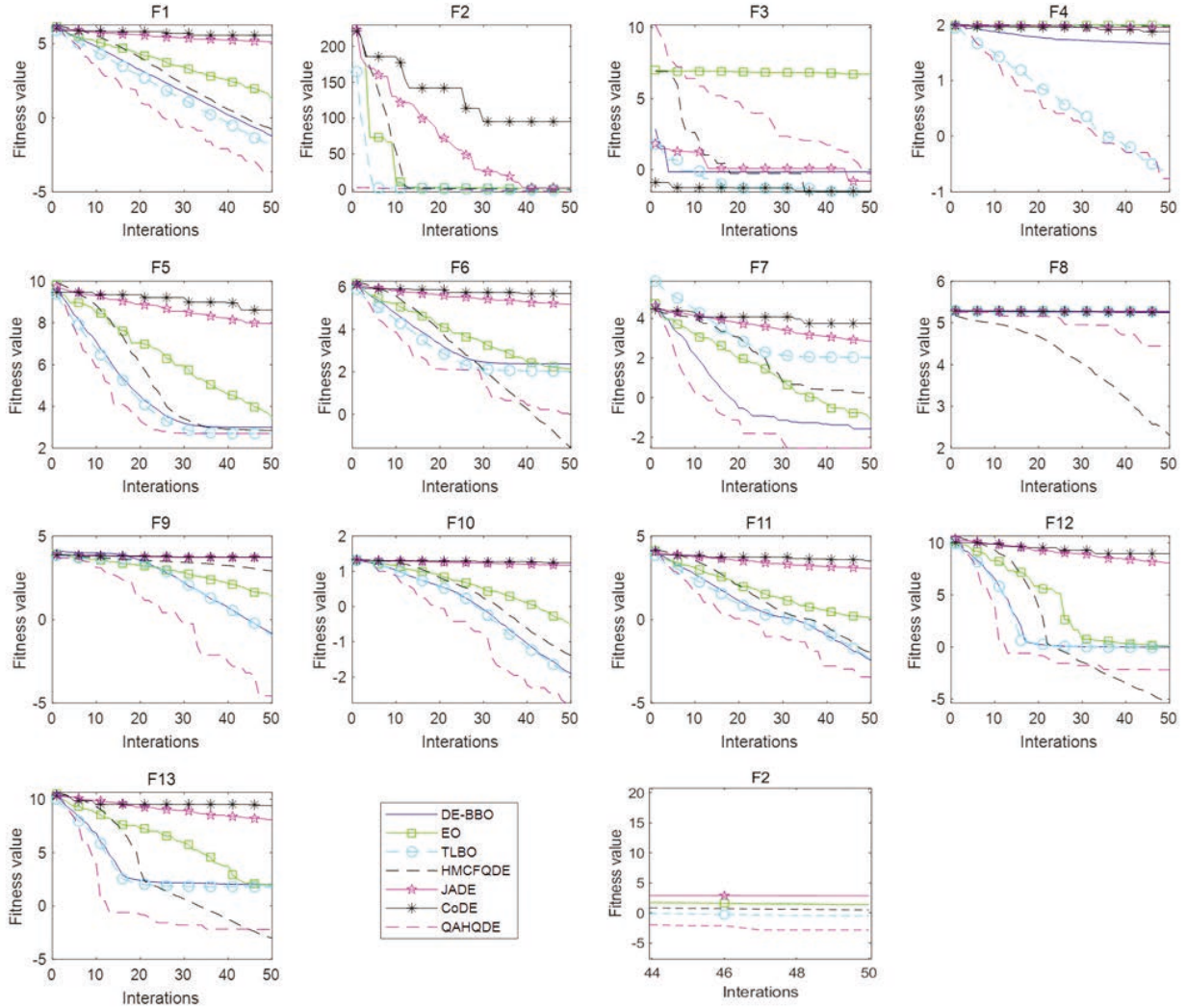


Figure 10. Optimization process curves for different algorithms (D=1000)

cations due to its significant advantage in running time.

5.3 Performance Evaluation of Algorithms for Ultra-High Dimensional Functions

Here, we will further compare QAHQDE with EO, DBO, L-SHADE[76], CJADE[13], SHADE-ILS[78] and MLSHADE.SPA[79] to further evaluate QAHQDE (D=3000). These algorithms all demonstrate exceptional performance in handling large-scale function computations. The obtained experiment results are shown in Table 9.

From the Table 9, when the dimension is set to D=3000, the QAHQDE algorithm demonstrates a significant advantage in overall optimization performance. Notably, its hybrid mutation strategy effectively frees the algorithm from the local optimal trap and successfully finds the optimal solutions of F1, F2, F5, F7, F8, F9, F10 and F11. In addition, the robustness of the QAHQDE algorithm is stable. In contrast, the comparison algorithms tend to suffer from search stagnation when dealing with some high-dimensional problems. Although QAHQDE fails to find optimal solutions for functions F3, F4, F6, F12, and F13, its performance is still among the best compared to other algorithms. For the remaining functions, the performance of the QAHQDE is second only to the current best algorithm, fully demonstrating its competitiveness in high-dimensional optimization problems.

5.4 Ranking Analysis

In order to reflect the overall optimization performance of the QAHQDE, these algorithms are ranked for test results in dimensions 100, 500, and 1000. The obtained results are shown in Table 10.

From the Table 10, the QAHQDE achieved the global best ranking in the majority of the investigated functions. (+ indicates the number of times QAHQDE won, – shows the quantity of times QAHQDE failed, and = shows the quantity of times the same performance was achieved). Compared to other algorithms, QAHQDE fails to achieve the best rankings in test functions F5, F6, F12 and F13 in the 100-dimensional case, and in the 500-dimensional case, QAHQDE fails to achieve the best rankings in test function F6, F8, F12 and F13. However, in any of the other dimensions and test functions,

QAHQDE achieves the best or second-best rankings, ranking first overall, showing that on most functions QAHQDE achieves better results, demonstrating its excellent search capabilities.

5.5 Rank Sum Test

The Friedman test is used as a multiple comparison tool to identify significant differences between the QAHQDE algorithm and other algorithms. The obtained adjusted p-values are for this Friedman analysis in Table 11 and Table 12.

Table 11. Adjusted p-value for the Friedman test with D = 100

Algorithm	Rank	p-value
DE-BBO	7.38	<0.05
EO	3.65	
DBO	3.35	
TLBO	4.35	
HMCFQDE	4.15	
JADE	5.73	
CoDE	4.65	
QAHQDE	2.73	

Table 12. Adjusted p-value for the Friedman test with D = 3000

Algorithm	Rank	p-value
EO	3.85	<0.05
DBO	2.27	
L-SHADE	6.23	
CJADE	6.15	
SHADE-ILS	4.42	
MLSHADE.SPA	3.27	
QAHQDE	1.81	

A reading bigger than 0.05 for each provided p-value indicates acceptance of the null assumption, which states that there is no statistically significant difference between QAHQDE and the comparable algorithms. The null hypothesis is rejected in favour of the alternative hypothesis when the p-value is less than or equal to 0.05, showing a statistically significant difference between QAHQDE and the comparable comparison method. Table 11 Table 12 discloses that QAHQDE exhibits significant dissimilarities from all the rival algorithms un-

Table 9. The experimental results of the five algorithms in 3000 dimensions

Functions	Index	EO	DBO	L-SHADE	CJADE	SHADE-ILS	MLSHADE-SPA	QAHQDE
F1	Avg.	0.0000E+00	0.0000E+00	1.5741E+05	1.5028E+05	2.4559E-51	1.7135E-64	0.0000E+00
	Std.	0.0000E+00	0.0000E+00	1.3631E+04	1.6020E+04	3.3621E-52	1.4226E-64	0.0000E+00
F2	Avg.	3.6299E-217	0.0000E+00	1.0000E+30	6.0746E+02	5.4622E-02	2.9557E-39	0.0000E+00
	Std.	0.0000E+00	0.0000E+00	0.0000E+00	4.4221E+01	1.3201E-02	1.8503E-39	0.0000E+00
F3	Avg.	4.4785E+03	3.7227E-110	1.4508E-12	9.6060E-11	8.0840E-05	3.3817E-16	9.6759E-14
	Std.	1.0833E+04	1.8613E-109	7.2538E-12	2.9691E-10	2.0560E-05	6.5900E-16	3.6779E-13
F4	Avg.	9.5889E+01	9.7002E-253	4.3135E+01	4.8977E+01	1.6583E+01	1.6583E+01	3.9493E-15
	Std.	6.0693E+00	0.0000E+00	2.9340E+00	2.3533E+00	1.7490E+00	2.2209E+00	7.2547E-15
F5	Avg.	2.9867E+03	2.9821E+03	4.2684E+07	3.9230E+07	6.3456E+04	4.7257E+03	2.5453E+03
	Std.	1.1936E+00	3.3159E+00	6.3892E+06	6.0974E+06	4.3524E+03	1.8657E+02	7.3637E+02
F6	Avg.	4.4218E+02	4.7560E+02	1.5905E+05	1.5414E+05	1.5250E+03	4.3345E-26	1.6636E+01
	Std.	5.9889E+00	8.4037E+01	1.0839E+04	1.5230E+04	9.2552E+02	9.0385E-26	6.2590E+00
F7	Avg.	1.7969E-04	1.7199E-04	2.2332E+03	1.6131E+03	1.9106E-02	1.0684E+01	4.9726E-05
	Std.	6.9949E-05	1.2799E-04	3.3071E+02	1.8426E+02	3.9965E-01	3.9965E-01	3.9819E-05
F8	Avg.	-5.2809E+05	-1.0242E+06	-8.4299E+05	-3.5743E+05	-4.7693E+05	-1.0984E+06	-1.2424E+06
	Std.	2.8875E+04	2.4100E+05	3.7312E+03	3.2730E+03	6.9536E+04	3.8420E+03	3.7301E+03
F9	Avg.	0.0000E+00	0.0000E+00	8.0816E+03	1.9333E+04	3.2890E+00	5.0345E+02	0.0000E+00
	Std.	0.0000E+00	0.0000E+00	3.1103E+02	2.9477E+03	4.5296E-01	3.9480E+01	0.0000E+00
F10	Avg.	7.7094E-15	8.8818E-16	1.3316E+01	1.3848E+01	6.9789E-10	1.3088E+00	8.8818E-16
	Std.	9.8370E-16	0.0000E+00	2.6669E-01	2.2122E-01	3.4900E-10	1.3191E-01	0.0000E+00
F11	Avg.	1.1102E-16	0.0000E+00	1.3821E+03	1.3057E+03	3.3083E-19	4.1966E-15	0.0000E+00
	Std.	0.0000E+00	0.0000E+00	9.7744E+01	7.9436E+01	2.5568E-20	1.1468E-15	0.0000E+00
F12	Avg.	4.1824E-01	4.7941E-01	2.6053E+07	2.4298E+07	4.4094E+03	4.1463E-04	5.3547E-03
	Std.	1.1326E-02	1.6094E-01	8.6618E+06	7.4173E+06	5.8560E+02	9.2715E-04	6.4794E-04
F13	Avg.	2.9786E+02	1.4833E+02	2.8708E+07	2.4366E+07	8.6334E-02	4.3949E-03	8.1164E-01
	Std.	2.8337E-01	5.1690E+01	7.9419E+06	7.1239E+06	1.1588E-02	6.0180E-03	4.6750E-01

Table 10. Ranking results

Functions	D	DE-BBO	EO	DBO	TLBO	HMCFQDE	JADE	CoDE	QAHQDE
F1	100	4	1	1	1	1	2	3	1
	500	4	1	1	1	1	3	2	1
	1000	4	1	1	1	1	3	2	1
F2	100	6	2	1	1	1	3	5	1
	500	5	2	1	1	1	4	3	1
	1000	5	2	1	1	1	4	3	1
F3	100	5	3	2	6	7	4	1	1
	500	5	7	2	4	6	3	1	1
	1000	5	7	2	6	4	3	1	1
F4	100	6	3	1	2	4	7	5	1
	500	5	8	2	3	4	4	7	1
	1000	3	6	8	7	4	5	2	1
F5	100	8	4	5	1	7	6	3	2
	500	8	2	4	3	5	7	6	1
	1000	7	3	2	4	5	8	6	1
F6	100	8	4	5	6	1	2	3	7
	500	8	5	6	7	3	4	1	2
	1000	8	4	6	5	1	7	3	2
F7	100	6	2	4	3	8	7	5	1
	500	7	4	2	3	6	8	5	1
	1000	7	3	4	2	5	8	6	1
F8	100	7	5	4	6	1	2	3	1
	500	7	5	3	6	1	4	8	2
	1000	6	4	3	7	1	5	8	2
F9	100	4	1	1	3	2	1	5	1
	500	4	1	1	1	2	3	5	1
	1000	4	1	1	1	2	3	5	1
F10	100	7	2	1	3	4	6	5	1
	500	6	2	1	4	3	7	5	1
	1000	6	2	1	4	3	7	5	1
F11	100	4	1	1	1	2	3	1	1
	500	6	1	1	2	3	5	4	1
	1000	7	2	1	3	4	6	5	1
F12	100	8	3	4	6	1	7	2	5
	500	8	3	4	5	1	7	6	2
	1000	7	3	5	4	1	8	6	2
F13	100	6	4	3	5	1	8	7	2
	500	7	4	3	5	1	8	6	2
	1000	7	4	3	5	1	8	6	2
Rank Sum		235	122	102	139	110	200	165	59
Overall Rank		8	4	2	5	3	7	6	1
+		39	29	21	28	22	37	31	#
=		0	8	16	9	7	1	4	#
-		0	2	2	2	10	1	4	#

Table 13. Comparison of different population sizes in 100 dimensions (F1-F6)

D=100	F1	F2	F3	F4	F5	F6
Sub1=0	0.0000E+00	4.3004E+00	2.1541E-28	7.3634E+01	1.0784E+03	6.1410E-02
Sub1=40	0.0000E+00	0.0000E+00	1.2037E-33	0.0000E+00	8.4714E+01	2.6794E-05
Sub1=45	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	8.0756E+01	1.3571E-05
Sub1=50	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	8.4191E+01	1.1571E-04
Sub1=90	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	5.9559E+01	5.9798E-02

Table 14. Comparison of different population sizes in 100 dimensions (F7-F13)

D=100	F7	F8	F9	F10	F11	F12	F13
Sub1=0	6.2934E-01	-4.1344E+04	1.3194E+02	4.9654E+00	4.2550E-02	2.9875E-01	1.7185E-01
Sub1=40	2.1733E-05	-4.1898E+04	0.0000E+00	8.8818E-16	0.0000E+00	4.4436E-07	3.7017E-04
Sub1=45	2.4902E-05	-4.1898E+04	0.0000E+00	8.8818E-16	0.0000E+00	2.5887E-07	1.9474E-04
Sub1=50	6.0943E-06	-4.1897E+04	0.0000E+00	8.8818E-16	0.0000E+00	1.7599E-06	8.0231E-05
Sub1=90	1.6854E-06	-4.1743E+04	0.0000E+00	8.8818E-16	0.0000E+00	7.9025E-04	3.3148E-03

der scrutiny. These experimental findings thus serve to reinforce the efficacy of the QAHQDE algorithm once more.

5.6 Analysis of population size in hybrid mutation strategy

The comparison of different population sizes in 100 dimensions is shown in Table 13 and Table 14.

The QAHQDE is very sensitive to the choice of mutation strategies. When faced with an optimization problem, the dispersion of subpopulations as well as aggregation can affect the algorithm to trap in a local optimum or to converge too slowly. Therefore, the number of populations of the QAHQDE algorithm is tested under the condition of D=100. From Tab.13 and Tab.14, with the increase of the number of populations from 0 to 45 lanes, the convergence accuracy of all the tests is improving, except for the results of F1 and F8 which are unchanged. As the number of populations increases from 45 to 90, the convergence accuracy of the results for F5, F6, and F12 all decrease. With population one of 45, for F7 and F13 the results are not the best compared to the other population numbers, but only second best. Therefore, the population numbers for each of the two variants were set to fixed values in the experiment (Sub1=Sub2=45).

5.7 Comparison with multiple algorithms in CEC2010 and CEC2013

Two performance analysis experiments were carried out to assess QAHQDE's performance [44]. First, CEC2010 was used to assess QAHQDE and compare it to eighteen cutting-edge algorithms. Second, 13 cutting-edge algorithms were compared with QAHQDE, which was assessed using CEC2013. Table 15 and Tab.16 display the experimental comparison of the state-of-the-art algorithms employing CEC2010 with QAHQDE.

The experimental comparisons between QAHQDE and state-of-the-art algorithms using CEC2013 is shown in Table 17 and Table 18.

It can be found from experiment results, although the convergence accuracy of QAHQDE with 18 state-of-the-art algorithms in CEC2010 is not the best, the convergence accuracies of functions F8, F9, F11, F12, F13, F14, F17 and F20 are ranked high. It can be found from Tab.16, the convergence accuracy of QAHQDE with 13 state-of-the-art algorithms in solving CEC2013 are ranked high for F2, F6, F10, F11, F15. In summary, in solving some functions from CEC2010 and CEC2013, the QAHQDE still has certain limitations. Therefore, we will continue to deeply study the QAHQDE algorithm in order to improve its optimization performance.

Table 15. Comparison results between QAHQDE and state-of-the-art algorithms for CEC2010

Func-tions	MLSHADEA -SPA	EADE	ANDE	LMDEa	SDENS	jDElsgo	DECC -DML	MA-SW chains	DISCC	DASA
F1	4.01E-23	4.70E-22	3.72E-26	1.35E-23	5.73E-06	8.86E-20	1.93E-25	2.10E-14	9.77E-25	1.52E-21
F2	4.70E+01	4.16E+02	3.72E+02	6.97E+02	2.21E+03	1.25E-01	2.17E+02	8.10E+02	5.07E+02	8.48E+00
F3	9.84E-14	6.25E-14	7.46E-14	6.44E-01	2.70E-05	3.81E-12	1.18E-13	7.28E-13	7.77E-14	7.20E-11
F4	1.84E+11	1.08E+11	5.13E+11	2.08E+11	5.11E+12	8.06E+10	3.58E+12	3.53E+11	1.26E+12	5.05E+11
F5	6.07E+07	8.79E+07	9.19E+07	6.62E+07	1.18E+08	9.72E+07	2.99E+08	1.68E+08	2.35E+08	6.20E+08
F6	6.60E-09	1.90E+01	1.64E+00	2.63E-01	2.02E-04	1.70E-08	7.93E+05	8.14E+04	2.13E+06	1.97E+07
F7	8.08E-06	2.11E-01	1.80E+00	2.45E-01	1.20E+08	1.31E-02	1.39E+08	1.03E+02	6.45E+05	7.78E+00
F8	4.43E+03	2.26E-04	1.25E+07	3.61E-04	5.12E+07	3.15E+06	3.46E+07	1.41E+07	7.54E+07	4.98E+07
F9	1.65E+07	3.67E+07	4.12E+07	2.64E+07	5.63E+08	3.11E+07	5.92E+07	1.41E+07	6.08E+07	3.60E+07
F10	5.01E+03	2.62E+03	3.06E+03	2.80E+03	6.87E+03	2.64E+03	1.25E+04	2.07E+03	2.27E+03	7.29E+03
F11	8.96E+01	1.14E+02	8.95E+01	1.19E+01	2.21E+02	2.20E+01	1.80E-13	3.80E+01	8.20E-01	1.98E+02
F12	4.35E+01	2.80E+04	4.89E+04	1.83E+04	4.13E+05	1.21E+04	3.80E+06	3.62E-06	3.34E+04	1.78E+03
F13	9.95E+01	1.01E+03	1.11E+03	5.95E+02	2.19E+03	7.11E+02	1.14E+03	1.25E+03	1.31E+03	1.21E+03
F14	6.02E+07	1.46E+08	1.87E+08	8.63E+07	1.88E+09	1.69E+08	1.89E+08	3.11E+07	2.08E+08	1.00E+08
F15	3.77E+03	3.18E+03	3.19E+03	5.63E+03	7.32E+03	5.84E+03	1.54E+04	2.74E+03	5.54E+03	1.45E+04
F16	2.49E+02	3.00E+02	3.04E+02	3.87E+02	4.08E+02	1.44E+02	5.08E-02	9.98E+01	1.98E+01	3.97E+02
F17	6.16E+02	1.52E+05	2.02E+05	2.14E+05	1.08E+06	1.02E+05	6.54E+06	1.24E+00	1.81E+05	1.03E+04
F18	5.73E+02	2.26E+03	2.35E+03	1.68E+03	3.08E+04	1.85E+03	2.47E+03	1.30E+03	5.16E+03	4.92E+03
F19	4.51E+05	1.29E+06	1.63E+06	4.42E+05	8.80E+05	2.74E+05	1.59E+07	2.85E+05	1.71E+06	8.34E+05
F20	1.26E+02	2.10E+03	2.20E+03	1.38E+03	9.90E+02	1.53E+03	9.91E+02	1.07E+03	1.94E+03	1.13E+03

6 Conclusion

The Quantum Differential Evolution (QDE) algorithm was proposed to improve the traditional Differential Evolution (DE), which struggles with high-dimensional and complex optimization problems. However, QDE still suffers from imbalanced initial search, low convergence accuracy, and poor stability. To address these issues, a novel algorithm, named QAHQDE, has been developed. In this algorithm, a chaotic initialization strategy, an adaptive parameter control mechanism, and a weighted mutation strategy are integrated. These components are designed to collaboratively enhance population diversity, local search ability, and overall convergence performance. An improved chaotic strategy was introduced to initialize the population. By increasing diversity, the search for optimal solutions was accelerated. A new adaptive parameter mechanism was designed to dynamically adjust the quantum mutation strength at different evolutionary stages, thereby improving convergence speed. A hybrid mutation strategy was proposed by combining a weighted mutation operator with the stan-

dard DE mutation scheme. Diverse mutation modes were applied to different subpopulations, which helped to improve the search capability of the algorithm.

The experimental results demonstrated that QAHQDE achieved stable performance and high accuracy on several high-dimensional benchmark functions (with dimensions $D = 100, 500, 1000, 3000$). Good scalability and practical application potential were observed, indicating the algorithm's suitability for large-scale optimization tasks. However, performance on the CEC2010 and CEC2013 test sets still showed room for improvement, suggesting limited adaptability to certain types of functions. Future work will focus on the following directions: (1) designing more targeted mutation and selection mechanisms for specific characteristics in the CEC2010 and CEC2013 functions; (2) investigating the integration of multiple intelligent strategies to further enhance generalization and robustness; and (3) optimizing the execution efficiency of quantum operations to reduce computational costs in high-dimensional scenarios. With these improvements, QAHQDE is expected to demonstrate

Table 16. Comparison results between QAHQDE and state-of-the-art algorithms for CEC2010 (continuation)

Functions	EOEA	jDEsps	MOS2012	DM-HDMR	HACC-D	CCGS	SEE	MMO-CC	QAHQDE
F1	2.20E-23	4.10E-23	0.00E+00	2.34E+01	1.99E-27	1.83E-22	6.99E-11	0.00E+00	1.85E-20
F2	3.62E-01	1.10E+02	1.97E+02	4.36E+03	1.43E-14	4.44E-02	8.77E+03	1.43E+03	1.68E+00
F3	1.67E-13	1.30E-13	1.12E+00	1.67E+01	3.45E-14	1.91E-01	1.99E+01	0.00E+00	2.10E-05
F4	2.86E+12	8.15E+11	1.91E+10	6.96E+11	1.55E+12	1.79E+12	2.58E+11	7.64E+06	1.55E+11
F5	2.24E+07	7.71E+07	6.81E+08	1.45E+08	1.96E+08	1.97E+07	5.85E+08	3.34E+08	6.98E+08
F6	3.85E+06	5.58E-03	1.99E+07	1.63E+01	3.55E-09	2.88E+06	1.99E+07	5.77E-01	2.00E+00
F7	1.24E+02	5.77E+05	0.00E+00	2.91E+05	3.87E-07	1.37E+02	3.14E-02	2.41E+10	2.63E-02
F8	1.01E+07	1.52E+06	1.12E+06	4.41E+07	7.44E+07	2.81E+07	1.82E+06	2.63E+08	3.84E+05
F9	4.63E+07	2.31E+04	5.75E+06	5.20E+07	3.32E+07	5.53E+07	2.67E+07	8.99E+01	2.26E+03
F10	1.00E+03	1.85E+03	7.86E+03	4.49E+03	1.30E+04	4.74E+03	1.27E+04	1.63E+03	1.71E+04
F11	3.18E+01	1.94E-05	1.99E+02	1.10E+01	7.82E-14	2.99E+01	2.19E+02	2.99E+00	2.31E-01
F12	2.61E+04	1.57E+04	0.00E+00	1.97E+03	1.31E+06	5.35E+03	2.60E+02	0.00E+00	1.63E+00
F13	1.24E+03	1.86E+02	1.36E+03	3.35E+06	1.96E+03	1.51E+03	7.12E+02	3.05E+04	6.82E+02
F14	1.65E+08	3.85E+05	1.52E+07	3.41E+08	9.21E+07	1.35E+08	9.88E+07	0.00E+00	2.45E+06
F15	2.14E+03	5.50E+03	1.54E+04	5.95E+03	1.56E+04	1.74E+03	1.50E+04	2.05E+03	1.73E+04
F16	8.26E+01	4.97E+00	3.97E+02	1.24E-06	1.95E-11	3.11E+01	3.97E+02	8.87E+00	4.18E+01
F17	7.93E+04	5.52E+04	4.66E-05	4.03E+04	1.42E+06	1.48E+04	7.40E+03	0.00E+00	5.06E+00
F18	2.94E+03	9.73E+02	3.91E+03	8.40E+03	4.02E+03	3.13E+03	3.14E+03	3.37E+04	1.47E+12
F19	1.84E+06	8.00E+05	3.41E+04	1.71E+06	1.87E+07	5.93E+05	7.13E+05	1.54E+07	8.84E+05
F20	1.97E+03	8.79E+02	5.31E+02	2.45E+06	1.51E+03	1.31E+03	1.43E+03	1.10E+03	1.64E+02

Table 17. Comparison results between QAHQDE and state-of-the-art algorithms using CEC2013

Func-tions	MLSHADE-SPA	MOS2013	DECC-CG	CBCC3-DG2	CCFR-IDG2	CCFR-1	CRO	IHDELS	VGDE	SACC
F1	1.94E-22	0.00E+00	2.03E-13	8.65E+05	2.00E-05	1.30E-05	1.84E+06	4.34E-28	0.00E+00	2.73E-24
F2	7.89E+01	8.32E+02	1.03E+03	1.41E+04	3.60E+02	5.50E-01	9.84E+02	1.32E+03	4.56E+01	7.06E+02
F3	9.96E-14	9.17E-13	2.87E-10	2.06E+01	2.10E+01	2.00E+01	2.01E+01	2.01E+01	3.98E-13	1.11E+00
F4	6.90E+08	1.74E+08	2.60E+10	3.39E+07	9.60E+07	4.50E+07	1.55E+10	3.04E+08	5.96E+08	4.56E+10
F5	1.80E+06	6.94E+06	7.28E+14	2.14E+06	2.80E+06	2.50E+06	2.38E+07	9.59E+06	3.00E+06	7.74E+06
F6	1.40E+03	1.48E+05	4.85E+04	1.05E+06	1.10E+06	1.10E+06	1.06E+06	1.03E+06	1.31E+05	2.47E+05
F7	5.31E+04	1.62E+04	6.07E+08	2.95E+07	2.00E+07	8.60E+06	2.78E+08	3.46E+04	1.85E+03	8.98E+07
F8	9.77E+12	8.00E+12	4.26E+14	6.74E+10	7.00E+10	9.60E+09	4.56E+14	1.36E+12	7.00E+14	1.20E+15
F9	1.61E+08	3.83E+08	4.27E+08	1.70E+08	1.90E+08	1.90E+08	5.27E+08	6.74E+08	2.31E+08	5.98E+08
F10	6.56E+02	9.02E+05	1.10E+07	9.28E+07	9.50E+07	9.50E+07	9.44E+07	9.16E+07	1.57E+02	2.95E+07
F11	4.04E+07	5.22E+07	2.46E+11	7.70E+08	4.00E+08	3.30E+08	2.91E+10	1.07E+07	7.52E+07	2.78E+09
F12	1.04E+02	2.47E+02	1.04E+03	5.81E+07	1.60E+09	6.00E+08	3.69E+03	3.77E+02	2.52E+03	8.73E+02
F13	7.21E+07	3.40E+06	3.42E+10	6.03E+08	1.20E+09	9.30E+08	5.33E+09	3.80E+06	1.36E+09	1.78E+09
F14	1.52E+07	2.56E+07	6.08E+11	1.11E+09	3.40E+09	2.10E+09	6.08E+10	1.58E+07	2.29E+10	1.75E+10
F15	2.76E+07	2.35E+06	6.05E+07	7.11E+06	9.80E+06	8.20E+06	1.88E+07	2.81E+06	3.44E+06	2.01E+06

Table 18. Comparison results between QAHQDE and state-of-the-art algorithms for CEC2013 (continuation)

Functions	SHADE-ILS	MOS	BICA	QAHQDE
F1	2.69E-24	0.00E+00	1.85E-12	1.94E-20
F2	1.00E+03	8.32E+02	8.46E-07	1.27E+00
F3	2.01E+01	9.17E-13	7.27E-01	3.90E-05
F4	1.48E+08	1.74E+08	8.85E+08	9.04E+08
F5	1.39E+06	6.94E+06	2.58E+06	7.64E+06
F6	1.02E+06	1.48E+05	1.46E+05	4.97E+04
F7	7.41E+01	1.62E+04	1.82E+05	4.06E+04
F8	3.17E+11	8.00E+12	3.78E+12	9.75E+10
F9	1.64E+08	3.83E+08	2.18E+08	2.29E+08
F10	9.18E+07	9.02E+05	1.24E+06	1.19E+04
F11	5.11E+05	5.22E+07	2.85E+07	1.16E+07
F12	6.18E+01	2.47E+02	1.40E+03	4.69E+03
F13	1.00E+05	3.40E+06	1.09E+07	3.33E+07
F14	5.76E+06	2.56E+07	4.27E+07	3.57E+07
F15	6.25E+05	2.35E+06	3.16E+06	2.21E+06

stronger competitiveness across a broader range of problem domains.

Acknowledgment

This work was supported by the National Natural Science Foundation of China, grant number U2133205, the Research and Innovation Funding Project for Postgraduates of Civil Aviation University of China under Grant 2022YJS015.

Data Availability Statement

Not applicable.

Conflicts of Interest

The authors declare no conflict of interest.

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