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EXPONENTIAL STATE ESTIMATION FOR DELAYED COMPETITIVE NEURAL NETWORK VIA STOCHASTIC SAMPLED-DATA CONTROL WITH MARKOV JUMP PARAMETERS UNDER ACTUATOR FAILURE

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Abstract

This article examines the problem of estimating the states of Markovian jumping competitive neural networks, where the estimation is done using stochastic sampled-data control with time-varying delay. Instead of continuously measuring the states, the network relies on sampled measurements, and a sampled-data estimator is proposed. The estimator uses probabilistic sampling during two sampling periods, following a Bernoulli distribution. The article also takes into account the possibility of actuator failure in real systems. To ensure the exponentially mean-square stability of the delayed neural networks, the article constructs a Lyapunov-Krasovskii functional (LKF) that includes information about the bounds of the delay. The sufficient conditions for stability are derived in the form of linear matrix inequalities (LMIs) by employing modified free matrix-based integral inequalities. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed method.

Keywords: stochastic sampled-data control, competitive neural networks, state estimation, Markov jump, actuator failure.

1 Introduction

Neural networks have grown in popularity over the last few decades due to their broad range of applications, which include pattern recognition, image processing, optimization issues, associative memory. Due to its many applications in various fields, such as information science, computer science, biology, environment the dynamics of neural network systems are receiving a lot of attention in the present [1, 2]. The various kinds of Neural Networks (NNs) include chaotic NNs, memristive NNs and bidirectional associative memory NNs. Cohen and Grossberg developed the competitive neural network model in 1983, which include two state variables. One is Short-term memory indicating the

quick neural activity and another one is long-term memory indicating gradual changes of synapses in the external stimuli. Previous studies [3, 4] have explored the dynamic behavior of competitive neural networks (CNNs) at different time frames. Many researchers are particularly interested in the stability and synchronization analysis for CNNs [5, 6].

Understanding neuron states plays a vital role in achieving practical performance in various applications. In fact, the network output may only contain partial information on the neuron states. Furthermore, due to network congestion, packet errors, and missing data, it is difficult to acquire the entire neuron state immediately. Due to the limited energy of rechargeable batteries and the cost of transmission in neural networks, it is usual practice to artificially manage the frequency of transmission between sensors and estimators. Estimating neuron states based on network output measurements is crucial. The state estimation problem for neural networks has gained a lot of interest in research. Recently, the state estimation problem has been studied for neural networks with time-varying delays using a new optimization technique [7].

Time delay is a common issue in real systems due to network congestion. This can lead to complex dynamic behaviors such as divergence, oscillation and instability. Recent studies have examined how time-varying delays affect system stability. The state estimation problem for switched time delay complex networks has been studied in [8]. In [9], the authors addresses the quasi synchronization problem for heterogeneous neural networks through impulsive control by considering transmittal delay. Meanwhile, dynamical systems may undergo certain structural or parametric changes an effect of unavoidable environmental disturbances, component failures, or repairs. To address these challenges, a subset of stochastic systems known as Markov jump systems has garnered significant study attention. These findings covered various problems for Markovian jump neural networks including stochastic stabilization [10], event-triggered synchronization [11] and state estimation [12]. Hence, Markov jump is essential for studying the state estimation of delayed CNNs and the research has received a lot of interest.

Networked control methods, such as sample-data control and event-triggered control, have recently acquired popularity and found use in automatic control systems because of their efficiency, variety, flexibility, and ease of implementation. However, the unpredictability of network conditions, including oscillations in network load and sampling mistakes, can cause variations in sample-data control sampling times. As a result, choosing a suitable sampling pattern is critical to achieve good control performance. In comparison with constant or variable sampling, stochastic sampled-data control (SSDC) effectively decreases the amount of transmitted data while conserving communication capacity. It also demonstrates enhanced adaptability to stochastic network settings. SSDC has gained substantial interest in recent years [13]. Despite its numerous benefits, SSDC faces a few challenges. One of the most difficult unaddressed issues is the state estimation problem of Markov jump CNNs (MJCNNs) with time-varying delays. This problem serves as an inspiration for the present work.

However, actuator failures are unavoidable in real-world systems, particularly in network environments. This will substantially reduce system performance. Moreover, the systems might become unreliable. To assess the reliability of sampled data, it is important to investigate its stability on NNs during actuator failure [14, 15]. Motivated by the above discussion, the current article investigates the SSDC state estimation problem for delayed MJCNNs under actuator failure conditions. The primary aspects of this article are mentioned in the following points:

1. To design the exponential state estimation problem of MJCNNs, the stochastic sampled-data approach is utilized in this paper. For simplicity, assume that there exist two sampling periods with certain occurrence probabilities.

2. By constructing suitable LKFs, exponential stability of the error system is achieved in the form of LMIs. The criterion incorporates a new modified Lemma 2 provides an enhanced condition in the augmented vector for state estimation of MJCNNs.

3. The proposed criteria provide adequate requirements to ensure mean square exponential stability and achieve the desired controller.

4. The proposed method is evaluated through simulations and the results are seen in the numerical section.

Notations: In this work, $\mathbb{R}^n$ indicates the n-dimensional real space. A matrix G with $m \times n$ dimension is denoted as $G_{m\times n}$. The identity matrix I with $n \times n$ dimension is denoted by I_n. $\Pr\{T\}$ represents the probability of occurance of event T. Matrix dimensions are considered compatible if not specified explicitly.

2 Mathematical Formulation and preliminaries

Let $\{r(\rho)_{\rho\geq 0}\}$ be the right-continuous markov process taking values from the set $\mathcal{M} = \{1,2,\ldots,l\}$. $\mathcal{Q} = (\pi_{pq})_{M\times M}$ is the matrix derived from the Markov process and it is denoted as follows:

$$P\{r(\rho+\Delta\rho) = q | r(\rho) = p\} = \begin{cases} \pi_{pq}\Delta\rho + o(\Delta\rho), & p \neq q, \\ 1+\pi_{pp}\Delta\rho + o(\Delta\rho), & p = q, \end{cases}$$

where $\Delta\rho > 0$ and $\lim_{\Delta\rho\to 0}\frac{o(\Delta\rho)}{\Delta\rho} = 0$, $\pi_{pq} \geq 0$ is the transition rate from p to q and if $p \neq q$,
$\pi_{pp} = -\sum_{q=1,\, q\neq p}^{M} \pi_{pq}$.
Consider the following delayed MJCNNs:

$$\begin{aligned} \text{STM:} \quad \dot{z}(\rho) &= -A(r(\rho))z(\rho) + B(r(\rho))F(z(\rho)) \\ &\quad + \bar{B}(r(\rho))F(z(\rho-ß(\rho))) \\ &\quad + D(r(\rho))g(\rho) + J \\ \text{LTM:} \quad \dot{g}(\rho) &= -C(r(\rho))g(\rho) + M(r(\rho))F(z(\rho)), \end{aligned} \tag{1}$$

where $z(\rho) = (z_1(\rho), z_2(\rho), \ldots, z_n(\rho))^T \in \mathbb{R}^n$ is the state vector of the MJCNNs, $g(\rho) = (g_1(\rho), g_2(\rho), \ldots, g_n(\rho))^T$ represents the synaptic efficiency. The activation function is represented as $F(z(\rho)) = (F_1(z_1(\rho)), F_2(z_2(\rho)), \ldots, F_n(z_n(\rho)))^T \in \mathbb{R}^n$. The time-varying delay is denoted by $ß(\rho)$ and satisfies $0 < ß(\rho) \leq \bar{ß}$, $\dot{ß}(\rho) \leq \mu$, where $\bar{ß}$ and μ are constants. $A(r(\rho)) = \text{diag}\ \{a_1(r(\rho)), a_2(r(\rho)), \ldots, a_n(r(\rho))\} > 0$ denotes the neuron time constant, the connection weight matrix is denoted by $B(r(\rho)) = (b(r(\rho))_{pq})_{n\times n}$, and the delayed connection weight matrix is represented in the form $\bar{B}(r(\rho)) = (\bar{b}(r(\rho))_{pq})_{n\times n}$. $D(r(\rho)) = (d(r(\rho))_{pq})_{n\times n}$ serves as the strength of the exterior stimulus, $J = (J_1, J_2, \ldots J_n)^T \in \mathbb{R}^n$ denotes the external input. $C(r(\rho)) = \text{diag}\ \{c_1(r(\rho)), c_2(r(\rho)), \ldots, c_n(r(\rho))\} > 0$ represents disposable scaling constant, $M(r(\rho)) = (m(r(\rho))_{pq})_{n\times n}$ is the constant external stimulus. The output measurements $y_z(\rho)$ and $y_g(\rho)$ are defined as follows:

$$y_z(\rho) = Ez(\rho), \qquad y_g(\rho) = Gg(\rho), \tag{2}$$

where $E, G \in \mathbb{R}^{m\times n}$ are known constant matrices.

Assumption 1: The activation function $F_p(.)$ satifies the following inequality:

$$0 \leq \frac{F_p(c) - F_p(d)}{c-d} \leq \hat{\rho}_p, \ p = 1,2,\ldots,n, \tag{3}$$

where $c, d \in \mathbb{R}$ and $c \neq d$.
Consider the following state estimator for the MJCNNs (1):

$$\begin{aligned} \text{STM:} \quad \dot{\hat{z}}(\rho) &= -A(r(\rho))\hat{z}(\rho) + B(r(\rho))F(\hat{z}(\rho)) \\ &\quad + \bar{B}(r(\rho))F(\hat{z}(\rho-ß(\rho))) \\ &\quad + D(r(\rho))\hat{g}(\rho) + J + U_1(\rho) \\ \text{LTM:} \quad \dot{\hat{g}}(\rho) &= -C(r(\rho))\hat{g}(\rho) + M(r(\rho))F(\hat{z}(\rho)) \\ &\quad + U_2(\rho). \end{aligned} \tag{4}$$

Then, the estimated output measurements are given as follows:

$$\hat{y}_z(\rho) = E\hat{z}(\rho), \qquad \hat{y}_g(\rho) = G\hat{g}(\rho) \tag{5}$$

where $\hat{z}(\rho) \in \mathbb{R}^n$, $\hat{g}(\rho) \in \mathbb{R}^n$ are the state variables estimators for $z(\rho)$ and $g(\rho)$. $U_1(\rho) \in \mathbb{R}^n$, $U_2(\rho) \in \mathbb{R}^n$ represent the control input. $\hat{y}_z(\rho) \in \mathbb{R}^m$, $\hat{y}_g(\rho) \in \mathbb{R}^m$ are the output vector estimator. The state estimator error vector is defined as $e_z(\rho) = z(\rho) - \hat{z}(\rho)$, $e_g(\rho) = g(\rho) - \hat{g}(\rho)$. The expression of error system from (1) and (4) as follows:

$$\begin{aligned} \dot{e}_z(\rho) &= -A(r(\rho))e_z(\rho) + B(r(\rho))F(e_z(\rho)) \\ &\quad + \bar{B}(r(\rho))F(e_z(\rho-ß(\rho))) \\ &\quad + D(r(\rho))e_g(\rho) - U_1(\rho), \\ \dot{e}_g(\rho) &= -C(r(\rho))e_g(\rho) + M(r(\rho))F(e_g(\rho)) \\ &\quad - U_2(\rho), \end{aligned} \tag{6}$$

where $F(e_z(\rho)) = F(z(\rho)) - F(\hat{z}(\rho))$, $F(e_z(\rho - ß(\rho))) = F(z(\rho-ß(\rho))) - F(\hat{z}(\rho-ß(\rho)))$. According to (3), $F_p(.)$ satisfies $0 \leq \frac{F_p(s)}{s} \leq \rho_p$, $\forall\ s \in \mathbb{R}$

and $s \neq 0$.

Suppose that $\{\rho_k\}$ is the sampled-data instants holds the sequence of times $0 = \rho_0 < \rho_1 < \rho_2 < \cdots < \rho_k \cdots$ and $\lim_{k\to\infty} \rho_k = +\infty$. The control input is defined by a Zero-order-holder (ZOH). At sampled-data instant $\{\rho_k\}$, the measurements $y_z(\rho_k)$, $\hat{y}_z(\rho_k)$, $y_g(\rho_k)$ and $\hat{y}_g(\rho_k)$ are provided to update the controller. Following that, the sampled-data controller is constructed as

$$\begin{aligned} U_1(\rho) =& K(y_z(\rho_k) - \hat{y}_z(\rho_k)) = KEe_z(\rho_k), \\ U_2(\rho) =& \bar{K}(y_g(\rho_k) - \hat{y}_g(\rho_k)) = \bar{K}Ge_g(\rho_k), \end{aligned}$$

where K and $\bar{K}$ are the gain matrices to be designed. In real-world environment, actuator failure is a common phenomenon. Considering the failure problem of the actuator is crucial. The control $U_1(\rho)$ and $U_2(\rho)$ with failure can be described as

$$U_1^{\varpi}(\rho) = \varpi(\rho)KEe_z(\rho_k),\ U_2^{\varpi}(\rho) = \varpi(\rho)\bar{K}Ge_g(\rho_k),$$

where $\varpi(\rho) = \text{diag}\{\varpi_1(\rho), \ldots, \varpi_n(\rho)\}$ is the actuator failure level with $\varpi_\delta(\rho)$ $(\delta = 1, 2, \ldots, n)$ satisfying $\underline{\varpi}_\delta \leq \dot{\varpi}_\delta(\rho) \leq \overline{\varpi}_\delta$. $\overline{\varpi}_\delta, \underline{\varpi}_\delta \in (0, 1]$ are the upper and lower bound of $\varpi_\delta(\rho)$. Further, when $\underline{\varpi}_\delta = \overline{\varpi}_\delta = 1$, the actuator works properly. The actuator failure level $\varpi(\rho)$ can be represented as

$$\varpi(\rho) = \tilde{\varpi} + \Delta\varpi(\rho),$$

where $\tilde{\varpi} = \text{diag}\left\{\frac{\underline{\varpi}_1+\overline{\varpi}_1}{2}, \ldots, \frac{\underline{\varpi}_n+\overline{\varpi}_n}{2}\right\}$, $\Delta\varpi(\rho) = \text{diag}\{\Delta\varpi_1(\rho), \ldots, \Delta\varpi_n(\rho)\}$, $|\Delta\varpi_{\delta(\rho)}| \leq \frac{\overline{\varpi}_1-\underline{\varpi}_1}{2}$. Let the uncertainty term $\Delta\varpi(\rho) = \Gamma(\rho)\hat{\varpi}$ with $\hat{\varpi} = \text{diag}\left\{\frac{\overline{\varpi}_1-\underline{\varpi}_1}{2}, \ldots, \frac{\overline{\varpi}_n-\underline{\varpi}_n}{2}\right\}$, the function $\Gamma(\rho)$ satisfies $\Gamma^T(\rho)\Gamma(\rho) \leq I$. Thus, for $\rho \in [\rho_k, \rho_{k+1})$, the error system (6) can be rewritten as

$$\begin{aligned} \text{STM:}\quad \dot{e}_z(\rho) =& -A_p e_z(\rho) + B_p F(e_z(\rho)) \\ & + \bar{B}_p F(e_z(\rho - ß(\rho))) + D_p e_g(\rho) \\ & - \varpi(\rho)KEe_z(\rho_k), \\ \text{LTM:}\quad \dot{e}_g(\rho) =& -C_p e_g(\rho) + M_p F(e_z(\rho)) \\ & - \varpi(\rho)\bar{K}Ge_g(\rho_k). \end{aligned} \tag{7}$$

Assume that the condition $0 < \varsigma_1 < \varsigma_2$ is true for two sampling periods ς_1 and ς_2. Thus $\rho_{k+1} - \rho_k = \varsigma_k \in \{\varsigma_1, \varsigma_2\}$. Suppose that the probability occurrence of ς_1 and ς_2 are given as $\Pr\{\varsigma_k = \varsigma_1\} = \mathfrak{b}$, $\Pr\{\varsigma_k = \varsigma_2\} = 1 - \mathfrak{b}$. The input delay is defined as $c_k(\rho) = \rho - \rho_k$. Then for $\rho \in [\rho_k, \rho_{k+1})$ The error system (7) can be written as

$$\begin{aligned} \text{STM:}\quad \dot{e}_z(\rho) =& -A_p e_z(\rho) + B_p F(e_z(\rho)) \\ & + \bar{B}_p F(e_z(\rho - ß(\rho))) + D_p e_g(\rho) \\ & - \varpi(\rho)KEe_z(\rho - c_k(\rho)), \\ \text{LTM:}\quad \dot{e}_g(\rho) =& -C_p e_g(\rho) + M_p F(e_z(\rho)) \\ & - \varpi(\rho)\bar{K}Ge_g(\rho - c_k(\rho)). \end{aligned} \tag{8}$$

For the range of values of $c_k(\rho)$ the two cases are considered

Case 1: If $\rho_{k+1} - \rho_k = \varsigma_1$ $(\varsigma_k = \varsigma_1)$ then the probability of input delay $c_k(\rho) \in [0, \varsigma_1)$ is 1,
$\Pr\{0 \leq c_k(\rho) < \varsigma_1 | \varsigma_k = \varsigma_1\} = 1$.

Case 2: If $\rho_{k+1} - \rho_k = \varsigma_2$ $(\varsigma_k = \varsigma_2)$ then the probability of input delay $c_k(\rho) \in [0, \varsigma_1)$ or $[\varsigma_1, \varsigma_2)$ is 1,
$\Pr\{0 \leq c_k(\rho) < \varsigma_1 | \varsigma_k = \varsigma_2\} = \frac{\varsigma_1}{\varsigma_2}$;
$\Pr\{\varsigma_1 \leq c_k(\rho) < \varsigma_2 | \varsigma_k = \varsigma_2\} = \frac{\varsigma_2-\varsigma_1}{\varsigma_2}$.

According to the Law of total probability,

$$\begin{aligned} &\Pr\{0 \leq c_k(\rho) < \varsigma_1\} \\ &= \Pr\{0 \leq c_k(\rho) < \varsigma_1 | \varsigma_k = \varsigma_1\} \times \Pr\{\varsigma_k = \varsigma_1\} \\ &\quad + \Pr\{0 \leq c_k(\rho) < \varsigma_1 | \varsigma_k = \varsigma_2\} \times \Pr\{\varsigma_k = \varsigma_2\} \\ &= \mathfrak{b} + \frac{\varsigma_1}{\varsigma_2}(1 - \mathfrak{b}) \triangleq \mathfrak{a} \end{aligned}$$

and

$$\begin{aligned} &\Pr\{\varsigma_1 \leq c_k(\rho) < \varsigma_2\} \\ &= \Pr\{\varsigma_1 \leq c_k(\rho) < \varsigma_2 | \varsigma_k = \varsigma_2\} \times \Pr\{\varsigma_k = \varsigma_2\} \\ &= \frac{\varsigma_2 - \varsigma_1}{\varsigma_2}(1 - \mathfrak{b}) \triangleq 1 - \mathfrak{a}. \end{aligned}$$

By framing two stochastic variables $\mathfrak{a}(\rho)$ and $\mathfrak{b}(\rho)$ which meet the Bernoulli distribution, our probability sampling system is converted into a continuous time-delay system

$$\mathfrak{a}(\rho) = \begin{cases} 1, & 0 \leq c_k(\rho) < \varsigma_1 \\ 0, & \varsigma_1 \leq c_k(\rho) < \varsigma_2 \end{cases}$$

$$\mathfrak{b}(\rho) = \begin{cases} 1, & \varsigma_k = \varsigma_1 \\ 0, & \varsigma_k = \varsigma_2. \end{cases}$$

The probability occurrence of stochastic variables $\mathfrak{a}(\rho)$ and $\mathfrak{b}(\rho)$ are determined as

$$\begin{aligned} \Pr\{\mathfrak{a}(\rho) = 1\} =& \Pr\{0 \leq c_k(\rho) < \varsigma_1\} = \mathfrak{a}, \\ \Pr\{\mathfrak{a}(\rho) = 0\} =& \Pr\{\varsigma_1 \leq c_k(\rho) < \varsigma_2\} = 1 - \mathfrak{a} \\ \Pr\{\mathfrak{b}(\rho) = 1\} =& \Pr\{\varsigma_k = \varsigma_1\} = \mathfrak{b}, \\ \Pr\{\mathfrak{b}(\rho) = 0\} =& \Pr\{\varsigma_k = \varsigma_2\} = 1 - \mathfrak{b}. \end{aligned}$$

The expectations for the stochastic variables $\mathfrak{a}(\rho)$ and $\mathfrak{b}(\rho)$ are specified as

$$\mathbb{E}\{\mathfrak{a}(\rho)\} = \mathfrak{a},\ \mathbb{E}\{\mathfrak{a}(\rho)-\mathfrak{a}\} = 0,\ \mathbb{E}\{\mathfrak{b}(\rho)\} = \mathfrak{b}.$$

For the better characterization of the input delay $c_k(\rho)$, we introduced the following two input delays:

$$c_k(\rho) = \begin{cases} c_{k1}(\rho), & 0 \le c_k(\rho) < \varsigma_1 \\ c_{k2}(\rho), & \varsigma_1 \le c_k(\rho) < \varsigma_2. \end{cases}$$

Thus, by means of $\mathfrak{a}(\rho)$, $c_{k1}(\rho)$, and $c_{k2}(\rho)$, the error system (8) can be represented by

$$\begin{aligned}
\text{STM:}\quad \dot{e}_z(\rho) =& -A_p e_z(\rho) + B_p F(e_z(\rho)) \\
&+ \bar{B}_p F(e_z(\rho-\beta(\rho))) + D_p e_g(\rho) \\
&- \mathfrak{a}(\rho)\varpi(\rho) K E e_z(\rho - c_{k1}(\rho)) \\
&- (1-\mathfrak{a}(\rho))\varpi(\rho) K E e_z(\rho - c_{k2}(\rho)), \\
\text{LTM:}\quad \dot{e}_g(\rho) =& -C_p e_g(\rho) + M_p F(e_z(\rho)) \\
&- \mathfrak{a}(\rho)\varpi(\rho)\bar{K} G e_g(\rho - c_{k1}(\rho)) \\
&- (1-\mathfrak{a}(\rho))\varpi(\rho)\bar{K} G e_g(\rho - c_{k2}(\rho)).
\end{aligned} \tag{9}$$

By setting $\vartheta(\rho) = [e_z(\rho)\ e_g(\rho)]^T$, we get the subsequent system from (9)

$$\begin{aligned}
\vartheta(\rho) =& \hat{A}_p\vartheta(\rho) + \hat{B}_p F(H\vartheta(\rho)) \\
&+ \hat{C}_p F(H\vartheta(\rho-\beta(\rho))) \\
&- \hat{W}_p\vartheta(\rho - c_{k1}(\rho)) - \hat{V}_p\vartheta(\rho - c_{k2}(\rho))
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
\hat{A}_p &= \begin{bmatrix} -A_p & D_p \\ 0 & -C_p \end{bmatrix},\ \hat{B}_p = \begin{bmatrix} B_p \\ M_p \end{bmatrix},\ \hat{C}_p = \begin{bmatrix} \bar{B}_p \\ 0 \end{bmatrix}, \\
\hat{W}_p &= \begin{bmatrix} \mathfrak{a}(\rho)\varpi(\rho)KE & 0 \\ 0 & \mathfrak{a}(\rho)\varpi(\rho)\bar{K}G \end{bmatrix}, \\
H &= \begin{bmatrix} I & 0 \end{bmatrix}, \\
\hat{V}_p &= \begin{bmatrix} (1-\mathfrak{a}(\rho))\varpi(\rho)\bar{K}E & 0 \\ 0 & (1-\mathfrak{a}(\rho))\varpi(\rho)\bar{K}G \end{bmatrix}.
\end{aligned}$$

Definition 1 [15]: The error system (10) is said to be exponentially stable in the mean square, that is there exist two positive constants ε and δ such that

$$\mathbb{E}\{\|\vartheta(\rho)\|^2\} \le \varepsilon e^{-\delta\rho}\sup_{-d^*\le\theta\le 0}\mathbb{E}\{\|\psi(\theta)\|^2\}$$

where $d^* = \max\{\bar{\beta}, \varsigma_2\}$, $\psi(.)$ is the initial function of system (10) defined as $\psi(\rho) = \vartheta(\rho), \rho \in [-d^*, 0]$.

Lemma 1 [15]: For the positive scalar $\upsilon \in (0,1)$, a positive matrix $\mathcal{S} \in \mathbb{R}^{n\times n}$, two matrices $\mathcal{N}_1, \mathcal{N}_2 \in \mathbb{R}^{n\times m}$, for all vector $\chi \in \mathbb{R}^m$, the function $\Psi(\upsilon, \mathcal{S})$ is given by

$$\Psi(\upsilon,\mathcal{S}) = \frac{1}{\upsilon}\chi^T\mathcal{N}_1^T\mathcal{S}\mathcal{N}_1\chi + \frac{1}{1-\upsilon}\chi^T\mathcal{N}_2^T\mathcal{S}\mathcal{N}_2\chi,$$

that is there exist a matrix $\mathcal{Y} \in \mathbb{R}^{n\times n}$ such that $\begin{bmatrix} \mathcal{S} & \mathcal{Y} \\ * & \mathcal{Y} \end{bmatrix} > 0$, then the subsequent inequality can be obtained:

$$\min_{\chi\in(0,1)} \Psi(\chi,\mathcal{S}) \ge \begin{bmatrix} \mathcal{N}_1\chi \\ \mathcal{N}_2\chi \end{bmatrix}^T \begin{bmatrix} \mathcal{S} & \mathcal{Y} \\ * & \mathcal{Y} \end{bmatrix} \begin{bmatrix} \mathcal{N}_1\chi \\ \mathcal{N}_2\chi \end{bmatrix}.$$

Lemma 2 [16]: Suppose the symmetric matrix $\mathfrak{T} \in \mathbb{R}^{2n\times 2n}$ satisfying

$$\mathfrak{T} = \begin{bmatrix} \mathfrak{T}_{11} & \mathfrak{T}_{12} \\ * & \mathfrak{T}_{22} \end{bmatrix} \ge 0, \mathfrak{T}_{pq} \in \mathbb{R}^{n\times n},\ p,q = 1,2$$

the integral inequality given below is reliable:

$$\begin{aligned}
&-(\rho-\rho_k)\int_{\rho_k}^{\rho} \underline{\vee}^T(\mathfrak{P})\mathfrak{T}_{11}\underline{\vee}(\mathfrak{P})d(\mathfrak{P}) \\
&\le \begin{bmatrix} \underline{\vee}(\rho) \\ \underline{\vee}(\rho_k) \end{bmatrix}^T \begin{bmatrix} \mathbb{G}_{11} & \mathbb{G}_{12} \\ * & \mathbb{G}_{22} \end{bmatrix} \begin{bmatrix} \underline{\vee}(\rho) \\ \underline{\vee}(\rho_k) \end{bmatrix},
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{G}_{11} =& \mathfrak{T}_{12}\mathfrak{T}_{22}^{-1}\mathfrak{T}_{12}^T - \mathfrak{T}_{11} \\
&+ (\rho-\rho_k)^2\mathfrak{T}_{22} \\
&+ 2(\rho-\rho_k)\mathfrak{T}_{12}, \\
\mathbb{G}_{12} =& \mathfrak{T}_{12}\mathfrak{T}_{22}^{-1}\mathfrak{T}_{12}^T + \mathfrak{T}_{11} \\
&- (\rho-\rho_k)\mathfrak{T}_{12}^T, \\
\mathbb{G}_{22} =& \mathfrak{T}_{12}\mathfrak{T}_{22}^{-1}\mathfrak{T}_{12}^T - \mathfrak{T}_{11}.
\end{aligned}$$

3 Main Results

In this section, our main aim is to design the SSDC state estimator for MJCNNs (1) such that the error system is exponentially stable. For the sake of brevity the following symbols are denoted:

$$e_i = \left[\ 0_{n\times(i-1)n},\ I_n,\ 0_{n\times(12-i)n}\ \right],\ \mathcal{P}_p = \varsigma_p - \varsigma_{p-1},$$

$$\begin{aligned}\xi(\rho) =& \text{col}\Big\{\vartheta(\rho), \dot{\vartheta}(\rho), \vartheta(\rho-\bar{ß}), \vartheta(\rho-ß(\rho)),\\ &\int_{\rho-\varsigma_1}^{\rho}\vartheta(s)ds, \int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\vartheta(s)ds, \vartheta(\rho-\varsigma_1), \vartheta(\rho-\varsigma_2),\\ &F(H\vartheta(\rho)), F(H\vartheta(\rho-ß(\rho))), \vartheta(\rho-c_{k1}(\rho)),\\ &\vartheta(\rho-c_{k2}(\rho))\Big\}, \Pi_1 = [e_1-e_{11}, e_{11}-e_7],\end{aligned}$$

$$\begin{aligned}\Pi_2 =& [e_7-e_{12}, e_{12}-e_8], \mathcal{P}_1 = \varsigma_1,\ \mathcal{P}_2 = \varsigma_2-\varsigma_1,\\ \Pi_3 =& [\varsigma_1 e_1 - e_5], \Pi_4 = [(\varsigma_2-\varsigma_2)e_1 - e_6],\\ \tilde{\rho}_1 =& \frac{e^{\delta\varsigma_1}-1}{\delta},\ \tilde{\rho}_2 = \frac{\varsigma_1^2[e^{\delta\varsigma_1}-\delta\varsigma_1-1]}{2\delta^2},\\ \tilde{\rho}_3 =& \frac{e^{\delta\varsigma_2}-e^{\delta\varsigma_1}}{\delta}, \tilde{\rho}_5 = \frac{1-e^{\delta\bar{ß}}}{\delta},\\ \tilde{\rho}_4 =& \frac{(\varsigma_2^2-\varsigma_1^2)[e^{\delta\varsigma_2}-\delta(\varsigma_2-\varsigma_1)-e^{\delta\varsigma_1}]}{2\delta^2}.\end{aligned}$$

$$\begin{aligned}\breve{\Xi}(\rho) =& \text{sym}\{e_2^T P_p e_1\} + e_1^T\left(\sum_{q=1}^{l}\pi_{pq}P_q\right)e_1\\ &+ e_1^T H_1 e_1 - e^{\delta\bar{ß}}e_3^T(H_1+H_2)e_3\\ &- (1-\dot{ß}(\rho))e^{\delta ß(\rho)}e_4^T H_2 e_4 + \mathfrak{a}(e_1^T H_3 e_1\\ &- e^{\delta\varsigma_1}e_7^T H_3 e_7 + \tilde{\rho}_1 e_2^T H_4 e_2 + \tilde{\rho}_2 e_2^T H_5 e_2\\ &+ \Pi_3^T H_5 \Pi_3) + (1-\mathfrak{a})(e_7^T H_6 e_7\\ &- e^{\delta\varsigma_2}e_8^T H_6 e_8 + \tilde{\rho}_3 e_2^T H_7 e_2 + \tilde{\rho}_4 e_2^T H_8 e_2\\ &+ \Pi_4^T H_8 \Pi_4) + \mathfrak{a}\Pi_1 Q_1 \Pi_1^T\\ &+ (1-\mathfrak{a})\Pi_2 Q_2 \Pi_2^T + \sum_{p=1}^{4}\Phi_p + \Phi_5 + \beth\\ \beth =& -e_2^T\mathfrak{L}_1 e_2 + \bar{ß}^2 e_2^T\mathfrak{L}_3 e_2 + 2\bar{ß}e_2^T\mathfrak{L}_2 e_2\\ &+ 2e_2^T\mathfrak{L}_1 e_3 - 2\bar{ß}e_2^T\mathfrak{L}_2 e_3 - e_3^T\mathfrak{L}_1 e_3.\end{aligned}$$

Theorem 1: For the given scalars $\mu > 0$, $\bar{ß} > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and matrices $U > 0$, $\Lambda = \text{diag}(\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_n) > 0$, the error system (10) is exponentially stable in mean square if there exist positive definite matrices $P_p (p \in \mathcal{M} = 1,2,3,\ldots,l)$, $H_k (k = 1,2,3,5,6,8)$, positive symmetric matrices H_4, H_7, $\mathfrak{L}_1$, $\begin{bmatrix}\mathfrak{L}_1 & \mathfrak{L}_2\\ * & \mathfrak{L}_3\end{bmatrix}$, matrices $\mathfrak{Z}_k (k = 1,2)$, such that the following inequality hold:

$$Q_1 = \left[\begin{smallmatrix} H_4 & \mathfrak{Z}_1 \\ * & H_4\end{smallmatrix}\right] > 0,\ Q_2 = \left[\begin{smallmatrix} H_7 & \mathfrak{Z}_2 \\ * & H_7\end{smallmatrix}\right] > 0, \quad (11)$$

$$\Xi(\rho) = \begin{bmatrix}\breve{\Xi}(\rho) & \sqrt{2}e_1^T\mathfrak{L}_2 - \sqrt{2}e_3^T\mathfrak{L}_2\\ * & -\mathfrak{L}_3\end{bmatrix} < 0, \quad (12)$$

where

$$\bar{W} = \begin{bmatrix}KE & 0\\ 0 & \bar{K}G\end{bmatrix}.$$

Proof: Consider the following LKFs:

$$V(\rho) = \sum_{p=1}^{5} V_p(\rho)$$

where

$$V_1(\rho) = e^{\delta\rho}\vartheta^T(\rho)P_p\vartheta(\rho) \quad (13)$$

$$\begin{aligned}V_2(\rho) &= \int_{\rho-\bar{ß}}^{\rho} e^{\delta s}\vartheta^T(s)H_1\vartheta(s)ds\\ &+ \int_{\rho-ß(\rho)}^{\rho-\bar{ß}} e^{\delta s}\vartheta^T(s)H_2\vartheta(s)ds\end{aligned} \quad (14)$$

$$
\begin{aligned}
&V_3(\rho)\\
&= \mathfrak{a}(\rho)\Bigg[\int_{\rho-\varsigma_1}^{\rho} e^{\delta s}\vartheta^T(s)H_3\vartheta(s)ds\\
&\quad + \mathcal{P}_1\int_{-\varsigma_1}^{0}\int_{\rho+\theta}^{\rho} e^{\delta(s-\theta)}\dot{\vartheta}^T(s)H_4\dot{\vartheta}(s)dsd\theta\\
&\quad + \frac{\varsigma_1^2}{2}\int_{-\varsigma_1}^{0}\int_{\theta}^{0}\int_{\rho+\lambda}^{\rho} e^{\delta(s-\lambda)}\dot{\vartheta}^T(s)H_5\dot{\vartheta}(s)dsd\lambda d\theta\Bigg]
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&V_4(\rho)\\
&= (1-\mathfrak{a}(\rho))\Bigg[\int_{\rho-\varsigma_2}^{\rho-\varsigma_1} e^{\delta s}\vartheta^T(s)H_6\vartheta(s)ds\\
&\quad + \mathcal{P}_2\int_{-\varsigma_2}^{-\varsigma_1}\int_{\rho+\theta}^{\rho} e^{\delta(s-\theta)}\dot{\vartheta}^T(s)H_7\dot{\vartheta}(s)dsd\theta\\
&\quad + \frac{\varsigma_2^2-\varsigma_1^2}{2}\int_{-\varsigma_2}^{-\varsigma_1}\int_{\theta}^{0}\int_{\rho+\lambda}^{\rho} e^{\delta(s-\lambda)}\dot{\vartheta}^T(s)H_8\dot{\vartheta}(s)dsd\lambda d\theta\Bigg]
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&V_5(\rho)\\
&= 2\bar{\beta}\int_{-\bar{\beta}}^{0}\int_{\rho+\theta}^{\rho} e^{\delta(s-\theta)}\dot{\vartheta}^T(s)\mathfrak{L}_1\dot{\vartheta}(s)dsd\theta.
\end{aligned}
\tag{17}
$$

The weak infinitesimal operator $\mathcal{L}V(\rho)$ is calculated along the trajectories of the error system (13) gives:

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_1(\rho)\}\\
&= \mathbb{E}\Bigg\{e^{\delta\rho}\Bigg[\mathrm{sym}\{\vartheta^T(\rho)P_p\dot{\vartheta}(\rho)\}\\
&\quad + \vartheta^T(\rho)\left(\sum_{q=1}^{l}\pi_{pq}P_q\right)\vartheta(\rho)\Bigg]\Bigg\},
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_2(\rho)\}\\
&\le \mathbb{E}\{e^{\delta\rho}[\vartheta^T(\rho)H_1\vartheta(\rho)\\
&\quad - e^{-\delta\bar{\beta}}\vartheta^T(\rho-\bar{\beta})(H_1+H_2)\vartheta(\rho-\bar{\beta})\\
&\quad - (1-\mu)e^{-\delta\bar{\beta}}\vartheta^T(\rho-\beta(\rho))H_2\vartheta(\rho-\beta(\rho))]\},
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_3(\rho)\}\\
&= \mathbb{E}\Bigg\{\mathfrak{a}e^{\delta\rho}\Bigg\{\vartheta^T(\rho)H_3\vartheta(\rho)\\
&\quad - e^{-\delta\varsigma_1}\vartheta^T(\rho-\varsigma_1)H_3\vartheta(\rho-\varsigma_1)\\
&\quad + \tilde{\rho}_1\dot{\vartheta}^T(\rho)H_4\dot{\vartheta}(\rho) - \int_{\rho-\varsigma_1}^{\rho}\dot{\vartheta}^T(s)H_4\dot{\vartheta}(s)ds\\
&\quad + \tilde{\rho}_2\dot{\vartheta}^T(\rho)H_5\dot{\vartheta}(\rho)\\
&\quad - \left(\varsigma_1\vartheta(\rho) - \int_{\rho-\varsigma_1}^{\rho}\vartheta(s)ds\right)^T H_5\\
&\quad \left(\varsigma_1\vartheta(\rho) - \int_{\rho-h_1}^{\rho}\vartheta(s)ds\right)\Bigg\}\Bigg\},
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_4(\rho)\}\\
&= \mathbb{E}\Bigg\{(1-\mathfrak{a})e^{\delta\rho}\Bigg\{\vartheta^T(\rho-\varsigma_1)H_6\vartheta(\rho-\varsigma_1)\\
&\quad - e^{-\delta\varsigma_2}\vartheta^T(\rho-\varsigma_2)H_6\vartheta(\rho-\varsigma_2)\\
&\quad + \tilde{\rho}_3\dot{\vartheta}^T(\rho)H_7\dot{\vartheta}(\rho) - \int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\dot{\vartheta}^T(s)H_7\dot{\vartheta}(s)ds\\
&\quad + \tilde{\rho}_4\dot{\vartheta}^T(\rho)H_8\dot{\vartheta}(\rho)\\
&\quad \times -\left((\varsigma_2-\varsigma_1)\vartheta(\rho) - \int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\vartheta(s)ds\right)^T\\
&\quad \times H_8\left((\varsigma_2-\varsigma_1)\vartheta(\rho) - \int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\vartheta(s)ds\right)\Bigg\}\Bigg\},
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_5(\rho)\}\\
&= \mathbb{E}\Bigg\{e^{\delta\rho}\Bigg\{2\bar{\beta}^2\tilde{\rho}_5\dot{\vartheta}^T(\rho)\mathfrak{L}_1\dot{\vartheta}(\rho)\\
&\quad - 2\bar{\beta}\int_{\rho-\bar{\beta}}^{\rho}\dot{\vartheta}^T(s)\mathfrak{L}_1\dot{\vartheta}(s)ds\Bigg\}\Bigg\}.
\end{aligned}
\tag{22}
$$

To evaluate the integral terms in (20) and (21) Lemma 1 is utilized,

$$
\begin{aligned}
&-\int_{\rho-\varsigma_1}^{\rho}\dot{\vartheta}^T(s)H_4\dot{\vartheta}(s)ds \le -\xi^T(\rho)\Pi_1 Q_1\Pi_1^T\xi(\rho)\\
&-\int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\dot{\vartheta}^T(s)H_7\dot{\vartheta}(s)ds \le -\xi^T(\rho)\Pi_2 Q_2\Pi_2^T\xi(\rho)
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_3(\rho)\}\\
&\le \mathbb{E}\Big\{\mathfrak{a}e^{\delta\rho}\Big\{\vartheta^T(\rho)H_3\vartheta(\rho)\\
&\quad -e^{-\delta\varsigma_1}\vartheta^T(\rho-\varsigma_1)H_3\vartheta(\rho-\varsigma_1)\\
&\quad +\tilde{\rho}_1\dot{\vartheta}^T(\rho)H_4\dot{\vartheta}(\rho)\\
&\quad +\xi^T(\rho)\Pi_1 Q_1\Pi_1^T\xi(\rho)+\tilde{\rho}_2\dot{\vartheta}^T(\rho)H_5\dot{\vartheta}(\rho)\\
&\quad -\Big(\varsigma_1\vartheta(\rho)-\int_{\rho-\varsigma_1}^{\rho}\vartheta(s)ds\Big)^T H_5\\
&\quad \times\Big(\varsigma_1\vartheta(\rho)-\int_{\rho-\varsigma_1}^{\rho}\vartheta(s)ds\Big)\Big\}\Big\},
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V_4(\rho)\}\\
&\le \mathbb{E}\Big\{(1-\mathfrak{a})e^{\delta\rho}\Big\{\vartheta^T(\rho-\varsigma_1)H_6\vartheta(\rho-\varsigma_1)\\
&\quad -e^{-\delta\varsigma_2}\vartheta^T(\rho-\varsigma_2)H_6\vartheta(\rho-\varsigma_2)\\
&\quad +\tilde{\rho}_3\dot{\vartheta}^T(\rho)H_7\dot{\vartheta}(\rho)+\xi^T(\rho)\Pi_2 Q_2\Pi_2^T\xi(\rho)\\
&\quad +\tilde{\rho}_4\dot{\vartheta}^T(\rho)H_8\dot{\vartheta}(\rho)\\
&\quad -\Big((\varsigma_2-\varsigma_1)\vartheta(\rho)-\int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\vartheta(s)ds\Big)^T H_8\\
&\quad \times\Big((\varsigma_2-\varsigma_1)\vartheta(\rho)-\int_{\rho-\varsigma_2}^{\rho-\varsigma_1}\vartheta(s)ds\Big)\Big\}\Big\}.
\end{aligned}
\tag{25}
$$

We use Lemma 2 for evaluating the integral term in (22), one can obtain

$$
\begin{aligned}
&-\int_{\rho-\bar{\beta}}^{\rho}\dot{\vartheta}^T(s)\mathfrak{L}_1\dot{\vartheta}(s)ds\\
&\le \xi^T(\rho)\begin{bmatrix} e(\rho)\\ e(\rho-\bar{\beta})\end{bmatrix}^T\Upsilon\begin{bmatrix} e(\rho)\\ e(\rho-\bar{\beta})\end{bmatrix}\xi(\rho)\\
&\mathbb{E}\{\mathcal{L}V_5(\rho)\}\\
&\le \mathbb{E}\Big\{e^{\delta\rho}\Big\{2\bar{\beta}^2\tilde{\rho}_5\dot{\vartheta}^T(\rho)\mathfrak{L}_1\dot{\vartheta}(\rho)\\
&\quad +\xi^T(\rho)\begin{bmatrix} e(\rho)\\ e(\rho-\bar{\beta})\end{bmatrix}^T\Upsilon\begin{bmatrix} e(\rho)\\ e(\rho-\bar{\beta})\end{bmatrix}\xi(\rho)\Big\}\Big\},
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
\Upsilon &= \begin{bmatrix}\Upsilon_{11} & \Upsilon_{12}\\ * & \Upsilon_{22}\end{bmatrix},\\
\Upsilon_{11} &= \mathfrak{L}_2\mathfrak{L}_3^{-1}\mathfrak{L}_2^T-\mathfrak{L}_1+(\bar{\beta}^2)\mathfrak{L}_3+2\bar{\beta}\mathfrak{L}_2,\\
\Upsilon_{22} &= \mathfrak{L}_2\mathfrak{L}_3^{-1}\mathfrak{L}_2^T-\mathfrak{L}_1,\\
\Upsilon_{12} &= -\mathfrak{L}_2\mathfrak{L}_3^{-1}\mathfrak{L}_2^T+\mathfrak{L}_1+\bar{\beta}\mathfrak{L}_2.
\end{aligned}
$$

The following inequalities can be obtained from Assumption 1, for a diagonal matrix Λ

$$
\begin{aligned}
\Phi_1 =& e^{\delta\rho}[F^T(H\vartheta(\rho))F(H\vartheta(\rho))\\
&-\vartheta^T(\rho)H^T\Lambda^T\Lambda H\vartheta(\rho)]\le 0\\
=& e^{\delta\rho}(e_9^Te_9-e_1^TH^T\Lambda^T\Lambda He_1)\le 0,
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\Phi_2 =& e^{\delta\rho}[F^T(H\vartheta(\rho-\beta(\rho)))F(H\vartheta(\rho-\beta(\rho)))\\
&-\vartheta^T(\rho-\beta(\rho))H^T\Lambda^T\Lambda H\vartheta(\rho-\beta(\rho))]\le 0\\
=& e^{\delta\rho}(e_{10}^Te_{10}-e_1^TH^T\Lambda^T\Lambda He_1)\le 0.
\end{aligned}
\tag{28}
$$

For the scalars $\varepsilon_1>0$ and $\varepsilon_2>0$, there exist

$$
\begin{aligned}
\Phi_3 =& e^{\delta\rho}\varepsilon_1[\vartheta^T(\rho)H^T\Lambda^T\Lambda H\vartheta(\rho)\\
&-F^T(H\vartheta(\rho))F(H\vartheta(\rho))]\ge 0\\
=& e^{\delta\rho}\varepsilon_1[e_1^TH^T\Lambda^T\Lambda He_1-e_9^Te_9]\ge 0,
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\Phi_4 =& e^{\delta\rho}\varepsilon_2[\vartheta^T(\rho-\beta(\rho))H^T\Lambda^T\Lambda H\vartheta(\rho-\beta(\rho))\\
&-F^T(H\vartheta(\rho-\beta(\rho)))F(H\vartheta(\rho-\beta(\rho)))]\ge 0\\
=& e^{\delta\rho}\varepsilon_2[e_4^TH^T\Lambda^T\Lambda He_4-e_{10}^Te_{10}]\ge 0.
\end{aligned}
\tag{30}
$$

The following equation holds, for any matrix U with appropriate dimension,

$$
\begin{aligned}
0 =& \mathbb{E}\{e^{\delta\rho}2[\vartheta^T(\rho)U+\dot{\vartheta}^T(\rho)U][-\dot{\vartheta}(\rho)+\hat{A}_p\vartheta(\rho)\\
&+\hat{B}_pF(H\vartheta(\rho))+\hat{C}_pF(H\vartheta(\rho-\beta(\rho)))\\
&-\hat{W}_p\vartheta(\rho-c_{k1}(\rho))-\hat{V}_p\vartheta(\rho-c_{k2}(\rho))]\}.
\end{aligned}
\tag{31}
$$

From the equation (31) we can obtain,

$$\begin{aligned}
&\Phi_5\\
&=\mathbb{E}\{e^{\delta\rho}2[\vartheta^T(\rho)U+\dot{\vartheta}^T(\rho)U][-\dot{\vartheta}(\rho)+\hat{A}_p\vartheta(\rho)\\
&\quad+\hat{B}_pF(H\vartheta(\rho))+\hat{C}_pF(H\vartheta(\rho-ß(\rho)))]\\
&\quad-2\mathfrak{a}\varpi(\rho)\vartheta^T(\rho)U\bar{W}\vartheta(\rho-c_{k1}(\rho))\\
&\quad-2\mathfrak{a}\varpi(\rho)\dot{\vartheta}^T(\rho)U\bar{W}\vartheta(\rho-c_{k1}(\rho))\\
&\quad-2(1-\mathfrak{a})\varpi(\rho)\vartheta^T(\rho)U\bar{W}\vartheta(\rho-c_{k2}(\rho))\\
&\quad-2(1-\mathfrak{a})\varpi(\rho)\dot{\vartheta}^T(\rho)U\bar{W}\vartheta(\rho-c_{k2}(\rho))\}=0\\
&=\mathbb{E}\{e^{\delta\rho}2[e_1^TU+e_2^TU][-e_2+\hat{A}_pe_1++\hat{B}_pe_9\\
&\quad+\hat{C}_pe_{10}-2\mathfrak{a}\varpi(\rho)(e_1^TU\bar{W}e_{11}+e_2^TU\bar{W}e_{11})\\
&\quad-2(1-\mathfrak{a})\varpi(\rho)(e_1^TU\bar{W}e_{12}+e_2^TU\bar{W}e_{12})]\}=0.
\end{aligned}\tag{32}$$

Then, combining with (18), (19), (24)-(32), we have

$$\mathbb{E}\{\mathcal{L}V(\rho)\}\leq\mathbb{E}\{\xi^T(\rho)\breve{\Xi}(\rho)\xi(\rho)\}.\tag{33}$$

From (33), we get

$$\begin{aligned}
V(\rho)\leq&e^{-\delta(\rho-\rho_k)}V(\rho_k)\\
\leq&e^{-\delta(\rho-\rho_{k-1})}V(\rho_{k-1})\\
\leq&\cdots\\
\leq&e^{-\delta\rho}V(0).
\end{aligned}\tag{34}$$

Moreover, from the definition of $V(\rho)$, we obtain

$$\begin{aligned}
e^{\delta\rho}\mathbb{E}\{\|\vartheta(\rho)\|^2\}\lambda_{\min}(P)\leq&e^{\delta\rho}\mathbb{E}\{V(\rho)\}\\
\leq&\mathbb{E}\{V(0)\}.
\end{aligned}\tag{35}$$

Futhermore,

$$\mathbb{E}\{V(0)\}\leq\tilde{\mathfrak{a}}\sup_{d^*\leq\theta\leq0}\mathbb{E}\{\|\psi(\theta)\|^2\}\tag{36}$$

where $\tilde{\mathfrak{a}}=\lambda_{\max}(P_p)+\lambda_{\max}(H_1)\frac{(1-e^{\delta\bar{ß}})}{\delta}+\lambda_{\max}(H_2)\frac{(1-e^{\delta\bar{ß}})}{\delta}+\lambda_{\max}(\mathfrak{L}_1)\frac{(1-e^{\delta\bar{ß}})}{\delta}$. From (34)-(36) we get that

$$\mathbb{E}\{\|\vartheta(\rho)\|^2\}\leq\tilde{\mathfrak{a}}\frac{e^{-\delta\rho}}{\lambda_{\min}(P)}\sup_{-\bar{ß}\leq\theta\leq0}\mathbb{E}\{\|\psi(\theta)\|^2\}\tag{37}$$

Then with help of Definition 1 the system (10) is exponentially mean-square stable. Thus the proof is accomplished.

Remark 1: In the absence of Markov jump particularly when $\underline{\varpi}_\delta=\overline{\varpi}_\delta=1$, which means actuator works properly, then the MJCNNs (1) and the state estimator for (2) is transformed to the following form:

$$\begin{aligned}
\text{STM:}\quad&\dot{z}(\rho)=-Az(\rho)+Bf(z(\rho))\\
&\qquad+\bar{B}f(z(\rho-ß(\rho)))+Dg(\rho)+J\\
\text{LTM:}\quad&\dot{g}(\rho)=-Cg(\rho)+Mf(g(\rho)),\\
\text{STM:}\quad&\dot{\hat{z}}(\rho)=-A\hat{z}(\rho)+Bf(\hat{z}(\rho))\\
&\qquad+\bar{B}f(\hat{z}(\rho-ß(\rho)))+D\hat{s}(\rho)\\
&\qquad+J+U_1(\rho)\\
\text{LTM:}\quad&\dot{\hat{g}}(\rho)=-C\hat{g}(\rho)+Mf(\hat{g}(\rho))+U_2(\rho)
\end{aligned}\tag{38}\tag{39}$$

Then the corresponding error system for (38) and (39) will be

$$\begin{aligned}
\vartheta(\rho)=&\hat{A}\vartheta(\rho)+\hat{B}F(H\vartheta(\rho))+\hat{C}F(H\vartheta(\rho-ß(\rho)))\\
&-\tilde{W}\vartheta(\rho-c_{k1}(\rho))-\tilde{V}\vartheta(\rho-c_{k2}(\rho))
\end{aligned}\tag{40}$$

where

$$\begin{aligned}
&\hat{A}=\begin{bmatrix}-A&D\\0&-C\end{bmatrix},\ \hat{B}=\begin{bmatrix}B\\M\end{bmatrix},\ \hat{C}=\begin{bmatrix}\bar{B}\\0\end{bmatrix},\\
&\tilde{W}_p=\begin{bmatrix}\mathfrak{a}(\rho)KE&0\\0&\mathfrak{a}(\rho)\bar{K}G\end{bmatrix},\ H=\begin{bmatrix}I&0\end{bmatrix},\\
&\tilde{V}_p=\begin{bmatrix}(1-\mathfrak{a}(\rho))\bar{K}E&0\\0&(1-\mathfrak{a}(\rho))\bar{K}G\end{bmatrix}.
\end{aligned}$$

For the error system (40), the stability conditions can be represented by the following corollary.

Corollary 1: For the given scalars $\mu>0$, $\bar{ß}>0$, $\varepsilon_1>0$, $\varepsilon_2>0$, and matrices $U>0$, $\Lambda=\text{diag}(\hat{\rho}_1,\hat{\rho}_2,\ldots,\hat{\rho}_n)>0$, the error system (40) is exponentially stable in mean square if there exist positive definite matrices $H_k(k=1,2,3,5,6,8)$, symmetric matrices H_4,H_7, $\mathfrak{L}_1$, $\begin{bmatrix}\mathfrak{L}_1&\mathfrak{L}_2*&\mathfrak{L}_3\end{bmatrix}$, matrices $\mathfrak{Z}_k(k=1,2)$, such that the following inequality hold:

$$Q_1=\begin{bmatrix}H_4&\mathfrak{Z}_1*&H_4\end{bmatrix}>0,Q_2=\begin{bmatrix}H_7&\mathfrak{Z}_2*&H_7\end{bmatrix}>0,\tag{41}$$

$$\Xi^l(\rho)=\begin{bmatrix}\breve{\Xi}^l(\rho)&\sqrt{2}e_1^T\mathfrak{L}_2-\sqrt{2}e_3^T\mathfrak{L}_2*&-\mathfrak{L}_3\end{bmatrix}<0,\tag{42}$$

where

$$\begin{aligned}\breve{\Xi}^{l}(\rho) =& e_1^T H_1 e_1 - e^{\delta\bar{\beta}} e_3^T (H_1 + H_2) e_3 \\ & - (1-\dot{\beta}(\rho)) e^{\delta\beta(\rho)} e_4^T H_2 e_4 + \mathfrak{a}(e_1^T H_3 e_1 \\ & - e^{\delta\varsigma_1} e_7^T H_3 e_7 + \tilde{\rho}_1 e_2^T H_4 e_2 + \tilde{\rho}_2 e_2^T H_5 e_2 \\ & + \Pi_3^T H_5 \Pi_3) + (1-\mathfrak{a})(e_7^T H_6 e_7 - e^{\delta\varsigma_2} e_8^T H_6 e_8 \\ & + \tilde{\rho}_3 e_2^T H_7 e_2 + \tilde{\rho}_4 e_2^T H_8 e_2 + \Pi_4^T H_8 \Pi_4) \\ & + \mathfrak{a}\Pi_1 Q_1 \Pi_1^T + (1-\mathfrak{a})\Pi_2 Q_2 \Pi_2^T \\ & + \sum_{p=1}^{4} \Phi_p + \beth + \tilde{\Phi}_5.\end{aligned}$$

$$\begin{aligned}\tilde{\Phi}_5 =& e^{\delta\rho} 2[e_1^T U + e_2^T U][-e_2 + \hat{A} e_1 + + \hat{B} e_9 \\ & + \hat{C} e_{10} - 2\mathfrak{a}\varpi(\rho)(e_1^T U \bar{W} e_{11} + e_2^T U \bar{W} e_{11}) \\ & - 2(1-\mathfrak{a})\varpi(\rho)(e_1^T U \bar{W} e_{12} + e_2^T U \bar{W} e_{12})].\end{aligned}$$

Proof: The proof is similar to that of Theorem 1, therefore we omit it.

Remark 2: It should be noted that the novel modified free matrix-based integral inequalities (MFMBII) represents in Lemma 2 can be directly involved in the integral inequality $-\int_{\rho-\bar{\beta}}^{\rho} \dot{\vartheta}^T(s)\mathfrak{L}_1\dot{\vartheta}(s)ds$. In contrast to the existing results using free matrix methods, the revised free matrix inequalities utilized in this paper yield less computational complexity when deriving practical stability conditions expressed in terms of LMIs. However, while we can achieve less computational outcomes in SSDC scenarios, obtaining similar results in continuous control systems remains a challenging task. The novel MFMBII Lemma 2 yields a free matrix with only 3 variables $\begin{bmatrix} \mathfrak{L}_{11} & \mathfrak{L}_{12} \\ * & \mathfrak{L}_{22} \end{bmatrix}$. This simply demonstrates the reduced computational burden associated with our results.

Remark 3: This article simplifies the scenario by assuming the existence of two sampling periods, ς_1 and ς_2, each with certain probabilities of occurrence. When designing the error system (9), the input delay approach is employed. The distribution of input delay $c_k(\rho)$ falls within the interval $[0,\varsigma_1)$ or $[\varsigma_1,\varsigma_2)$ and this is determined by the random nature of sampled-data periods ς_1 and ς_2. To quantify the probability distribution of $c_k(\rho)$, a stochastic variable $\mathfrak{a}(\rho)$ which holds the condition of Bernoulli distribution, is introduced. The $\mathfrak{a}(\rho)$ specifies the event $c_k(\rho)$ lies in the interval $[0,\varsigma_1)$ and $[\varsigma_1,\varsigma_2)$ by taking the values 1 and 0. Further, two input delays, $c_{k1}(\rho)$ and $c_{k2}(\rho)$, are introduced for a better description of the error system (9).

Remark 4: Compared to the prior work [17], the current work is based on a sampled data control with stochastic sampling, which is cost-effective and reasonable. Additionally, in contrast to the work [18], this paper considers the actuator failure scenario. Therefore, the results presented here expand upon and enrich the findings from the previous literature. It is important to note that Theorem 1's LMIs (11) and (12) must be met, which somewhat restricts the extent of the solution. So far, state estimation for several types of neural networks has been researched. However, there is a scarcity of work on the SSDC state estimation problem for MJCNNs subject to actuator failure. In doing so, the theoretical results presented here significantly enrich the study of the exponential state estimation problem in this domain.

4 Numerical simulation

The viability as well as efficacy of the findings are demonstrated by a numerical example in this section.

Example : In this example, using the error system (10) controller gain matrix was calculated with the help of Theorem 1.

Consider the MJCNNs (1) and output measurement (2), with the corresponding parameters are given as

$$A_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & 0 \\ 0.1 & -0.4 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 2.55 & -0.1 \\ -0.16 & 3.5 \end{bmatrix}, B_2 = \begin{bmatrix} 1.98 & -0.1 \\ -0.2 & 2.7 \end{bmatrix},$$
$$\bar{B}_1 = \begin{bmatrix} -2.1 & -0.52 \\ -0.3 & -2 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} -1.86 & -0.90 \\ 0 & -1.3 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1.28 & 0 \\ 0 & 2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1.4 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1.2 & -0.1 \\ 0.2 & 0.3 \end{bmatrix}, D_2 = \begin{bmatrix} 2 & -0.45 \\ 0.2 & 0.3 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 2 & -0.16 \\ -0.1 & 1.5 \end{bmatrix}, M_2 = \begin{bmatrix} 2 & -0.1 \\ -0.2 & 1.46 \end{bmatrix},$$
$$Q = \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}.$$

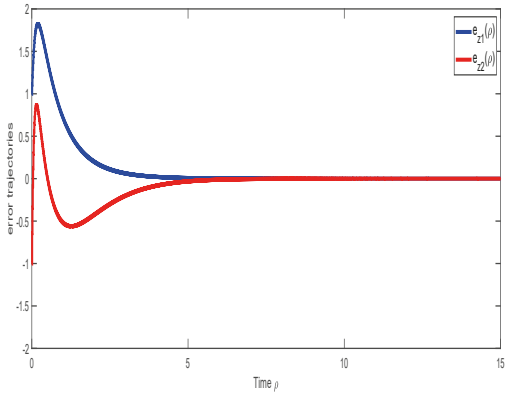


Figure 1. The error trajectories of $z(\rho)$ and its estimated state $\hat{z}(\rho)$

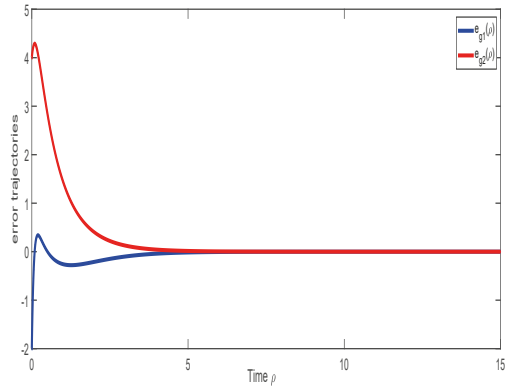


Figure 2. The error trajectories of $g(\rho)$ and its estimated state $\hat{g}(\rho)$

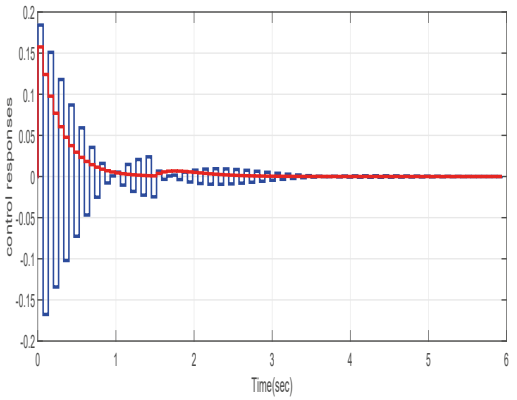


Figure 3. Stochastic sampled-data control

The nonlinear activation function $F(z) = \tanh(z)$, $ß(\rho) = 0.2(1+\sin(\rho))$, $\mu = 0.2, \bar{ß} = 0.4, \varsigma_1 = 0.2, \varsigma_2 = 0.3, ƀ = 0.3, \varepsilon_1 = 1.1, \varepsilon_2 = 1.2$. We assume that no failure happens to the controllers $U_1(\rho), U_2(\rho)$ so the actuator failure values become $\underline{\varpi}(\rho) = \overline{\varpi}(\rho) = 1$. Let $W = U\bar{W}$, from the LMI conditions (11), (12) and the parameters defined above, we get the subsequent solution:

$$U = \begin{bmatrix} 0.0382 & 0.0018 & 0 & 0 \\ 0.0018 & 0.0364 & 0 & 0 \\ 0 & 0 & 0.2009 & 0.0082 \\ 0 & 0 & 0.0082 & 0.1697 \end{bmatrix},$$
$$W = \begin{bmatrix} 0.0747 & 0.0019 & 0 & 0 \\ 0.0011 & 0.0797 & 0 & 0 \\ 0 & 0 & 0.2051 & 0.0431 \\ 0 & 0 & 0.0431 & 0.1094 \end{bmatrix},$$
$$\bar{W} = U^{-1}W$$
$$= \begin{bmatrix} 1.9554 & -0.0451 & 0 & 0 \\ -0.0396 & 2.2083 & 0 & 0 \\ 0 & 0 & 1.0092 & 0.1882 \\ 0 & 0 & 0.2082 & 0.6337 \end{bmatrix}.$$

Let $E = G = I$, one can obtain the gain matrices as follows:

$$K = \begin{bmatrix} 1.9554 & -0.0451 \\ -0.0396 & 2.2083 \end{bmatrix}, \bar{K} = \begin{bmatrix} 1.0092 & 0.1882 \\ 0.2082 & 0.6337 \end{bmatrix}$$

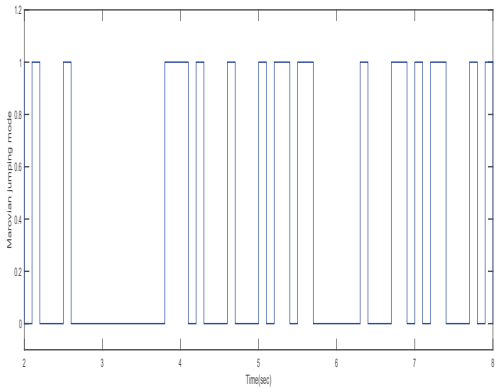


Figure 4. Markovian jumping mode

Figures 1 and 2 represent the error trajectories of $z(\rho)$, $g(\rho)$ for its corresponding estimated states $\hat{z}(\rho)$, $\hat{g}(\rho)$ respectively. Figure 3 represents the control response of stochastic sampled-data control. Figure 4 illustrates the Markovian jumping mode. It is seen that the trajectories of the error true states, $e_z(\rho), e_g(\rho)$, eventually go to zero which also verifies the effectiveness of our method.

Conclusion

In conclusion, this research addresses the challenging problem of exponential state estimation for MJCNNs, with a specific focus on the presence of actuator failures. By proposing a sampled-data control approach with two sampling periods and employing probabilistic sampling, the paper successfully ensures the exponential stability of the error system. The derived stability conditions using LMIs provide a solid foundation for the design of the sampled-data controller. Additionally, the transformation of the state estimator into a continuous time-delay system with stochastic delays allows for effective handling of the system dynamics. Through the construction of appropriate LKFs and the application of basic inequality techniques, the problem of exponential stability is effectively solved by determining suitable feedback gains and sampling intervals. The numerical example presented in the paper further demonstrates the validity and effectiveness of the proposed methodology.

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