

RANKING OF ALTERNATIVES DESCRIBED BY ATANASSOV'S INTUITIONISTIC FUZZY SETS – RECONCILING SOME MISUNDERSTANDINGS

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Abstract

Atanassov's intuitionistic fuzzy sets (IFSs) are a very convenient tool for describing alternatives/options while making decisions because they make it possible to naturally represent the pros, cons, and hesitation. The IFSs have attracted a significant interest and have been applied in various fields. Of course, their use poses some challenges. One of the main challenges is the ranking of alternatives/options described by the intuitionistic fuzzy sets, to be called for brevity the intuitionistic fuzzy alternatives. This is a crucial issue, notably for the applications, for instance, in decision making. We first present in detail and analyze the benefits of a method we introduced previously (cf. Szmidt and Kacprzyk [1]). For this method, we augment the original assumptions with an additional assumption, which is justified and inherently reasonable. As a result, we obtain formulas which are better justified than those previously used as they explicitly consider the arguments in favor (pro), against (con), and hesitance. Since the intuitionistic fuzzy alternatives can not be linearly ranked, then the additional assumptions during the ranking process are necessary. We address these issues and analyze examples to clarify our new approach. We examine some other methods discussed in the literature and analyze their results, and show that the new assumptions reconcile some misconceptions raised by those other papers.

Keywords: Atanassov's intuitionistic fuzzy sets, intuitionistic fuzzy options, ranking, geometrical representations

1 Introduction

Atanassov's intuitionistic fuzzy sets (cf. Atanassov [2, 3, 4]), abbreviated as the IFSs, are an extension of the fuzzy sets introduced by Zadeh [5]. They provide a valuable framework for representing and handling imperfect information. The IFSs have been applied in various domains, particularly in decision making, which is a fundamental challenge in human activities. One of the most critical issues while decision making is the ranking of alternatives/options. This paper focuses on the ranking of alternatives described using the IFSs, referred to here as the "intuitionistic fuzzy alternatives". Each alternative is associated with two membership degrees, denoted as $\mu(\cdot)$ (indicating the extent to which it satisfies a set of criteria) and $\nu(\cdot)$ (indicating the extent to which it does not satisfy these criteria). This interpretation using Atanassov's intuitionistic fuzzy sets does provide a powerful tool for an effective representation of alternatives/options.

The ranking of intuitionistic fuzzy alternatives is a challenge as it requires the introduction of some additional assumptions. This is necessary because there is no linear order among the intuitionistic fuzzy alternatives/options.

In this paper, we clarify the circumstances under which the ranking method proposed by us, cf. Szmidt and Kacprzyk [1], is appropriate and effective. This method is here augmented with an additional, natural assumption which has not been mentioned in our paper. Interestingly enough, the absence of this assumption in our previous work has raised some doubts in the literature which has already been addressed in our next paper, cf. Szmidt and Kacprzyk [6]. Now, we examine more profoundly the problem, and propose a transparent and effective solution.

Moreover, we conduct a critical examination of some other existing methods for ranking the intuitionistic fuzzy alternatives and indicate their limitations. We assume a new, powerful and comprehensible geometric representations of these ranking methods to clearly show their characteristics.

2 Brief introduction to the IFSs

Zadeh's [5] fuzzy set A' in X can conveniently be represented as

$$A' = \{\langle x, \mu_{A'}(x) \rangle | x \in X\} \quad (1)$$

where $\mu_{A'}(x) \in [0, 1]$ is the degree of membership of element x in A' .

Its powerful extension is Atanassov's [2, 3, 4, 7] Intuitionistic Fuzzy Set (IFS), A , represented as

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\} \quad (2)$$

where: $\mu_A : X \rightarrow [0, 1]$ is a degree of membership and $\nu_A : X \rightarrow [0, 1]$ is a degree of non-membership of x in A such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

For a method for the determination of the assigning memberships and non-memberships for the IFSs from data, cf. [8].

Moreover, there is also a quantity, called a *hesitation margin* $\pi_A : X \rightarrow [0, 1]$, which is directly derived from (2) and (3)

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \quad (3)$$

The hesitation margin represents a lack of knowledge of whether or not x belongs to A ; $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The two-term representation (without a hesitation margin) and the three-term representation (with a hesitation margin) of the IFSs were presented in [9, 10, 11].

The concept of a hesitation margin was found very useful in the definition and analyses of many concepts and properties of the IFSs, e.g. the distances, entropy and similarity (cf. [12, 13, 14, 15, 16, 17, 18, 19, 20], etc.). Moreover, it has found numerous applications in many fields like: image processing [21], classification with imbalanced and overlapping classes [22, 23, 24], classification through intuitionistic fuzzy trees (cf. [25]), multiagent decisions, negotiations, voting, group decision making, etc. (cf. [26, 27, 28, 29, 30]), [31, 32, 33, 34, 35, 36, 37, 38, 39]. Sometimes the concept of the hesitation margin is just indispensable, for example, for a proper definition of

the Hausdorff distance [40] or attribute selection [37, 41, 42]. Research on IFSs belongs to a broad trend of research on generalized uncertain fuzzy systems, such as type-2 fuzzy logic systems [43, 44] or fuzzy-rough systems [45], which deal with data uncertainty occurring due to lack of data, class overlap or the presence of noise in the data (cf. [46, 47, 48]).

3 Representation of the intuitionistic fuzzy alternatives

The elements specified by an IFS can be described either by two terms, i.e. the membership values μ and the non-membership values ν , or the three terms, i.e. the membership values μ , non-membership values ν and the hesitation margins π . As it was pointed out in Section 2, the three-term model can often yield more reliable results. However, in our critical review, we examine in the context of ranking of the intuitionistic fuzzy alternatives/options both methods, i.e. with two and three terms. In Figure 1 both types of their geometric representation are presented.

The triangle NOM with its interior represents all possible elements belonging to an IFS. In other words, the elements of the triangle NOM represent all possible alternatives (options, solutions) which may be considered by a decision maker. Point $M = \langle \mu, \nu, \pi \rangle = \langle 1, 0, 0 \rangle$ represents the elements fully belonging to the IFS, i.e. alternatives fulfilling all the requirements as the membership value is equal to 1 ("ideal positive alternatives"). Point $N = \langle 0, 1, 0 \rangle$ represents the alternatives which do not fulfill any requirement as the non-membership value is equal to 1. Point $O = \langle 0, 0, 1 \rangle$ represents the alternatives about which the decision maker knows nothing (the hesitation margin is equal to 1). The elements on the line (hypotenuse) MN represent the alternatives with the hesitation margin equal to 0. For example, element $A = \langle 0.5, 0.5, 0 \rangle$ represents the fuzzy alternatives for which the arguments for, represented by the membership function ($\mu = 0.5$), and the arguments against, represented by the non-membership function ($\nu = 0.5$), are equal. The equal values of μ and ν occur on an imagined line (Figure 1) ABO . The closer to the point O the greater the values of the hesitation margins π (π is equal to: zero for A , 0.2 for B , 0 for O). The paral-

lel lines to MN indicate the elements with the same values of the hesitation margin. In Figure 1 there are visible parallel lines for which the values of the hesitation margins π are equal to: 0, 0.2, 0.4. For example, point $F = \langle 0.5, 0.1, 0.4 \rangle$ represents the alternatives with 50% of "for", 10% of "against", and 40% for the hesitation.

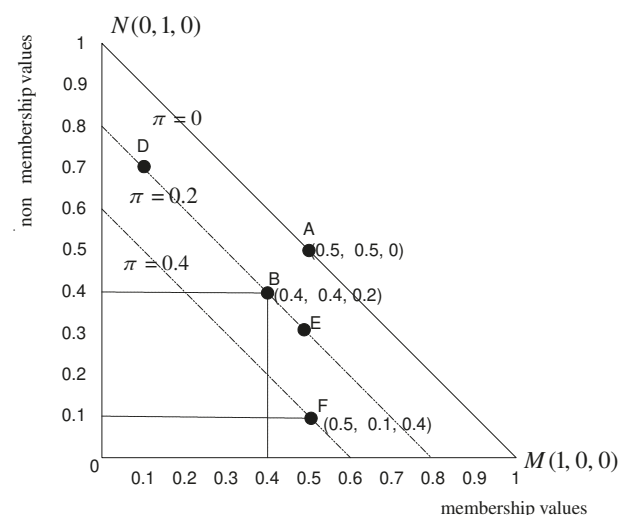


Figure 1. Representation of alternatives described by the IFSs

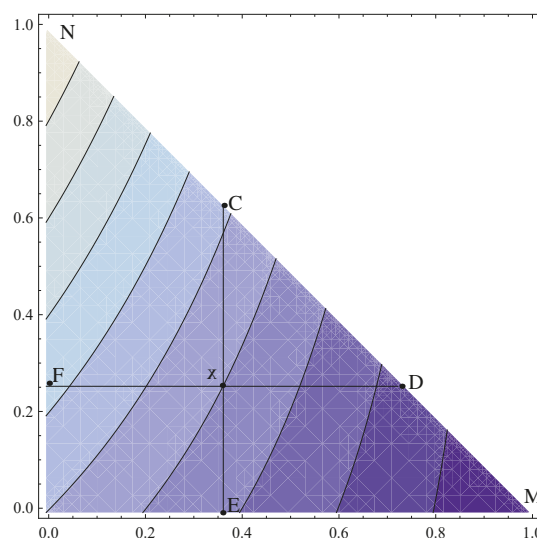


Figure 2. Illustration of the regions where Definition 1 can/can't be used

We would like to stress that we consider here the a general description of the IFS alternatives. We do not consider specific forms of the membership/non-membership functions exemplified by trapezoidal, triangular, etc., see e.g. [49].

The question arises about how to rank the IFS alternatives. We will consider the problem in the next sections.

4 Ranking

The best way to rank order the intuitionistic fuzzy alternatives/options is the following definition.

Definition 1 [3, 4] For two intuitionistic fuzzy alternatives $x_1 = \langle \mu_1, \nu_1 \rangle$ and $x_2 = \langle \mu_2, \nu_2 \rangle$, $x_1 \leq x_2$ if

$$\mu_1 \leq \mu_2 \quad \text{and} \quad \nu_1 \geq \nu_2. \quad (4)$$

If these "ideal" conditions of Definition 1 are fulfilled, then we obtain an order which is well justified and acceptable without any doubts.

The idea behind Definition 1 is: the bigger the membership value (representing advantages of an alternative/option) and the lower the non-membership value (representing the disadvantages of an alternative/option) the better an alternative/option. The simplest function expressing the idea is:

$$De(x) = 0.5(1 - \mu_x)(1 + \nu_x) \quad (5)$$

where $0 \leq De(x) \leq 1$. The lower the values of $De(x)$ the better an option.

In Figure 2 we see the contour plot of $De(x)$ (5). We can see an element x (representing an option considered) and two regions where Definition 1 can be applied. Namely, an option x can be compared with the elements belonging to the two regions. In the first region, $xFNC$, there are options worse than the option x as their membership values are lower and non-membership values are bigger. In the second region, $xDME$, we have options better than the option x as their membership values are bigger and non-membership values are smaller.

It is worth mentioning that point C in Figure 2 is expressed by the necessity operator, i.e., $\Box A$ (6), point D is expressed by the possibility $\Diamond A$ (7), following the modal extensions proposed in [4], [7, 50]:

– the necessity operator

$$\Box A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E \}, \quad (6)$$

– the possibility operator

$$\Diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle | x \in E \}. \quad (7)$$

The two regions where the conditions of Definition 1 are fulfilled, i.e. the regions $xFNC$ and $xDME$, are pointed out by operators $H_{\alpha,\beta}^*(A)$ (8) and $J_{\alpha,\beta}^*(A)$ (9), respectively, cf. Atanassov[50]:

$$H_{\alpha,\beta}^*(A) = \{ \langle x, \alpha \cdot \mu_A(x), \nu_A(x) + \beta \cdot (1 - \alpha \cdot \mu_A(x) - \nu_A(x)) \rangle | x \in E \}, \quad (8)$$

$$J_{\alpha,\beta}^*(A) = \{ \langle x, \mu_A(x) + \alpha \cdot (1 - \mu_A(x) - \beta \cdot \nu_A(x)), \beta \cdot \nu_A(x) \rangle | x \in E \} \quad (9)$$

where $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and $\alpha + \beta = 1$.

A weak point of Definition 1 is that lots of elements belonging to an IFS do not fulfill the condition (4). There are two regions in which the conditions of Definition 1 are not fulfilled. In the region xCD (Figure 2) any element has both its membership value and non-membership value bigger than those of element x . In the region $xEOF$ (Figure 2) any element has both its membership value and non-membership value smaller than those of element x .

For example, having the alternatives/options $x_1 = \langle 0.3, 0.4, 0.3 \rangle$ and $x_2 = \langle 0.4, 0.5, 0.1 \rangle$ the Definition 1 is not a proper choice.

Remark – later we will use notation $x(\mu, \nu, \pi)$ instead of $x = \langle \mu, \nu, \pi \rangle$.

It is not possible to perform total ordering among all alternatives based on Definition 1. For this reason, to rank intuitionistic fuzzy alternatives, we have defined additional measure R (10), cf. [1].

In [6] we presented geometrical representations of R and other well-known methods, explaining the advantages of R . Here we even deeper explain why R works well when Definition 1 can not be applied.

The measure R for the ordering of the alternatives $x(\mu_x, \nu_x, \pi_x)$ is given as [1]:

$$R(x) = 0.5(1 + \pi_x)l_{IFS}(M, x) \quad (10)$$

where $l_{IFS}(M, x)$ is the Hamming distance x from ideal positive alternative $M(1, 0, 0)$.

The Hamming distance between two intuitionistic fuzzy sets A and B in $X = \{x_1, x_2, \dots, x_n\}$ is

equal to [14]:

$$l_{IFS}(A, B) = \frac{1}{2n} \sum_{i=1}^n (|\mu_A(x_i) - \mu_B(x_i)| \quad (11)$$

$$+ |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|), \quad (12)$$

thus

$$l_{IFS}(M, x) = \frac{1}{2} (|1 - \mu_x| + |0 - \nu_x| + |0 - \pi_x|) = 1 - \mu_x \quad (13)$$

As a result, (10) is given as:

$$R(x) = 0.5(1 + \pi_x)(1 - \mu_x) \quad (14)$$

Equation (14) expresses the “quality” of an alternative. The lower the value of $R(x)$, (14), the better the alternative. From (14) we conclude that for the best alternatives the amount of positive information included μ_x is as big as possible, and the hesitation margin π_x is as small as possible.

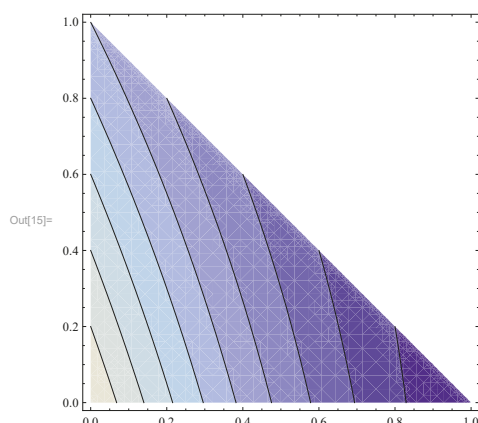


Figure 3. Contour plot of (10)

The values of function R (14) for any intuitionistic fuzzy alternative are presented in Figure 3. By considering the numbers obtained via R , (14), we may notice that the value 0.25 obtained for alternative $(0.5, 0.5, 0)$ constitutes the “border” of the “interesting” alternatives – in the sense of the amount of the positive knowledge.

Next, an important observation is that the same values of R , (14), which are on any contour line in Figure 3, are perfectly well ordered by Atanassov's Definition 1 (when membership values increase the same time non-membership values decrease on the contour lines).

Returning to the alternatives mentioned above: $x_1(0.3, 0.4, 0.3)$, $x_2(0.4, 0.5, 0.1)$, from (14) we obtain $R(x_1) = 0.455$, $R(x_2) = 0.33$ which means that

x_1 is a worse alternative than x_2 (the membership value of x_2 is bigger and the hesitation margin of x_2 is smaller than the respective values of x_1).

It is necessary to stress that when the conditions of Definition 1 are fulfilled, then we should not use the measure R (14). We should pay attention to the equal values of the membership values and non-membership values in the conditions of Definition 1. The equal values mean that Definition 1 should be used.

For example, for $x_1(0.3, 0.5, 0.2)$, $x_2(0.3, 0.6, 0.1)$, we have equal membership values so that from Definition 1, as the non-membership value of x_2 , i.e., 0.6, is bigger than the non-membership value of x_1 , i.e., 0.5, then we conclude that x_1 is better than x_2 .

On the other hand, if we use (14) and calculate R , we obtain $R(x_1) = 0.42$, $R(x_2) = 0.385$ which would suggest the opposite answer and is not justified. Using Definition 1 is fully justified by its conditions. However, we can show also in another way that the answer obtained by Definition 1 is correct. To do so, we recall that the three-term representation of the IFSs can be expressed by two intervals, namely (Szmidt and Kacprzyk [9]):

$$[\mu(x), \mu(x) + \alpha\pi(x)] \quad (15)$$

$$[\nu(x), \nu(x) + (1 - \alpha)\pi(x)] \quad (16)$$

where $\alpha \in [0, 1]$.

Table 1. Chosen scenarios (15)–(16) for $x_1(0.3, 0.5, 0.2)$, and $x_2(0.3, 0.6, 0.1)$

α	$x_1(0.3, 0.5, 0.2)$	$x_2(0.3, 0.6, 0.1)$	result
$\alpha = 1$	(0.5, 0.5)	(0.4, 0.6)	$x_1 > x_2$
$\alpha = 0$	(0.3, 0.7)	(0.3, 0.7)	$x_1 = x_2$
$\alpha = 0.5$	(0.4, 0.6)	(0.35, 0.65)	$x_1 > x_2$

Making use of the above representation (15)–(16) we can consider different scenarios for $x_1(0.3, 0.5, 0.2)$, and $x_2(0.3, 0.6, 0.1)$ (cf. Table 1). We can notice from Table 1 that both for the highest possibility ($\alpha = 1$) and for the lowest possibility ($\alpha = 0$), the alternative x_1 is better than x_2 . When the hesitation margin is divided equally between the membership values and non-membership values of x_1 and x_2 , i.e., for $\alpha = 0.5$, both alternatives become equal. Then we obtain the same as

given by Definition 1. When the conditions of Definition 1 are fulfilled, we should not therefore use measure R (14).

A similar situation repeats when we consider the alternatives with equal non-membership values. An example for $x_1(0.6, 0.2, 0.2)$ and $x_2(0.5, 0.2, 0.3)$ is presented in Table 2. The result is again the same as that obtained from Definition 1, i.e., as the non-membership values are equal and x_1 has membership value bigger than x_2 ($0.6 > 0.5$), then we conclude that x_1 is a better alternative than x_2 . However, in this case we have the alternatives which not only fulfill the conditions of Definition 1 but the same time it is true that $\mu(x_1) > \mu(x_2)$ and $\pi(x_1) < \pi(x_2)$. It means that the conditions for R are fulfilled, too. As a result we obtain $R(x_1) < R(x_2)$ (i.e., $0.24 < 0.32$) which means that x_1 is a better alternative than x_2 . It is valid for the specific data. However, a general rule is that if it is possible to use Definition 1 we should do it.

Table 2. Chosen scenarios (15)–(16) for $x_1(0.6, 0.2, 0.2)$, and $x_2(0.5, 0.2, 0.3)$

α	$x_1(0.6, 0.2, 0.2)$	$x_2(0.5, 0.2, 0.3)$	result
$\alpha = 1$	(0.8, 0.2, 0)	(0.8, 0.2)	$x_1 = x_2$
$\alpha = 0$	(0.6, 0.4, 0)	(0.5, 0.5)	$x_1 > x_2$
$\alpha = 0.5$	(0.7, 0.3)	(0.65, 0.35)	$x_1 > x_2$

Table 3. Chosen scenarios (15)–(16) for $x_1(1, 0, 0)$, and $x_2(0, 0, 1)$

α	$x_1(1, 0, 0)$	$(0, 0, 1)$	result
$\alpha = 1$	(1, 0, 0)	(1, 0, 0)	$x_1 = x_2$
$\alpha = 0$	(1, 0, 0)	(0, 1, 0)	$x_1 > x_2$
$\alpha = 0.5$	(1, 0, 0)	(0.5, 0.5, 0)	$x_1 > x_2$

Table 4. Chosen scenarios (15)–(16) for $x_1(0, 1, 0)$, and $x_2(0, 0, 1)$

α	$x_1(0, 1, 0)$	$(0, 0, 1)$	result
$\alpha = 1$	(0, 1, 0)	(1, 0, 0)	$x_1 < x_2$
$\alpha = 0$	(0, 1, 0)	(0, 1, 0)	$x_1 = x_2$
$\alpha = 0.5$	(0, 1, 0)	(0.5, 0.5, 0)	$x_1 < x_2$

It is worth verifying the special alternatives, i.e., $x_1(1, 0, 0)$ and $(0, 0, 1)$, i.e., the best possible option and the option with hesitation equal to one (Table 3). As the conditions of Definition 1 are fulfilled, the results in Table 3 confirm that $x_1 > x_2$.

Also measure R gives the same result $R(x_1) < R(x_2)$ (i.e., $0 < 1$) which means that x_1 is better than x_2 .

Another interesting case concerns alternatives $x_1(0, 1, 0)$ (the worst alternative) and $(0, 0, 1)$, i.e., the option with hesitation margin equal to one. The conditions of Definition 1 are fulfilled so that under the same values of the memberships and bigger values of the non-membership for x_1 , we conclude that $x_1 < x_2$. The results are also confirmed in (Table 4). However, while ranking the alternatives using measure R we obtain $R(x_1) = 0.5$, $R(x_2) = 1$, i.e., $R(x_1) < R(x_2)$ which means that x_1 is better than x_2 . This result is not justified. But it is not a drawback of measure R but of the fact that measure R should not be used when the conditions of Definition 1 are fulfilled.

The above examples indicate the source of some doubts concerning measure R (14) pointed out in some papers. The authors do not apply Definition 1 when its conditions are fulfilled but, instead, use R (14) obtaining sometimes good, sometimes wrong results. However, it is not a drawback of measure R (14) but of the use of this measure when it should not be used. The measure R (14) works properly when Definition 1 can not be applied.

The conclusion is that, first, we verify the conditions of Definition 1 and use this definition if possible. Only if it is not possible, we use measure R (14). An option with a bigger membership value and a lower hesitation margin is always pointed out as a better one by measure R (14). It can happen that at the same time the conditions of Definition 1 and of measure R are fulfilled. But both approaches not always give the same answer. As a rule, whenever possible, we should use Definition 1.

In our previous paper [6] we pointed out the drawbacks of some well-known methods for ranking the intuitionistic fuzzy alternatives. We have presented a general geometrical representation of these methods and gave some numerical examples showing that these methods do not work properly. Now we will show that our method using measure R (14) gives proper results in the same situations.

5 Some selected popular methods for ranking the intuitionistic fuzzy alternatives

We consider here briefly several methods for ranking the intuitionistic fuzzy alternatives known from the literature. They seem to be different but under a close scrutiny using a very illustrative graphical representation, they exhibit an important common feature that has a detrimental effect on their effectiveness and efficiency. These methods will now be briefly discussed.

5.1 Chen and Tan's [51] method

Chen and Tan [51] proposed a score function for the vague sets [52]. However, in [53] it is proved that vague sets are equivalent to the IFSs up to the isomorphism so that their methods can also be employed for the ranking of the intuitionistic fuzzy alternatives as it is often done.

The score function for an intuitionistic fuzzy alternative $x(\mu, \nu)$ [51] is:

$$S(x) = \mu - \nu, \quad (17)$$

where $S(x) \in [-1, 1]$.

It is easy to notice from (17) that the score function $S(x)$ can fail to properly evaluate the intuitionistic fuzzy alternatives as for quite different intuitionistic fuzzy alternatives it can produce the same result. For example, for $x(\mu, \nu)$ equal to: $(0.7, 0.3)$, $(0.6, 0.2)$, $(0.5, 0.1)$, $(0.4, 0)$ we obtain for all of them (17) $S(x) = 0.4$ which does not meet our expectations. On the other hand, when using measure R (14) we obtain: $(0.7, 0.3) > (0.6, 0.2) > (0.5, 0.1) > (0.4, 0)$ which is justified.

5.2 Hong and Choi's [54] method

Hong and Choi [54] consider, in addition to the score function (17), an accuracy function H :

$$H(x) = \mu + \nu, \quad (18)$$

where $H(x) \in [0, 1]$.

Xu [55] proposed a method of ranking the intuitionistic fuzzy alternatives making use of (17) and (18). The idea in the case of two alternatives x_i and x_j boils down to the following algorithm [55]:

Algorithm 1 Score and Accuracy

- if $S(x_i) \leq S(x_j)$, then x_i is smaller than x_j ;
 - if $S(x_i) = S(x_j)$, then:
 - if $H(x_i) = H(x_j)$, then x_i and x_j represent the same information (are equal);
 - if $H(x_i) \leq H(x_j)$, then x_i is smaller than x_j .
-

However, the above ranking can often produce counter-intuitive results. For example, for two intuitionistic fuzzy alternatives, $x_1 = (0.5, 0.4)$ and $x_2 = (0.3, 0.1)$, we obtain $S(x_1) = 0.5 - 0.4 = 0.1$, $S(x_2) = 0.3 - 0.1 = 0.2$, which indicates that x_1 is smaller than x_2 . But the positive and negative information supplied by x_1 (i.e. $0.5 + 0.4 = 0.9$) is for sure greater than those supplied by x_2 (i.e. $0.3 + 0.1 = 0.4$). Moreover, the hesitation margin of x_2 equal to $\pi_2 = 1 - 0.3 - 0.1 = 0.6$ is greater than the hesitation margin $\pi_1 = 1 - 0.5 - 0.4 = 0.1$ in the case of x_1 . In other words, it is difficult to agree that x_1 is smaller than x_2 .

Other disadvantages of the Algorithm 1 are twofold. The procedure indicates not a proper trend while ranking the alternatives, and produces not continuous results. A trend means here that, e.g., when the membership values increase and the hesitation margins decrease, one could expect that the alternatives are better. Unfortunately, the results generated are counter-intuitive.

Example 1 Consider the following intuitionistic fuzzy alternatives $a = (\mu, \nu, \pi)$:

$$\begin{aligned} x_1 &= (0.1, 0, 0.9), \\ x_2 &= (0.2, 0.11, 0.69), \\ x_3 &= (0.3, 0.22, 0.48), \\ x_4 &= (0.4, 0.33, 0.27), \\ x_5 &= (0.5, 0.44, 0.06), \end{aligned}$$

for which the scores (17) are:

$$\begin{aligned} S(x_1) &= 0.1 - 0 = 0.1, \\ S(x_2) &= 0.2 - 0.11 = 0.09, \\ S(x_3) &= 0.3 - 0.22 = 0.08, \\ S(x_4) &= 0.4 - 0.33 = 0.07, \\ S(x_5) &= 0.5 - 0.44 = 0.06, \end{aligned}$$

Thus, from Algorithm 1 we have:

$$x_1 > x_2 > x_3 > x_4 > x_5$$

The result is counterintuitive as the hesitation margin $\pi = 0.9$ is the biggest for the alternative x_1 pointed out as the “best”. For the “worst” alternative x_5 the hesitation margin is the smallest, equal to 0.06. The membership values increase from 0.1 (for x_1) to 0.5 (for x_5). In spite of it, the alternative a_5 with the biggest membership value and the smallest hesitation margin is ranked as the worst one. It is difficult to agree with such a result.

By changing a little the non-membership values in the above example, i.e. by considering:

$$\begin{aligned} x_1 &= (0.1, 0, 0.9), \\ x_2 &= (0.2, 0.1, 0.7), \\ x_3 &= (0.3, 0.2, 0.5), \\ x_4 &= (0.4, 0.3, 0.3), \\ x_5 &= (0.5, 0.4, 0.1), \end{aligned}$$

we obtain the same score (17) for each $a_i, i = 1, \dots, 5$, and from the second part of the ranking procedure, i.e., after calculating $H(a)$ (18), we obtain the reverse order, i.e.:

$$x_5 > x_4 > x_3 > x_2 > x_1$$

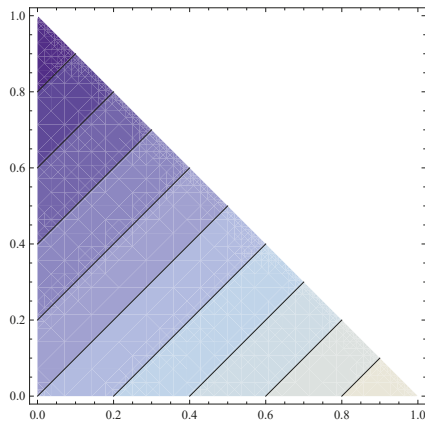


Figure 4. Contour plot of the method “Score and Accuracy”

Certainly, a ranking procedure should not be so sensitive to such small changes of the parameters.

The above examples show that Algorithm 1 should not be used, especially in real world tasks.

The method [55] is illustrated in Figure 4. The contour plot indicates that Definition 1, which defines the ordering in a natural way, is not assumed.

The lines in Figure 4 pointing out the same values for the different alternatives are directed up and right. It means that at the same time the membership values and non-membership values of the alternative represented on the same line increase. As a result, there is not a natural ordering among such alternatives.

It is worth emphasizing that measure R (14) gives the same result in both cases (before and after the change of the parameters), i.e.:

$$x_5 > x_4 > x_3 > x_2 > x_1$$

which means that the alternatives with bigger membership values and smaller hesitation margins are better.

Remark: in Figures 4 – 7, where contour plots of different methods are given, on axes X and Y, the membership values, and the non-membership values are presented, respectively.

5.3 Zhang and Xu [56] method

In Zhang and Xu [56] the ranking of the alternatives is based on the following definition [56].

Definition 2 For each intuitionistic fuzzy alternative: $x(\mu, \nu)$ a measure $L(x)$ is defined:

$$L(x) = \frac{1 - \nu}{2 - \mu - \nu} \quad (19)$$

The order L between the intuitionistic fuzzy alternatives is defined as

- if $L(x_1) \leq L(x_2)$, then x_1 is smaller than x_2 ;
- if $L(x_1) = L(x_2)$, then:
 - if $H(x_1) = H(x_2)$, then x_1 and x_2 represent the same information (are equal);
 - if $H(x_1) \leq H(x_2)$, then x_1 is smaller than x_2 .

In Figure 5 we have a contour plot of the order given by Definition 2.

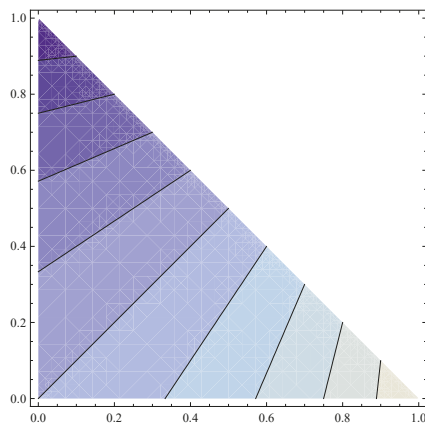


Figure 5. Contour plot of the method given by measure (19)

However, for $x_1(0.1, 0.1)$ and for $x_2(0.4, 0.6)$ we have:

$$L(x_1) = \frac{1 - 0.1}{2 - 0.2} = \frac{0.9}{1.8} = 0.5$$

$$L(x_2) = \frac{1 - 0.6}{2 - 1} = \frac{0.4}{1} = 0.4$$

As $L(x_1) > L(x_2)$ we conclude using measure $L(x)$ (19) that $x_1 > x_2$. In fact neither x_1 nor x_2 are good choices but in the case of x_1 our knowledge is small and the hesitation margin is equal to 0.8, that is, is high. In the case of x_2 the hesitation margin is equal to zero.

Moreover, measure $L(x)$ (19) is sensitive to very small changes of the membership and non-membership values.

Consider now $x_1(0.1, 0.1)$ and $x_2(0.5, 0.5)$. As from (19) $L(x_1) = L(x_2) = 0.5$, we calculate $H(x_1) = 0.2$ and $H(x_1) = 1$. As $H(x_1)$ is smaller than $H(x_2)$, we conclude that $x_2(0.5, 0.5)$ is better than $x_1(0.1, 0.1)$. Now we change a little bit the non-membership value in x_1 assuming that $x_1(0.1, 0.05)$, and we only change the non-membership value a little bit, from 0.1 to 0.05. Now from (19) we have:

$$L(x_1) = \frac{1 - 0.05}{2 - 0.15} = \frac{0.95}{1.85} = 0.514$$

which means that this time $x_1(0.1, 0.05)$ is better than $x_2(0.5, 0.5)$ (for which we have $L(x_2) = 0.5$).

From the point of view of decision making such big changes concerning the ranking of alternatives

caused by so small changes of option parameters are difficult to accept.

It is worth emphasizing that measure R (14) gives in both cases the same answer: $x_1 < x_2$.

5.4 Guo's [57] method

In Guo [57] the following definition is presented:

Definition 3 For each intuitionistic fuzzy alternative $x(\mu, \nu)$ a measure $Z(x)$ is defined

$$Z(x) = 0.25(1 + \mu + \nu)(1 + \mu - \nu) \quad (20)$$

The order Z is then defined as

$$x_1 \leq x_2$$

iff $Z(x_1) \leq Z(x_2)$.

In Figure 6 we have a contour plot of the order given by Definition 3.

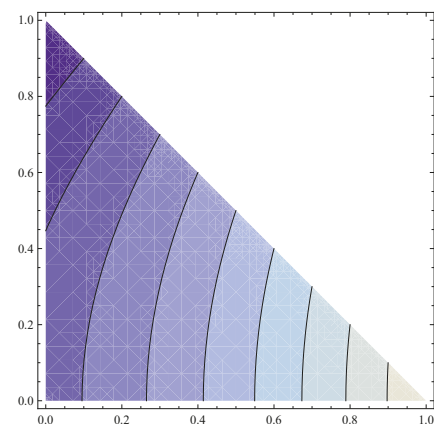


Figure 6. Contour plot of the method given by measure (20)

It is easy to notice that for quite different elements, which are on the same contour line, the values of measure (20) are the same so that we are not able to distinguish quite different elements. For example for $x_1(0.42, 0)$ and for $x_2(0.5, 0.5)$ we have:

$$Z(x_1) = 0.25(1 + 0.42)(1 + 0.42) = 0.5$$

$$Z(x_2) = 0.25(1 + 1)(1) = 0.5$$

but x_1 is not equal to x_2 . In fact they are quite different but (20) cannot distinguish them.

As the lines of the contour plot of the method given by measure (20) are directed up and right (cf. Figure 6), i.e., the membership values and the non-membership values of the elements increase at the same time, it is not possible to use Definition 1, cf. [3, 4].

From R (14) we obtain that $x_1 < x_2$ which is justified (as the membership value of x_2 is bigger than that of x_1 and the hesitation margin of x_2 is smaller than that of x_1).

5.5 Xing, Xiong, Liu [58] method

In Xing, Xiong, Liu [58] the following definition is presented:

Definition 4 [58]: For each intuitionistic fuzzy alternative: $x(\mu, \nu)$ a measure $P(x)$ was defined

$$P(x) = 1 - \sqrt{0.5((1 - \mu))^2 + \nu^2} \quad (21)$$

The order P is defined as

$$x_1 \leq x_2$$

iff $P(x_1) \leq P(x_2)$.

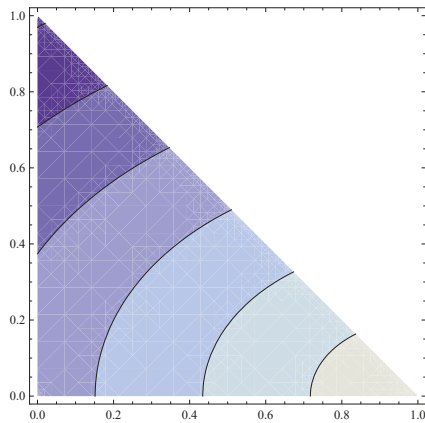


Figure 7. Contour plot of the method given by measure (21)

However, for $x_1(0.2, 0)$ and for $x_2(0.53, 0.47)$ we have:

$$P(x_1) = 1 - \sqrt{0.5((1 - 0.2))^2 + 0^2} = 0.43$$

and also

$$P(x_1) = 1 - \sqrt{0.5((1 - 0.53))^2 + 0.47^2} = 0.43$$

but x_1 is not equal to x_2 . In fact they are quite different but (21) is not able to distinguish them.

In Figure 7 we have the contour plot of the order given by Definition 4. Again, it is easy to notice the same fact as in the case of the previously considered methods, the lines of the contour plot are so directed that it is not possible to use Definition 1 and to solve the problem of equal values of P (21).

However, from the proposed R (14) we obtain that $x_1 < x_2$ which is justified as the membership value of x_2 (equal to 0.53) is bigger than that of x_1 (equal to 0.2) and the hesitation margin of x_2 (equal to 0) is smaller than that of x_1 (equal to 0.8).

6 A brief comparison of the methods

Our method presented in Section 4 makes use of Definition 1. However, when the conditions of Definition 1 are not fulfilled, then measure R , (14), is applied. Moreover, for two different alternatives we can obtain the same values of R , (14), as it can be seen observing the contour lines of R in Figure 3. The values of R on the contour line are the same for all alternatives represented on the line. However, an important observation is that the same values of R , (14), which are on any contour line in Figure 3 are perfectly well ordered by Definition 1. The contour lines are directed up and left (Figure 3) which means that when the membership values increase, then the same time non-membership values decrease on the contour lines.

The method is transparent and easy to explain, as the bigger the membership value and the smaller the hesitation margin, the better the alternative is.

The methods presented in Section 5 do not take into account Definition 1. Counterexamples were presented in the context of each method in Section 5. The contour lines in Figures 4 – 7, are directed up and right which means that when the membership values increase on a contour line, then the same time the no-membership values increase as well. It means that it is not possible to order the alternatives represented on the same contour line, i.e., the alternatives for which the formulas from Section 5 give the same values. These formulas are less transparent and not always easy to explain.

7 Conclusions

We presented and critically analyzed revisited our method for ranking the intuitionistic fuzzy alternatives which takes into account both the positive and negative information associated with the alternative, measured by a distance to the positive ideal alternative, and the reliability of the information, measured by the alternative's hesitation margin. The method consists of two steps. In the first step, the method uses Definition 1. When it is not possible to use this definition because the assumptions are not fulfilled, an approach that ranks higher the alternatives with bigger membership values and lower hesitation values is applied. The results are promising and the transparency of this augmented method is increased.

We also critically examined several methods from the literature which suggest how to rank the intuitionistic fuzzy alternatives but yield some doubtful or not intuitively appealing results. Trying to reconcile the misunderstanding, we explained some deficiencies of these methods by showing examples and counter-examples using mainly geometrical interpretations.

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