

PREDICTION OF THE NON-LINEAR RUPTURE OF A SECTION UNDER COMPOUND DEVIATED BENDING

D. Atlaoui^{a,*}, Y. Bouafia^a, A. Mansouri^b, H. Ait Aider^b

^a Department of Civil Engineering, Faculty of Construction Engineering, Laboratory of Modeling of Materials and Structures in Civil Engineering (L2MSGC), University 'Mouloud Mammeri' of Tizi Ouzou, 15000 Tizi Ouzou, Algeria;
E-mails: *atlaoudjamel2005@yahoo.fr, youcef.bouafia2012@yahoo.com

^b Department of Civil Engineering, Faculty Of Construction Engineering, University 'Mouloud Mammeri' of Tizi Ouzou, 15000 Tizi Ouzou, Algeria; E-mails: mans_arez@yahoo.fr, h_aitaider@yahoo.fr

Received: 04.10.2023 / Accepted: 10.11.2023 / Revised: 10.02.2024 / Available online: 31.05.2024

DOI: [10.2478/jaes-2024-0002](https://doi.org/10.2478/jaes-2024-0002)

KEY WORDS: numerical simulation, reinforced concrete, cracking, failure, bearing capacity, mesh.

ABSTRACT:

Under stresses close to the ultimate stresses, a reinforced concrete section yields and cracks. The calculation in linear elasticity no longer makes it possible to evaluate the real deformations of the section. One is then led to make a study in nonlinear elasticity. The object of this work is the elaboration of a calculation method in nonlinear elasticity, allowing the simulation until the failure of reinforced concrete sections subjected to a compound deviated bending taking into account the real behavior laws (nonlinear) of the materials (steel and concrete). A computer program is developed following FORTRAN standards, and then confronted with the obtained experimental results on failure tests of sections subjected to deflected bending. The comparison between the numerical simulation and the experimental test/calculation shows the achievement of satisfactory results.

1. INTRODUCTION

Most sections in reinforced concrete, pre-stressed concrete or mixed section are designed according to the linear formalism of the theory of elasticity established in the 19th century. However, numerous tests carried out on these sections and the constituent materials suggest that it is not possible to obtain, by linear calculation, an exact representation. The use of increasingly slender shapes in construction, combined with the current trend of designing at ultimate limit states, leads to consider the non-linear behavior of structures. Consequently, the development of analysis methods, on computer, taking into account the real behavior of materials and their connection, is essential because of the complexity of the modeling.

In this sense, several non-linear methods are developed for the study of the flat frames behavior loaded in their plane (Bouafia, 1987). Currently, few digital models make it possible to characterize the behavior of a reinforced, pre-stressed or mixed concrete section in a three-dimensional plane. Each of these was determined based on experimental results (Fouré, 1985; Nait raba, 1990; Espion, 1986; Kachi, 1997; Ren et al., 2015). They are generally endowed with coefficients the determination of which often raises difficulties. Moreover, no model is considered general. Several studies have been carried out around the world in the field of study of reinforced concrete structures in nonlinear elasticity. One can cite the studies

carried out by (Islam and Shahria, 2013; Bouafia et al., 2000; Bouafia et al., 1998; Sargin, 1968; Virlogeux and M'rad, 1986; Virlogeux, 1986; Grelat and Nait rabah, 1988; Bratina, 2005), also, the calculation of reinforced concrete sections is a problem well known by several authors (Bouafia et al., 2002; Daunys and Rimovkis, 2006; Florent, 1999; Bonet et al., 2006; Choa and Kwonb, 2008; Liang and Fragomeni, 2009; Ronagh and Baji, 2014; Adjrad et al., 2016; Kachi et al., 2015; Adjrad et al., 2015). If all these studies carried out were focused on the simulation of the structural elements behavior in nonlinear elasticity, it was in an attempt to answer all the questions that have been chosen to be focused on in our research work, i.e. the development of a calculation method based on nonlinear elasticity, allowing simulation until the RC sections failure in a three-dimensional plane (XYZ) while taking into account the nonlinear behavior laws of the materials. On the basis of this study, a computer program is produced in accordance with FORTRAN standards, and then confronted with the experimental results of the three-dimensional sections.

In this study, algorithms are developed allowing the calculation of the internal deformations of the section knowing the external forces; and the establishment of the loading curves, which make it possible to observe the evolution of the section behavior until its bearing capacity is exhausted. In the optics of an application to reinforced concrete, in the study of this section, the contribution of the tense concrete located between two

successive bending cracks is considered, thus a study of the average behavior of a zone of beam located between two successive cracks was performed. The comparison between the results of the numerical simulation and the experimental results is made on the tests of (Fouré, 1985) carried out at the center of expertise for building and public works (CEBTP).

2. POSITION OF THE PROBLEM: HYPOTHESES

In linear elasticity, the calculation of the deformations ($\delta u, \delta w$) developed by loads (N, M) does not present any difficulty and this from the traditional relations of the resistance of materials. On the other hand, in nonlinear elasticity, it is easy to calculate the internal stresses developed by given strains, but no direct method exists to calculate the strains from given internal stresses. The use of numerical methods is therefore essential (Bouafia, 1987). This study falls within this framework. It is a question of developing a soft ware allowing simulating the average behavior of a zone of beam in nonlinear elasticity until failure. This area is defined as a section with a bending crack. One is interested in the average behavior of this zone of beam of which the contribution of the tense concrete located between two successive cracks is taken into account. The study of a cross section of a reinforced concrete beam element, stressed in bending along the two directions perpendicular to the axis of the beam is meticulously performed. The section is defined in the main reference $Gxyz$ (Figure 1). The study was carried out according to the hypothesis of small deformations taking into account the nonlinear elastic behavior of the materials. The non-linear analysis essentially concerns the stresses due to an axial load N and two bending moments M_y and M_z acting in the xy and xz planes respectively. The longitudinal deformation at a given point in the cross-section is governed by the law of conservation of plane sections. A perfect adhesion between steel and concrete has also been assumed. The section is broken down into several trapeziums (mesh) in both directions (Figure 1). The average strains in each trapezium, along the two directions y and z , are given by the expressions in Equation 1.

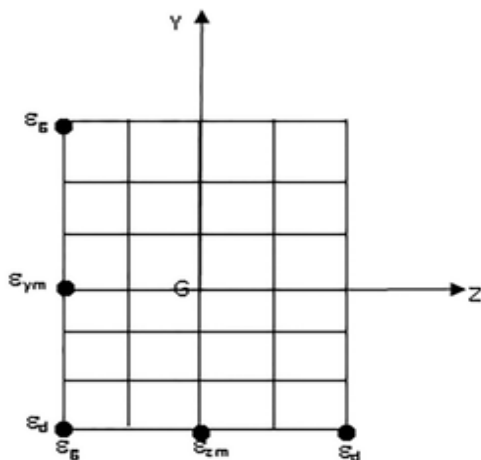


Figure 1. Section mesh

$$\begin{cases} \varepsilon_{zm} = \delta u + \delta w_y \cdot y + \delta w_z \cdot z \\ \varepsilon_{ym} = \delta u + \delta w_y \cdot ym + \delta w_z \cdot z \end{cases} \quad (1)$$

3. SECTION MODELING

3.1 Discretization of the concrete section

The concrete section was considered as a succession of trapeziums. Each trapezium was defined by the dimensions of its lower and upper bases $b_{yj}, b_{yj+1}, b_{zj}, b_{zj+1}$ and its ordinates $y_j, y_{j+1}, z_j, z_{j+1}$, with respect to the reference axis passing through the center of gravity of the section (Kachi et al., 2014) see Figure 2. The integration process is numerical, it is necessary to express the width of the trapezium of order j at a dimension y and the width of the trapezium of order j at a dimension z by relations compatible with the numerical calculation Equation 2.

$$\begin{cases} b(y) = b_{yj} + (b_{yj+1} - b_{yj}) \frac{(y - y_j)(y_{j+1} - y_j)}{(y_{j+1} - y_j)} \\ b(z) = b_{zj} + (b_{zj+1} - b_{zj}) \frac{(z - z_j)(z_{j+1} - z_j)}{(z_{j+1} - z_j)} \end{cases} \quad (2)$$

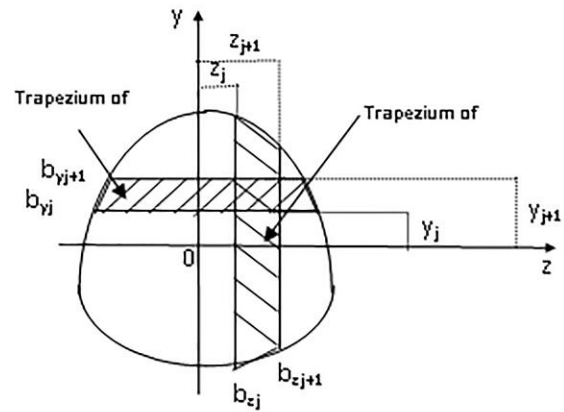


Figure 2. Section modeling

3.2 Calculation of internal stresses

The stress in a given point of the section is related to the deformation, through the behavior law of the corresponding material; $\sigma = \varphi(\varepsilon)$ see Figure 3. The value of the function $\varphi(\varepsilon)$ was considered to be zero beyond the compressive and tensile strength of the material, for the deformations ε_{cu} and ε_{rt} respectively.

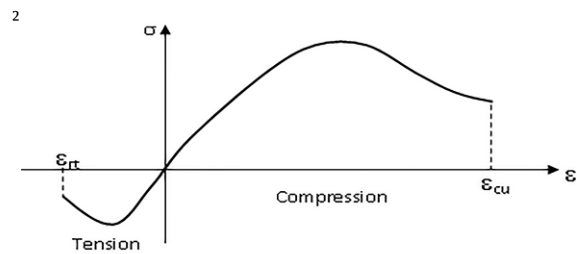


Figure 3. Behavior law of a material

4. Calculation of internal forces

in nonlinear elasticity; taking into account the real behavior of the different materials, some details are given by (Bouafia et al., 2000; Adjrad et al., 2015). The internal forces developed in the concrete by the deformations, based on the works of

(Naitrabah, 1990), (Espion, 1986) and (Bouafia et al., 2000), are given by the following expressions Equation 3.

$$\begin{cases} N_{int} = \iint \sigma \cdot dy \cdot dz = \iint \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \cdot dy \cdot dz \\ M_{yint} = \iint \sigma \cdot y \cdot dy \cdot dz = \iint \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \cdot y \cdot dy \cdot dz \\ M_{zint} = \iint \sigma \cdot z \cdot dy \cdot dz = \iint \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \cdot z \cdot dy \cdot dz \end{cases} \quad (3)$$

$b(y)$ represents the width of the section at the ordinate y , and $b(z)$ the width of the section at the ordinate z . The equations Equation 3 then becomes Equation 4:

$$\begin{cases} N_{int} = \sum_{j=1}^{na} [\phi_a (\delta u + \delta w_y \cdot y_{aj} + \delta w_z \cdot z_{aj}) \cdot A_{y_{aj}}] \\ M_{yint} = \sum_{j=1}^{na} y_{aj} \cdot [\phi_a (\delta u + \delta w_y \cdot y_{aj} + \delta w_z \cdot z_{aj}) \cdot A_{y_{aj}}] \\ M_{zint} = \sum_{j=1}^{na} z_{aj} \cdot [\phi_a (\delta u + \delta w_y \cdot y_{aj} + \delta w_z \cdot z_{aj}) \cdot A_{z_{aj}}] \end{cases} \quad (4)$$

Equation 4 can be written in the following matrix form as shown in Equation 5:

$$\begin{bmatrix} N_{int} \\ M_{yint} \\ M_{zint} \end{bmatrix} \begin{pmatrix} \delta u, \delta w_y, \delta w_z \end{pmatrix} = \int \begin{pmatrix} \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \\ \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \cdot y \\ \phi (\delta u + \delta w_y \cdot y + \delta w_z \cdot z) \cdot z \end{pmatrix} \begin{bmatrix} b(y) \cdot dy \\ b(y) \cdot dy \\ b(z) \cdot dz \end{bmatrix} \quad (5)$$

5. RESULTS COMPARISON AND NUMERICAL APPLICATIONS

5.1 Parametric study

In order to fix the parameter K'_b , which characterize the descending branch of the curve representing the behavior law of concrete in compression, a parametric study has been carried out on Sargin's law given by the expression of Equation 6.

$$\sigma = f_{cj} \frac{K_b \bar{\varepsilon} + (K'_b - 1) \bar{\varepsilon}^2}{1 + (K_b - 2) \bar{\varepsilon} + K'_b \bar{\varepsilon}^2} \quad (6)$$

The results of the "OG" fouré test allowed determining the values of the coefficients k_b and k'_b , corresponding to the curve which is closest to the experimental curve (Figure 4). This approach allows introducing the behavior law with coefficients which make it possible to better model the behavior of the concrete in simple compression, in the design software of the section subjected to compound loads.

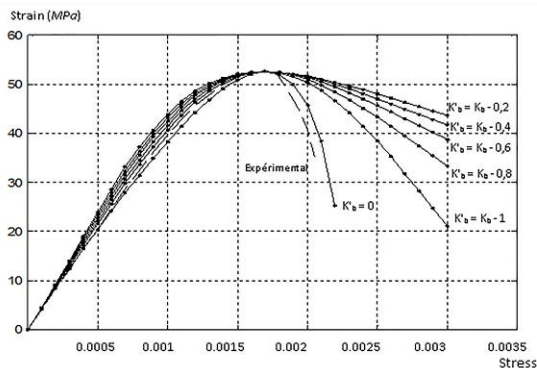


Figure 4. Behavior law of the concrete "TestOG"

5.2 Mesh influence

The numerical simulation have been compared with the experimental results produced by (Fouré, 1985).

5.3 Fouré "OG" test

A reinforced concrete beam section 245 mm high and 151 mm wide, tested at the C.E.B.T.P (Fouré, 1985) has been studied. It is reinforced with two bars of 16 mm ($\phi 16$) in diameter (16) in the tense zone, and two bars of 6 mm ($\phi 6$) in diameter (6) in the compressed zone (Figure 5).

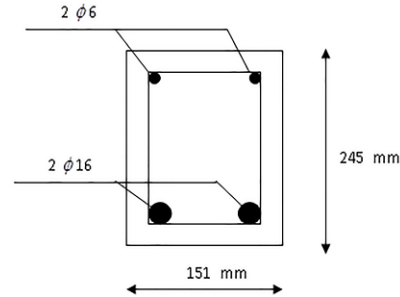


Figure 5. Reinforced concrete section "OG Test"

The characteristics of the concrete and the reinforcements are given in tables 1 and 2.

E_c MPa	F_{tj} MPa	ε_0	ε_{cu}	F_{b0}	K_b	K'_b
52.5	3.35	0.0017	0.0035	41875	1.292	0.292

Table 1. Characteristics of the concrete

Bar layer	σ_e (MPa)	σ_r (MPa)	E_a (MPa)	A (mm ²)
ayer 1	575	700	2×10^5	402
Layer 2	575	700	2×10^5	56

Bar layer	V_y (mm)	V_z (mm)	ε_u	K_s (Type)
Layer 1	25	25	0.005	2
Layer 2	225	226	0.005	2

Table 2. Characteristics of the steel bars

a. Mesh along the y direction

The mesh of the section according to the direction y is given by the Figure 6.

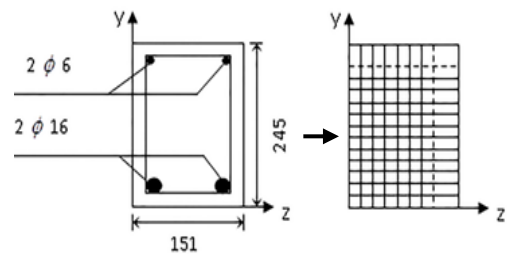


Figure 6. Mesh of the section according to the direction

The mesh of this section along the y direction gives the moment-curvature curve $M_y=f(w_y)$ shown (Figure 7).

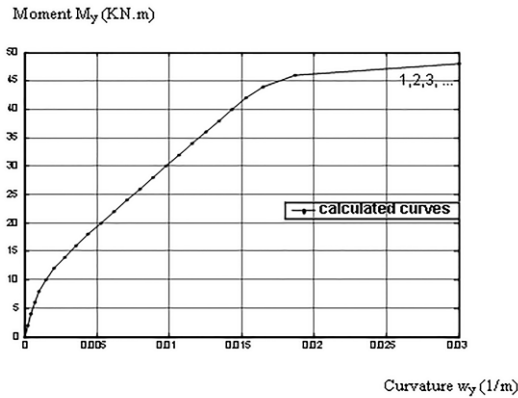


Figure 7. Calculated curves $M_y=f(w_y)$ after meshing the section

From Figure 7, it can be seen that the mesh of the section, along the y direction, has no influence on the calculated curve $M_y=f(w_y)$. By increasing the number of trapeziums (from 1 to 3), the curves obtained become merged.

b. Mesh along the z direction

The mesh of the section according to the direction z is given by the Figure 8.

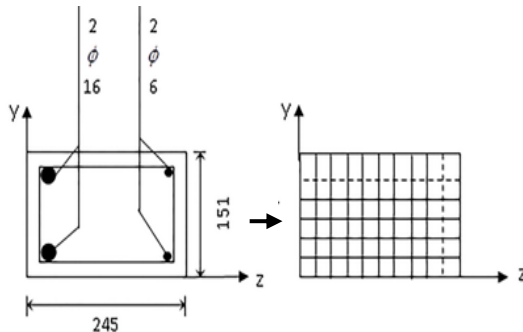


Figure 8. Mesh of the section according to the direction z

The mesh of this section along the z direction gives the moment-curvature curve $M_z=f(w_z)$ shown in Figure 9.

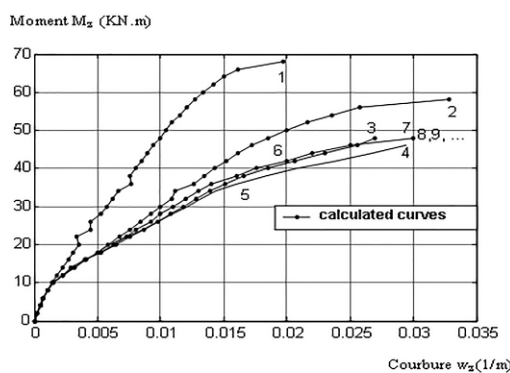


Figure 9. Calculated curves $M_z=f(w_z)$ after meshing the section

From Figure 9, it can be seen that the mesh of the section has a significant influence. Along the z direction, after increasing the number of trapeziums, the behavior curves converge towards a single curve. Indeed, they overlap for a number of trapezoids equal or greater than 3.

5.3 Superposition of curves calculated along y and z

Figure 10 illustrates the superposition of the calculated curves $M_y=f(w_y)$, $M_z=f(w_z)$.

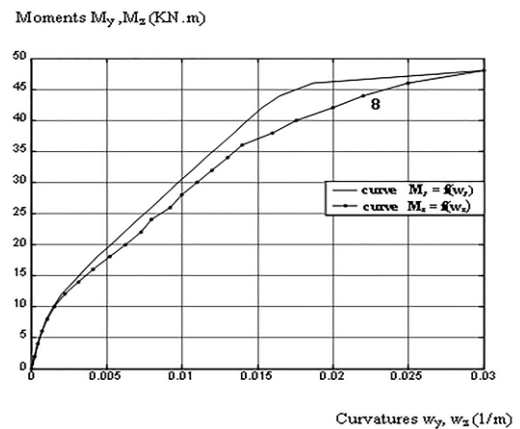


Figure 10. Superposition of calculated curves $M_y=f(w_y)$, $M_z=f(w_z)$

The mesh adopted, since it has no influence along y, is of only 1 element along the y direction. Along the z direction, the number of elements is 8, for the "OG" test. The difference between the two curves can be explained by the fact that there is an interaction between the average deformations considered in the calculation according to the two directions (see Equation 1).

5.4 Superposition of calculated and experimental curves

The superposition of the curves $N=f(\epsilon)$, $M_y=f(w_y)$ et $M_z=f(w_z)$ with the experimental curve gives Figures (11,12 and 13) respectively.

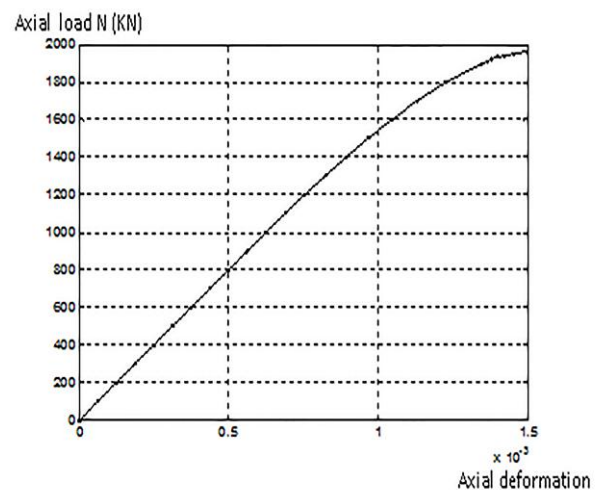


Figure 11. Axial load – axial deformation

$N_{calculé} = 1960 \text{ KN} - N_{exp} = 2000 \text{ KN}$, soit une erreur de 2 %.

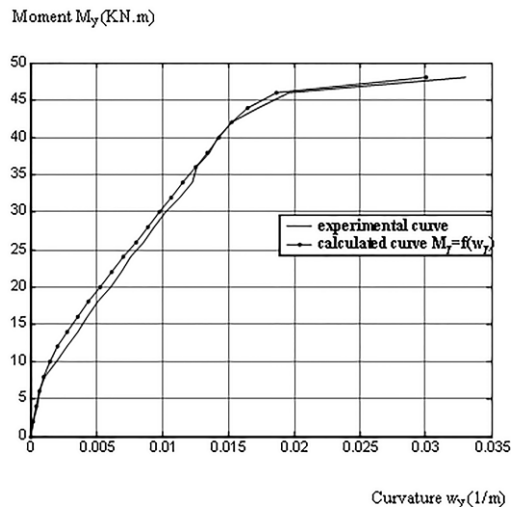


Figure 12. Superposition of the calculated curve $M_y=f(w_y)$ with the experimental curve

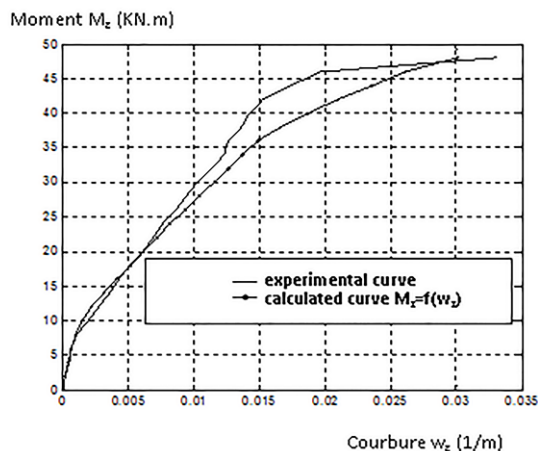


Figure 13. The calculated curve $M_z=f(w_z)$ with the experimental curve

According to Figure 13, the calculated curve $M_z=f(w_z)$ and the experimental curve coincide before the cracking of the section. At cracking, a gap between the two curves can be seen. This difference can be explained by the fact that the interaction between the stresses in the two directions y and z are not taken into account in this calculation. Near the failure point, the two curves coincide. The experimental and simulation comparison are given in Table 3 and Figures 12 and 13.

Moment exp (KN.m)	Moment cal/y M_y (KN.m)	Moment cal/z M_z (KN.m)	curvature exp	curvature cal/y (w_y)
48.8	49.9	49.9	0.033	0.030

curvature cal/z (w_z)	M_y cal /exp	M_z cal /exp	W_y cal / W_{exp}	W_z cal / W_{exp}
0.030	1.00	1.00	0.91	0.91

Table 3. Comparison of moments and max curvatures calculation/test

6. CONCLUSIONS

The resolution method presented is applicable to reinforced concrete sections subjected to a compound deviated bending. Its programming ended with the development of non-linear calculation software, until failure. The behavior laws considered, for each material (steel and concrete) are real laws. The result of the calculation makes it possible to follow the behavior of a section and to plot the curves representing the loads as a function of the deformations.

Sargin's law has been used to describe the behavior of concrete in compression. The parametric study, carried out, makes it possible to adjust the calculated curve to the one obtained during the test.

The various calculations carried out allowed noting that the mesh of the section influences the behavior curves, and mainly according to the direction z . The mesh adopted, since it has no influence according to y , is of 1 single element. Along the direction z , however, the number of elements necessary is 8 in order to compare it to the Fouré test.

The difference between the two curves, calculated according to the two directions, can be explained by the fact that there is an interaction between the averages deformations considered in the calculation (see expressions Equation 1).

Indeed, the accepted hypothesis is quite simplifying. Stresses in both directions have been decoupled. However, the results of the comparison are quite satisfactory; the difference observed appears, mainly, in the zone of opening and propagation of the cracks. It becomes null in the zone of plasticization and failure of the section.

The comparison of the numerical simulation results with the experimental ones is made on the "OG" Fouré tests carried out at the CEBTP. The test/calculation comparison gives satisfactory results; the maximum error obtained is of the order of 2% in the case of loads, of the order of 7% in the case of deformations along the y direction, and of the order of 13% in the case of deformations along the z direction. The deviation observed along the z direction is visible in the post-cracking domain (at the generation and propagation of cracking). This is due to negligence of the interaction between the two loads M_y and M_z . Of course, this approach is rather simplistic. On the other hand, this difference decreases in the vicinity of the failure.

References

- Bouafia, Y., (1987). Simulation numérique du comportement moyen d'une zone de poutre en béton armé, précontraint et béton armé de fibres d'acier. Mémoire de DEA. Université Paris 6.
- Fouré, B., (1985). Béton à haute résistance. Rapport sur les essais de poutres et de poteaux OG. Rapport interne C.E.B.T.P.
- Nait Rabah, O., (1990). Simulation du comportement non linéaire des ossatures spatiales. Application aux structures en béton armé, béton précontraint et mixte. Thèse de docteur, E.C.P.

- Espion, B., (1986). Contribution à l'analyse non linéaire des ossatures planes. Thèse de Docteur. Université libre de BRUXELLES.
- Kachi, M. S., (1997). Calcul non linéaire, jusqu'à rupture d'une section en béton armé de fibres métalliques. Thèse de magister. UMMTO.
- Ren, W. Sneed, L. H. Gai, Y. Kang, X., (2015). Test results and nonlinear analysis of RC T-beams strengthened by bonded steel plates. *International Journal of Concrete Structures and Materials* 9(2): 133–143.
- Islam, M. S. Shahria, A., (2013). Principal component and multiple regression analysis for steel fiber reinforced concrete (SFRC) beams. *International Journal of Concrete Structures and Materials* 7(4): 303–317.
- Bouafia, Y. Kachi, M. S. Fouré, B., (2000). Numerical modeling of the behavior of steel fiber reinforced concrete. II. international Symposium on Cement and Concrete Technology. Turkish Cement Manufacturers Association – European Cement Association. Istanbul – Turkiye. Sep. 6-10. Vol.2 : pp.582-591.
- Bouafia, Y. Kachi, M. S. Fouré, B., (1998). Relation effort-ouverture dans le cas du béton armé de fibre d'acier. *Annales du BTP*, n°2.
- Sargin, M., (1968). Stress-strain Relationship for concrete and the analysis of structural concrete sections. PHD thesis presented to the University of Waterloo, Waterloo, Ontario, Canada; 334p.
- Virlogeux, M. M'rad, A., (1996). Etude d'une section de poutre en élasticité non linéaire. Application au béton armé, précontraint ou section mixte. *Annales de L'I.T.B.T.P.* N°444.
- Virlogeux, M., (1986). Calcul des structures en élasticité non linéaire [1]. *Annales des ponts et chaussées* 3 trim.
- Grelat, A. Nait Rabah, O., (1988). Simulation Numérique du comportement des structures en béton armé ou précontraint hors du domaine linéaire. Rapport du SES – CEBTP.
- Bratina, S. C. Saje, M. Planine I., (2005). Numerical modeling of behaviour of reinforcing concrete columns in fire and comparison with eurocode 2. University of Ljubljana. Faculty of civil and geodetic engineering, Jamova 2, SI-1115 Ljubljana, Slovenia. *International journal of solids and structures* 42: 5715-5733.
- Bouafia, Y. Kachi, M. S. Fouré, B., (2002). Relation stress-strain in the case of steel fiber reinforced concrete In ESKA (Eds. *Annales of ITB*), N° 3.
- Daunys, M. Rimovkis, S., (2006). Analysis of circular cross-section element, loaded by static and cyclic elastic-plastic pure bending. *International journal of Fatigue* 28: 211-222.
- Florent, R., (1999). Contribution à l'analyse non linéaire géométrique et matérielle des ossatures spatiales en génie civil. Thèse de doctorat devant l'institut national des sciences appliqué de Lyon.
- Bonet, J. L. Baros, M. H. F. M. Romero, M. L., (2006). Comparative study of analytical and numerical algorithms for designing reinforced concrete sections under biaxial bending. *Computers & Structures* 84: 2184–2193.
- Choa, C.G. Kwonb, M., (2008). Prediction of non linear bending behavior for FRP Concrete beams based on multiaxial constitutive laws. *Engineering Structures* 30: 2311–2320.
- Liang, Q. Q. Fragomeni, S., (2014). Non linear analysis of circular concrete-filled steel tubular short columns under axial loading. *Journal of Constructional Steel Research* 65: 2186–2196.
- Ronagh, H. R. Baji, H., (2014). On the FE modeling of FRP Retrofitted beam-column subassemblies. *International Journal of Concrete Structures and Materials* 8(2): 141–155.
- Adjrad, A. Bouafia, Y. Kachi, M. S. Ghazi, F., (2016). Prediction of the Rupture of Circular Sections of Reinforced Concrete and Fiber Reinforced Concrete. *International Journal of Concrete Structures and Materials, Int J Concr Struct Mater* 10:373-381.
- Kachi M. S. Bouafia, Y. Saad, M. Dumontet, H. Bouhrat, S., (2014). Contribution to the calculation of reinforced concrete circular sections by discrete reinforcement. *International Journal of Engineering and Technology* 6(1): 38–42.
- Adjrad, A. Bouafia, Y. Kachi, M. S. Dumontet, H., (2015). Modeling of externally prestressed beams until fracture in non linear elasticity. *Applied Mechanics and Materials* 749: 379–385.