

Barrier Function-Based Integral Sliding Mode Controller Design for a Single-Link Rotary Flexible Joint Robot

Ahmed Mohsen Mohammad^{1,2},
Shibly Ahmed AL-Samarraie¹ and
Alaa Abdulhady Jaber^{3,*}

¹Control and Systems Engineering
Department, University of
Technology - Iraq, Baghdad, Iraq

²Ministry of Oil, Baghdad, Iraq

³Mechanical Engineering
Department, University of
Technology - Iraq, Baghdad, Iraq

*E-mail: alaa.a.jaber@uotechnology.
edu.iq

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Abstract

This paper proposes and evaluates a novel control approach for trajectory tracking, stability enhancement, and vibration reduction of a flexible joint robot (FJR). The FJR is a 2-degree-of-freedom underactuated nonlinear system that is challenging to control due to vibration, underactuation, uncertainties, and external disturbances. The control objectives are high trajectory tracking performance together with suppressing vibration. The proposed control approach uses integral sliding mode control (ISMC) combined with a barrier function based on back-stepping. This ensures robust and smooth performance by eliminating the reaching phase where sliding mode control (SMC) is typically not robust. Robustness is thus guaranteed from the start. The FJR is modeled as a 4th-order system using Lagrangian mechanics and decomposed into two 2nd-order subsystems for control design. Sliding variables are defined for each subsystem. Using these variables, the proposed control achieves robust trajectory tracking, stability, and vibration reduction. Numerical simulations in MATLAB validate the superior performance of the proposed ISMC-barrier function control compared to conventional ISMC for the FJR. This novel control approach addresses the challenges of controlling this FJR.

Keywords

Flexible Joint Robot, Integral Sliding Mode Control, Barrier Function, Trajectory Tracking, Vibration Reduction

1. Introduction

Robotic arms, consisting of flexible rotary joints, can offer many attractive features, such as low cost, smaller actuators, lightweight, larger work volume, better transportability, higher operational speed, and maneuverability. Flexibility in the robotics mechanical joints could be introduced due to different transmission components such as springs, gears, bearings, hydraulic lines, etc. Fast response requirements in robotic arms amplify the significance of joint elasticity due to the consequential torsional vibrations. Therefore, an efficient controller design that minimizes the effects of joint flexibility is highly desirable in practical scenarios. Flexibility affects the performance

of the system more than rigidity. Research on tracking control for flexible joint robot (FJR) manipulators has been an area of focus. With appropriate control mechanisms, rotary flexible joints can be operated safely at high performance and faster speeds, and with enhanced accuracy and precision. The goal is maintaining the link tip at a desired angular position while minimizing vibration. However, actuator saturation presents a constraint for regulating the FJR link tip since the actuator can only supply a fixed torque amount to the arm, restricting performance. Large, powerful motors are impractical due to their size and weight. Furthermore, large motors are typically more expensive and demand higher power consumption. In summary, actuator limitations pose control challenges

for precise positioning and vibration reduction of FJR manipulators. This necessitates innovative control solutions that work within the torque constraints of compact, affordable actuators.

Moreover, the FJR models certainly contain nonlinearities, parameter variations, unmodelled dynamics parameters, and unknown time-varying disturbances, which industriously affect the system control performance. Ignoring joint flexibility will increase the system's potential unmodelled dynamic parameters, thus affecting the performance and precision of the trajectory tracking of a manipulator.

The robot arm manipulators having joint flexibility are underactuated systems because the number of control inputs is less than the number of degrees of freedom; therefore, the control function of such systems becomes difficult. In recent years, there has been an essential increase in the number of realizations on the control of FJR manipulators. As a part of the design, the arm's end-effector was aligned with the reference direction to reduce vibration. Modeling and control of the FJR manipulator were proposed by Akyuz and Bingul [1], considering vibration and trajectory tracking control with Proportional–Integral–Derivative (PID) and state feedback controllers (SFCs). To control an FJR, a method in the study of Aziz and Iqbal [2] based on the 1st and 2nd-order su2 per-twisting sliding mode control (SMC) is proposed as a nonlinear control strategy. Alam et al. [3] used two approaches based on SMC, traditional SMC and ISMC, to design a robust nonlinear controller for an FJR problem. Direct adaptive back-stepping control with tuning functions approach for a single-link FJR based on the tracking error-based parameter adaptation law was proposed in Soukkou et al.'s study [4] as well as in that of Yim [5]. A PID control technique was used to control flexible manipulators. Trajectory tracking results and reducing undesirable vibrations witnessed the effectiveness of the developed control strategy in the studies of Alam et al. [6] and Duong et al. [7]. SMC techniques for the tracking trajectory problem of an FJR arm and the altitude of quadrotor Unmanned Aerial Vehicle (UAV) with feedback linearization or disturbance observer have been proposed in the literature [8–10]. Performance comparison between two controllers is made in terms of dynamic behavior and robustness capability. Linear active disturbance rejection control (LADRC) and nonlinear active disturbance rejection control (NADRC) were proposed by Humaidi and Badr [11] to control the nonlinear system structure of single-link FJR without linearization and using the Particle Swarm

Optimization (PSO) technique to improve the system performance. An intelligent mobile robot system to guide passengers was presented in Tran et al.'s study [12], where the distance between the robot and passengers is predicted with the use of Multi-task Cascaded Convolutional Networks (MTCNN). Also, a real-time monitoring deviation against seismic effects and a robust fiber Bragg grating (FBG) sensor network are presented in Saad's study [13].

In Raza et al.'s study [14], a sampled-data control law based on SMC using two different approaches was designed. The first involves designing the sampled-data SMC based on the continuous time system, and the second approach the discrete SMC based on the obtained approximate discrete model of the system. The nonlinear model of FJR is considered to estimate the unknown states using a high-gain observer (HGO). Furthermore, in the presence of model uncertainties and matched disturbances of FJR, an SMC-based output feedback controller (OFC) was proposed by Ullah et al. [15]. Also, two sliding mode controllers were designed for a gearbox-connected two-mass system in Mohammed et al.'s study [16], where the first was employed for tracking position, while the other was used for stability purposes. Panchal et al. [17] designed and experimentally validated an Extended State Observer (ESO) based robust predictive controller for an FJR. ESO was utilized to estimate the output derivatives, uncertainties, and disturbances. The estimates were then employed in the predictive controller designed for the nominal system. In the researches of Al-Saggaf et al. [18] and Andaluz et al. [19], a state feedback-based fractional integral control scheme was designed to guarantee robustness and disturbance rejection. To realize the evaluation of the trajectory tracking performance of an FJR, reduced joint angle error and minimized vibration were achieved. To minimize vibrations and to optimize the performance of the position tracking of the FJR system, Dharavath and Ohri [20] designed two controllers, namely SFC with pole-placement and Linear Quadratic Regulator (LQR) by utilizing the Riccati equation. Robust and converse dynamic (RCD) control was proposed by Iskanderani and Mehedi [21] to solve the control problem of an FJR under parametric uncertainty and perform control trajectory tracking. Because SMC has many benefits, including high accuracy and insensitivity to the system perturbations, guaranteed stability, and robustness to system uncertainties and disturbances, various SMC schemes have been applied to the FJR, such as cascaded extended state observer (CESO) [22]. The optimal 2nd-order

integral sliding mode control (OSOISM) is proposed in the studies of Rsetam et al. [23] and Momeni and Bagchi [24]. In Rsetam et al.'s researches [25, 26], a hierarchical sliding mode control (HSMC) and hierarchical non-singular terminal sliding mode controller (HNTSMC) were proposed.

Rsetam et al. [27] designed super-twisting-based integral sliding mode control (ISMC) for the FJR. Thus, many researchers worldwide are interested in the complexity reduction process in controlling the vibration and stiffness of an FJR manipulator by applying the following linear and nonlinear control techniques. In Jin et al. [28], a new dynamic parameter identification method was developed based on an adaptive control algorithm for flexible joints. In the research of Le-Tien and Albu-Schäffer [29], robust adaptive tracking control is based on an SFC with integrator terms for elastic joint robots with uncertain parameters. Dynamic learning from adaptive neural control with prescribed tracking error performance for FJR included unknown dynamics proposed in Chen et al.'s study [30]. To achieve trajectory tracking and vibration control, Kandroodi et al. [31] and Oleiwi et al. [32] proposed control of FJR via reduced rule-based fuzzy logic control with experimental validation. σ -Stabilization of FJR via delayed controllers was proposed in Ochoa-Ortega et al.'s study [33]. In Cheng et al.'s study [34], sliding mode tracking control and its chattering suppression method were proposed for FJR manipulators with only state measurements of joint actuators. The fault-tolerant tracking control problem was conducted for a single-link flexible joint manipulator (SFJM) system with fault, uncertainty including nonlinear function, and unmatched disturbance. An observer-based sliding mode fault-tolerant tracking control was proposed in the study of Chen and Guo [35].

In a comparative study against controllers with similar structures [17, 20–22, 27], the proposed ISMC-based control method outperformed other, producing smoother deflection angles (α) and reduced tracking errors (e_t). This superior performance was attributed to the elimination of chattering in the control signal without compromising tracking precision, ensuring better end-effector trajectory adherence.

The control objectives in this investigation are to achieve high tracking performance and suppress the vibration for the FJR manipulator with external disturbance and parameter uncertainties considered. Thus, the present research's contribution can be highlighted via the following points:

1. Two integral sliding mode controllers designed using a barrier function were proposed

to control a 4th-order, 2-degree-of-freedom (2DOF) system. This enabled the position of the FJR system's connected link to track a desired position precisely.

2. Two subsystems, an actuated strategy and an unactuated strategy, have been proposed for the FJR system. The two integral sliding mode controllers designed earlier are applied to each subsystem in a coordinated manner using a back-stepping approach.

The proposed controller in this investigation will use the sliding mode differentiator (SMD) to estimate the required derivatives in its formula.

The rest of the article is organized as follows. Section 2 introduces the problem statement and dynamic model; Section 3 summarizes the controller design for the considered FJR; Section 4 applies the proposed method by simulating the FJR model; and Section 5 concludes the paper.

2. Problem Formulation and Dynamic Modelling

A rotary FJR manipulator developed by Quanser [36] is considered in this research. FJR utilizes flexible joints in robotics to demonstrate vibration analysis and resonance control concepts. A typical single-link FJR experimental platform is presented here to exhibit the behavior of an FJR arm. Due to joint flexibility, natural frequencies can be easily excited. To avoid this, maneuver accelerations are reduced, and so vibrations decay naturally. However, this takes an unacceptable amount of time due to the system's limited natural frequency. Various control techniques have been applied to this system, as reviewed in Section 1. However, regulating such a nonlinear system precisely using ISMC based on a barrier function still poses challenges.

2.1. Platform description

As seen in Fig. 1, it consists of double rotary arms mounted on a base platform – a 29.8 cm extended arm attached with springs holding a short arm. The short arm's placement on the long arm determines the flexible joint's vibration effects. The arm's flexibility comes from the spring stiffness. The base is fixed on a Quanser SRV02 rotary servo motor, which has a DC motor enclosed in an aluminum frame connected to a gearbox. This complete servo system contains



Figure 1: Rotary flexible joint platform.

gears, a potentiometer, a tachometer, encoders, and a DC motor. A tachometer measures the rotary arm's angular displacement, while a potentiometer measures the load gear orientation to obtain angular velocity. Two encoders measure joint rotation and servo load shaft angle.

2.2. Dynamic model

In Quanser's study [36], the mathematical model of Quanser's rotary single-link FJR is presented. The schematic of the rotary flexible joint arm module is shown in Fig. 2, where it operates in a horizontal plane; thus, gravity terms are not considered. Consequently, θ and α represent the angular position of the motor and the vibration/deflection angle of the link (flexible joint), respectively. Euler-Lagrange's method is often utilized for more complex systems, such as a robot manipulator with multiple joints. The dynamics of the FJR can be modeled using differential equations representing the link and motor dynamics. The dynamical model of the system is presented as follows [31]:

$$J_{eq}\ddot{\theta} + J_{arm}(\ddot{\theta} + \ddot{\alpha}) = \tau - B_{eq}\dot{\theta} \quad (1)$$

$$J_{arm}(\ddot{\theta} + \ddot{\alpha}) + K_s\alpha - d(t) = 0 \quad (2)$$

In this dynamic model, there are two kinds of inertia. The equivalent moment of inertia of the rotary arm is expressed by J_{eq} , whereas the inertia of the link is expressed by J_{arm} . Further, B_{eq} and K_s represent the viscous friction coefficient of the servo and linear spring stiffness, respectively, and $d(t)$ is the external disturbance acting on the unactuated dynamics.

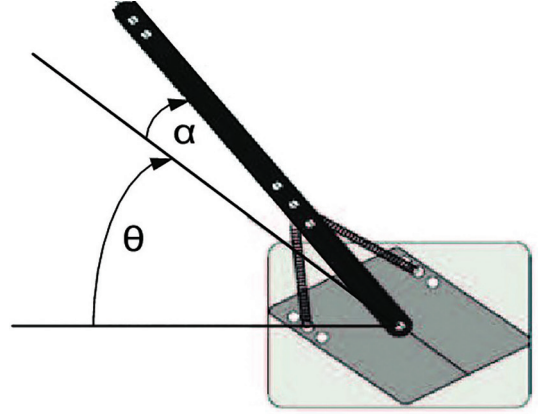


Figure 2: Schematic diagram of the FJR system.

Table 1: Nominal parameters of the FJR manipulator

Symbol	Description	Value
J_{eq}	Equivalent moment of inertia of servo motor ($\text{kg} \cdot \text{m}^2$)	0.0018
J_{arm}	Moment of inertia of the link ($\text{kg} \cdot \text{m}^2$)	0.0033
B_{eq}	The viscous friction coefficient of the manipulator ($\text{N} \cdot \text{m} \cdot \text{s}/\text{rad}$)	0.015
K_s	Joint stiffness (N/m)	1.3
K_m	Motor back-EMF constant $\text{N}/(\text{rad}/\text{s})$	0.0077
K_g	High gear ratio	70
K_t	Motor torque constant ($\text{N} \cdot \text{M}/\text{A}$)	0.00767
R_m	Motor torque constant (Ω)	2.6
η_m	Motor efficiency	0.69
η_g	Gearbox efficiency	0.90

EMF, Electromotive Force

The torque (τ) that is enforced at the base of the rotary manipulator arm is generated while the input voltage is applied, which is define as:

$$\tau = \frac{K_t K_g \eta_m \eta_g (V_m - K_g K_m \dot{\theta})}{R_m} \quad (3)$$

All the symbols in the above equation are defined, and their numerical values are presented in Table 1 below [20].

Hence, by substituting Eq. (3) into Eq. (1), Eqs. (4) and (5) can be written as:

$$\ddot{\alpha} = -\ddot{\theta} - \frac{K_s}{J_{arm}} \alpha - \frac{d(t)}{J_{arm}} \quad (4)$$

$$\ddot{\theta} = \frac{K_s}{J_{eq}} \alpha - F_1 \dot{\theta} + F_2 V_m \quad (5)$$

where

$$F_1 = \frac{B_{eq} R_m + \eta_m \eta_g K_t K_m K_g^2}{J_{eq} R_m}$$

$$F_2 = \frac{\eta_m \eta_g K_t K_g}{J_{eq} R_m}$$

V_m is the motor control voltage; the target is to design a controller to track $y = \alpha + \theta$ its desired trajectory. Therefore, Eqs. (4) and (5) are rewritten as

$$\ddot{y} = \frac{-K_s}{J_{arm}} (y - \theta) - \frac{d(t)}{J_{arm}} \quad (6)$$

$$\ddot{\theta} = \frac{K_s}{J_{eq}} (y - \theta) - F_1 \dot{\theta} + F_2 V_m \quad (7)$$

For the convenience of controller design, the following state vector is defined:

$$x_1 = y, x_2 = \dot{y}, x_3 = \theta, x_4 = \dot{\theta}, y = x_1$$

So, the dynamics of Eqs. (6) and (7) can be written in state space form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 (x_1 - x_3) + T_L \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= b_1 (x_1 - x_3) - F_1 x_4 + F_2 V_m \end{aligned} \quad (8)$$

where

$$a_1 = \frac{-K_s}{J_{arm}}, b_1 = \frac{K_s}{J_{eq}}, \text{ and } T_L = -\frac{d(t)}{J_{arm}}.$$

The uncertainty in the above equations consists of the joint stiffness value, i.e., K_s , which can be obtained as:

$$K_s = K_{sn} + \Delta K_s \text{ then, } a_1 = a_{1n} + \Delta a_1, \text{ and } b_1 = b_{1n} + \Delta b_1$$

Also, uncertainties in the system model coefficients, F_1 and F_2 , can be obtained as:

$$F_1 = F_{1n} + \Delta F_1, F_2 = F_{2n} + \Delta F_2$$

Thereby, the system dynamics of FJR, as it is the basis for the designed controller in the next section, accordingly becomes:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{1n} (x_1 - x_3) + \delta_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= b_{1n} (x_1 - x_3) - F_{1n} x_4 + F_{2n} V_m + \delta_2 \end{aligned} \quad (9)$$

where δ_1 and δ_2 are the unactuated and actuated perturbations, respectively, and can be written as:

$$\begin{aligned} \delta_1 &= \Delta a_1 (x_1 - x_3) + T_L \\ \delta_2 &= \Delta b_1 (x_1 - x_3) - \Delta F_1 x_4 + \Delta F_2 V_m \end{aligned} \quad (10)$$

3. Controller Design

This section discusses the design methodology and implementation of ISMC based on barrier function [37, 38] for angular tracking and vibration control of FJR.

The controller's design methodology involves dividing Eq. (9)'s system into actuated and unactuated subsystems. Illustrated in Fig. 3 is the proposed controller's block diagram, employing two distinct controllers for each subsystem. These controllers, interconnected through a back-stepping method, operate as follows: the first, acting as a virtual controller, is responsible for tracking the FJR link's desired angle within the unactuated subsystem. The second functions as an actual controller for the actuated subsystem, represented by the motor control voltage V_m , aimed at tracking the input provided by the virtual controller. The introduction of the barrier function concept is detailed in the initial part of this section, followed by the introduction of ISMC based on the barrier function, utilizing the system described in Eq. (9).

3.1. Preliminaries (Barriers Functions [BFs])

Definition [39]: Let us assume that some $\varepsilon > 0$ is given and fixed; a BF is defined as an even continuous

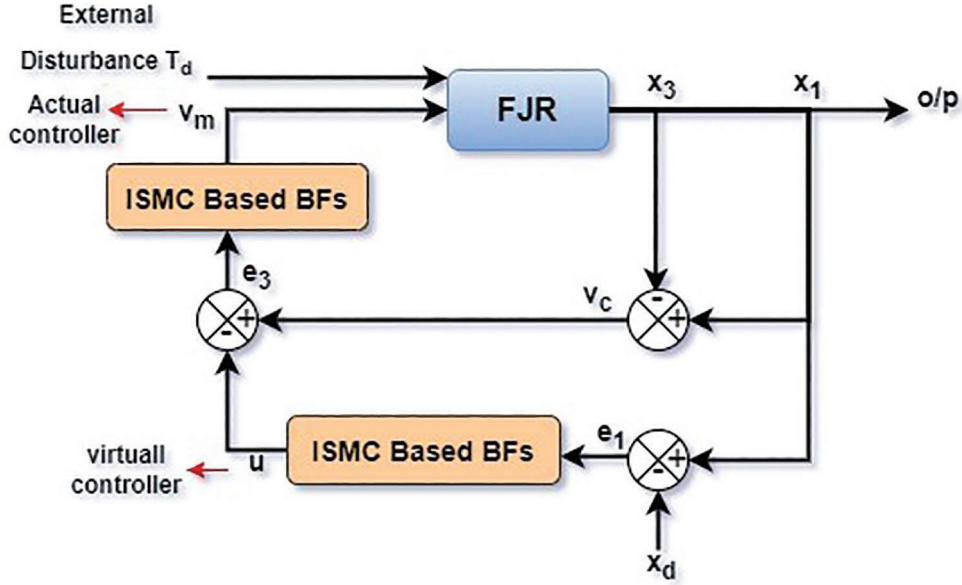


Figure 3: Overall block diagram of the proposed control system.

function $f: z \in [-\varepsilon, \varepsilon] \rightarrow g(z) \in [b, \infty]$ strictly increasing on $[0, \varepsilon]$.

- $\lim_{|z| \rightarrow \varepsilon} g(z) = \infty_+$
- $g(z)$ has a unique minimum at zero and $g(0) = b \geq 0$.

In this paper, two different classes of BFs are considered:

- Positive-definite BFs (PBFs): $g_p(z) = \frac{\varepsilon F}{\varepsilon - |z|}$, $g_p(0) = F$.
- Positive semi-definite BFs (PSBFs): $g_{ps}(z) = \frac{|z|}{\varepsilon - |z|}$, $g_{ps}(0) = 0$.

The PBFs $g_{ps}(z)$ in (b) were chosen and will be used when simulating the FJR system in this paper.

3.2. ISMC design based on barrier function

The problem with the robustness of the conventional SMC is that it is not guaranteed during the reaching phase, although the parametric variations and external disturbances during the sliding mode alone can be achieved. However, this problem is addressed via the proposed controller in this research, which is ISMC based on the barrier function approach previously proposed in the researches of Abd and Al-Samarraie [38] and Mohammadridha and Al-Samarraie [40]. This controller was able to eliminate the reaching phase

and all parameter uncertainties from the first instant to ensure robustness from the beginning of a process.

As mentioned above, the ISMC is designed for each subsystem; so, the unactuated subsystem, according to Eq. (9), is

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{in}(x_1 - x_3) + \delta_1 \end{aligned} \quad (11)$$

Let us define the following state transformation.

$$v_c = (x_1 - x_3) \quad (12)$$

Accordingly, the mathematical model in Eq. (11) becomes as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{in}v_c + \delta_1 \end{aligned} \quad (13)$$

where v_c is a new state that can be defined as a controller.

By introducing the new state, the back-stepping approach can be applied to the system dynamics in Eq. (14), which is regarded as a virtual control, as seen later in this section.

Now, let us consider the unactuated subsystem with $v_c = u$, where u is the virtual controller.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{in}u + \delta_1 \end{aligned} \quad (14)$$

The error functions e_1 and e_2 are formally defined, respectively, as the quantitative difference or discrepancy between the intended/target input (i.e., reference value) and the real/actual measured values of the output. This definition is demonstrated through the mathematical Eq. (15) below.

$$\begin{aligned} e_1 &= x_1 - x_d \\ e_2 &= x_2 - \dot{x}_d \end{aligned} \quad (15)$$

where x_d is the desired (or reference) signal; hence, Eq. (16) in terms of the error functions is given by

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= a_{in}u + \delta_1 - \ddot{x}_d \end{aligned} \quad (16)$$

In this work, the proposed virtual controller u for the unactuated subsystem in Eq. (14) is the ISMC based on the barrier function, which is given by

$$u = \frac{1}{a_{in}}(u_o + u_s) \quad (17)$$

where u_o is the nominal control, which will be determined later, while u_s is PBFs. For the unactuated subsystem, the sliding variable s_1 is defined as

$$s_1 = e_2 + z_1. \quad (18)$$

It consists of two main parts: e_2 is the derivative of the tracking error and z_1 is the integral term, with $s_1, z_1 \in R$. Then, the integral sliding manifold derivative is

$$\dot{s}_1 = \dot{e}_2 + \dot{z}_1. \quad (19)$$

By substituting Eqs. (16) and (17) in Eq. (19), the following equation can be obtained:

$$\dot{s}_1 = u_o + u_s + \delta_1 - \ddot{x}_d + \dot{z}_1 \quad (20)$$

where $\ddot{x}_d$ is the second derivative of the desired input.

Now, the derivative of the integral part is defined as

$$\dot{z}_1 = \ddot{x}_d - u_o \quad (21)$$

Accordingly, $\dot{s}_1$ becomes

$$\dot{s}_1 = u_s + \delta_1. \quad (22)$$

The equivalent system dynamics [41], which represent the system dynamics during the sliding mode, can be obtained from the system dynamics described in Eq. (23) by substituting $u_s = -\delta_1$. So, following the substitution of u and u_s in Eq. (17), the system dynamic from Eq. (16) would assume the following form:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= u_o - \ddot{x}_d \end{aligned} \quad (23)$$

Note that the unactuated subsystem is equivalent to the subsystem given in Eq. (24); just after v_c , it becomes equal to the virtual controller u , which is the task of the actual controller. Therefore, a nominal controller u_o can be selected as in Eq. (24), which makes the origin of the error dynamics in Eq. (24) globally asymptotically stable.

$$u_o = \ddot{x}_d - c_1 e_1 - c_2 e_2 \quad (24)$$

where c_1 and c_2 are positive constant values chosen based on the desired characteristic. Consequently, the proposed ISMC law based on using barrier function is a continuous control law, where u_s can take the following form:

$$u_s = \frac{-\lambda_1 s_1}{\varepsilon_1 - |s_1|} \quad (25)$$

where $\varepsilon_1 > 0$ is the thickness of the boundary layer, while $\lambda_1 > 0$ represents the control gain, which is not required to be computed as the gain in the classical ISMC because the bond of perturbation must be known. So, only a suitable value for ε_1 according to the wanted accuracy has to be selected.

Thereby, the chattering and reaching phase will be eliminated from the beginning of the process when selecting ε_1 as small as possible. Furthermore, the steady-state error is a function to ε_1 , and so it becomes smaller for smaller ε_1 . To this end, the actuated subsystem is given according to Eq. (9):

$$\begin{aligned} \dot{x}_3 &= x_4 \\ \dot{x}_4 &= b_{1n}(x_1 - x_3) - F_{1n}x_4 + F_{2n}V_m + \delta_2 \end{aligned} \quad (26)$$

The controller's task V_m in the actuated subsystem is to make the horizontal link position θ track the desired angle while reducing the vibration angle α of the rotary FJR. Similarly, the following steps are executed

to design the actual controller by considering the state transformation v_c dynamic in Eq. (12). The error functions e_3 and e_4 are defined as

$$\begin{aligned} e_3 &= v_c - u \\ e_4 &= \dot{v}_c - \dot{u} \end{aligned} \quad (27)$$

where u is the desired input (virtual controller) for the unactuated subsystem; accordingly, Eq. (27) in terms of the error functions (Eq. (28)) is provided by

$$\begin{aligned} \dot{e}_3 &= e_4 \\ \dot{e}_4 &= F_n + \delta_1 - F_{2n} V_m - \delta_2 - \ddot{u} \end{aligned} \quad (28)$$

where

$$F_n = a_{1n}(x_1 - x_3) - b_{1n}(x_1 - x_3) + F_{1n} x_4 \quad (29)$$

Thus, the proposed controller for the actuated subsystem in Eq. (26) is the ISMC based on the barrier function. Therefore, the V_m controller is given as follows:

$$V_m = \frac{-1}{F_{2n}}(V_{mn} + V_{ms}) \quad (30)$$

where V_{mn} is the nominal control and V_{ms} is PBFs. For the actuated subsystem, the sliding variable s_2 is defined as:

$$s_2 = e_4 + z_2 \quad (31)$$

It consists of two main parts: e_4 is the derivative of the tracking error and z_2 is the integral term with s_2 , $z_2 \in R$. Then, the integral sliding manifold derivative is

$$\dot{s}_2 = \dot{e}_4 + \dot{z}_2 \quad (32)$$

By substituting Eqs. (28) and (30) into Eq. (32), the following is obtained:

$$\dot{s}_2 = F_n + V_{mn} + V_{ms} - \tilde{\delta} + \dot{z}_2 \quad (33)$$

where $\tilde{\delta} = \delta_1 - \delta_2 - \ddot{u}$.

Now, let the derivative of the integral part be defined as

$$\dot{z}_2 = -F_n - V_{mn} \quad (34)$$

Accordingly, $\dot{s}_2$ becomes

$$\dot{s}_2 = \tilde{\delta} + V_{ms} \quad (35)$$

Then, in the equivalent mode, $V_{ms} = -\tilde{\delta}$. Therefore, from Eq. (28), we obtain the following equivalent dynamics:

$$\begin{aligned} \dot{e}_3 &= e_4 \\ \dot{e}_4 &= F_n + V_{mn} \end{aligned} \quad (36)$$

where the nominal control V_{mn} is selected as:

$$V_{mn} = -F_n + v \quad (37)$$

where v is the equivalent controller.

Consequently, the proposed ISMC law based on using a barrier function for the actuated subsystem is also a continuous control law where V_{ms} can take the following form:

$$V_{ms} = \frac{-\lambda_2 s_2}{\varepsilon_2 - |s_2|} \quad (38)$$

where $\varepsilon_2 > 0$ is the thickness of the boundary layer, while $\lambda_2 > 0$ represents the control gain. Thereby, only a suitable value of ε_2 needs to be selected according to the desired accuracy so that the chattering and reaching phases are eliminated from the initiation of the process due to selecting ε_2 as small as possible. Therefore, the steady-state error is a function of ε_2 , and so it becomes smaller for smaller ε_2 .

The nominal control part of the motor voltage is designed based on the design of V_{mn} above. As a first step, the dynamic of FJR in terms of the error functions is introduced in the following way, that is to say from Eqs. (17) and (27), we have:

$$v_c = e_3 + u = e_3 + \frac{1}{a_{1n}}(u_o + u_s)$$

Then Eq. (16) becomes:

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= u_o + u_s + a_{1n} e_3 + \hat{\delta}_1 \end{aligned} \quad (39)$$

Eq. (39) is the unactuated subsystem in terms of the error functions. In the next step, the actuated subsystem in Eqs. (28) and (30) are rewritten in the following form:

$$\begin{aligned} \dot{e}_3 &= e_4 \\ \dot{e}_4 &= F_n + V_{mn} + V_{ms} + \tilde{\delta} \end{aligned} \quad (40)$$

And from Eq. (37), $V_{mn} = -F_n + v$, the dynamics of FJR in terms of the error functions is outlined using the following mathematical expression:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{1n} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} 0 \\ u_s + \hat{\delta}_1 \\ 0 \\ V_{ms} + \tilde{\delta} \end{bmatrix} \quad (41)$$

Since the sliding mode is introduced in the ISMC design from the first instant, one can use the equivalent concept to design v by writing the error model, which is given in Eq. (42) in the equivalent form, i.e.,

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{1n} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} 0 \\ u_s + \hat{\delta}_1 \\ 0 \\ V_{ms} + \tilde{\delta} \end{bmatrix}_{\text{equivalent}} \quad (42)$$

But

$$\begin{bmatrix} 0 \\ u_s + \hat{\delta}_1 \\ 0 \\ V_{ms} + \tilde{\delta} \end{bmatrix}_{\text{equivalent}} = \begin{bmatrix} 0 \\ [u_s]_{\text{equivalent}} + \hat{\delta}_1 \\ 0 \\ [V_{ms}]_{\text{equivalent}} + \tilde{\delta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $[u_s]_{\text{equivalent}} = -\hat{\delta}_1$ and $[V_{ms}]_{\text{equivalent}} = -\tilde{\delta}$, resulting in the dynamics of FJR in the equivalent form from the first instant given by:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{1n} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v = Ae + Bv \quad (43)$$

One can easily show that A and B are controllable pairs by using linear state feedback for v .

$$v = -Ke \quad (44)$$

where $K = [c_3 \ c_4 \ c_5 \ c_6]^T$. The matrix $A - BK$ is Hurwitz with the desired characteristics. Accordingly, the nominal voltage control signal becomes

$$V_{mn} = -F_n - Ke \quad (45)$$

which will render the origin $e = 0$ asymptotically stable.

3.3. Design SMD

In this section, a sliding mode observer (SMO) is proposed [42]. The first derivative of the virtual controller u ($\dot{u}$) is required in the error function to design the controller of the actuated subsystem. So, the SMD is used to estimate the first derivatives of u . The SMD is constructed as in Eq. (46):

$$\begin{aligned} s_o &= u - \eta \\ \dot{\eta} &= k_o * \tan^{-1}(\gamma_o s_o) \\ \dot{v}_o &= \frac{1}{\tau_o} (-v_o + k_o * \tan^{-1}(\gamma_o s_o)) \end{aligned} \quad (46)$$

where s_o is the SMD variable, and k_o and γ_o are the differentiator parameters. The second line in Eq. (46) is the SMD dynamics, while the third line is the low-pass filter with time constants τ_o . Thus, the low-pass filter outputs the estimation of the first derivatives for u .

Remark: In Eq. (46), the subscript (o) refers to the observer.

4. Simulation Results

This section presents the simulation results of applying the ISMC-based barrier function to Quanser's FJR system. The controller is designed to control position tracking (θ) and reduce the deflection angle (α) in the FJR system. In the presence of disturbance $T_L = 0.35\sin(0.63t)$, we evaluate the controller's performance in terms of trajectory tracking, vibration response, control effort, and robustness. The controller parameters and observer used in simulations are given in Table 2. During simulations, the FJR system is assumed to be initially situated at the following conditions: $x_i = [0 \ 0 \ 0 \ 0]^T$. The controller reference input is assigned to two types of inputs: square-wave and sinusoidal profiles. The tip angle $y = (\theta + \alpha)$ is the output response represented by x_1 , which has to be controlled.

Furthermore, in this study, the uncertain element originates from the joint stiffness, specifically the coefficient k_s in the system model Eqs. (6) and (7), with a parametric variation of $\pm 20\%$.

In this work, two scenarios are considered to carry out simulations to demonstrate the proposed control's effectiveness and robustness.

4.1. Scenario I

In this case, the desired position of the link of FJR is sinusoidal input at $x_d = 35\sin(2\pi 0.1t)$ deg. Figs. 3 and 4 show the sliding variable response for the virtual (unactuated) and actual (actuated) ISMC-based barrier functions. It can be seen that the sliding variable of the two controllers is smooth and does not suffer from the chattering phenomenon. As the figures demonstrate, the controllers work to regulate the sliding variable to zero levels after less than 0.15 s, and do not exceed the predefined neighborhood around zero ($\varepsilon_1, \varepsilon_2$). The suggested controller successfully removes the reaching phase and disturbances, ensuring robustness right from the start of a process. This highlights that the FJR system, operating under the ISMC-based barrier function, accomplishes effective stabilization. The position link of the FJR system and the vibration angle are shown in Fig. 5, where

Table 2: Controllers and observer parameters

Controllers	Design parameters
Unactuated controller (virtual controller)	$\varepsilon_1 = 0.01, \lambda_1 = 1, c_1 = 16, c_2 = 8$
Actuated controller	$\varepsilon_2 = 0.01, \lambda_2 = 0.01, c_3 = 12, c_4 = 7$
Observer	$K_o = 1000, \gamma_o = 100, \tau_o = 0.0001$

the tracking performance of the proposed controller was demonstrated when the desired trajectory to be tracked is a sinusoidal signal. It is obvious that the proposed controller needs less than 0.1 s to make the system follow the input waveform and be very close to it after that. Another note from Fig. 5 in the zoomed view is that during the time interval between 0 s and 0.2 s, there is a small increase in the amplitude of vibration angle (α), but it is satisfactory, and then the rest of this angle goes to less than (thereafter, as illustrated in Fig. 6. The position tracking error $e_1 = x_1 - x_d$ is plotted for this scenario, and the maximum position tracking error does not overtake 0.1, as shown in Fig. 7.

Additionally, the control effort represented by V_m was required for the tracking process, as clarified in Fig. 8. It is obvious in the zoomed version of this figure that the proposed controller is smooth due to using the barrier function with ISMC instead of the signum function because the ISMC-based barrier function does not produce chattering in the invariant neighborhood of s, which is the undesirable behavior in SMC. Therefore, it does not need continuous approximation (arctan) or a low-pass filter.

4.2. Scenario II

To investigate the ability and effectiveness of the designed controller, a square-wave profile with an amplitude of $\pm 35^\circ$ is used as another desired trajectory that the arm of FJR should track. Figs. 9 and 10

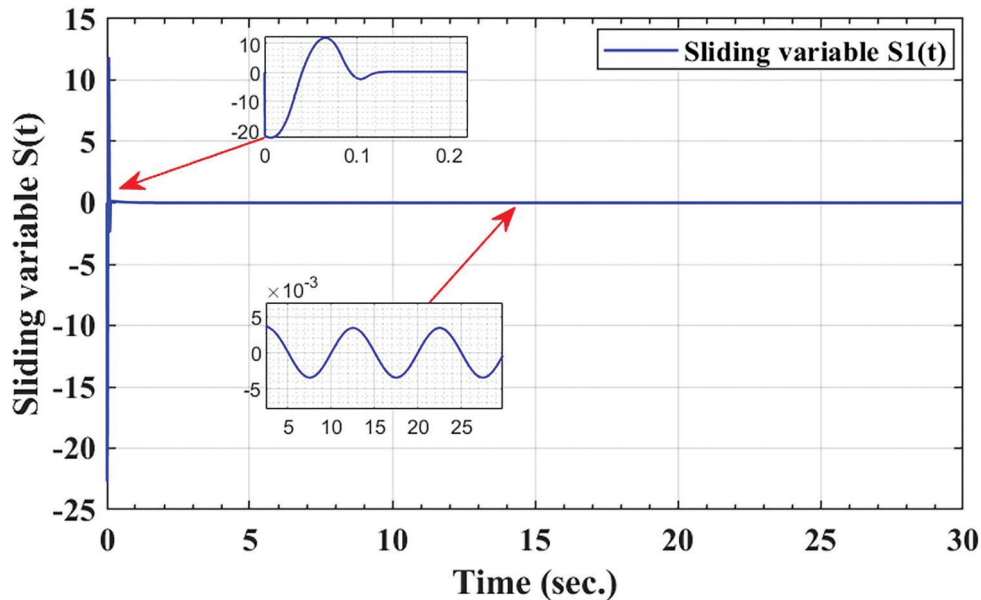


Figure 3: The sliding variable of unactuated controller vs. time.

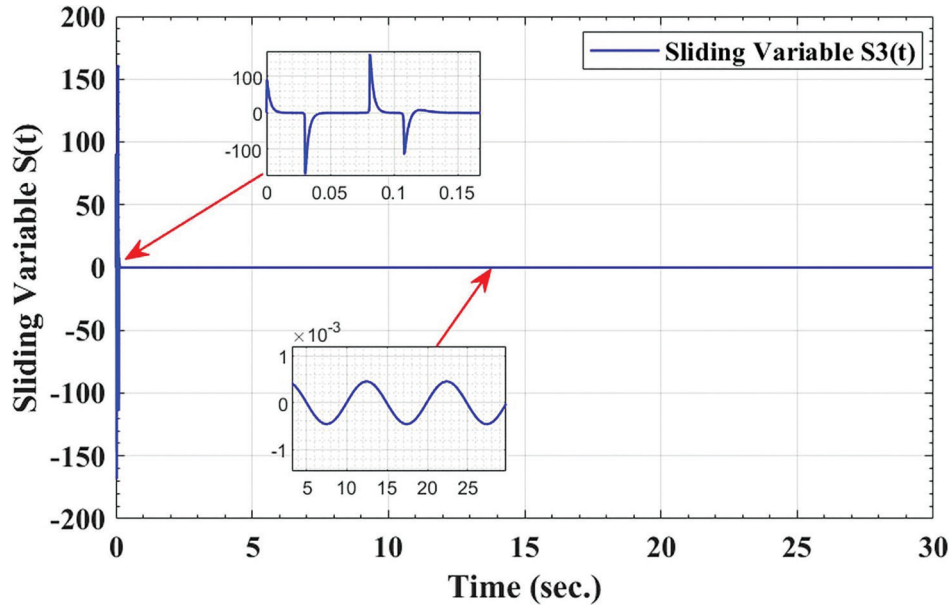


Figure 4: The sliding variable of actuated controller vs. time.

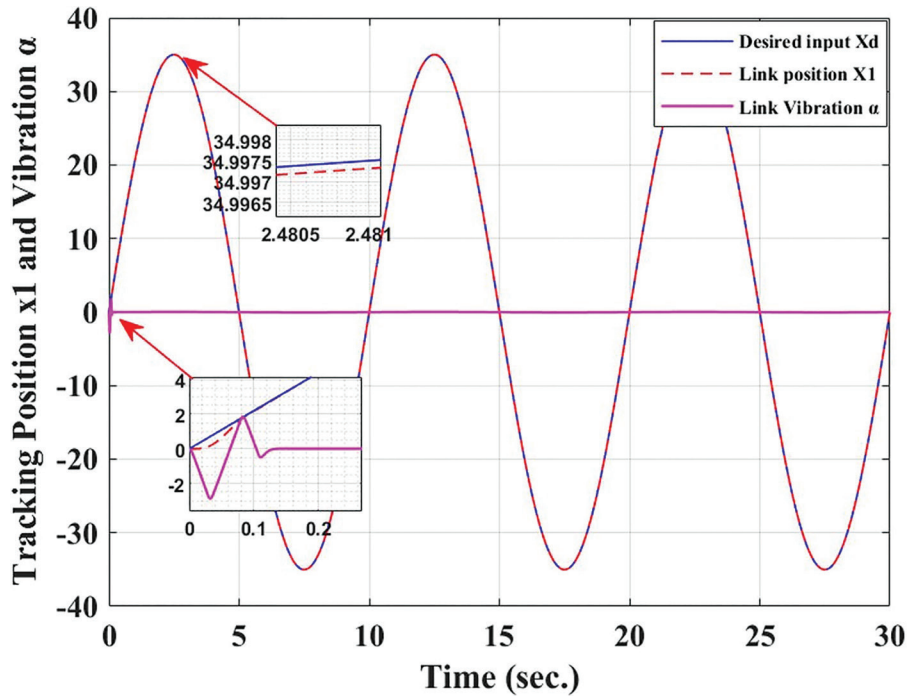


Figure 5: Tracking response (x_1) of the FJR and vibration angle (α) vs. time. FJR, flexible joint robot.

below depict the tracking performance of the FJR utilizing the designed controller. Fig. 9 illustrates the position of the FJR link, while Fig. 10 depicts the angular displacement (α) caused by vibrations in the FJR link.

Fig. 9 demonstrates that the link position of the FJR adheres to the desired trajectory satisfactorily, exhibiting a maximum tracking error of approximately 1° . The vibration angle of the FJR link is

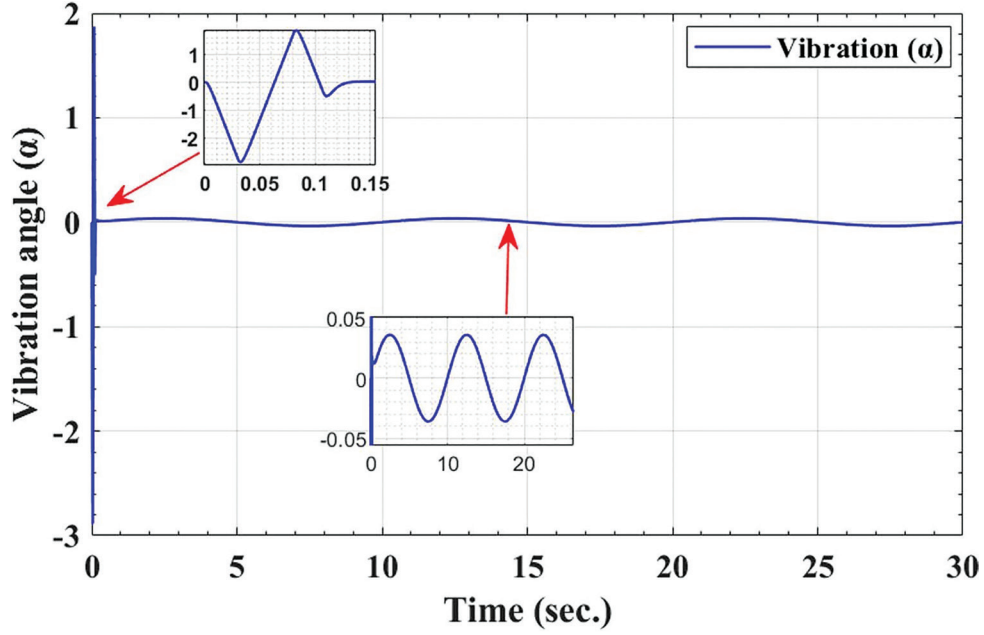


Figure 6: Vibration angle of the FJR vs. time. FJR, flexible joint robot.

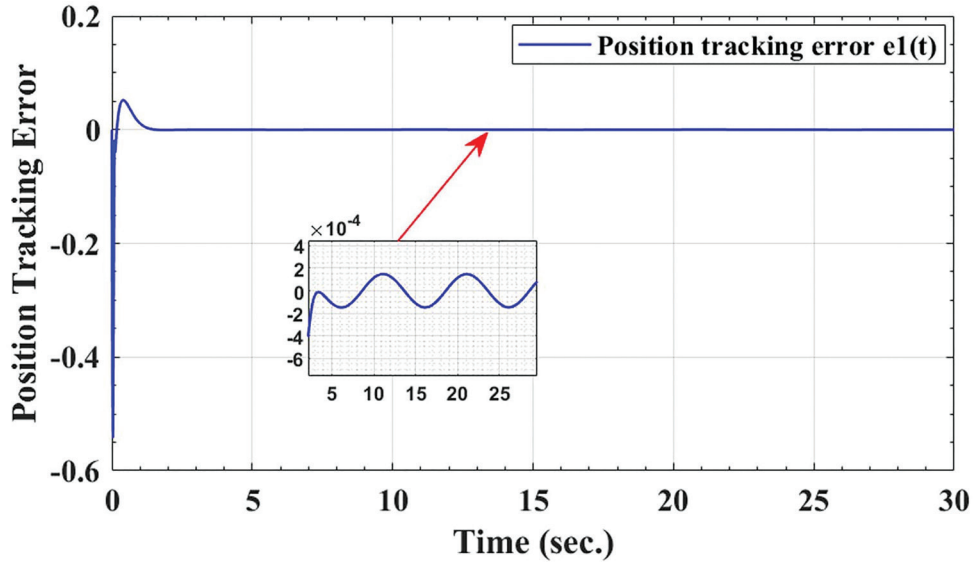


Figure 7: Position tracking error e_1 vs. time.

within the allowed range, with a maximum value of around 0.005° . This observation suggests that the designed controller has successfully followed the trajectory and effectively reduced the vibration of the FJR link.

However, Fig. 2 illustrates the oscillation of the vibration angle of the FJR connection in response to the switching of the square wave. This phenomenon

occurs due to the controller's need to rapidly adjust its output to follow the specified trajectory. The subsequent part of the response exhibits a diminishing oscillatory pattern with an amplitude of less than 0.005° . The oscillation decrease can be attributed to the intrinsic dynamics of the FJR system. Nevertheless, its impact on the tracking position of the tip of the FJR system is not substantial.

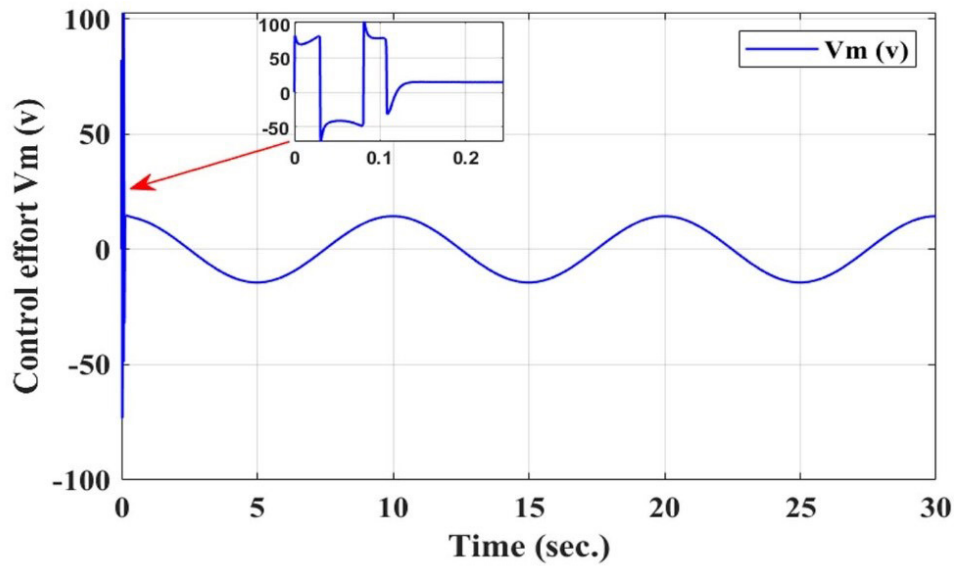


Figure 8: Controlled voltage (V_m) vs. time.

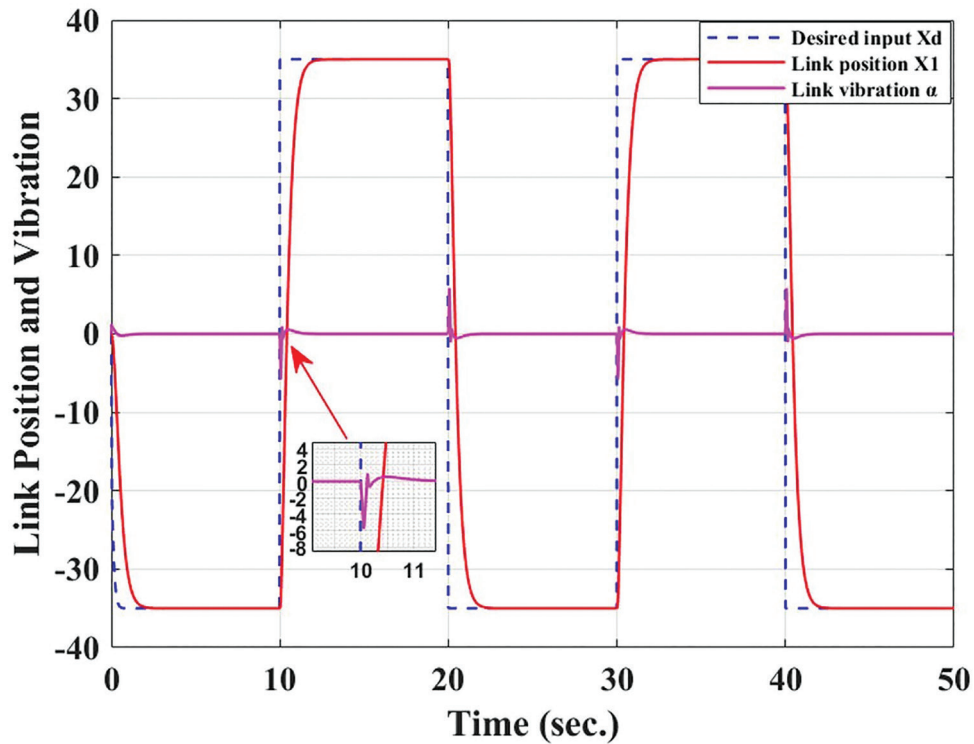


Figure 9: Tracking response (x_1) of the FJR and vibration angle (α) vs. time. FJR, flexible joint robot.

Broadly speaking, the results illustrated in the two figures showcase the effectiveness of the applied controller in precise trajectory tracking and vibration

reduction for the FJR link. However, room for improvement exists, particularly in reducing the vibration angle during the square-wave transition.

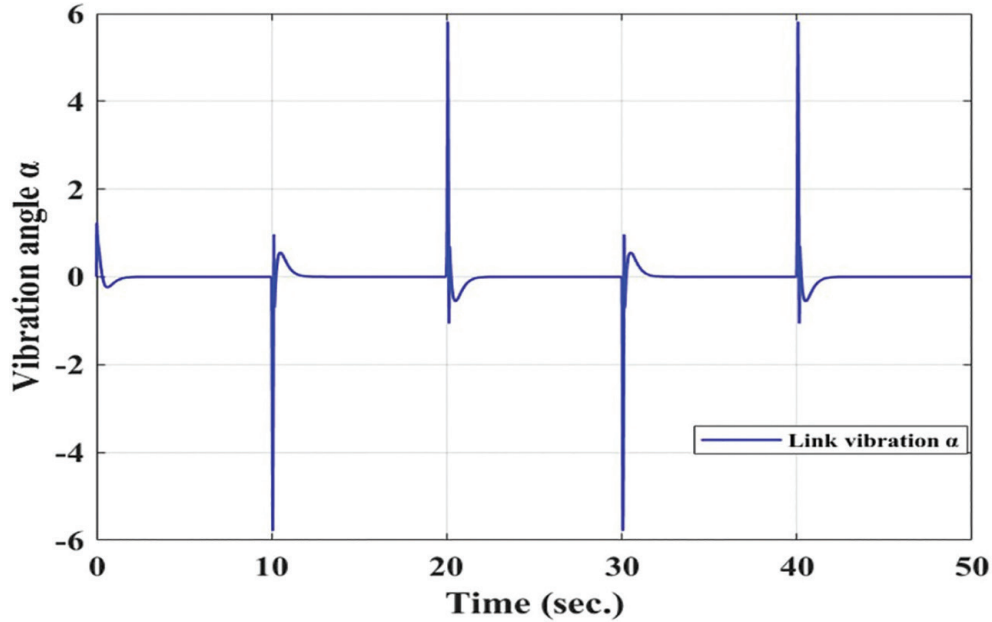


Figure 10: Vibration angle of the FJR vs. time. FJR, flexible joint robot.

Finally, a comparative study is conducted between the proposed method and other controllers with similar structures as in multiple researches in the literature [17, 20–22, 27]. Clearly, it is shown from the zoomed version of (α) and (e_i) in Figs. 6, 7, and 10 that the deflection angle (α) and tracking error (e_i) are lesser and smoother than that obtained in the researches of Dharavath and Ohri [20] and Rsetam et al. [27]. This is because the proposed ISMC-based barrier function can smooth out the control signal by eliminating the harmful chattering that can be noticed in the control input designed by Panchal et al. [17] and Rsetam et al. [22] without losing the precision of the tracking, as shown in Figs. 5 and 9. However, the tracking trajectory of the system encountered in multiple studies in the literature [17, 20–22, 27] deviates from the desired reference, and the sliding manifold in the researches of Rsetam et al. [22, 27] suffers from the chattering problem, whereas these problems or limitations are not encountered pursuant to use of the proposed control employed in the present research. More importantly, the proposed controller can impose the end-effector of the link to follow the predefined trajectory better than others, as shown in Figs. 5 and 9.

5. Conclusions

This paper presented a robust ISMC utilizing a barrier function for an FJR to handle matched

perturbations and modeling uncertainties. The designed continuous, chattering-free controller was chosen for its high robustness and simple design. The proposed method can reject perturbations from the start at $t = 0$ to the end, unlike classical ISMC, which requires knowing uncertainty bounds initially, causing control input chattering. Two simulations were performed with sinusoidal and square-wave desired positions. Position tracking accuracy and steady-state error were tuned by choosing a small positive parameter defining the barrier invariant set. The ISMC-based barrier function showed superior position tracking error reduction and significant vibration angle (α) oscillation minimization of the FJR. Additionally, an SMD estimated the virtual controller's first derivative. Presented simulations demonstrated the controller's effectiveness in improving tracking response, reducing vibration, and robustness against perturbations. Future work can further evaluate the ISMC-based barrier function experimentally and numerically to demonstrate robustness beyond just FJR position tracking. Comparative assessment of the proposed technique confirms that the ISMC with barrier function provides a good performance of 90% in comparison with the previous researches involving SMC.

Competing Interests

The authors declare that no competing interest exists.

Ethical Approval

Not applicable.

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