



Original Study

Extension of Vedic mathematics sutras for square calculation

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Abstract

Square is one of the basic arithmetic operations. Referring to the earlier contributions provided by ancient Indian mathematicians concerning square calculation, various techniques have been suggested by Aryabhata, Bhaskara I, Sridhara, Mahavira and Bhaskara II to quote a few. Besides, P. Swami Bharati Krishna Tirthji discussed various techniques for square calculating based on various sutras. In this paper, we summarise the works done by ancient Indian mathematicians and discuss the applications of calculating squares using different sutras. In addition, we propose a new method of calculating square based on the drawbacks identified and that can be applied for the square calculation of any number. Our method splits the number and hence makes the calculation simpler. Moreover, for bigger values, nested calculations can also be applied.

Keywords: Ancient Indian mathematicians, dvandvayoga, ekadhikena purvena, square, Vedic mathematics, Vedic sutras.

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1 Introduction

Square is a special case of repeated multiplication. In Sanskrit, square is referred to as Varga. The literal meaning of Varga is rows or troops (of similar things). Referring to the works done by ancient Indian Mathematicians, lots of references for different methods of calculating squares are available. Some of the earlier works include the contribution of Aryabhata in Aryabhatiyam (499 AD) [1], Bhaskara I in (628 AD) [2], Mahavira in Ganita-sara-sangraha (850 AD) [3], Bhaskara II in Lilavati (1150 AD) [4], Tirthaji et al. [5], Sridhara in Patiganita (750 AD) [6] and Narayan Pandita in Ganitakaumudi (1356) [7].

Referring to Sulva Sutras (800 BC or much before that up to 3200 BC), Vedanga Jyotisa (1350 BC) of Lagadha and Aryabhatiya (499 AD) of Aryabhata are some of the initial references for the works done for square computation. In these works, explicit references of calculation of Square were not mentioned because it was considered too elementary to be part of geometry or astronomy. Some of the initial references for calculation of square are available in Aryabhatiyam [1], which quotes that a square figure is of four equal sides (and the number representing its area) called varga.

The product of these two equal quantities is also called varga.

vargassamacaturasrah phalanca sadrsadvayasya samvargah

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To add, as the calculation of square root was mentioned, it was assumed that the concept of calculation of square was clearly known. Going through the work of Brahmagupta, definite rules for calculating square are available in Brahmasphutasiddhanta [2]. Brahmagupta quotes 3 methods of calculating square. The initial method for calculating square is quoted as

raserunam dvigunam vahutaragunamunakrtiyutam vargah

which means combining the product, twice the digit in the less (lowest) place into the several others (digits) with its (i.e. of the digit in the lowest place) square (repeatedly) gives the square.

Another method quoted by Brahmagupta can be understood as -The product of the sum and the difference of the number (to be squared) and an assumed number plus the square of the assumed number gives square. As long as you square, so long also, you double. As long as you multiply, so long also, you square. Three methods of calculating square have been clearly discussed by Mahavira (850 AD) in Ganitasarasamgraha [3]. The first method can be defined as- Having squared the last digit, multiply the rest by the digits by twice the last, which is moved forward (by one place). Then moving the remaining digits to continue the same operation for calculating square. As quoted by Mahavira, the multiplication of two equal quantities; or the multiplication of the two quantities obtained (from the given quantity) by the subtraction (therefrom) and the addition of any chosen quantity, together with the addition of the square of that chosen quantity (to that product) or the sum of a series in arithmetic progression, of which 1 is the first term, 2 is common difference, and the number of terms wherein is that (of which the square is) required: gives rise to the (required) square.

The third method quoted by Mahavira is stated as- The square of numbers consisting of two or more places is (equal to) the sum of the squares of all the numbers (in all the places) combined with twice the product of those (numbers) taken (two at a time) in order. Get the square of the last figure (in the number, the order of counting the figures being from right to left), and then multiply this last (figure), after it is doubled and pushed on (to the right by one notational place), by (the figures found in) the remaining places. Each of the remaining figures (in the number) are to be pushed on (by one place) and then dealt with similarly.

Referring to Lilavati by Bhaskara II [4], three methods for calculating squares has been discussed. The product of a number with itself is called its square.

*samadvighatah krtirucyatetha sthapyontyavom dvigunantyanighnah
svasvoparisthacca tathapare nkastyaktvantyamutsarya punasca rasim*

To square a number- first write the square of the extreme left hand digit on its top. Then multiply the next digit by the double of first digit and write the result on the top. Next multiply the third digit by the double of the first digit and write the result on the top. In this way, we arrive at the unit's place. Next cross the first digit and shift the number so formed one place to the right. Then, we repeat the same procedure. Finally, we add all the products written at the top and the sum is the required square.

The second method is

*khandaddhayasyabhihatirdvinighni tatkhandaavagaikyayuta krtirva
istonayugrasivadhah krtih syadistasya vargena samanvito va*

Split the given number into two parts. To the sum of the squares of the two parts, we add twice the product of the two parts.

The third method is -Add and subtract a suitable number from the given number. Take the product of the two numbers thus obtained and add the square of the suitable number chosen above. The result is the required square.

*sakhe navanam ca caturdasanam bruhii trihinasya satatrayasya
pamcottarasyapyayutasya varga janasi cedvargavicaramargam*

And on the basis of the methods of finding squares, we find the squares of 9, 14, 297 and 100005.

2 Preliminaries

Looking into the approach used in Vedic mathematics for calculation of Square, various Sutras as quoted by Swami Bharati Krishna Tirathji Maharaj [5] are available for calculation. In this subsection of the paper, we present the various sutras which will bring deeper ideas for the main aims of this paper.

2.1 Ekadhikena Purvena sutra

The meaning of this sutra is "One more than the previous one". This sutra can be applied for calculating squares of numbers ending with 5. For example 85, 985, 65 etc. The details of the calculation is depicted in Instance 1.

Instance-1: 75^2

$$\text{LHS} = 7 \times (7 + 1) = 7 \times 8 = 56$$

$$\text{RHS} = 5^2 = 25$$

LHS/RHS

$$56/25$$

$$\text{Answer} = 5625.$$

2.2 Sankalana Vyavakalanabhyam sutra

The meaning of this sutra is "By addition and subtraction". This sutra can be applied for calculating squares of numbers either one above or below base value. For example 101, 999, 401, 899, 601, 801 etc. The details of the calculation are depicted in Instance 2.

Instance-2: 399^2 and 401^2

The given number 399 and 401 is close to 400.

$$399^2 = 400^2 - 400 - 399 = 159201$$

$$401^2 = 400^2 + 400 + 401 = 160801.$$

2.3 Duplex method/Dvandvayoga sutra

This approach can be applied for calculating squares of any number. The details of the calculation are depicted in Instance 3.

Instance-3: 491^2

$$D(4)|D(49)|D(491)|D(91)|D(1)$$

$$4^2|2*4*9|9^2 + 2*4*1|2*9*1|1^2$$

$$16|72|81 + 8|18|1$$

$$16|72|89|18|1$$

$$16|72|89 + 1|8|1$$

$$16|72 + 9|0|8|1$$

$$16 + 8|1|0|8|1$$

$$24|1|0|8|1$$

$$\text{Answer} = 241081.$$

2.4 Nikhilam Navtascharam Dashtah

The meaning of this sutra is "All from 9, last from 10". This method can be used for squaring values near base value. The details of the calculation are depicted in Instance-4.

Instance-4: 99^2

As this value is close to 100, it can be written as

$$99 - 1/01 * 01$$

$$98/01$$

$$\text{Answer} = 9801$$

3 Proposed methodology

The method presented in this paper draws an inspiration from the Vedic Sutras discussed above and tries to come off with limitations. Like Ekadhikena Purvena Sutra, our proposed method splits the calculation hence

simplifying the steps. Moreover, the limitation of Ekadhikena Purvena comes off, where it can be applied only for squaring the numbers ending with 5 but our approach can be applied for calculating squares of any number. Like Nikhilam Navatascharam Dashatah, our method splits calculation into LHS and RHS and also takes reference of the series value calculation for LHS. However, our approach can be applied for calculation of square of number consisting of any number of digits unlike Nikhilam Navatascharam Dashatah which can be applied only for values close to base value. Like Duplex, our method splits the values for calculating squares and can be applied for calculating squares of any number. However, when dealing with bigger values, the calculation steps involved in Duplex become complex. This shortcoming is handled by our method, where the square is calculated only for partial value.

In this paper, we are presenting a novel approach for calculation of square which can be considered as an extension of the existing methods and reduces the number of steps of calculation. This approach can be applied to any number. The methodology for finding squares is as follows:

1. Consider any number P for square calculation with n positions

$$P = p_n p_{n-1} p_{n-2} p_{n-3} p_{n-4} p_{n-5} \dots p_3 p_2 p_1 p_0.$$

2. Calculate length $L = \text{length of number } P$.
3. Identify the range value of P in terms of ones position and locate the position of the given number.
4. Split the calculations into LHS and RHS.
5. RHS part = Take two last positions (i.e. $p_1 p_0$ from right and calculate the square using Nikhilam Navatascharam Dashatah, Ekadhikena Purvena or Duplex Method).
6. LHS part = It consists of two segments i.e. Initial value and Series Value
 - 6.1 Initial value = $IV = (p_n p_{n-1} p_{n-2} p_{n-3} p_{n-4} p_{n-5} \dots p_3 p_2 p_1)^2$
 - 6.2 Series values Series value = $SV = \text{Calculate first term and difference for the series.}$
 - 6.2.1 First term = $a_0 = p_1^2$
 - 6.2.2 Difference = $d = 2 \times (p_n p_{n-1} p_{n-2} p_{n-3} p_{n-4} p_{n-5} \dots p_3 p_2)$
 - 6.2.3 $a_n = -a_0 + (n-1)d$
7. LHS part = $IV + SV$.
8. Merge together LHS|RHS. Keep only right most two places in RHS and remaining will be taken as carry and added to LHS.

4 Applications

On the basis of the proposed approach, applications are discussed for a clear understanding. Application 1 for 3 digit number, Application 2 for 4 digit number, Application 3 for 5 digit number, Application 4 for 6 digit number is solved using the proposed methodology. The value of n depends on the number of digits in the given value.

4.1 Application 1.

998^2 (3 digit)

Let $P = 998$,

Position values $p_2 = 9, p_1 = 9, p_0 = 8$,

Length of number $L = 3$

Range of values = 990 to 999

Position of digit = 9^{th} in the given range

RHS = $(p_1 p_0)^2 = (98)^2$

Table 1 Table summarizes calculation for series of values using proposed methodology.

Value	LHS (Initial Value) $(p_2 p_1)^2$	LHS(Series Value) $(a_0 + nd)$	LHS= Initial Value+ Series Value	RHS $(p_1 p_0)^2$	Answer
140^2	$10 * 14^2 = 196$	$a_0 = -4^2 = -16$	$19616 = 180$	$40^2 = 1600$	$180 1600 = 196 00$
141^2	$10 * 14^2 = 196$	$a_0 + d = -4^2 + 2 = -14$	$196 - 14 = 182$	$41^2 = 1681$	$182 1681 = 198 81$
142^2	$10 * 14^2 = 196$	$a_0 + 2d = -4^2 + 2 * 2 = -12$	$196 - 12 = 184$	$42^2 = 1764$	$184 1764 = 201 64$
143^2	$10 * 14^2 = 196$	$a_0 + 3d = -4^2 + 3 * 2 = -10$	$196 - 10 = 186$	$43^2 = 1849$	$186 1849 = 204 49$
144^2	$10 * 14^2 = 196$	$a_0 + 4d = -4^2 + 4 * 2 = -8$	$196 - 8 = 188$	$44^2 = 1936$	$188 1936 = 207 36$
145^2	$10 * 14^2 = 196$	$a_0 + 5d = -4^2 + 5 * 2 = -6$	$196 - 6 = 190$	$45^2 = 2025$	$190 2025 = 210 25$
146^2	$10 * 14^2 = 196$	$a_0 + 6d = -4^2 + 6 * 2 = -4$	$196 - 4 = 192$	$46^2 = 2116$	$192 2116 = 213 16$
147^2	$10 * 14^2 = 196$	$a_0 + 7d = -4^2 + 7 * 2 = -2$	$196 - 2 = 194$	$47^2 = 2209$	$194 2209 = 216 09$
148^2	$10 * 14^2 = 196$	$a_0 + 8d = -4^2 + 8 * 2 = 0$	$196 - 0 = 196$	$48^2 = 2304$	$196 2304 = 219 04$
149^2	$10 * 14^2 = 196$	$a_0 + 9d = -4^2 + 9 * 2 = 2$	$196 + 2 = 198$	$49^2 = 2401$	$198 2401 = 222 01$
150^2	$10 * 15^2 = 225$	$a_0 = -5^2 = -25$	$22525 = 200$	$50^2 = 2500$	$200 2500 = 225 00$
151^2	$10 * 15^2 = 225$	$a_0 + d = -5^2 + 2 = -23$	$225 - 23 = 202$	$51^2 = 2601$	$202 2601 = 228 01$
152^2	$10 * 15^2 = 225$	$a_0 + 2d = -5^2 + 2 * 2 = -21$	$225 - 21 = 204$	$52^2 = 2704$	$204 2704 = 231 04$
153^2	$10 * 15^2 = 225$	$a_0 + 3d = -5^2 + 3 * 2 = -19$	$225 - 19 = 206$	$53^2 = 2809$	$206 2809 = 234 09$
154^2	$10 * 15^2 = 225$	$a_0 + 4d = -5^2 + 4 * 2 = -17$	$225 - 17 = 208$	$54^2 = 2916$	$208 2916 = 237 16$
155^2	$10 * 15^2 = 225$	$a_0 + 5d = -5^2 + 5 * 2 = -15$	$225 - 15 = 210$	$55^2 = 3025$	$210 3025 = 240 25$
156^2	$10 * 15^2 = 225$	$a_0 + 6d = -5^2 + 6 * 2 = -13$	$225 - 13 = 212$	$56^2 = 3136$	$212 3136 = 243 36$
157^2	$10 * 15^2 = 225$	$a_0 + 7d = -5^2 + 7 * 2 = -11$	$225 - 11 = 214$	$57^2 = 3249$	$214 3249 = 246 49$
158^2	$10 * 15^2 = 225$	$a_0 + 8d = -5^2 + 8 * 2 = -9$	$225 - 9 = 216$	$58^2 = 3364$	$216 3364 = 249 64$
159^2	$10 * 15^2 = 225$	$a_0 + 9d = -5^2 + 9 * 2 = -7$	$225 - 7 = 218$	$59^2 = 3481$	$218 3481 = 252 81$
160^2	$16^2 = 256$	$-6^2 = -36$	$256 - 36 = 220$	$60^2 = 3600$	$220 3600 = 256 00$

Here either Duplex method/ Nikhilam Navtascharma can be used for calculating square.

Use Duplex method for calculating $(98)^2$

$$(98)^2 = D(9)|D(99)|D(9) = 9^2|2 * 9 * 9|9^2 = 9801$$

SV calculation

$$\text{First Term} = a_0 = p_1^2 = 9^2 = 81$$

$$\text{Difference} = d = 2 \times (p_n, \dots, p_3, p_2) = 2 \times 9 = 18$$

Position of digit = 9 in the given range.

$$a_n = -a_0 + (n - 1)d$$

$$a_9 = -081 + 8 \times 18 = 63$$

$$IV = 99^2$$

$$\text{LHS part} = IV + SV = 980163 = 9864$$

$$\text{RHS part} = 9604,$$

$$\text{LHS} | \text{RHS}$$

$$9864|9604 = 9864 + 96|04 = 996|004$$

$$\text{Thus } 998^2 = 996004.$$

The summarizing of the details of calculation of series of values from 140 to 160 is presented in Table 1.

4.2 Application 2.

$$1042^2 \text{ (4 digit)}$$

$$\text{Let } P = 1042$$

$$\text{Position of values } p_3 = 1, p_2 = 0, p_1 = 4, p_0 = 2$$

$$\text{Length of number } L = 4$$

$$\text{Range of value} = 1040 \text{ to } 1049$$

$$\text{Position of digit} = 3^{\text{rd}} \text{ in the given range.}$$

$$\text{RHS} = (p_1 p_0)^2 = (42)^2$$

Use Duplex method for calculating $(42)^2$

$$(42)^2 = D(4)|D(42)|D(2)$$

$$4^2|2 * 4 * 2|2^2 = 16|16|4 = 1764.$$

$$\text{LHS} = \text{IV} + \text{SV}$$

$$\text{IV} = (p_3 p_2 p_1)^2 = 104^2$$

$$104^2 = D(1)|D(10)|D(104)|D(04)|D(4)$$

$$1|2*1*0|0^2 + 2*1*4|2*0*4|4^2 = 10816.$$

SV calculation

$$\text{First Term} = a_0 = p_1^2 = 4^2 = 16$$

$$\text{Difference} = d = 2 \times (p_3 p_2) = 2 \times 10 = 20.$$

Range of value = 1040 to 1049.

Position of digit = 3rd in the given range.

$$a_n = -a_0 + (n-1)d$$

$$a_3 = -16 + 2 \times 20 = 24$$

$$\text{LHS part} = \text{IV} + \text{SV} = 10816 + 24 = 10840$$

$$\text{RHS part} = 1764$$

LHS|RHS

$$10840|1764 = 10840 + 17|64 = 10857|64$$

$$\text{Thus } 1042^2 = 1085764.$$

4.3 Application 3.

$$61879^2 \text{ (5 digit)}$$

$$\text{Let } P = 61879$$

$$\text{Position of values } p_4 = 6, p_3 = 1, p_2 = 8, p_1 = 7, p_0 = 9$$

$$\text{Length of number } L = 5$$

$$\text{Range of value} = 61870 \text{ to } 61879$$

Position of digit = 10th in the given range.

$$\text{RHS} = (p_1 p_0)^2 = (79)^2$$

Use Duplex method for calculating $(79)^2$

$$(79)^2 = D(7)|D(79)|D(9)$$

$$7^2|2*7*9|9^2 = 49|126|81 = 6241.$$

$$\text{LHS} = \text{IV} + \text{SV}$$

$$\text{IV} = (p_4 p_3 p_2 p_1)^2 = 6187^2$$

$$6187^2 = D(6)|D(61)|D(618)|D(6187)|D(187)|D(87)|D(7)$$

$$36|2*1*6|1^2 + 2*6*8|2(1*8 + 6*7)|8^2 + 2*1*7|2*8*7|7^2 = 38278969.$$

SV calculation:

$$\text{First Term} = a_0 = p_1^2 = 7^2 = 49$$

$$\text{Difference} = d = 2 \times (p_4 p_3 p_2) = 2 \times 618 = 1236$$

Position of digit = 10th in the given range.

$$a_n = -a_0 + (n-1)d$$

$$a_{10} = -49 + 9 \times 1236 = 11075$$

$$\text{LHS part} = \text{IV} + \text{SV} = 38278969 + 11075 = 38290044.$$

$$\text{RHS part} = 6241$$

LHS|RHS

$$38290044|6241 = 38290044 + 62|41 = 38290106|41$$

$$\text{Thus } 61879^2 = 3829010641.$$

4.4 Application 4.

$$444444^2 \text{ (6 digit)}$$

$$\text{Let } P = 444444$$

$$\text{Position of values } p_5 = p_4 = p_3 = p_2 = p_1 = p_0 = 4$$

$$\text{Length of number } L = 6$$

$$\text{Range of value} = 444440 \text{ to } 444449$$

Position of digit = 5th in the given range.

$$\text{RHS} = (p_1 p_0)^2 = (44)^2$$

Use Duplex method for calculating $(44)^2$

$$(44)^2 = D(4)|D(44)|D(4)$$

$$4^2|2 \times 4 \times 4|4^2 = 16|32|16 = 1936.$$

$$\text{LHS} = +\text{IV} + \text{SV}$$

$$\text{IV} = (p_4 p_3 p_2 p_1)^2 = 44444^2$$

$$44444^2 = D(4)|D(44)|D(444)|D(4444)|D(44444)|D(4444)|D(444)|D(44)|D(4) = 1975269136.$$

SV calculation

$$\text{First Term} = a_0 = p_1^2 = 4^2 = 16.$$

$$\text{Difference} = d = 2 \times p_4 p_3 p_2 = 2 \times 4444 = 8888.$$

$$\text{Range of value} = 444440 \text{ to } 444449$$

Position of digit = 5th in the given range.

$$a_n = -a_0 + (n-1)d, a_5 = -16 + 4 \times 8888 = 35536.$$

$$\text{LHS part} = \text{IV} + \text{SV} = 1975269136 + 35536 = 1975304672.$$

$$\text{RHS part} = 1936$$

LHS|RHS

$$1975304672|1936 = 1975304672 + 19|36$$

$$\text{Thus } 44444^2 = 197530469136.$$

Our proposed method can be extended for calculation of squares of bigger values and nested calculations can be implemented to make calculation easier.

4.5 Application 5.

61879² (5 digit Nested Calculation)

$$6187^2 + (-49 + 2 \times 618)|79^2$$

$$= 618^2 + 11075|6241$$

$$= 618^2 + 790|87^2 + 11075|6241 \text{ (Calculate } 618^2 \text{)}$$

$$= 61^2 + 95|18^2 + 790|87^2 + 11075|6241 \text{ (Calculate } 618^2 \text{)}$$

$$= 3721 + 95|324 + 790|7569 + 11075|6241$$

$$= 3816|324 + 790|7569 + 11075|6241 \text{ (Add overflow from RHS to LHS)}$$

$$= 3819|24 + 790|7569 + 11075|6241$$

$$= 381924 + 790|7569 + 11075|6241$$

$$= 382714|7569 + 11075|6241 \text{ (Add overflow from RHS to LHS)}$$

$$= 382789|69 + 11075|6241$$

$$= 38278969 + 11075|6241$$

$$= 38290044|6241 \text{ (Add overflow from RHS to LHS)}$$

$$= 38290106|41.$$

$$\text{Thus } 61879^2 = 3829010641.$$

All the applications discussed above present a unique approach. The method presented in this paper draws inspiration from the Vedic sutras discussed above and tries to come off with the limitations. Like Ekadhikena Purvena Sutra, our proposed method splits the calculation hence simplifying the steps. Moreover, the limitation of Ekadhikena Purvena occurs, where it can be applied only for squaring the numbers ending with 5 but our approach can be applied for calculating squares of any number. Like Nikhilam Navatascharam Dashatah, our method splits calculation into LHS and RHS and also takes reference of the series value calculation for LHS. However, our approach can be applied for calculation of square of number consisting of any number of digits unlike Nikhilam Navatascharam Dashatah which can be applied only for values close to base value. Like Duplex, our method splits the values for calculating squares and can be applied for calculating squares of any number. However, when dealing with bigger values, the calculation steps involved in Duplex become complex. This shortcoming is handled by our method, where the square is calculated only for partial value.

5 Conclusion

In this paper, we have summarized the different methods of calculating square quoted by various ancient Indian mathematicians. Besides that, we have also summarized the various Sutras for calculating squares as quoted by P. Swami Bharati Krishna Tirathji Maharaj. Moreover, we have presented a novel computational approach for calculating square by taking into consideration the various sutras. An effort has been done to remove the limitations of the Sutras in our proposed method. Our method splits the calculation into LHS and RHS which makes the computation simpler. Moreover, nested calculations can be applied for handling squares of bigger values. New methods of square can be explored based on the existing work. Moreover, the same can be analysed so as to understand its application for other arithmetic operations.

6 Declarations

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The authors hereby declare that there is no conflict of interests regarding the publication of this paper.

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6.3 Author's contributions:

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All data that support the findings of this study are included within the article.

6.6 Using of AI tools:

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