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Original Study

Dispersive modified Benjamin-Bona-Mahony and Kudryashov-Sinelshchikov equations: non-topological, topological, and rogue wave solitons

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Abstract

This study delves into the exploration of three distinct envelope solitons within the nonlinear dispersive modified Benjamin Bona Mahony (NDMBBM) equation, originating from seismic sea waves, and the Kudryashov-Sinelshchikov (KS) equation. The solitons emerge naturally during the derivation process, and their existence is scrutinized using the ansatz approach. The findings reveal the presence of non-topological (bright), topological (dark) solitons, and rogue wave (singular) solitons, presenting significant applications in applied research and engineering. Additionally, two-dimensional and three-dimensional revolution plots are employed with varying parameter values to scrutinize the physical characteristics of these solitons.

Keywords: Nonlinear dispersive modified Benjamin-Bona-Mahony equation, KS equation, non-topological solitons, rogue waves, topological solitons.
AMS 2020 codes: 35C07; 35D; 45K; 65M; 45K05.

1 Introduction

Nonlinear differential equations arise in modeling several physical phenomena. These equations help handle the dynamics of the specific models [1–3]. Particularly interesting circumstances are modelled by using nonlinear partial differential equations (NPDEs). Nonlinear wave phenomena are studied in a wide range of scientific and engineering fields, such as optical fibers, computational fluid dynamics, plasma physics, solid-state physics chemical dynamics, biological, and chemical-physical science, geochemistry, and shallow waves [4–6]. Nonlinear wave processes involving dissipation, dispersion, responses, convection, and propagation are crucial in understanding nonlinear wave equations [7–9]. Many approaches have been applied in recent years to examine nonlinear problems. Some of these are the Jacobi elliptic function method [10–12], the sub-equation approach [13], the exponential rational function method [14], the new extended direct algebraic method [15] as well as symmetry approaches [16–21], and others [22–26]. Solitons, which are nonlinear confined solitary waves that preserve their shape while traveling at a constant speed, are a part of numerous nonlinear physical

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systems. The theory of soliton propagation has been utilized for many important oceanic phenomena, including that of the dispersion of nonlinear shallow as well as deep sea waves. In addition to gravity waves in a related domain and nonlinear wave motion rescaling, the wave motion includes wave propagation in dispersive media such as gas bubbles, liquid flow in lakes, rivers, ocean, and fluid flow in elastic tubes. This type of evolution equation may have a substantial impact on studies of fluid flow and ocean wave motion. It is a common practice to use nonlinear evolution equations to simulate ocean and atmospheric dynamics. Ocean engineering is concerned with the size and frequency of the waves in the ocean as well as their temperature and density (such as tsunamis and wind waves). The nonlinear evolution equations [27–30] can be used to describe nonlinear sea wave propagation. One of the main manifestations of nonlinear wave phenomena is the spread of nonlinear waves in a dynamical system, which has attracted increasing attention.

In this study, the rogue wave, topological, and non-topological soliton solutions are derived by utilizing the solitary wave ansatz method. We categorically acknowledge the reliability of this approach, which has recently been successfully applied. The improved simple equation method was utilized by Khan et al. [31] to identify accurate traveling wave solutions for the coupled Klein-Gordon equations and the NDMBBM equation. To create novel, accurate solutions to the NDMBBM problem and the mKdV-Burgers equation, Filiz and Arzu [32] used the consistent Riccati expansion approach. Kumar et al. [33] used the sinh-Gordon function method to validate the stability analysis and discover numerical and fresh closed-form trigonometric function solutions of the NDMBBM equation. They carried out numerical simulations of the discovered solutions using the proper parameters. The exact and numerical approximations of the nonlinear NDMBBM equation were also tackled using the finite forward difference method. To evaluate the stability of the numerical technique, they used the Fourier-Von Neumann analysis. In a table format, they presented the outcomes for both the numerical and exact solutions.

The rest of the paper is organized as follows. Section 2 presents the ansatz approach to NDMBBM equation. Section 3 describes the ansatz approach to KS equation. The models, which includes the development of topological, non-topological, and rogue wave soliton solutions are also introduced. Section 4 introduces the physical interpretation of the results. Finally, the novelties of this paper are reported through the discussion and conclusions in Section 5.

2 Ansatz approach to NDMBBM equation

The nonlinear NDMBBM equation is defined as follows

$$Y_t + Y_x - \alpha Y^2 Y_x + Y_{xxx} = 0, \tag{1}$$

where α is a nonzero real constant. The nonlinear NDMBBM equation was originally formulated to model surface wave propagation in nonlinear dispersive media, but it also describes hydromagnetic waves in plasma, acoustic waves in unique form crystals, and acoustic gravitational waves in compressible fluids [34].

Solitons and their topological characteristics provide an intriguing subject of study. Dark solitons, labeled as topological solitons, and bright solitons, termed non-topological solitons, showcase unique traits. This topological classification enhances our grasp of these phenomena, finding applications in diverse physical systems. The interplay between topology and solitons opens exciting avenues for investigating and controlling complex structures across various scientific domains.

2.1 Non-topological (bright) soliton solutions

The following non-topological wave ansatz [35] has been used in this section to find non-topological solutions for the equation (1),

$$Y(x,t) = \frac{E}{\cosh^p(\kappa)}, \tag{2}$$

in which $\kappa = C(x - vt)$. In the aforementioned equations, the variables E and C are referred to as unconstrained parameters. E stands for amplitude of soliton, C is the inverse width, and v denotes the speed. Later, the homogeneous balance principle is used to determine how to evaluate the exponent p. From (2), it is possible to follow

$$Y^n(x,t) = \frac{E^n}{\cosh^{pn}(\kappa)}, \tag{3}$$

so that

$$(Y(x,t))_t = ECpv\frac{\tanh(\kappa)}{\cosh^p(\kappa)}, \tag{4}$$

$$(Y(x,t))_x = -ECp\frac{\tanh(\kappa)}{\cosh^p(\kappa)}, \tag{5}$$

and

$$(Y(x,t))_{xxx} = EC^3p\tanh(\kappa)\left(-\frac{p^2}{\cosh^p(\kappa)} + \frac{(p+1)(p+2)}{\cosh^{p+2}(\kappa)}\right). \tag{6}$$

Eq (1) undergoes

$$ECpv\frac{\tanh(\kappa)}{\cosh^p(\kappa)} - ECp\frac{\tanh(\kappa)}{\cosh^p(\kappa)} + \alpha\left(\frac{E}{\cosh^p(\kappa)}\right)^2 ECp\frac{\tanh(\kappa)}{\cosh^p(\kappa)} + EC^3p\tanh(\kappa)\left(-\frac{p^2}{\cosh^p(\kappa)} + \frac{(p+1)(p+2)}{\cosh^{p+2}(\kappa)}\right) = 0.$$

Through the division of $ECp\tanh(\kappa)$ and subsequent manipulation, we reach the following result

$$\frac{v-1-C^2p^2}{\cosh^p(\kappa)} + \left(\frac{\alpha E^2}{\cosh^{3p}(\kappa)}\right) + \frac{C^2(p+1)(p+2)}{\cosh^{p+2}(\kappa)} = 0. \tag{7}$$

By applying the homogeneous balancing principle, we may assume that $3p = p+2$ and $C^2(p+1)(p+2) \neq 0$. This results in $p = 1$. The following system is obtained from (7) as

$$\begin{aligned} v - 1 - C^2p^2 &= 0, \\ \alpha E^2 + C^2(p+1)(p+2) &= 0. \end{aligned} \tag{8}$$

We solve the aforementioned system of equations for $p = 1$ and obtain

$$v - 1 - C^2 = 0, \quad \alpha E^2 + 6C^2 = 0. \tag{9}$$

For $\alpha < 0$, we can compute the velocity of the soliton,

$$v = 1 - C^2. \tag{10}$$

Subsequently, we ascertain the amplitude (E) of the soliton.

$$E = C\sqrt{-\frac{6}{\alpha}}. \tag{11}$$

Hence, the NDMBBM equation (1) possesses a bright soliton solution

$$Y(x,t) = \frac{C\sqrt{-\frac{6}{\alpha}}}{\cosh\left(C(x-(1-C^2)t)\right)}. \tag{12}$$

2.2 Rogue wave solutions

In this section, the rogue wave solutions for (1) have been generated using the wave ansatz [35]

$$Y(x,t) = \frac{E}{\sinh^p(\kappa)}, \tag{13}$$

in which $\kappa = C(x - vt)$. In equation (13), the variables E and C are denoted as unconstrained parameters. Here, E represents the wave's amplitude, C signifies their inverse width, and v indicates their speed. Subsequently, the homogeneous balance principle is employed to establish the evaluation of the exponent p. The progression can be traced from (13)

$$(Y(x,t))_t = ECpv\frac{\coth(\kappa)}{\sinh^p(\kappa)}, \tag{14}$$

$$(Y(x,t))_x = -ECp\frac{\coth(\kappa)}{\sinh^p(\kappa)}, \tag{15}$$

and

$$(Y(x,t))_{xxx} = EC^3p\coth(\kappa)\left(-\frac{p^2}{\sinh^p(\kappa)} + \frac{(p+1)(p+2)}{\sinh^{p+2}(\kappa)}\right). \tag{16}$$

In consideration of this, equation (1) can be expressed as

$$\begin{aligned} ECpv\frac{\coth(\kappa)}{\sinh^p(\kappa)} - ECp\frac{\coth(\kappa)}{\sinh^p(\kappa)} + \alpha\left(\frac{E}{\sinh^p(\kappa)}\right)^2 ECp\frac{\coth(\kappa)}{\sinh^p(\kappa)} + \\ EC^3p\coth(\kappa)\left(-\frac{p^2}{\sinh^p(\kappa)} + \frac{(p+1)(p+2)}{\sinh^{p+2}(\kappa)}\right) = 0. \end{aligned} \tag{17}$$

Upon dividing by $ECp\tanh(\kappa)$, we obtain

$$\begin{aligned} \frac{v}{\sinh^p(\kappa)} - \frac{1}{\sinh^p(\kappa)} + \left(\frac{E}{\sinh^p(\kappa)}\right)^2\frac{\alpha}{\sinh^p(\kappa)} \\ +C^2\left(-\frac{p^2}{\sinh^p(\kappa)} + \frac{(p+1)(p+2)}{\sinh^{p+2}(\kappa)}\right) = 0. \end{aligned} \tag{18}$$

$$\frac{v-1-C^2p^2}{\sinh^p(\kappa)} + \frac{\alpha E^2}{\sinh^{3p}(\kappa)} + \frac{C^2(p+1)(p+2)}{\sinh^{p+2}(\kappa)} = 0. \tag{19}$$

Following the homogeneous balancing principle, we assume that $\alpha E^2 \neq 0$ and $C^2 \neq (p+1)(p+2) \neq 0$ possess equal powers. Consequently, we deduce that $p = 1$. The subsequent system is derived from (19)

$$\begin{aligned} v - 1 - C^2p^2 = 0, \\ \alpha E^2 + C^2(p+1)(p+2) = 0. \end{aligned} \tag{20}$$

Upon solving the aforementioned set of equations for $p = 1$, we reach

$$v - 1 - C^2 = 0, \quad \alpha E^2 + 6C^2 = 0. \tag{21}$$

For $\alpha < 0$, we determine the speed of the wave

$$v = 1 - C^2. \tag{22}$$

Subsequently, we ascertain the amplitude E of the soliton.

$$E = C\sqrt{-\frac{6}{\alpha}}. \tag{23}$$

Therefore, the NDMBBM equation (1) features a rogue wave solution represented by

$$Y(x,t) = \frac{C\sqrt{-\frac{6}{\alpha}}}{\sinh\left(C(x-(1-C^2)t)\right)}. \tag{24}$$

2.3 Topological (dark) soliton solution

In this section, topological solutions to equation (1) have been derived by employing the topological wave ansatz [35]

$$Y(x,t) = E\tanh^{y}(\kappa), \tag{25}$$

in which $\kappa = D(x-vt)$. In the given equations, the variables E and D are denoted as unconstrained parameters. Here, E represents the amplitude of solitons, D signifies their inverse width, and v indicates their velocity. Subsequently, the homogeneous balance principle is employed to establish the evaluation of the exponent y. The progression can be traced from (25)

$$(Y(x,t))_t = -EDyv(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)), \tag{26}$$

$$(Y(x,t))_x = EDy(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)), \tag{27}$$

and

$$\begin{aligned}(Y(x,t))_{xxx} = ED^3y\big((y-1)(y-2)\tanh^{y-3}(\kappa) - (3y^2-3y+2)\tanh^{y-1}(\kappa)\\ +(3y^2+3y+2)\tanh^{y+1}(\kappa) - (y+1)(y+2)\tanh^{y-3}(\kappa)\big).\end{aligned} \tag{28}$$

Eq (1) consequently transforms into

$$\begin{aligned}-EDyv\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big) + EDy\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big)\\ -\alpha EDy\big(E\tanh^{y}(\kappa)\big)^2\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big)\\ +ED^3y\Big((y-1)(y-2)\tanh^{y-3}(\kappa) - (3y^2-3y+2)\tanh^{y-1}(\kappa)\\ +(3y^2+3y+2)\tanh^{y+1}(\kappa) - (y+1)(y+2)\tanh^{y-3}(\kappa)\Big) = 0.\end{aligned} \tag{29}$$

Upon division by EDy, we get

$$\begin{aligned}-v\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big) + \big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big)\\ -\alpha\big(E\tanh^{y}(\kappa)\big)^2\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big)\\ +D^2\Big((y-1)(y-2)\tanh^{y-3}(\kappa) - (3y^2-3y+2)\tanh^{y-1}(\kappa)\\ +(3y^2+3y+2)\tanh^{y+1}(\kappa) - (y+1)(y+2)\tanh^{y-3}(\kappa)\Big) = 0.\end{aligned} \tag{30}$$

We further simplify it in the form

$$\begin{aligned}-v\big(\tanh^{y-1} - \tanh^{y+1}(\kappa)\big) + v\big(\tanh^{y-1}(\kappa) - \tanh^{y+1}(\kappa)\big)\\ -\alpha\big(E\tanh^{y}(\kappa)\big)^2\big(\tanh^{y-1}(\xi) - \tanh^{y+1}(\kappa)\big)\\ +D^2\Big((y-1)(y-2)\tanh^{y-3}(\kappa) - (3y^2-3y+2)\tanh^{y-1}(\kappa)\\ +(3y^2+3y+2)\tanh^{y+1}(\kappa) - (y+1)(y+2)\tanh^{y-3}(\kappa)\Big) = 0.\end{aligned} \tag{31}$$

By separating different powers of tanh, we obtain

$$\begin{aligned}\tanh^{y-1}(\kappa)\big(1-v-D^2(3y^2-3y+2)\big) + \tanh^{y+1}(\kappa)\big(v-1+D^2(3y^2+3y+2)\big)\\ -\alpha E^2\tanh^{3y-1}(\kappa) + \alpha E^2\tanh^{3y+1}(\kappa) + D^2(y-1)(y-2)\tanh^{y-3}(\kappa)\\ -D^2(y+1)(y+2)\tanh^{y+3}(\kappa) = 0.\end{aligned} \tag{32}$$

Given that $\alpha E^2 \neq 0$, $D^2(y+1)(y+2) \neq 0$, and $D^2(y-1)(y-2) \neq 0$, we assume their powers to be equal. By setting $3y-1 = y-3$, we find $y = -1$. However, since y cannot be negative, we explore alternatives. Considering $3y+1 = y+3$, we arrive at $y = 1$. Equation (32) for $y = 1$ implies

$$\left(1 - v - 2D^2\right) + \tanh^2(\kappa)\left(v - 1 + 8D^2 - \alpha E^2\right) + \tanh^4(\kappa)\left(\alpha E^2 - 6D^2\right) = 0. \tag{33}$$

The system derived from (33) is as follows

$$\begin{aligned} 1 - v - 2D^2 &= 0, \\ v - 1 + 8D^2 - E^2\alpha &= 0, \\ \alpha E^2 - 6D^2 &= 0. \end{aligned} \tag{34}$$

The speed of the soliton, for arbitrary values of D and $\alpha > 0$, is

$$v = 1 - 2D^2. \tag{35}$$

Subsequently, we determine the amplitude E of the soliton as

$$E = \pm D\sqrt{\frac{6}{\alpha}}. \tag{36}$$

Therefore, the NDMBBM equation (1) features the dark soliton solution given by

$$Y_1(x,t) = D\sqrt{\frac{6}{\alpha}}\tanh\left(D(x - (1-2D^2)t)\right), \tag{37}$$

and

$$Y_2(x,t) = -D\sqrt{\frac{6}{\alpha}}\tanh\left(D(x - (1-2D^2)t)\right). \tag{38}$$

3 Ansatz approach to KS equation

The KS equation is given by

$$Y_t + \alpha Y Y_x + \beta Y_{xxx} + \gamma (Y Y_x)_{xx} + \sigma Y_x Y_{xx} = 0. \tag{39}$$

The equation under consideration is defined by the constants α, β, γ, and σ, as proposed by Kudryashov and Sinelshchikov to elucidate pressure waves in a mixture of liquid and gas bubbles, accounting for fluid viscosity and heat transfer [36]. In our exploration, we delve into topological, non-topological, and rogue wave solutions for this equation. While Akram et al. [37] previously identified kink, periodic, dark, and bright soliton solutions, our current investigation focuses on dark, bright, and rogue wave solutions. The addition of rogue wave solutions contributes to the richness of the KS equation's theoretical framework.

3.1 Non-topological (bright) soliton solutions

In this section, we have employed the following non-topological wave ansatz to derive solutions for equation (39). The chosen ansatz takes the form

$$Y(x,t) = \frac{E}{\cosh^y(\kappa)}, \tag{40}$$

in which $\kappa = C(x - vt)$. In the provided equations, the variables E and C are denoted as unconstrained parameters. Upon substituting equation (40) into equation (39) and carrying out some manipulation, we reach

$$v - C^2\beta(y^2+6y+4) - \frac{C^2\beta(y+1)(y+2)}{\cosh^2(\kappa)} + \frac{C^2Ey(y+2)+C^2\gamma E(y^2+6y+4)}{\cosh^y(\kappa)} - \frac{C^2Ey(y+1)+C^2\gamma E(y+1)(y+2)}{\cosh^{y+2}(\kappa)} = 0. \tag{41}$$

Applying the homogeneous balancing principle, we assume that $y = 2$ and $C^2(y+1)(y+2) \neq 0$. Upon setting $y = 2$, we obtain equation (41) as

$$v - 20C^2\beta - \frac{12C^2\beta + 8C^2E + 20C^2\gamma E}{\cosh^2(\kappa)} - \frac{6C^2E + 12C^2\gamma E}{\cosh^4(\kappa)} = 0. \tag{42}$$

Consequently, we obtain the following system of equations

$$\begin{aligned} v - 20C^2\beta &= 0, \\ 12C^2\beta + 8C^2E + 20C^2\gamma E &= 0, \\ 6C^2E + 12C^2\gamma E &= 0. \end{aligned} \tag{43}$$

The speed of the soliton, for arbitrary values of C and β, can be calculated as

$$v = -20C^2\beta. \tag{44}$$

Subsequently, we determine the amplitude of the soliton, denoted as E

$$E = -6\beta. \tag{45}$$

Therefore, the bright soliton solution for the KS equation (39) is given by

$$Y(x,t) = -\frac{6\beta}{\cosh^2\left(C(x+20C^2\beta t)\right)}. \tag{46}$$

3.2 Rogue wave solutions

In this section, the rogue wave solutions for equation (39) have been generated using the following wave ansatz

$$Y(x,t) = \frac{E}{\sinh^p(\kappa)}, \tag{47}$$

in which $\xi = C(x - vt)$. In the provided equations, the variables E and C are denoted as unconstrained parameters. Substituting equation (47) into equation (39) and performing some manipulation, we obtain

$$v + C^2\beta p^2 + \frac{C^2\beta(p+1)(p+2)}{\cosh^2(\kappa)} + \frac{E\alpha - C^2Ep^2(3\gamma+\sigma) + EC^2\gamma p^2}{\sinh^p(\kappa)} + \frac{E\gamma C^2(p+1)(p+2) - C^2Ep(p+1)(3\gamma+\sigma)}{\sinh^{p+2}(\kappa)} = 0. \tag{48}$$

Applying the homogeneous balancing principle, we assume that $p = 2$. Consequently, equation (48) transforms into

$$v + 4C^2\beta + \frac{12C^2\beta + E\alpha - 4C^2E\sigma - 8EC^2\gamma}{\sinh^2(\kappa)} - \frac{6EC^2(\gamma+\sigma)}{\sinh^{p+2}(\kappa)} = 0. \tag{49}$$

By comparing powers of $\sinh\kappa$, we obtain the following system from equation (49)

$$v+4C^2\beta=0,\quad 12C^2\beta+E\alpha-4C^2E\sigma-8EC^2\gamma=0,\quad 6EC^2(\gamma+\sigma)=0. \tag{50}$$

The speed of the wave, for arbitrary values of C and α, is obtained as

$$v=4C^2\beta. \tag{51}$$

Subsequently, we determine the amplitude of the rogue wave

$$E=-\frac{12C^2\beta}{\alpha+4C^2\sigma}. \tag{52}$$

Therefore, the rogue wave solution for the KS equation (39) is given by

$$Y(x,t)=-\frac{12C^2\beta}{\alpha+4C^2\sigma}\operatorname{csch}^2\big(C(x+4C^2\beta t)\big). \tag{53}$$

3.3 Topological (dark) soliton solution

By employing the topological wave ansatz explained subsequently, this section has successfully established dark soliton solutions for equation (39). We follow the ansatz

$$Y(x,t)=E\tanh^p(\kappa), \tag{54}$$

in which $\kappa=C(x-\beta t)$. In the given equations, the parameters E and C are labeled as unconstrained variables. Upon substituting equation (54) into equation (39) and conducting some manipulations, we deduce $q=2$ along with the corresponding parameter values

$$v=4C^2\beta. \tag{55}$$

Subsequently, we determine the amplitude of the soliton as

$$E=\frac{16C^2\sigma}{12\gamma+164C^2\gamma+32C^2\sigma}. \tag{56}$$

Therefore, the KS equation (39) features a dark soliton solution given by

$$Y(x,t)=\frac{16C^2\sigma}{12\gamma+164C^2\gamma+32C^2\sigma}\tanh^2\big(C(x+4C^2\beta\, t)\big). \tag{57}$$

4 Physical interpretation

The comprehension of solutions to the NPDEs is essential in order to grasp the underlying physical phenomena. Solitons, maintaining their shape and constant velocity during propagation, play a crucial role in mathematics and physics. Distinguishing between topological (dark) and non-topological (bright) solitons is relevant to the modulation instability of the carrier wave train. Topological solitons arise when the carrier wave is unstable to long-wave modulations, while non-topological solitons emerge when the carrier wave is modulationally stable. Rogue waves, also known as freak or killer waves, have evolved from marine folklore to recognized phenomena. These waves, surpassing twice the size of surrounding waves, are highly unpredictable and often appear unexpectedly from different directions than the prevailing wind and waves. The exploration of rogue waves contributes to a deeper understanding of extreme events in fluid dynamics. The study features Figures 1 and 4 presenting non-topological soliton solutions, Figures 2 and 5 showcasing rogue wave solutions, and Figures 3 and 6 illustrating topological soliton solutions. These visual depictions provide valuable insights

into the physical structures and behaviors of these solitons and rogue waves, enhancing our comprehension of their real-world manifestations.

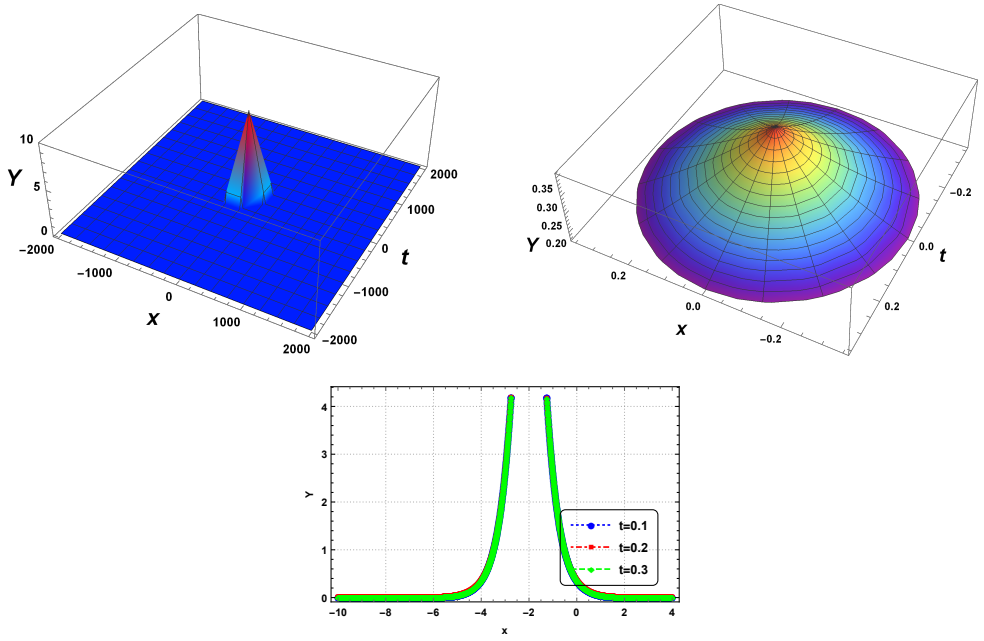


Fig. 1 Non-topological (bright) soliton nature of surface wave propagation by (12) with $C = 1,\ \alpha = -1$ and $t = 0.1, 0.2, 0.3$.

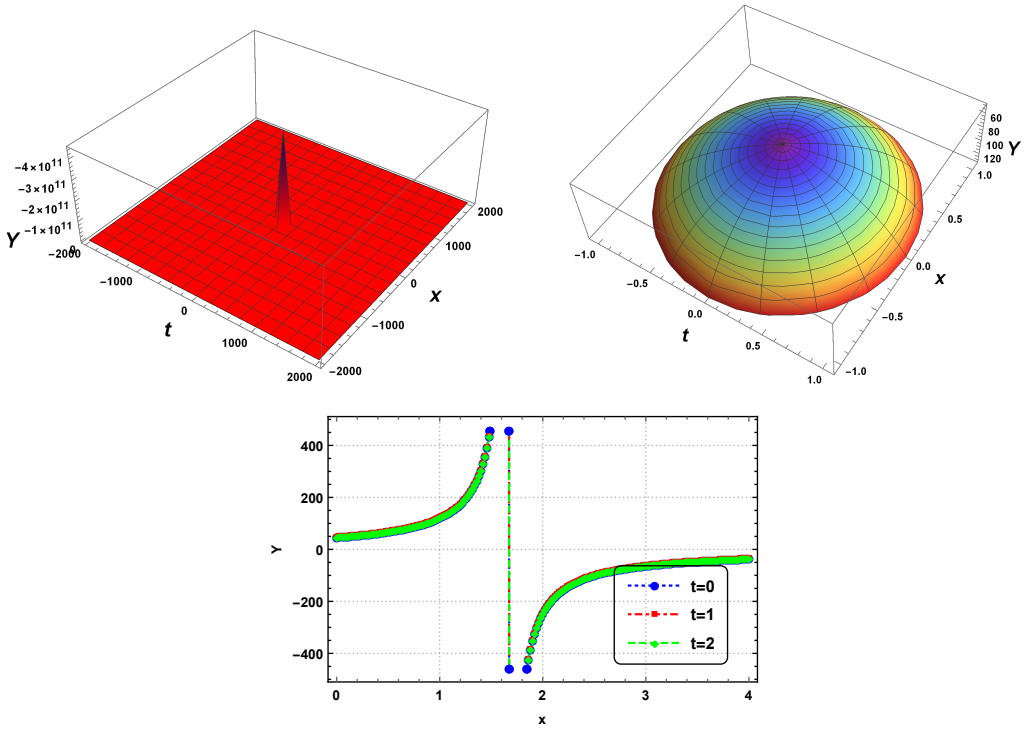


Fig. 2 Rogue wave solutions of the equation (24) with $C = 0.2,\ \alpha = -1$ and $t = 0, 1, 2$.

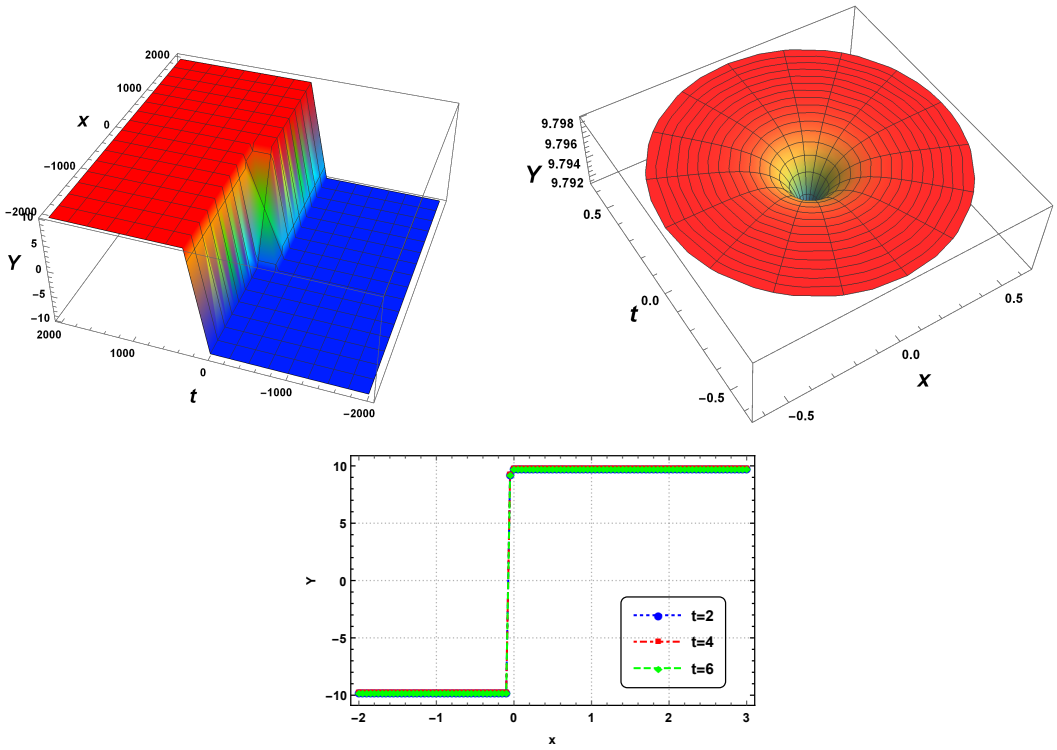


Fig. 3 Topological (dark) soliton nature of surface wave propagation by (37) with $D = 4,\ \alpha = 1$ and $t = 2, 4, 6$.

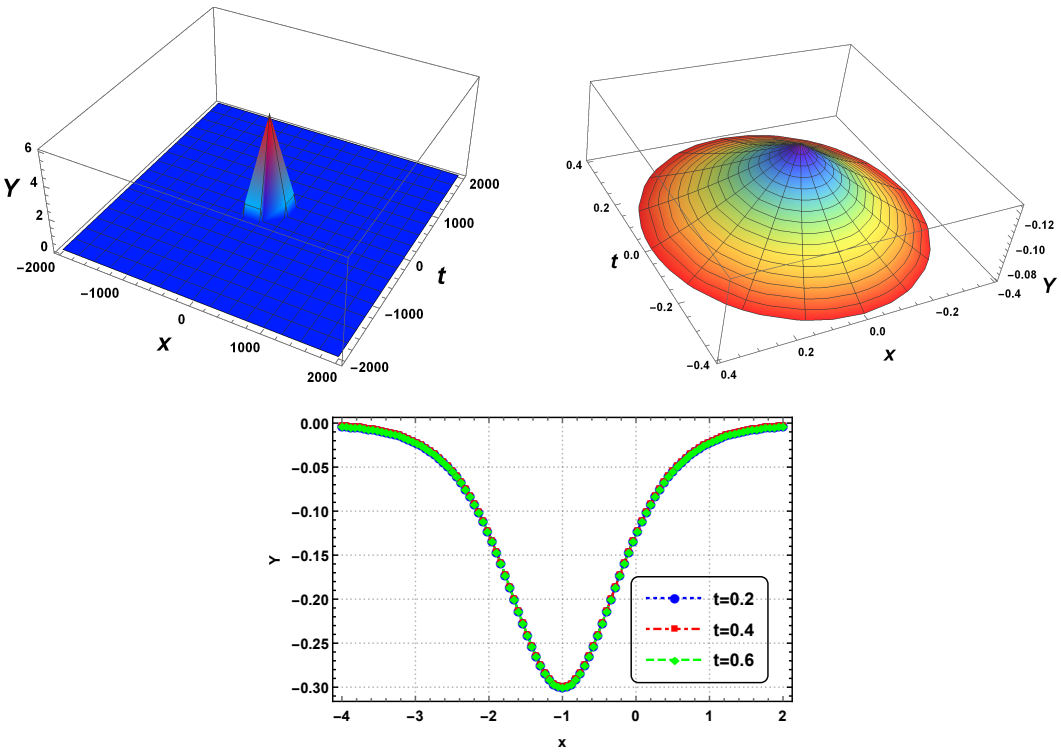


Fig. 4 Non-topological (bright) soliton solutions of the equation (46) with $C = 0.05,\ \beta = 1$ and $t = 0.2, 0.4, 0.6$.

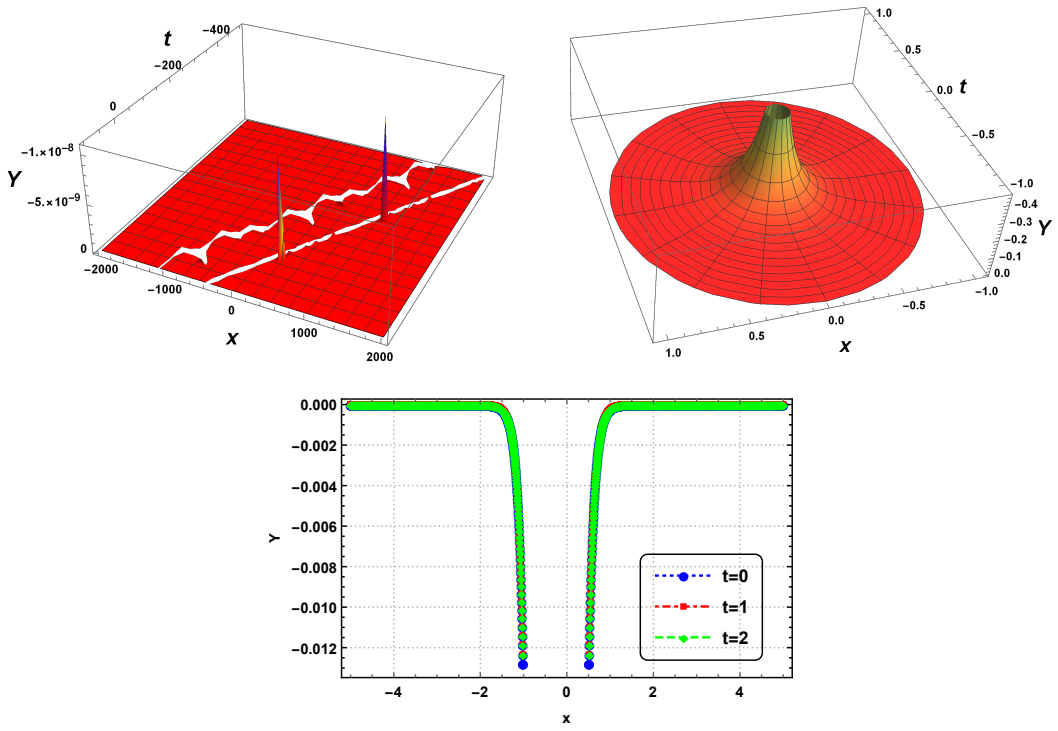


Fig. 5 Rogue wave solutions of the equation (53) with $C = 1,\ \alpha = 1,\ \sigma = 1,\ \beta = 1$ and $t = 0, 1, 2$.

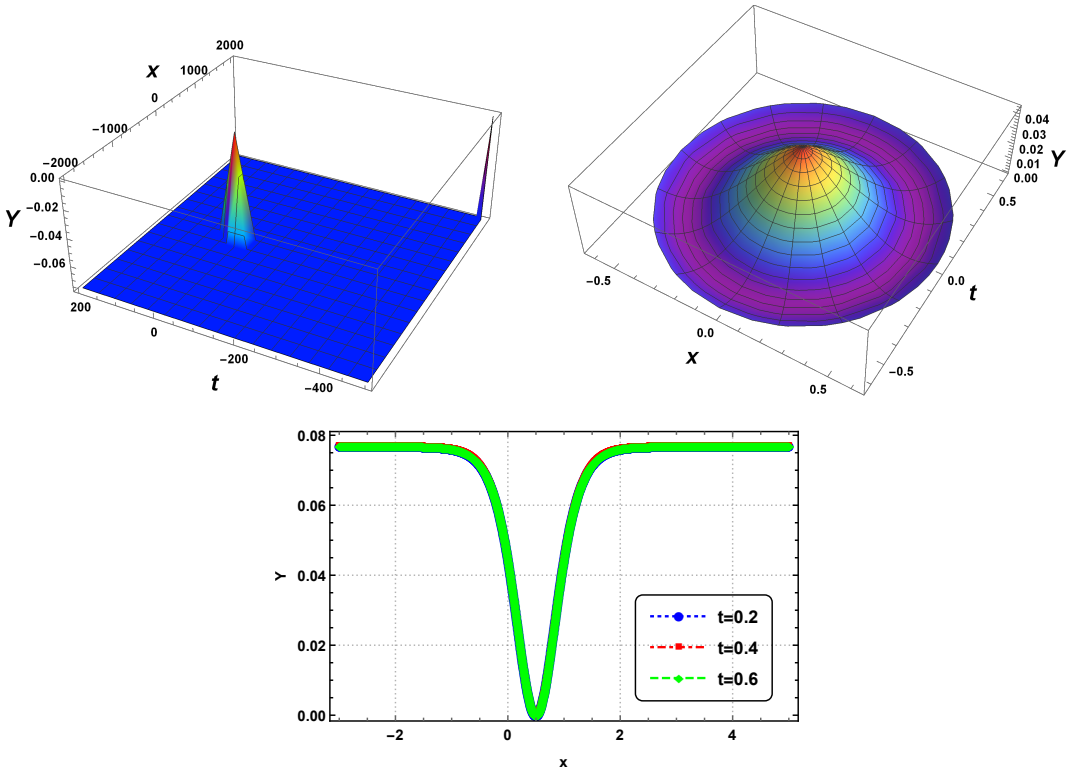


Fig. 6 Topological (dark) soliton solutions of the equation (57) with $C = 1,\ \sigma = 1,\ \gamma = 1,\ \beta = -0.5$ and $t = 0.2, 0.4, 0.6$.

5 Discussion and conclusions

The study of transmissions within dynamic systems is currently a subject of emerging fascination. Utilizing the ansatz approach for the NDMBBM equation and KS equation, we have developed novel elucidations of rogue wave solitons, topological, and non-topological solitons, showing the efficacy of this recently successful method. Existing literature, exemplified by Khan et al. [31], has discussed characteristics of trigonometric and hyperbolic type solutions with dark and bright solitons. Moreover, hyperbolic solutions have been reported by [32, 33]. However, our findings stand out as we discovered dark, bright, and rogue wave solutions, setting them apart from earlier research. Akram et al. [37] identified kink, periodic, dark, and bright soliton solutions, but our study uniquely includes dark, bright, and rogue wave solutions, making the discussion of rogue wave solutions intriguing in the context of the NDMBBM and KS equations. We plan to apply the same ansatz to explore nonlinear evolution equations in mathematical physics for future investigations.

6 Declarations

6.1 Conflict of interests:

The authors declare that there is no conflict of interest regarding the publication of this paper.

6.2 Author's contributions:

M.U.-Methodology, Resources. A.H.-Methodology, Writing-Review, Software and Editing and Supervision. H.A.-Resources, Writing-Original Draft and Methodology. F.Z.-Validation, Supervision, Conceptualization and Formal Analysis. N.A.-Data Curation, Investigation, Plotting and Visualization. All authors read and approved the final submitted version of this manuscript.

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6.5 Data availability statement:

All data that support the findings of this study are included within the article.

6.6 Using of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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