



Original Study

Comparing some plant communities in a region of Türkiye via fuzzy similarity

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Abstract

In this study, the results obtained from forest vegetation via the research project on plant sociology which was conducted in the Black Sea region of Türkiye is evaluated with the help of a fuzzy similarity measures approach. Via this project, the plant sociology in an area which has not been studied in the Black Sea region of Türkiye is performed to investigate the plant communities and ecological and sociological relationships with each other. The similarity relations among the plant communities and relevés (sampling areas) which they covered are investigated. The issue of fuzzy similarity of sets and elements in sets is studied. According to this point of view, the fuzzy similarity among the plant communities and among the relevés is introduced. This joint study is carried out in a fuzzy environment, considering the classical results found in the project in question to be obtained in more detail for application. It is also understood that such studies can only be best performed with an interdisciplinary working group.

Keywords: Fuzzy similarity, plant community, phytosociology, forest vegetation, ecology, metric.

AMS 2020 codes: 92C80; 92D40; 03B52; 92B10.

1 Introduction

The fuzzy theory is one of the most studied fields of science. This theory has been used in various fields of science by Gu et al. [1], Gong et al. [2], Rasham et al. [3] and Kausar et al. [4]. We know that the works of Moraczewski [5,6], Feoli et al. [7], Pedersen et al. [8], Srivastava and Shukla [9], Feoli [10], Rapinel et al. [11], Akdeniz Şafak et al. [12], Bingöl et al. [13], Akdeniz Şafak et al. [14], Akın et al. [15], have examined plant communities in fuzzy environment.

In nature, plants can live individually or in groups. The plant community is a dynamical form and part of vegetation [16]. A plant community can also be defined as the gathering of living plants in mutual and continuous relationship among themselves and with their environment [17]. Plants can be examined and identified both individually and in communities. It is possible to identify plant groups quantitatively and qualitatively. A plant

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community can be identified with its components, phytosociological structure and its characteristics. Therefore, different researchers have developed different methods in different time periods [18–21]. As a result, analysis based on the similarities of plant communities show their better recognition, whether it is a newly identified or already existing one.

While conducting plant sociology studies, it can only be carried out by similarity function whether a newly identified union is a new community for the scientific world or is the same as the existing community found in different regions by different researchers. These degrees of similarity with this new study are in fuzzy logic. It could be specified with more sensitive expressions based on membership degrees. Some mathematical applications in the form of similarity indexes given by Sorensen [22] facilitate the understanding of the degree of relationships among the plant species and the similarity among the plant communities. When comparing two or more plant communities in terms of species composition, similarity or difference indexes provide some quantitative evaluation possibilities [23]. The degree of similarity or difference between any two plant populations identified in the field study can be demonstrated by the cluster analysis method [24]. Further summarizing of vegetation data in any study area can only be achieved by ordination [25, 26], and vegetation mapping [27]. Feoli [28] used fuzzy expressions that are partially present in vegetation tables in different biogeographic areas. Pedersen et al. [8] used fuzzy set clustering to examine the floristic and environmental similarity among relevés, whereas they used fuzzy set ordination to relate floristic patterns to environmental variables. We developed the similarity in our own vegetation tables with the help of fuzzy logic. When Zadeh [29] put forward this logic, he did not refer to whether a statement is true or false, but he referred to the degree of belonging.

In our study, the membership degrees in the tables were prepared according to the presence (1) or absence (0) of plant communities determined in the field with the Braun-Blanquet approach which has been calculated in a fuzzy sense beyond $\{0, 1\}$ logic. In other words, with our work, a classical similarity of plant sociology studies has been extended by fuzzy approach. New outcomes have been obtained in this method. Then, the similarity between fuzzy sets based on the similarity measure was used in order to compare the plant communities with each other. The conclusion that the fuzzy similarity theory can be considered as an extension of the similarity was expressed by Feoli and Orloci [30].

The first use of similarity functions in plant ecology was carried out by Jaccard [31]. Later, it gained significant momentum with the works of Greig-Smith [32], Sokal and Sneath [33], Pielou [34] and Orłóci [35]. The application of the "Similarity Theory (ST)" is now becoming widespread in many disciplines, particularly genetics [36], biomedicine [37], artificial intelligence [38], and forestry sciences [39]. ST has close links with the fuzzy systems theory (FST) as demonstrated by Roberts [40, 41] in plant ecology. For this reason, elaborating the link between ST and FST is the backbone of the study. Our main goal in this study was to compare the membership degrees of 5 communities and 2 sub-communities of forest vegetation, which were determined in the project [42]. That is we clearly demonstrate the similarity degree of them. We achieved this goal by using fuzzy similarity features, not classical logic (Aristotle logic). Another important goal of this study is to understand the nature of plant communities and to arrive at useful classifications of vegetation at different hierarchical levels. With this study, we have made sure that the communities identified in the research field are independent, pure and homogeneous communities. In addition, the fuzzy approach based on similarity and dissimilarity criteria is extremely important in order to create national or international databases of plant communities with correct data. This method is important because repetitive communities will be prevented and correct data flow will be provided. These results help the protection and planning of a country's economy.

The outcomes obtained in this joint study are given in the results section, together with tables and explanations. The region studied in the project [42] on which this study is based covers Sakarat Mountain and its surroundings; forest, steppe and sub-alpin vegetation types were determined in the research area. Here 5 communities and 2 sub-communities belonging to forest vegetation are evaluated. Sakarat Mountain is located within the provincial borders of Amasya and Tokat. It is also located in the transition zone between the Central Anatolia and Middle Black Sea Region.

The conclusions obtained in the previous project on the Sakarat Mountain are generalized with the above

mentioned approach (i.e. fuzzy similarity). The new results are shown in the tables in comparison. Finally, the results achieved and what can be done in the future are explained.

This research paper is organized as follows: Section 2 gives background information about the scheme. Section 3 presents metric 1 with fuzzy similarity of two fuzzy sets. Section 4 introduces Metric 2 with fuzzy similarity of two elements. Section 5 contains some results and discussions about the novelties of this paper. And finally in section 6, the differences between the crisp approximation and fuzzy similarity and the advantages of extending the classical approximation are given.

2 The method

It is known that in the literature some similarity measures between fuzzy sets and between elements of the sets have been used. The new similarity measures are also used in plant communities and relevés of them. Some advantages of using fuzzy similarity measures rather than crisp measures are getting comprehensive knowledge of communities and their elements, making classifications, grouping, checking the accuracy of the results.

The main aim of our joint work is to fuzzify the results obtained in the project [42] of A Phytoecological and Phytosociological Research on the Sakarat Mountain (Amasya). The reason for this is to get some benefits which are explained in the introduction part.

In our work, we have five plant communities and sixty two relevés for extending results obtained in the project [42] mentioned above. Our plant communities are shown as AS_1 (*Pino sylvestri-Fagetum orientali ass.nova*), AS_2 , (*Acero campestri-Pinetum sylvestri ass.nova*), AS_3 , (*Quercu pubescenti-Pinetum sylvestri ass.nova*), AS_4 (*Carpino orientali-Quercetum boissieri ass.nova*), AS_5 (*Rhododendro lutei-Fagetum orientali ass.nova*; AS_{5a} (*carpino orientali-quercetosum macrantherae subass.nova*); AS_{5b} (*vaccinio arctostaphylli-loniceretosum orientali subass.nova*) and relevés as $r_1, r_2, r_3, \dots, r_{62} := r_1(1)r_{62}$. A member is involved in communities with the membership degrees as shown in Table 3.

2.1 Why is fuzzy similarity necessary?

Let us consider plant communities of $\{AS_1, AS_2, AS_3, AS_4, AS_5\}$ and relevés $\{r_1(1)r_{62}\}$ involved in these plants communities. A relevé in $\{r_1(1)r_{62}\}$ is involved in plant communities with the membership degrees as shown in Table 4-A1, Table 4-A2, Table 4-B1 and Table 4-B2.

After the observation on plant communities and relevés, it would be useful to ask the following 2 types of questions in behaviour analysis:

Type 1 question: At what degree can the plant communities $\{AS_1, AS_2, AS_3, AS_4, AS_5\}$ be cooperated? or What is the guaranteed minimum level of gathering between communities AS_i and AS_j ? Where $i, j = 1(1)5$.

Type 2 question: At what degree can the relevés r_i and r_j be the same communities? or What is the level of their friendship? Where $i, j = 1(1)62$.

To answer these two fundamental questions in plant sociology we need to give the following two main fuzzy metrics separately for fuzzy similarity with their properties. The reason for this is to establish the new similarity between plant communities (fuzzy sets) and their relevés (fuzzy elements) (See also Lee-Kwang et al. [43]).

3 Metric 1: Fuzzy similarity of two fuzzy sets

Let $X = \{AS_1, AS_2, AS_3, AS_4, AS_5\}$ be the set of plant communities we deal with.

Let a fuzzy set AS_i in the universal set X is defined by the membership function $AS_i(r_k)$, for $r_k \in AS_i \subset X; k = 1(1)62$. Here $AS_i; i = 1(1)5$ be any plant community taken from X . Then we can give the following equality

called as a fuzzy similarity (FS) of AS_i and AS_j .

$$FS(AS_i, AS_j) = \max\{\min\{\mu_{AS_i}(r_k), \mu_{AS_j}(r_k)\}\}; i, j = 1(1)5; k = 1(1)62, r_k \in AS_i \subset X,$$

$$= \max\{\min[\mu_{AS_i}(r_1), \mu_{AS_j}(r_1)], \min[\mu_{AS_i}(r_2), \mu_{AS_j}(r_2)], \dots, \min[\mu_{AS_i}(r_{62}), \mu_{AS_j}(r_{62})]\}. \quad (1)$$

Namely, $\min[\mu_{AS_i}(r_1), \mu_{AS_j}(r_1)] = m_1(i, j)$; $\min[\mu_{AS_i}(r_2), \mu_{AS_j}(r_2)] = m_2(i, j)$; $\dots$; $\min[\mu_{AS_i}(r_{62}), \mu_{AS_j}(r_{62})] = m_{62}(i, j)$; $i, j = 1(1)5$.

Clearly, it may be symbolized as follows:

$$FS(AS_1, AS_2) = \max\{m_1(1, 2), m_2(1, 2), \dots, m_{62}(1, 2)\}, FS(AS_1, AS_3) = \max\{m_1(1, 3), m_2(1, 3), \dots, m_{62}(1, 3)\},$$

$$FS(AS_1, AS_4) = \max\{m_1(1, 4), m_2(1, 4), \dots, m_{62}(1, 4)\}, FS(AS_1, AS_5) = \max\{m_1(1, 5), m_2(1, 5), \dots, m_{62}(1, 5)\},$$

$$FS(AS_2, AS_3) = \max\{m_1(2, 3), m_2(2, 3), \dots, m_{62}(2, 3)\}, FS(AS_2, AS_4) = \max\{m_1(2, 4), m_2(2, 4), \dots, m_{62}(2, 4)\},$$

$$FS(AS_2, AS_5) = \max\{m_1(2, 5), m_2(2, 5), \dots, m_{62}(2, 5)\}, FS(AS_3, AS_4) = \max\{m_1(3, 4), m_2(3, 4), \dots, m_{62}(3, 4)\},$$

$$FS(AS_3, AS_5) = \max\{m_1(3, 5), m_2(3, 5), \dots, m_{62}(3, 5)\}, FS(AS_4, AS_5) = \max\{m_1(4, 5), m_2(4, 5), \dots, m_{62}(4, 5)\}.$$

Therefore $m_k(i, j) = \min[\mu_{AS_i}(r_k), \mu_{AS_j}(r_k)]$, $i, j = 1(1)5$; $k = 1(1)62$.

We can see some properties of Metric 1 in the following:

- 1) $FS(AS_i, AS_j)$ is the maximum membership degree in the intersection $FS(AS_i \cap AS_j)$.
 - 2) The similarity degree is bounded $0 \leq FS(AS_i, AS_j) \leq 1$.
 - 3) If AS_i and AS_j are normalized, and $AS_i = AS_j$, $FS(AS_i, AS_j) = 1$. If $AS_i \cap AS_j = \emptyset$, $FS(AS_i, AS_j) = 0$.
 - 4) The measure is commutative; $FS(AS_i, AS_j) = FS(AS_j, AS_i)$.
 - 5) When the sets AS_i and AS_j are crisp sets, $FS = 0$ if $AS_i \cap AS_j = \emptyset$, $FS = 1$ if $AS_i \cap AS_j \neq \emptyset$.
- (See also Lee-Kwang et al. [43]).

3.1 Application 1

Consider similarity degrees of any two fuzzy sets in $X = \{AS_1, AS_2, AS_3\}$ by using relevés $\{r_1, r_2, r_3, r_4\}$ as shown in Table 1 below: (for the calculation of membership degrees, see Moraczewski [5])

Table 1 Fuzzy similarity degrees of any two plant communities.

X	AS_1	AS_2	AS_3
r_1	0.4	0.6	0
r_2	0.8	0.3	0.4
r_3	0.9	0	0.8
r_4	0	0.5	1

$$FS(AS_1, AS_2) = \max\{\min\{\mu_{AS_1}(r_1), \mu_{AS_2}(r_1)\}, \{\mu_{AS_1}(r_2), \mu_{AS_2}(r_2)\}, \{\mu_{AS_1}(r_3), \mu_{AS_2}(r_3)\}, \{\mu_{AS_1}(r_4), \mu_{AS_2}(r_4)\}\},$$

$$= \max\{\{\min(0.4, 0.6)\}, \{\min(0.8, 0.3)\}, \{\min(0.9, 0)\}, \{\min(0, 0.5)\}\},$$

$$= \max\{0.4, 0.3, 0, 0\} = \max\{0.4, 0.3, 0\},$$

$$= 0.4.$$

Similarly,

$$FS(AS_1, AS_3) = 0.8, FS(AS_2, AS_3) = 0.5 \text{ can be obtained easily.}$$

4 Metric 2: Fuzzy similarity of two elements

Here our elements are relevés of fuzzy sets (plant communities). The fuzzy similarity measure between two elements $r_k, r_l \in AS_i \subset X$ in fuzzy sets, $i = 1(1)5; k, l = 1(1)62$ is defined as follows:

$$\begin{aligned} FS_e(r_k, r_l) &= \max\{\min\{\mu_{AS_i}(r_k), \mu_{AS_i}(r_l)\}; i = 1(1)5; k, l = 1(1)62, \\ &= \max\{m_i(k, l)\}, \\ &= \max\{m_1(k, l), m_2(k, l), m_3(k, l), m_4(k, l), m_5(k, l)\}, \\ m_i(k, l) &= \{m_i(1, 2), m_i(1, 3), \dots, m_i(1, 62), m_i(2, 3), m_i(2, 4), \dots, m_i(2, 62), m_i(3, 4), m_i(3, 5), \dots, \\ &m_i(3, 62), \dots, m_i(61, 62)\}, i = 1(1)5. \end{aligned}$$

The fuzzy similarity measure between two elements $r_k, r_m \in AS_i \subset X$ in fuzzy sets, $i = 1(1)5; k, m = 1(1)62$ is defined as follows:

$$FS_e(r_k, r_m) = \max\{\min\{\mu_{AS_i}(r_k), \mu_{AS_i}(r_m)\}\}; i = 1(1)5; k, m = 1(1)62. \quad (2)$$

Metric 2 satisfies the following three properties:

- (1) $0 \leq FS_e(r_k, r_m) \leq 1$.
 - (2) If $r_k = r_m$ $FS_e(r_k, r_m) = 1$, if not $r_k \in AS_i, r_m \in AS_i$ then $FS_e(r_k, r_m) = 0$.
 - (3) $FS_e(r_k, r_m) = FS_e(r_m, r_k)$.
- (See also Lee-Kwang et al. [43]).

The measure of the fuzzy similarity between two elements satisfies the following properties:

- (1) $0 \leq FS_e(r_k, r_l) \leq 1$.
- (2) If $r_k = r_l$ $FS_e(r_k, r_l) = 1$, if $r_k \in AS_i, r_l \notin AS_i$; or $r_k \notin AS_i, r_l \in AS_i$; then $FS_e(r_k, r_l) = 0$.
- (3) $FS_e(r_k, r_l) = FS_e(r_l, r_k)$.

4.1 Application 2

Now, we give Table 2 as an example for Metric 2 as follows:

Table 2 Membership degrees of relevés $\{r_1, r_2, r_3, r_4\}$ in two fuzzy sets (plant communities) $\{AS_1, AS_2\}$ (for the calculation of membership degrees, see Moraczewski [5]).

X	AS_1	AS_2
r_1	0.2	0.5
r_2	1	1
r_3	0	0
r_4	0.5	1

By Formula 2, we have

$$FS_e(r_1, r_2) = \max\{\min(\mu_{AS_1}(r_1), \mu_{AS_1}(r_2)); i = 1, 2.$$

$$FS_e(r_1, r_2) = \max\{\{\min\{\mu_{AS_1}(r_1), \mu_{AS_1}(r_2)\}, \min\{\mu_{AS_2}(r_1), \mu_{AS_2}(r_2)\}\}.$$

$$\begin{aligned}
&= \max\{\min\{0.2, 1.0\}, \min\{0.5, 1.0\}\} \\
&= \max\{0.2, 0.5\} \\
&= 0.5.
\end{aligned}$$

Therefore we have the result of $FS_e(r_1, r_2) = 0.5$. Similarly, according to the Metric 2 we can easily obtain the following five equalities;

$$\begin{aligned}
FS_e(r_1, r_4) &= 0.5, \\
FS_e(r_1, r_3) &= 0, \\
FS_e(r_2, r_3) &= 0, \\
FS_e(r_2, r_4) &= 1, \\
FS_e(r_3, r_4) &= 0.
\end{aligned}$$

That is to say, if we need to find the similarity of r_1, r_2 and we are given $X = \{AS_1, AS_2\}$ then

$$FS_e(r_1, r_2) = \max\{\min\{\mu_{AS_1}(r_1), \mu_{AS_1}(r_2)\}, \min\{\mu_{AS_2}(r_1), \mu_{AS_2}(r_2)\}\}$$

is satisfied. These two metrics used here are a new approach for the similarity of phytosociology which we call *fuzzy similarity*. In condition, we have proposed two measures: one measures the similarity between fuzzy sets and the other between elements in fuzzy sets. The proposed measures can also work in the crisp sets. We have seen that the proposed measures can be useful in the behavior analysis in an organization.

5 Results and discussion

In a classical plant sociology works, if the similarity in the compared groups is 50% percent or more, they are considered as "the same community". If the similarity in the compared groups is less than 50% percent, these groups are evaluated as "the different community". In classical approaches, degrees of similarity are not mentioned. Whereas, in the fuzzy similarity approach, results for these details are given. This situation is important in the following aspects: it primarily indicates the degree of similarity of the community compared to each other, it also indicates which communities are closer or distant from each other, and that the relevés within the same community have the same or very close membership degrees within themselves is an indicator of homogeneity.

In this joint study, the fuzzy similarity analysis has been applied on five communities and two sub-communities belonging only to forest vegetation among three different vegetation types (forest, steppe and subalpine vegetation) identified in the project [42] titled "A Phytoecological and Phytosociological Research on the Sakarat Mountain (Amasya)". The main part of this joined work is given in this section. Here, we obtained two kinds of fuzzy similarity tables of which similarity of fuzzy sets and similarity of fuzzy elements in the sets in our approach.

In this study, fuzzy similarity measures were used. The fuzzy similarity methods used in our study are more general and comprehensive than classical phytosociological methods. The method, earlier used, in the mentioned project is quite classical. The relations of five communities and two sub-communities of forest vegetation with each other were analyzed by fuzzy similarity methods. Numerical analysis results are given in Table 3 below. As can be seen from this table, $FS(AS_1, AS_2)$ was calculated as 0.5. That is, the fuzzy similarity of community AS_1 to community AS_2 is 0.5. Here, $FS(AS_1, AS_2)$ expression is the degree of similarity of AS_1 and AS_2 .

How the values in Table 3 are calculated is explained in detail with an example in Metric 1.

Table 3 Fuzzy similarity of plant communities (FSPC) and their numerical values.

FSPC	Numerical values	FSPC	Numerical values
$FS(AS_1, AS_2)$	0.50	$FS(AS_2, AS_5)$	0.00
$FS(AS_1, AS_3)$	0.25	$FS(AS_3, AS_4)$	0.00
$FS(AS_1, AS_4)$	0.16	$FS(AS_3, AS_5)$	0.00
$FS(AS_1, AS_5)$	0.25	$FS(AS_4, AS_5)$	0.16
$FS(AS_2, AS_3)$	0.25	$FS(AS_{5a}, AS_{5b})$	0.16
$FS(AS_2, AS_4)$	0.00		

Remark 1:

If we used the crisp similarity in this table we would have only 1 value and the rest of them would be 0. The numerical values in Table 3 are the degrees to which the plant taxa seen in plant communities are common (the same). For the example, the result of $FS(AS_2, AS_3) = 0.25$ indicates that one-fourth of the taxa in AS_2 are the same as one-fourth of the taxa in AS_3 . In classical methods, 50% percent and above 1; less than 50% percent is given as 0. This shows that binary logic (Aristotle logic) is inadequate. The outcomes obtained with multi-valuable logic are more detailed and provide satisfactory information about the subject we are interested in. The above Table 3 results are obtained with multiple logic (fuzzy logic).

Descriptions of the relevés (Table 4-19: 4-A1.1, 4-A1.2, 4-A1.3, 4-A1.4, 4-A2.1, 4-A2.2, 4-A2.3, 4-A2.4; Table 4-B1.1, 4-B1.2, 4-B1.3, 4-B1.4, 4-B2.1, 4-B2.2, 4-B2.3, 4-B2.4):

The relevés in Table 4-19 (Table 4-A1, Table 4-A2, Table 4-B1 and Table 4-B2) include five communities. We can express them as follows: $r_1 - r_{10}$ is AS_1 ; $r_{11} - r_{20}$ is AS_2 ; $r_{21} - r_{30}$ is AS_3 ; $r_{31} - r_{40}$ is AS_4 and $r_{41} - r_{62}$ are within the scope of AS_5 ($AS_5 = (AS_{5a} \cup AS_{5b})$).

The distinct groupings in the parts where certain relevés are located show that the relevés forming the communities are regularly separated. It clearly shows that the similarities of the grouped relevés are in balance within themselves and that they are clearly different from other relevés. In addition, the fact that the relevés within the same community have the same or very close membership degrees indicates homogeneity.

How the values in Tables 4-A1, Tables 4-A2, Tables 4-B1 and Tables 4-B2 are calculated is explained in detail with an example in Metric 2.

Remark 2:

The reason for dividing Table 4 to several parts is due to the fact it is too large. Here, each table given below is quarter part of the main table of fuzzy similarity between elements.

In Tables 4-A1, it shows the similarity degrees of r_i, r_j relevés in the case of $i, j = 1(1)31$. Here, it is seen that the relevés with a similarity degree of 0.5 are dominant in number. In crisp logic, the similarity degree for the relevés of different plant communities is "0"; moreover, in the same plant communities, this similarity value is evaluated as "1".

Table 4 (4-A1.1) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i, j = 1(1)15$).

$FS_e(r_i, r_j)$	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}
r_1	-	1.00	1.00	0.75	1.00	0.75	1.00	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_2	1.00	-	1.00	0.75	1.00	0.75	1.00	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_3	1.00	1.00	-	0.75	1.00	0.75	1.00	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_4	0.75	0.75	0.75	-	0.75	0.75	0.75	0.75	0.75	0.75	0.50	0.50	0.50	0.50	0.5
r_5	1.00	1.00	1.00	0.75	-	0.75	1.00	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_6	0.75	0.75	0.75	0.75	0.75	-	0.75	0.75	0.75	0.75	0.50	0.50	0.50	0.50	0.5
r_7	1.00	1.00	1.00	0.75	1.00	0.75	-	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_8	1.00	1.00	1.00	0.75	1.00	0.75	1.00	-	1.00	1.00	0.50	0.50	0.50	0.50	0.5
r_9	1.00	1.00	1.00	0.75	1.00	0.75	1.00	1.00	-	1.00	0.50	0.50	0.50	0.50	0.5
r_{10}	1.00	1.00	1.00	0.75	1.00	0.75	1.00	1.00	1.00	-	0.50	0.50	0.50	0.50	0.5
r_{11}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	-	0.85	1.00	0.85	0.5
r_{12}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.85	-	0.85	0.50	0.5
r_{13}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	1.00	0.85	-	0.85	0.5
r_{14}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.85	0.50	0.85	-	0.5
r_{15}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	-

Table 5 (4-A1.2) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 1(1)15; j = 16(1)31$).

$FS_e(r_i, r_j)$	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}	r_{30}	r_{31}
r_1	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_2	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_3	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_4	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_5	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_6	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_7	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_8	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_9	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{10}	0.50	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{11}	0.71	1.00	0.85	0.85	0.57	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{12}	0.71	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{13}	0.71	1.00	0.85	0.85	0.57	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{14}	0.71	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{15}	0.43	0.50	0.50	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00

Table 6 (4-A1.3) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 16(1)31, j = 1(1)15$).

$FS_e(r_i, r_j)$	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}
r_{16}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.71	0.71	0.71	0.71	0.43
r_{17}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	1.00	0.85	1.00	0.85	0.50
r_{18}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.85	0.50	0.85	0.50	0.50
r_{19}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.85	0.50	0.85	0.50	0.50
r_{20}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.57	0.50	0.57	0.50	0.50
r_{21}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{22}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{23}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{24}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{25}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{26}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{27}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{28}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{29}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{30}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{31}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7 (4-A1.4) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i, j = 16(1)31$).

$FS_e(r_i, r_j)$	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}	r_{30}	r_{31}
r_{16}	-	0.71	0.71	0.71	0.57	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{17}	0.71	-	0.85	0.85	0.57	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{18}	0.71	0.50	-	0.50	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{19}	0.71	0.50	0.50	-	0.50	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{20}	0.57	0.57	0.50	0.50	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.00
r_{21}	0.25	0.25	0.25	0.25	0.25	-	0.85	0.85	0.71	0.85	0.57	0.57	0.85	0.28	0.57	0.00
r_{22}	0.25	0.25	0.25	0.25	0.25	0.85	-	0.85	0.71	0.85	0.57	0.57	0.85	0.28	0.57	0.00
r_{23}	0.25	0.25	0.25	0.25	0.25	0.85	0.85	-	0.71	0.85	0.57	0.57	0.85	0.28	0.57	0.00
r_{24}	0.25	0.25	0.25	0.25	0.25	0.71	0.71	0.71	-	0.71	0.57	0.57	0.85	0.28	0.57	0.00
r_{25}	0.25	0.25	0.25	0.25	0.25	0.85	0.85	0.85	0.71	-	0.57	0.57	0.85	0.28	0.57	0.00
r_{26}	0.25	0.25	0.25	0.25	0.25	0.57	0.57	0.57	0.57	0.57	-	0.57	0.57	0.28	0.57	0.00
r_{27}	0.25	0.25	0.25	0.25	0.25	0.57	0.57	0.57	0.57	0.57	0.57	-	0.57	0.28	0.57	0.00
r_{28}	0.25	0.25	0.25	0.25	0.25	0.85	0.85	0.85	0.85	0.85	0.57	0.57	-	0.28	0.57	0.00
r_{29}	0.25	0.25	0.25	0.25	0.25	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	-	0.57	0.00
r_{30}	0.25	0.25	0.25	0.25	0.25	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	-	0.00
r_{31}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-

Tables 4-A2 show the similarity degrees of the r_i, r_j relevés. In the case of $i = 1(1)31; j = 32(1)62$. Here, relevés with a degree of similarity of 0.25 seem to be dominant in number (For the comparison to crisp case, see the explanation for Table 4-A1). Moreover, Tables 4-A2 is constructed by $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (fuzzy similarity between $r_i, r_j; i = 1(1)31; j = 32(1)62$).

Table 8 (4-A2.1) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 1(1)15, j = 32(1)46$).

$FS_e(r_i, r_j)$	r_{32}	r_{33}	r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	r_{40}	r_{41}	r_{42}	r_{43}	r_{44}	r_{45}	r_{46}
r_1	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_2	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_3	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_4	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_5	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_6	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_7	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_8	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_9	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{10}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{11}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{12}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{13}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{14}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25
r_{15}	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25

Table 9 (4-A2.2) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 1(1)15, j = 47(1)62$).

$FS_e(r_i, r_j)$	r_{47}	r_{48}	r_{49}	r_{50}	r_{51}	r_{52}	r_{53}	r_{54}	r_{55}	r_{56}	r_{57}	r_{58}	r_{59}	r_{60}	r_{61}	r_{62}
r_1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_6	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_7	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_8	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{10}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{11}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{12}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{13}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{14}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{15}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 10 (4-A2.3) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 16(1)31, j = 32(1)46$).

$FS_e(r_i, r_j)$	r_{32}	r_{33}	r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	r_{40}	r_{41}	r_{42}	r_{43}	r_{44}	r_{45}	r_{46}
r_{16}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{17}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{18}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{19}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{20}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{21}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{22}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{23}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{24}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{25}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{26}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{27}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{28}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{29}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{30}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.25	0.25	0.25	0.25
r_{31}	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.16	0.16	0.16	0.16	0.16	0.16

Table 11 (4-A2.4) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 16(1)31, j = 47(1)62$).

$FS_e(r_i, r_j)$	r_{47}	r_{48}	r_{49}	r_{50}	r_{51}	r_{52}	r_{53}	r_{54}	r_{55}	r_{56}	r_{57}	r_{58}	r_{59}	r_{60}	r_{61}	r_{62}
r_{16}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{17}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{18}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{19}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{20}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{21}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{22}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{23}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{24}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{25}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{26}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{27}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{28}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{29}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{30}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{31}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16

The following Tables 4-B1 are for $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (fuzzy similarity between $r_i, r_j; i = 32(1)62; j = 1(1)31$). Tables 4-B1 show the similarity degrees of the r_i, r_j relevés. In the case of $i = 32(1)62; j = 1(1)31$. Here, it is seen that the relevés with a similarity degree of 0.25 are dominant in number (For the comparison to crisp case, see also the explanation for Tables 4-A1).

Table 12 (4-B1.1) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 32(1)46, j = 1(1)15$).

$FS_e(r_i, r_j)$	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}
r_{32}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{33}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{34}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{35}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{36}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{37}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{38}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{39}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{40}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
r_{41}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{42}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{43}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{44}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{45}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{46}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 13 (4-B1.2) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 32(1)46, j = 16(1)31$).

$FS_e(r_i, r_j)$	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}	r_{30}	r_{31}
r_{32}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{33}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{34}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{35}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{36}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{37}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{38}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{39}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66
r_{40}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16
r_{41}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{42}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{43}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{44}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{45}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{46}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16

Table 14 (4-B1.3) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 47(1)62, j = 1(1)15$).

$FS_e(r_i, r_j)$	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}
r_{47}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{48}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{49}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{50}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{51}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{52}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{53}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{54}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{55}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{56}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{57}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{58}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{59}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{60}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{61}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{62}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 15 (4-B1.4) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 47(1)62, j = 16(1)31$).

$FS_e(r_i, r_j)$	r_{16}	r_{17}	r_{18}	r_{19}	r_{20}	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}	r_{26}	r_{27}	r_{28}	r_{29}	r_{30}	r_{31}
r_{47}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{48}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{49}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{50}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{51}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{52}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{53}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{54}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{55}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{56}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{57}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{58}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{59}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{60}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{61}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16
r_{62}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.16

The following Tables 4-B2 are for $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (fuzzy similarity between $r_i, r_j; i, j = 32(1)62$). Tables 4-B2 show the similarity degrees of the r_i, r_j relevés. In the case of $i, j = 32(1)62$. Here, it is seen that the relevés with a similarity degree of 0.25 are dominant in number (For the comparison to crisp case, see also the explanation for Tables 4-A1). In addition, in these Tables (4-19), we can see also that the relevés taken during the field studies were carried out correctly.

Table 16 (4-B2.1) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i, j = 32(1)46$).

$FS_e(r_i, r_j)$	r_{32}	r_{33}	r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	r_{40}	r_{41}	r_{42}	r_{43}	r_{44}	r_{45}	r_{46}
r_{32}	-	0.83	0.66	0.83	1.00	0.82	0.66	1.00	0.83	0.16	0.16	0.16	0.16	0.16	0.16
r_{33}	0.83	-	0.66	0.83	0.83	0.83	0.83	0.66	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{34}	0.66	0.66	-	0.66	0.66	0.66	0.66	0.66	0.66	0.16	0.16	0.16	0.16	0.16	0.16
r_{35}	0.83	0.83	0.66	-	0.83	0.83	0.83	0.66	0.83	0.16	0.16	0.16	0.16	0.16	0.16
r_{36}	1.00	0.83	0.66	0.83	-	0.82	0.66	1.00	0.83	0.16	0.16	0.16	0.16	0.16	0.16
r_{37}	0.82	0.83	0.66	0.83	0.83	-	0.83	0.66	0.83	0.16	0.16	0.16	0.16	0.16	0.16
r_{38}	0.66	0.66	0.66	0.66	0.66	0.66	-	0.66	0.66	0.16	0.16	0.16	0.16	0.16	0.16
r_{39}	1.00	0.83	0.66	0.83	1.00	0.82	0.66	-	0.83	0.16	0.16	0.16	0.16	0.16	0.16
r_{40}	0.83	0.83	0.66	0.83	0.83	0.83	0.83	0.66	-	0.16	0.16	0.16	0.16	0.16	0.16
r_{41}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	-	0.25	0.25	0.25	0.25	0.25
r_{42}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	-	0.25	0.25	0.25	0.25
r_{43}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	-	0.25	0.25	0.25
r_{44}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	-	0.25	0.25
r_{45}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	-	0.25
r_{46}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	-

Table 17 (4-B2.2) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 32(1)46, j = 47(1)62$).

$FS_e(r_i, r_j)$	r_{47}	r_{48}	r_{49}	r_{50}	r_{51}	r_{52}	r_{53}	r_{54}	r_{55}	r_{56}	r_{57}	r_{58}	r_{59}	r_{60}	r_{61}	r_{62}
r_{32}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{33}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{34}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{35}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{36}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{37}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{38}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{39}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{40}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
r_{41}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{42}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{43}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{44}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{45}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{46}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 18 (4-B2.3) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i = 47(1)62, j = 32(1)46$).

$FS_e(r_i, r_j)$	r_{32}	r_{33}	r_{34}	r_{35}	r_{36}	r_{37}	r_{38}	r_{39}	r_{40}	r_{41}	r_{42}	r_{43}	r_{44}	r_{45}	r_{46}
r_{47}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{48}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{49}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{50}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{51}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{52}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{53}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{54}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{55}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{56}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{57}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{58}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{59}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{60}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{61}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25
r_{62}	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.25	0.25	0.25	0.25	0.25	0.25

Table 19 (4-B2.4) $FS_e(r_i, r_j)$, Fuzzy similarity of elements (relevés) (for $r_i, r_j; i, j = 47(1)62$).

$FS_e(r_i, r_j)$	r_{47}	r_{48}	r_{49}	r_{50}	r_{51}	r_{52}	r_{53}	r_{54}	r_{55}	r_{56}	r_{57}	r_{58}	r_{59}	r_{60}	r_{61}	r_{62}
r_{47}	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{48}	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{49}	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{50}	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{51}	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{52}	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{53}	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{54}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{55}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25	0.25
r_{56}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25	0.25
r_{57}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25	0.25
r_{58}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25	0.25
r_{59}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25	0.25
r_{60}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25	0.25
r_{61}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-	0.25
r_{62}	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	-

6 Conclusion

So far, until the analysis in the sense of fuzzy logic, in the classical methods used in new syntaxon researches, if the similarity features to the known syntaxa are more than fifty percent, it is stated that the examined species are similar to the previous ones; otherwise, if the common features were less than fifty percent, it was decided that the examined species were a new species. In the examinations made using fuzzy logic, the similarity degrees are given exactly (e.g. 57 % instead of 1 i.e. the similarity degree belongs to the closed interval [50,100] or 49 % instead of 0 i.e. the similarity degree belongs to [0, 50)), that is, in detail, more than the classical methods. This study has a superiority over classical methods as it gives more detailed information than previous studies, in other words it gives us more precise information on which group is closer (not a new syntaxon) and which is

farther away (it is a new syntaxon). This study brings a different perspective to new researches in plant sociology.

7 Declarations

7.1 Conflict of interest:

According to the authors of this paper, there are no conflicts of interest to report regarding the article that is being presented.

7.2 Funding:

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7.3 Author's contribution:

M.Ü.B.-Conceptualization, Methodology, Software. S.A.Ş.-Formal Analysis, Writing-Review Editing, Validation, Writing-Original Draft. Ö.A.-Supervision. All authors read and approved the final submitted version of this manuscript.

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7.5 Data availability statement:

All data that support the findings of this study are included within the article.

7.6 Using of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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