sciendo International Journal of Mathematics and Computer in Engineering 2(2) (2024) 251–262



International Journal of Mathematics and Computer in Engineering

https://sciendo.com/journal/IJMCE

Original Study

A hierarchy of double, quadruple and octuple primes

Jon Rokne[1]†

[1]Department of Computer Science, University of Calgary, Alberta, T2N 1N4, Canada

Communicated by Haci Mehmet Baskonus; Received: 13.10.2023; Accepted: 25.12.2023; Online: 10.01.2024

Abstract

In this paper double, quadruple and octuple primes are discussed. Computational and theoretical properties as well as examples of quadruple and octuple primes are presented, where the octuple primes in most cases have primes between the defining quadruples although the existence of a pure quadruple prime has also been demonstrated computationally. The possibility of sixtentuple primes is also briefly discussed.

Keywords: Prime, double prime, quadruple prime, octuple prime, calculations.
AMS 2020 codes: 11A41; 11A51; 11Y05; 11Y40.

1 Introduction

Number theory is one of the oldest mathematical disciplines. It deals with the properties of integers [1, 2]. One of the central topics in number theory is the factorization of the numbers and if a number cannot be factored into a product of two or more integers then it is a prime number. A prime is, therefore, an integer that is only divisible by integers 1 and the number itself. Methods for establishing whether a number is a prime or a composite number in a consecutive sequence of integers include several methods such as Eratosthenes, Selberg, Brun and others [3].

Primes have many interesting properties that have been studied since antiquity and a number of papers and books have been written about them [4–10]. A computational perspective on primes is provided in the book *Prime Numbers, a Computational Perspective* by Crandall and Pomerance [11]. A prime pair (also called a double prime in this paper) is two primes separated by an even number. The conjecture that there is an infinite number of such prime pairs is still unsolved although substantial progress has been made [12]. A numerical analysis approach to the twin prime conjecture is found in [13]. Interesting computational results in number theory are also found in the book *Experimental Number Theory* by Villegas [14]. Some areas of current interest are primality testing, that is checking if a natural number is a prime number, and factoring non-primes into its prime factors [15–17]. The paper [12] discusses the gaps that might occur between successive primes.

Prime patterns have been of interest to a number of authors [18, 19] and prime constellations have been discussed by, for example [20]. Both prime patterns and prime constellations are concerned with particular configurations of primes. In this paper a new pattern forming a hierarchy of groups of primes is depicted as a

†Corresponding author.
Email address: rokne@ucalgary.ca

sciendo
ISSN 2956-7068
doi:10.2478/ijmce-2024-0019
sciendo
Open Access.© 2024 Rokne, published by Sciendo.
This work is licensed under the Creative Commons Attribution alone 4.0 License.

binary tree as shown in Figure 1 where each level of the tree represents particular constellations.

The hierarchy consists of constellations of double primes some of which form the basis for quadruple primes which are then, in some cases, the basis for octuple primes where the exact nature of these constellations will be discussed in the sequel. Known and new properties for these constellations of primes are proven and examples of quadruple and octuple primes are exhibited. A conjecture for the existence of 16-tuple primes is stated as well.

The paper is in parts experimental mathematics [21, 22] where interesting numerical facts are established using a computer and in parts theoretical mathematics when some properties of these prime constellations are proven.

Applications of the results in this paper might include cryptography.

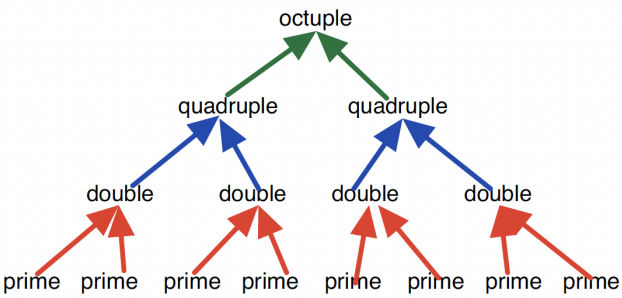


Fig. 1 The hierarchy formed from primes, double primes, qudruple primes and an octuple prime.

2 Definitions

By "center" of a prime n-tuple is meant the midpoint between the "centers" two $(n-1)$tuples defining the n-tuple.

- The letter E denotes "even".
- The letter P will denote "prime".
- The term "index" denotes a number being considered.
- The abbreviation "divis" is short for divisor.
- "dcenter" denotes the center of a double prime.
- "qcenter" denotes the center of a quadruple prime.
- "ocenter" denotes the center of octuple primes.
- "scenter" denotes the center of a sextuple prime if it exists.
- "tuple" denotes any of the double, quadruple or octuple primes.
- "span" denotes the distance between centers of defining tuples in the hierarchy of tuples.

3 Double primes

A double prime, also called a prime pair or twin primes, is a set of two primes separated by an even non-prime which is called the dcenter of the double prime. The first double primes are $d_1 = (1,3), d_2 = (3,5), d_3 =$

$(5,7), d_4 = (11,13)$ where d_2 is a special case since it uses the second number from the previous double prime together with the first number from the following double prime.

The following simple result is well known, but established here as well.

Theorem 1. *Assume $d > 4$. If d =dcenter is the center of a double prime then $d+3$ is divisible by 3.*

Proof.

Table 1 Double prime example.

index	$d-3$	$d-2$	$d-1$	d	$d+1$	$d+2$	$d+3$	$d+4$	$d+5$
divis		E	P	dcenter	P	E		E	

In Table 1 $d > 4$ is the dcenter of a double prime and it follows that $d-2$ cannot be divisible by 3 since it would imply that $d+1$ was composite. Together with the fact that $d-1$ is prime it follows that $d-3$ is divisible by 3. Similarly d and $d+3$ are also divisible by 3 since they form a sequence $d-3$, d, $d+3$.

From this theorem it follows that if the dcenter of a double prime is d then the first possible prime after the double prime would be at $d+5$ since $d+3$ is divisible by 3. Hence a possible succeeding double prime would have the center at $d+6$.

Double primes have been studied by a number of authors and for some background on these primes see for example [23] and [24, 25].

4 Quadruple primes

A quadruple prime is a set of two double primes with dcenters d_1 and d_2 separated by 3 non-primes which is the smallest distance possible between two double primes (assuming that the dcenters are greater than 6). The center of the quadruple prime is called the qcenter=$q = (d_1 + d_2)/2$.

Computationally exhibiting the following case of a two double primes with dcenters at $d_1 = 12$ and $d_2 = 18$ separated by 3 non-primes provides an example of a quadruple prime with qcenter at $q = 15$ as shown in Table 2.

Table 2 Quadruple prime example.

index	10	11	12	13	14	15	16	17	18	19	20
divis	E	P	d_1	P	E	q-center	E	P	d_2	P	E

Note that it is easily shown that two double primes separated by an even number is not possible hence the separation would have to be at least 3 numbers.

The first case of a quadruple prime has dcenters $d_1 = 6$ and $d_2 = 12$ with qcenter at $q = 9$ which overlaps with the case considered above. It is conjectured that this is the only case for which there is such an overlap.

Examples of quadruple primes are found, for example, in [26].

Before continuing it should be noted that given a consecutive list of k positive integers, exactly one of the integers is divisible by k. (For a proof see for example [27].)

There are some results for quadruple primes that are now discussed in the following theorem.

Theorem 2. *If q =qcenter is the center of a quadruple prime then*

1. *$q-6$ is divisible by 3.*

2. $q-5$ *is divisible by* 5.

3. $q-6$ *is not divisible by* 5.

Proof.

1. This follows from the remark above for double primes. As a result the sequence numbers from $q-9$ to $q+4$ have the divisibility properties shown in Table 3.

Table 3 Divisibility by 3.

index	$q-9$	$q-8$	$q-7$	$q-6$	$q-5$	$q-4$	$q-3$	$q-2$	$q-1$	q	$q+1$	$q+2$	$q+3$	q+4
divis	2,3		2	3	2	P	2,3	P	2	3	2	P	2,3	P

2. Consider the numbers $q-5, q-6, q-7, q-8, q-9$. Since this is a continuous sequence of 5 numbers exactly one of the numbers is divisible by 5.

   Viewing Table 3 it follows that:

   - $q-6$ cannot be divisible by 5 since it would imply that $q+4$ would be divisible by 5, a contradiction since $q+4$ is prime.

   - $q-7$ cannot be divisible by 5 since it would imply that $q-2$ would be divisible by 5, a contradiction since $q-2$ is prime.

   - $q-8$ cannot be divisible by 5 since it would imply that $q+2$ would be divisible by 5, a contradiction since $q+2$ is prime.

   - $q-9$ cannot be divisible by 5 since it would imply that $q-4$ would be divisible by 5, a contradiction since $q-4$ is prime.

   Hence $q-5$ is divisible by 5. By symmetry $q+5$ is also divisible by 5.

3. It follows immediately from 2) that $q-6$ is not divisible by 5 and hence any divisor has to be 3 together with 7 or larger.

Divisibility by 2, 3 and 5 is now shown in Table 4.

Table 4 Divisibility by 2, 3 and 5 around qprime q.

index	$q-7$	$q-6$	$q-5$	$q-4$	$q-3$	$q-2$	$q-1$	q	$q+1$	$q+2$	$q+3$	$q+4$
divis	2	3	2,5	P	2,3	P	2	3,5	2	P	2,3	P

Consider now the quadruple prime with qcenter $q = 195$ with known primes and the numbers divisible by 2, 3 and 5 adjacent to 195. These numbers are listed in Figure 2.

index	185	186	187	188	189	190	191	192	193	194
divi	5	2,3		2	3	2,5	P	2,3	P	2
index	195	196	197	198	199	200	201	202	203	204
divi	3,5	2	P	2,3	P	2,5	3	2		2,3
index	205	206	207	208	209	210	211	212	213	214
divi	5	2	3	2		2,3,5	P	2	3	2
index	215	216	217	218	219	220	221	222	223	224
divi	5	2,3		2	3	2,5		2,3	P	2
index	225	226	227	228	229	230	231	232	233	234
divi	3,5	2	P	2,3	P	2,5	3	2	P	2,3
index	235	236	237	238	239	240	241	242	243	244
divi	5	2	3	2	P	2,3,5	P	2	3	2

Fig. 2 Divisors around specific qcenter $q = 195$.

For a general quadruple prime with qcenter q the table is as in Figure 3.

index	q - 10	q - 9	q - 8	q - 7	q - 6	q - 5	q - 4	q - 3	q - 2	q - 1
divis	5	2,3		2	3	2,5	P	2,3	P	2
index	q	$q+1$	$q+2$	$q+3$	$q+4$	$q+5$	$q+6$	$q+7$	$q+8$	$q+9$
divis	3,5	2	P	2,3	P	2,5	3	2		2,3
index	q +10	$q+11$	q +12	$q+13$	$q+14$	$q+15$	$q+16$	$q+17$	$q+18$	$q+19$
divis	5	2	3	2		2,3,5		2	3	2
index	$q+20$	$q+21$	$q+22$	$q+23$	$q+24$	$q+25$	$q+26$	$q+27$	$q+28$	$q+29$
divis	5	2,3		2	3	2,5		2,3		2
index	$q+30$	$q+31$	$q+32$	$q+33$	$q+34$	$q+35$	$q+36$	$q+37$	$q+38$	$q+39$
divis	3,5	2		2,3		2,5	3	2		2,3

Fig. 3 Divisors around general qcenter q.

In the figures there are two framed sequences. The red-framed sequence centered on the qcenter 195 runs from 190 to 199 see Figure 2, or in terms of distances to a general qcenter q, the sequence runs from $q-4$ to $q+4$, see Figure 3. These red frames define the quadruple primes with qcenter 195 (or in general q). The notations in the lines "divis" indicate both the divisors and the primes that are present in this case. The primes at 191, 193, 197 and 199 (or in terms of the general case $q-4$, $q-2$, $q+2$ and $q+4$) are present for each of the cases of quadruple primes. The P's in Figure 2 at 221, 223, 227, 229, 233, 239 and 241 are specific to the case where $q = 195$.

Given the sequence of divisors shown in Figure 2 it is clear that the first possible quadruple prime following the quadruple prime with qcenter 195 would have to have a qcenter at 225. In this particular case 221 is undefined, 223 a prime, 227 a prime and 229 a prime. This means that if 221 had been prime then 225 would have been the qcenter of a new quadruple prime as indicated by the green frame.

The green frame in table in Figure 3 shows the general case indicating that if $q+30$ was the qcenter of a quadruple prime then the fact that $q+26$, $q+28$, $q+32$ and $q+34$ are undefined opens up the possibility that they might be prime and hence if they happened to be prime then the sequence $q+26$ to $q+34$ would be a

quadruple prime. In general, for a quadruple prime with qcenter q the entries $q+26$, $q+28$, $q+32$ and $q+34$ may be prime or composite as shown in Table 5. The entries at $q+14$ and $q+16$, which will be discussed later, may also be prime or composite.

Table 5 Primality of $q+26$, $q+28$, $q+32$ and $q+34$ for general q-center=q.

q-center	$q+26$	$q+28$	$q+32$	$q+34$
195	NO	YES	YES	YES
825	NO	YES	YES	YES
1485	YES	NO	NO	NO
1875	NO	YES	YES	NO
2085	YES	YES	NO	NO
21015	NO	NO	NO	NO

The following result places some restrictions on the possible quadruple primes with qcenter $q+30$ following a quadruple prime with qcenter q.

Theorem 3. *Let q be the qcenter of a quadruple prime where $q+28$ and $q+34$ are primes such as is the case for the example $q=195$.*

Then $q-6$ is divisible by 7 and the table in Figure 2 can be expanded with added information to a new Table 6.

Proof.

Consider now the sequence $q-7,\cdots,q-1$. Then exactly one number in the sequence is divisible by 7.

$q-7$ cannot be divisible by 7 since it would imply $q+28$ would be divisible by 7, a contradiction since $q+28$ is prime.

$q-5$ cannot be divisible by 7 since it would contradict the primality of $q+2$.

$q-3$ cannot be divisible by 7 since it would imply $q+4$ was composite which is not the case.

$q-1$ cannot be divisible by 7 since it would contradict the primality of $q+34$.

$q-4$ and $q-2$ are prime.

This leaves $q-6$ which therefore has to be divisible by 7.

Note that the theorem does not imply that if $q-6$ is divisible by 7 then $q+28$ and $q+34$ are prime. An example verifying this is provided by $q=1003365$.

The table in Figure 2 can now be expanded with added information to the table in Figure 4.

index	191	192	193	194	195	196	197	198	199	200
divis.	P	2,3	P	2	3,5	2,7	P	2,3	P	2,5
index	201	202	203	204	205	206	207	208	209	210
divis.	3	2	7	2,3	5	2	3	2		2,3,5,7
index	211	212	213	214	215	216	217	218	219	220
divis.	P	2	3	2	5	2,3	7	2	3	2,5
index	221	222	223	224	225	226	227	228	229	230
divis.		2,3	P	2,7	3,5	2	P	2,3	P	2,5

Fig. 4 Expanded table in the neighborhood of the quadruple prime with qcenter 195, now including divisibility by 7.

Table 6 Divisors in the neighborhood of the quadruple prime with qcenter 195, now including 7.

index	185	186	187	188	189	190	191	192	193	194
divis	5	2,3		2	3,7	2,5	P	2,3	P	2
index	195	196	197	198	199	200	201	202	203	204
divis	3,5	2,7	P	2,3	P	2,5	3	2	7	2,3
index	205	206	207	208	209	210	211	212	213	214
divis	5	2	3	2		2,3,5,7	P	2	3	2
index	215	216	217	218	219	220	221	222	223	224
divis	5	2,3	7	2	3	2,5		2,3	P	2,7
index	225	226	227	228	229	230	231	232	233	234
divis	3,5	2	P	2,3	P	2,5	3,7	2	P	E,3
index	235	236	237	238	239	240	241	242	243	244
divis	5	2	3	2,7	P	2,3,5	P	2	3	2

5 Octuple primes

In the table in Figure 2 when starting from the quaduple prime center $q = 195$ the first possible following quadruple prime could have been with the numbers $221 = q+26$, $223 = q+28$, $227 = q+32$ and $229 = q+34$ as in Table 6. In this specific case 223, 226 and 229 are primes and 221 is composite. Hence in this case there is no second quadruple prime, however, due to the compositedness of 221. It is noted that since 223 and 229 are primes 189 is divisible by 7.

Fig. 5 Table of divisors for a general octuple where first quadruple has qcenter$= q$.

index	q - 4	q - 3	q - 2	q - 1	q	q + 1	q + 2	q + 3	q + 4	q + 5
divis	P	2,3	P	2	3,5	2,7	P	2,3	P	2,5
index	q + 6	q + 7	q + 8	q + 9	q +10	q + 11	q +12	q + 13	q + 14	q + 15
divis	3	2	7	2,3	5	2	3	2	?	2,3,5,7
index	q + 16	q + 17	q + 18	q + 19	q + 20	q + 21	q + 22	q + 23	q + 24	q + 25
divis	?	2	3	2	5	2,3	7	2	3	2,5
index	q + 26	q + 27	q + 28	q + 29	q + 30	q + 31	q + 32	q + 33	q +34	q+ 35
divis	P	2,3	P	2,7	3,5	2	P	2,3	P	2,5

The question is therefore, are there quadruple primes q where the factors $q+26$, $q+28$, $q+32$ and $q+34$ are all prime? If so, then the sequence $q-26$ to $q+34$ would form a new quadruple prime with center $q_n = q+30$. The two quadruple primes with centers q and q_n would then form what could be called an octuple prime with an o-center=$o = q+15$. This would look like the table in Figure 5. Note that all of the entries in the table are specified with respect to divisibility by 2, 3, 5 and/or 7 except for the entries $q+14$ and $q+16$. The divisibility by 7 is a consequence of the primality requirements for $q+26$ and $q+34$ which implies the divisibility of $q-6$ and hence any of the further entries $q-6+k*7, k = 1,2,\cdots$.

The question is now: do octuple primes defined in this manner exist and if so, what specific properties do they have? It should be noted that the term octuple prime has been used before in a slightly different context, see for example [24]. It is used in this context to denote the progression of particular tuples from primes to double primes then quadruple primes and finally octuple primes.

As shown computationally in [26] octuple primes exist and the first octuple prime has o-center=$o = 1006320$. The details of this octuple prime are listed in Table 7.

Table 7 The octuple with o-center= $o = 1006320$.

type of center	center label	center value	factored center
d-center	d_{11}	1006302	(2,1),(3,1),(11,1),(79,1),(193,1)
q-center	q_1	1006305	(3,1),(5,1),(73,1),(919,1)
d-center	d_{12}	1006308	(2,2),(3,2),(27953,1)
o-center	o	1006320	(2,4),(3,1),(5,1),(7,1),(599,1)
d-center	d_{21}	1006332	(2,2),(3,1),(17,1),(4933,1)
q-center	q_2	1006335	(3,2),(5,1),(11,1),(19,1),(107,1)
d-center	d_{22}	1006338	(2,1),(3,1),(179,1),(937,1)

In the case of double primes there was one non-prime separating the two primes and in the case of quadruple primes there were three non-primes separating the double primes. In a sense the "gaps" between the previous sets of primes were small, i.e one and three steps. In the case of octuple primes the gap is much larger. For a given octuple center o there are 30 numbers between the constituent quadruple primes with centers q_1 and q_2. Most of the gap entries are defined by their divisibility. From the results above only the quantities $o-1$ and $o+1$ are not specified in the table in Figure 6. The octuple primes may therefore be classified into three cases depending on whether these quantities are prime or not as in Table 8.

Fig. 6 Octuple with ocenter o between $o-26$ and $o+24$.

index	$q-4$	$q-3$	$q-2$	$q-1$	q	$q+1$	$q+2$	$q+3$	$q+4$	$q+5$
divi	P	2,3	P	2	3,5	2,7	P	2,3	P	2,5
index	$q+5$	$q+7$	$q+8$	$q+9$	$q+10$	$q+11$	$q+12$	$q+13$	$q+14$	$q+15$
divi	3	2	7	2,3	5	2	3	2	?	2,3,5,7
index	$q+16$	$q+17$	$q+18$	$q+19$	$q+20$	$q+21$	$q+22$	$q+3$	$q+24$	$q+25$
divi	?	2	3	2	5	2,3	7	2	3	2
index	$q+26$	$q+27$	$q+28$	$q+29$	$q+30$	$q+31$	$q+32$	$q+33$	$q+34$	$q+35$
divi	P	2,3	P	2,7	3,5	5	P	2,3	P	2,7

Table 8 Primality $o-1$ and $o+1$ for select cases of octuple primes.

o =o-center	prime $o-1$	prime $o+1$
1006320	False	False
2594970	False	True
3919230	True	False
9600570	False	True
10531080	False	False
157131660	False	True
179028780	True	False
211950270	True	False
255352230	False	False
267587880	False	False
724491390	True	False
871411380	False	False

The octuples with composite $o-1$ and $o+1$ might be denoted as pure octuples and the remaining octuples as impure octuples.

Table 9 Pure and impure octuples up to 10^{10}.

Number of octuple primes	Pure octuple primes	Octuple prime with $o-1$ prime	Ocuple primes with $o+1$ prime	Ocuple primes with $o-1$ and $o+1$ prime
65	42	9	14	0

There are not many prime octuples. The results shown in Table 9 show the number of octuple primes of the different kinds for numbers up to 10^{10}.

The table shows that there are no octuple primes where both $o-1$ and $o+1$ are prime. Because of this one might be tempted to conjecture that there are no octuple primes where the primes on both sides of the octuple center are prime.

However, when the computations are performed up to 10^{11} the results are as in Table 10 disproving the conjecture.

Table 10 Pure and impure octuples up to 10^{11}.

Number of octuple primes	Pure octuple primes	Octuple primes with $o-1$ prime	Octuple primes with $o+1$ prime	Octuple primes with $o-1$ and $o+1$ prime
267	187	34	47	3

The three examples where $o-1$ and $o+1$ are both prime have ocenters $o_1 = 39713433690$, $o_2 = 66419473050$ and $o_3 = 71525244630$.

sciendo

6 Conclusion

The previous sections have shown that the centres of the double, quadruple and octuple primes form a sequence as shown in Table 11.

Table 11 Divisors of centers.

	centers divisible by	span divisible by	comment
dcenter	1	1*2	verified
qcenter	3	1*2*3	verified
ocenter	15	1*2*3*5	verified
scenter	105	1*2*3*5*7	hypothetical

Since the ocentres of octuple are all divisible by $15 = 3*5$ one might conjecture that there are sixtentuple primes composed of two octuple primes whose ocentres are separated by $210 = 2*3*5*7$ and with a scenter divisible by $3*5*7 = 105$. This table shows an unusual regularity in the first three entries rarely found in the literature on primes.

Considering that a specific octuple prime has the properties in the neighborhood as in Table 12 a possible second ocenter at a distance of 210 from the given ocenter would be as in Table 13.

Table 12 Divisors in the neighborhood of the first o-center o of a possible sixtentuple prime.

index	$q_{n-1}-4$	$q_{n-1}-3$	$q_{n-1}-2$	$q_{n-1}-1$	q_{n-1}	$q_{n-1}+1$	$q_{n-1}+2$	$q_{n-1}+3$	$q_{n-1}+4$
divis	unsp.	2,3	unsp.	2	3,5	2,7	unsp.	2,3	unsp.

Table 13 Divisors in the neighborhood of the second o-center o_n of a possible sixtentuple prime.

index	$q_{n+1}-4$	$q_{n+1}-3$	$q_{n+1}-2$	$q_{n+1}-1$	q_{n+1}	$q_{n+1}+1$	$q_{n+1}+2$	$q_{n+1}+3$	$q_{n+1}+4$
divis	unsp.	2,3	unsp.	2,7	3,5	2	unsp.	2,3	unsp.

In both o-center cases there are entries that are unspecified as far as the divisors 3, 5, 7 are concerned. This means that the possibility of these entries all being prime is not excluded.

For the specific case of o-center $o = 100320$, if it were the first o-center for a sixtentuple prime then the second o-center would be at $o_n = 100530$. Since o_n is not the o-center of an octuple it does not show the existence of a sixtentuple prime, however. Further numerical exploration would be needed to demonstrate the possible existence of sixtentuple primes.

7 Declarations

7.1 Conflict of interest:

There is no conflict of interest.

7.2 Author's contributions:

J.R.-Conceptualization, Methodology, Formal analysis, Writing-Review and Editing, Validation, Writing-Original Draft, Resources. All authors read and approved the final submitted version of this manuscript.

7.3 Funding:

No funding was received to assist with the preparation of this manuscript.

7.4 Acknowledgement:

Not applicable.

7.5 Data availability statement:

All data that support the findings of this study are included within the article.

7.6 Using of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

References

[1] Keng H.L., Introduction to Number Theory, Springer-Verlag Berlin Heidelberg, Germany, 1982.
[2] Andrews G.E., Number Theory, W.B. Saunders Co., Dover Publication Inc., New York, USA, 1971.
[3] Halberstam H., Richert H.E., Sieve Methods, Dover Publications, USA, 2011.
[4] Caragiu M., Sequential Experiments with Primes, Springer, USA, 2017.
[5] Stepney S., Euclid's proof that there are an infinite number of primes, https://www-users.cs.york.ac.uk/susan/cyc/p/primeprf.htm, Accessed: September 3, 2022.
[6] Uselton S.C., A study of semiprime arithmetic sequences, Honors Theses, 67, https://repository.belmont.edu/cgi/viewcontent.cgi?article=1081&context=honors_theses, Accessed: September 7, 2022.
[7] Faber X., Granville A., Prime factors of dynamic sequences, Journal Für Die Reine und Angewandte Mathematik, 661, 189–214, 2011.
[8] Numbers Aplenty, Semiprimes, https://www.numbersaplenty.com/set/semiprime/, Accessed: September 9, 2022.
[9] Borne K., Abdenim O.H., 20 Best prime numbers books of all time, https://bookauthority.org/books/best-prime-numbers-books.
[10] Niven I., Zuckerman H.S., Montgomery H.L., An Introduction to the Theory of Numbers, John Wiley & Sons, USA, 1991.
[11] Crandall R., Pomerance C., Prime Numbers A Computational Perspective (2nd Ed.), Springer, USA, 2005.
[12] Zhang Y., Bounded gaps between primes, Annals of Mathematics, 179(3), 1121–1174, 2014.
[13] Pietro G.D., Numerical analysis approach to twin primes conjecture, Notes on Number Theory and Discrete Mathematics, 27(3), 175–183, 2021.
[14] Villegas F.R., Experimental Number Theory, Oxford University Press, UK, 2007.
[15] Hamiss K., A simple algorithm for prime factorization and primality testing, Journal of Mathematics, 2022(ID:7034529), 1–10, 2022.
[16] Hang P., Sun Z., Wang S., A pattern of prime numbers and its application in primality testing, Journal of Physics: Conference Series, 2381, 2022 6th International Conference on Mechanics, Mathematics and Applied Physics (ICMMAP 2022), 19–21 August 2022, Qingdao, China.
[17] Riesel H., Prime Numbers and Computer Methods for Factorization (2nd Ed.), Boston, MA: Birkhäuser, Switzerland, 2012.
[18] Goudsmit S.A., Unusual prime number sequences, Nature, 214, 1164, 1967.
[19] Engelsma T.J., k-tuple permissible patterns, http://www.pi-e.de/ktuplets.htm, Accessed: September 12, 2023.
[20] Ericksen L., Primality Testing and Prime Constellations, Siauliai Mathematical Seminar, 3(11), 61–77, 2008.
[21] McEvoy M., Experimental mathematics, computers and the a priori, Synthese, 190, 397–412, 2013.
[22] Experimental mathematics, https://en.wikipedia.org/wiki/Experimental_mathematics, Ac-

cessed: October 5, 2022.
[23] Weisstein E., Twin primes, https://mathworld.wolfram.com/TwinPrimes.html, Accessed: September 12, 2023.
[24] Goldston D.A., Pintz J., Yildirim C.Y., Primes in tuples I, Annals of Mathematics, 170(2), 819–862, 2009.
[25] Goldston D.A., Pintz J., Yildirim C.Y., Primes in tuples II, Acta Mathematica, 204(1), 1–47, 2010.
[26] Rokne J., Some observations on prime pairs, quadruples and octuples, IEEE Canadian Review, 92, 8–11, 2023.
[27] Sato N., Number Theory, https://artofproblemsolving.com/articles/files/SatoNT.pdf, Accessed: September 10, 2023.