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Review Study

A numerical approach for an epidemic SIR model via Morgan-Voyce series

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Abstract

This study presents the problem of spreading non fatal disease in a population by using the Morgan-Voyce collocation method. The main aim of this paper is to find the exact solutions of the SIR model with vaccination. The problem may be modelled mathematically with a nonlinear system of ordinary differential equations. The presented method reduces the problem into a nonlinear algebraic system of equations by using unknown coefficient Morgan-Voyce polynomials and expanding approximate solutions. Morgan-Voyce polynomials are used. These unknown coefficients are calculated via the collocation method and matrix operation derivations. Two examples are given to show the feasibility of the method. To calculate the solutions, MATLAB R2021a is used. Additionally, comparing our method to the Homotopy perturbation method (HPM) and the Laplace Adomian decomposition method (LADM) proves the accuracy of the solution. The method studied can be seen as effective from these comparisons. So, it is essential to find solutions for the governing model. The study will contribute to literature since we also discuss the vaccination situation. The results of this study are valuable for controlling an epidemic.

Keywords: SIR model with vaccine, Morgan-Voyce polynomials, collocation points, epidemic model.
AMS 2020 codes: 34A30; 35A24; 35G20; 35F20.

1 Introduction

As the ordinary differential system of equations is considerably used to explain mathematical properties, they are used to represent the real-world problems arising in the field of science such as biology, physics, engineering, finance, or sociology. It is well-known that many models in these areas can be described by using a system of ordinary differential equations. Analytically, it is generally very hard or impossible to solve these models. Moreover, as the exact solutions of these models cannot be found, we need to use the numerical methods to find approximate solutions. Many researchers used methods for solving such equations. Hojjati et al. [1] investigated the multistep method. Mastorakis [2] presented the collocation method. Shawagfed et al. [3] proposed the Adomian decomposition method. While Yüzbaşı ve Karaçayır [4] handled the Galerkin method, Yüzbaşı [5] proposed the exponential collocation method. Al-Omari et al. [6] used the Galerkin finite element method to observe numerical properties of models.

In modern times, many experts studied the epidemic models. Iwa et al. studied malaria and cholera models by using the Caputo-Fabrizio derivative of order $a \in (0,1)$ varied with some notable parameters in the frac-

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tional system [7]. Atede et al. considered a fractional-order vaccination model for the novel Coronavirus 2019 (COVID-19) incorporating environmental transmission and analyzed using tools of fractional calculus [8]. Joshi et al. proposed a non-singular SIR model with the Mittag-Leffler law [9]. Odionyenma et al. investigated the fractional derivative model of Chlamydia-Gonorrhea co-infection via Caputo derivative [10]. Sabbar proposed an epidemic model with three intervention measures: media coverage, isolation, and medical therapy [11]. Joshi and Yavuz presented the fractional-order coinfection model for the transmission dynamics of COVID-19 and tuberculosis [12].

This paper considers the SIR model being an epidemic model. This model is used to study the spread of infectious diseases in a population. Unlike the SIR model, there is an additional factor for vaccination. In a vaccination-inclusive model, controlling the epidemic can be achieved by forecasting its future direction. Because biologists can use it to deal with natural initial conditions to design and conduct experiments in observing the spreading of diseases, with the help of these experiments and modeling, the way of controlling the spreading of the epidemic can be learned. Getting exact solutions for the problems representing such natural events is extremely difficult. Searching for convenient methods is an excellent mission for the science community.

The SIR model is composed of three members: those who are susceptible to disease, those who might be infected with the disease, and those who recovered from or become immune to it. Kermack and McKendrick earlier [13] discovered the SIR model.

Rachah and Torres improved the SIR model [14]. They expressed the susceptible-infectious-recovery (SIR) model with vaccination and the initial conditions as follows,

$$\begin{aligned} \frac{dS}{dt} &= -\beta S(t) I(t) - \nu S(t) \\ \frac{dI}{dt} &= \beta S(t) I(t) - \gamma I(t) \\ \frac{dR}{dt} &= \gamma I(t) + \nu S(t) \end{aligned} \tag{1}$$

with initial conditions given by

$$S(0) = N_S,\ I(0) = N_I,\ R(0) = N_R$$

where ν is the percentage of individuals vaccinated every day [15].
N_S= The number of susceptible people in the population at t time.
N_I = The number of infected people in the population at t time.
N_R = The number of recovered people in the population at t time.
N = The size of the population.
β = Transmissivity rate.
γ = Recovery rate.
A person can only be in one of the three groups in any time.

$$N_S + N_I + N_R = N.$$

Recently, some researchers have analyzed the SIR model using various methods to obtain an approximate solution. Argub and El-Ajou [16] applied the Homotopy analysis method for different parameter values for the SIR model. Biazar [17] presented the Adomian decomposition method. While Rafai et al. [18] studied the homotopy perturbation method, İbrahim and Ismail [19] used a differential transformation approach. This model was also solved using the Laplace-Adomian decomposition method [20]. Harman ve Johnston [21] solved by using the stochastic Galerkin method. This epidemic model was solved by Kousar et al. [22] by using the 4th-order Runge Kutta method. It was also solved by the Hussain et al. [23] Runge Kutta-2 ve Runge Kutta-4 methods. Kousar solved this problem via the Euler method [22].

In this paper, the solutions of the SIR model have been found using the Morgan-Voyce collocation method (M-VCM). Many researchers have recently utilized the collocation method to solve this differential equation. Taylor [24–28], Pell-Lucas [29], Morgan-Voyce [30–34], Dickson [35], ortho-exponential [36], and

Bernoulli [37] matrix methods have been used to solve the linear differential, integro-differential equations, fractional differential equations [38], nonlinear differential equations, multi-pantograph [39], delay differential equations and difference and Fredholm integro-difference equation [40]. This method can significantly aid in reducing the complexity of solutions for large-scale epidemic models and can contribute to obtaining results that closely approximate the exact solutions of ordinary differential equation systems.

This study is organized as follows. Section 2 describes the method. We give the error estimation of the method in Section 3. Several applications are provided in Section 4. Finally, we introduce the results of this paper in Conclusion 5.

2 The Morgan-Voyce collocation method

This section presents the M-VCM in clear. This method is used to obtain the approximate solution of the equation (1). Equation (1) is expressed in the truncated Morgan-Voyce series form as follows,

$$S(t)=\sum_{l=0}^{L}A_{1,l}B_l(t),\ I(t)=\sum_{l=0}^{L}A_{2,l}B_l(t),\ R(t)=\sum_{l=0}^{L}A_{3,l}B_l(t). \tag{2}$$

Here, $A_{1,l}$, $A_{2,l}$ and $A_{3,l}$ $(l=0,\ 1,\ 2,\cdots,\ L)$ are the unknown Morgan-Voyce coefficients, L is an arbitrary positive integer such as $(L\geq 2)$. Swamy earlier defined the Morgan-Voyce polynomials $(B_l(t),\ (l=0,\ 1,\ 2,\cdots,\ L))$ [41]. $B_l(t),\ (l=0,\ 1,\ 2,\cdots,\ L)$ are presented by,

$$B_L(t)=\sum_{k=0}^{L}\binom{L+k+1}{L-k}t^k.$$

To write equation (1) in matrix form, we can first write the approximate solutions as follows,

$$S(t)=B(t)A_1,\ I(t)=B(t)A_2,\ R(t)=B(t)A_3, \tag{3}$$

where,

$$B(t)=\left[B_1(t)\ B_2(t)\cdots B_{L-1}(t)\ B_L(t)\right],\ A_1=\left[a_{1,0}\ a_{1,1}\cdots a_{1,L}\right]^T,$$

$$A_2=\left[a_{2,0}\ a_{2,1}\cdots a_{2,L}\right]^T,\ A_3=\left[a_{3,0}\ a_{3,1}\cdots a_{3,L}\right]^T,$$

$$B^T(t)=\begin{bmatrix}B_0(t)\\B_1(t)\\B_2(t)\\\vdots\\B_L(t)\end{bmatrix}=\begin{bmatrix}\binom{1}{0}&0&0&\cdots&0\\\binom{2}{1}&\binom{3}{0}&0&\cdots&0\\\binom{3}{2}&\binom{4}{1}&\binom{5}{0}&\cdots&0\\\vdots&\vdots&\vdots&\cdots&\vdots\\\binom{L+1}{L}&\binom{L+2}{L-1}&\binom{L+3}{L-2}&\cdots&\binom{2L+1}{0}\end{bmatrix}\begin{bmatrix}1\\t\\t^2\\\vdots\\t^n\end{bmatrix},$$

here we can write,

$$F(t)=\left[1\ t\ t^2\cdots t^L\right],\ R=\begin{bmatrix}\binom{1}{0}&0&0&\cdots&0\\\binom{2}{1}&\binom{3}{0}&0&\cdots&0\\\binom{3}{2}&\binom{4}{1}&\binom{5}{0}&\cdots&0\\\vdots&\vdots&\vdots&\cdots&\vdots\\\binom{L+1}{L}&\binom{L+2}{L-1}&\binom{L+3}{L-2}&\cdots&\binom{2L+1}{0}\end{bmatrix}.$$

Therefore, we can write the following equations,

$$S(t) = F(t)R^T A_1,\ I(t) = F(t)R^T A_2,\ R(t) = F(t)R^T A_3. \tag{4}$$

The relationship between matrix $F(t)$ and its derivative matrix $F^{(1)}(t)$ is as follows,

$$F^{(1)}(t) = F(t)T^T \tag{5}$$

and here,

$$T^T = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & L \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

From equations (4) and (5), we can get the following equations,

$$S^{(1)}(t) = F(t)T^T R^T A_1,\ I^{(1)}(t) = F(t)T^T R^T A_2,\ R^{(1)}(t) = F(t)T^T R^T A_3.$$

Thus, we can obtain the matrices $y(t)$ and $y^{(1)}(t)$ as follows,

$$y(t) = \overline{FR}A,\ y^1(t) = \overline{FTR}A,$$

where,

$$y(t) = \begin{bmatrix} S(t) \\ I(t) \\ R(t) \end{bmatrix},\ y^{(1)}(t) = \begin{bmatrix} S^{(1)}(t) \\ I^{(1)}(t) \\ R^{(1)}(t) \end{bmatrix},\ \overline{F}(t) = \begin{bmatrix} F(t) & 0 & 0 \\ 0 & F(t) & 0 \\ 0 & 0 & F(t) \end{bmatrix},\ \overline{T} = \begin{bmatrix} T^T & 0 & 0 \\ 0 & T^T & 0 \\ 0 & 0 & T^T \end{bmatrix},$$

and,

$$\overline{R} = \begin{bmatrix} R^T & 0 & 0 \\ 0 & R^T & 0 \\ 0 & 0 & R^T \end{bmatrix},\ A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}.$$

Equation (1) can be written in matrix form as follows,

$$y^1(t) - Dy(t) - Ny(t) - Ey_{1,2}(t) = g, \tag{6}$$

where,

$$g = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & \gamma & 0 \end{bmatrix},\ E = \begin{bmatrix} -\beta \\ \beta \\ 0 \end{bmatrix},\ N = \begin{bmatrix} -\nu & 0 & 0 \\ 0 & 0 & 0 \\ \nu & 0 & 0 \end{bmatrix},\ y_{1,2} = [S(t)I(t)].$$

Now, to determine the unknown coefficients of $A_{1,I}$, $A_{2,I}$ and $A_{3,I}$, we can define the collocation points for the interval $a \le t \le b$ as follows,

$$t_i = a + \frac{b-a}{L} i, \quad i = 0,\ 1,\ \cdots,\ L.$$

Substituting the collocation points in equation (6), we get the following system,

$$y^{(1)}(t_i) - Dy(t_i) - Ny(t_i) - Ey_{1,2}(t_i) = g,$$

where,

$$y(t_i) = \overline{F}(t_i)\overline{R}A, \tag{7}$$

$$y^{(1)}(t_i) = \overline{F}(t_i)\overline{TR}A, \tag{8}$$

here,

$$F = \left[\overline{F}(t_0)\ \overline{F}(t_1)\cdots\overline{F}(t_L)\right]^T,\ \overline{F}(t_i) = \begin{bmatrix} F(t_i) & 0 & 0 \\ 0 & F(t_i) & 0 \\ 0 & 0 & F(t_i) \end{bmatrix}.$$

So, equation (1) can be written in the following matrix form utilizing the upper matrices,

$$Y^{(1)} - \overline{D}Y - \overline{N}Y - \overline{\widetilde{E}Y} = G, \tag{9}$$

where,

$$Y^{(1)} = \begin{bmatrix} y^{(1)}(t_0) \\ y^{(1)}(t_1) \\ \vdots \\ y^{(1)}(t_L) \end{bmatrix},\ \overline{D} = \begin{bmatrix} D & 0 & \cdots & 0 \\ 0 & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D \end{bmatrix}_{(L+1)(L+1)},\ \overline{N} = \begin{bmatrix} N & 0 & \cdots & 0 \\ 0 & N & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & N \end{bmatrix}_{(L+1)(L+1)},$$

$$Y = \begin{bmatrix} y(t_0) \\ y(t_1) \\ \vdots \\ y(t_L) \end{bmatrix},\ G = \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix}_{(L+1)1},\ \widetilde{\overline{Y}} = \begin{bmatrix} y_{1,2}(t_0) \\ y_{1,2}(t_1) \\ \vdots \\ y_{1,2}(t_L) \end{bmatrix},\ \overline{E} = \begin{bmatrix} E & 0 & \cdots & 0 \\ 0 & E & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E \end{bmatrix}_{(L+1)(L+1)}.$$

Similarly, the following matrix form is obtained by substituting the collocation points in $y_{1,2}(t)$,

$$\widetilde{\overline{Y}} = \begin{bmatrix} y_{1,2}(t_0) \\ y_{1,2}(t_1) \\ \vdots \\ y_{1,2}(t_L) \end{bmatrix} = \begin{bmatrix} I(t_0) & 0 & \cdots & 0 \\ 0 & I(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I(t_L) \end{bmatrix} \begin{bmatrix} S(t_0) \\ S(t_1) \\ \vdots \\ S(t_L) \end{bmatrix} = \overline{IS},$$

where,

$$\overline{I} = \widetilde{\overline{F}}\widetilde{\overline{R}}\overline{A_2} \quad \text{and} \quad \overline{S} = \widetilde{\overline{T}}\widetilde{\overline{R}}A, \tag{10}$$

and,

$$\widetilde{F} = \begin{bmatrix} F(t_0) & 0 & \cdots & 0 \\ 0 & F(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F(t_L) \end{bmatrix},\ \overline{A_2} = \begin{bmatrix} A_2 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_2 \end{bmatrix}_{(L+1)(L+1)},\ \widetilde{R} = \begin{bmatrix} R^T & 0 & \cdots & 0 \\ 0 & R^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R^T \end{bmatrix}_{(L+1)(L+1)},$$

$$\widetilde{\overline{F}} = \begin{bmatrix} F(t_0) \\ F(t_1) \\ \vdots \\ F(t_L) \end{bmatrix},\ \widetilde{\overline{R}} = \begin{bmatrix} R^T\ K\ K \end{bmatrix},\ K = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(L+1)(L+1)}.$$

The fundamental matrix equation from equations (7)-(10) is as follows,

$$\left\{F\overline{BR} - \overline{D}F\overline{R} - \overline{N}F\overline{R} - \overline{E}\widetilde{F}\widetilde{\overline{R}}\overline{A_2}\widetilde{\overline{F}}\widetilde{\overline{R}}\right\}A = G. \tag{11}$$

Briefly, equation (11) can be written as,

$$WA = G \text{ or } [W;G],$$

where,

$$W = F\overline{TR} - \overline{D}F\overline{R} - \overline{N}F\overline{R} - \overline{E}\widetilde{F}\widetilde{R}\overline{A_2}\widetilde{\widetilde{F}}\widetilde{R}. \tag{12}$$

Equation (12) refers to a nonlinear system of $3(L+1)$ algebraic equations with the unknown Morgan-Voyce coefficients $A_{1,I}$, $A_{2,I}$ and $A_{3,I}$. The matrix form of the initial conditions for $t \longrightarrow 0$ in equation (3) can be written as follows,

$$\begin{aligned} S(t) &= B(0)A_1 = [N_S] \\ I(t) &= B(0)A_2 = [N_I] \\ R(t) &= B(0)A_3 = [N_R]. \end{aligned}$$

Thus, these matrix forms can be written as,

$$\begin{aligned} U_1 &= S(0) = \left[a_{1,0}\ a_{1,1} \cdots a_{1,L}\right] \\ U_2 &= I(0) = \left[a_{2,0}\ a_{2,1} \cdots a_{2,L}\right] \\ U_3 &= R(0) = \left[a_{3,0}\ a_{3,1} \cdots a_{3,L}\right]. \end{aligned} \tag{13}$$

Finally, we replace the row matrices in equation (13) with any three rows of the matrix in equation (12), and we get a solution for equation (1) under initial conditions. Finally, we get the augmented matrix as,

$$\widetilde{W}A = G.$$

First of all, this system is solved and the unknown coefficients $a_{i,0},\ a_{i,1},\ \cdots,\ a_{i,L}, (i = 1,\ 2,\ 3)$ are found. Then, these coefficients are substituted in equation (2) and, approximate solutions are found.

3 Error estimation

In this section, we investigate the accuracy of the presented method. $S_L(t),\ I_L(t),\ R_L(t)$ are the approximate solutions of equation (1). Therefore, these functions approximately satisfy the equation (1). Once we substitute these functions and the first derivatives of these functions in the equation (1), we get the error estimations for $t = t_r \in [0,R],\ r = 0,\ 1,...$ as following,

$$\begin{aligned} E_{1,L}(t_r) &= |S'(t_r) + \beta S(t_r)I(t_r) + \nu S(t_r)| \cong 0 \\ E_{2,L}(t_r) &= |I'(t_r) - \beta S(t_r)I(t_r) + \gamma I(t_r)| \cong 0 \\ E_{3,L}(t_r) &= |R'(t_r) - \gamma I(t_r) - \nu S(t_r)| \cong 0, \end{aligned}$$

and $E_{i,L} \leq 10^{-k_r},\ i = 1,\ 2,\ 3$ (k_r is a positive number). If we get $max 10^{-k_r} = 10^{-k}$, as L increases, error estimation $E_{i,L}(t_r),\ (t = 1,\ 2,\ 3)$ at each of the points decreases and becomes smaller than 10^{-k}.

4 Numerical applications

This section supports the accuracy and efficiency of the presented method. We offer two numerical examples, one comparing the errors and solutions with the other methods. In the second sample, the results for different vaccination rates are compared. The tables and figures show the comparisons at specific points within the given interval. Numerical computations are performed using MATLABR2021a software.

Example 1. *In this example, we consider the SIR model without vaccination for interval* $0 \leq t \leq 1$. *The following parameter values are used,*

$$N_S = 20,\ N_I = 15,\ N_R = 10,\ \beta = 0.01,\ \gamma = 0.02,\ \nu = 0. \tag{14}$$

We compared the results obtained by the proposed method with those in [42]. The approximate solutions for $L = 5$ *are obtained using the presented method as follows,*

$$\begin{aligned}
S_5(t) &= 20.00000000 - 2.999999999t - 0.04499957685t^2 + 0.02804671200t^3 \\
&\quad + 0.0008058035642t^4 - 0.0003329149155t^5 \\
I_5(t) &= 15.00000000 + 2.699999999t + 0.01799970470t^2 - 0.02816759777t^3 \\
&\quad - 0.0006626352597t^4 + 0.0003329022387t^5 \\
R_5(t) &= 10.00000000 + 0.300000000t + 0.2699987216t^2 + 0.0001208857706t^3 \\
&\quad - 0.0001431683045t^4 + 0.126768711610 \times 10^{-7}t^5.
\end{aligned}$$

It is known that there is no exact solution for equation (1). Therefore, the obtained solutions using the presented method are compared with the approximation solutions obtained by using the Hermite collocation method (HCM) [42], HPM rafei and LADM [20] in Tables 1, 3 and 5. Besides, the residual errors are presented in Tables 2, 4 and 6 and plotted in Figures 1, 2, 3 for $L = 5$. *The errors obtained with the presented method are smaller than those obtained with HPM and LADM and similar to those obtained with HCM. So, we can say that the presented method improves the results. As the plots and tables show, the M-VCM is effective and usable in solving a nonlinear system of equations. In addition, considerable errors are obtained even for the small value of L.*

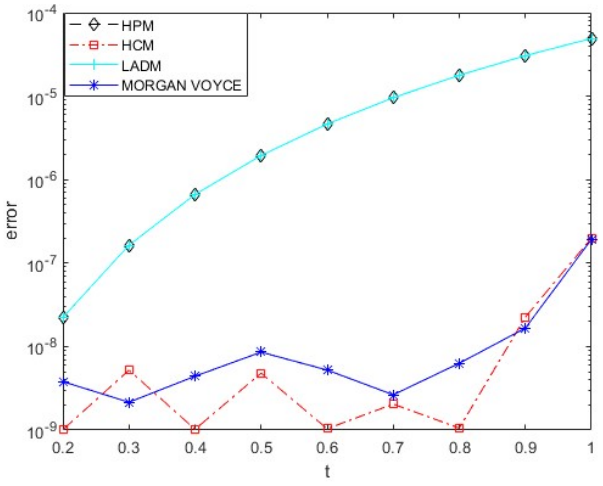

Fig. 1 Comparison of the residual errors of $S(t)$ (without vaccination) for equation (1).

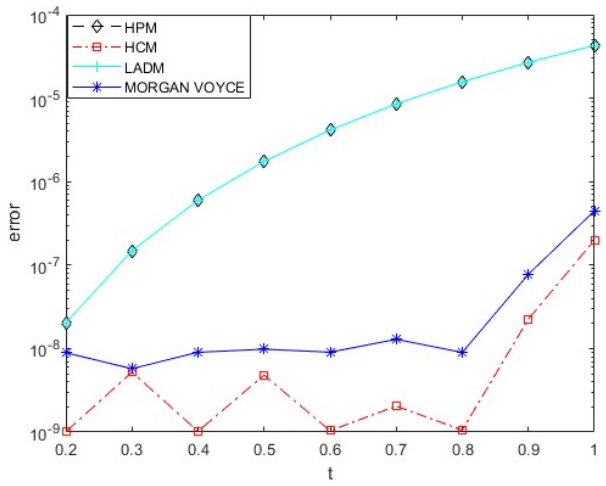

Fig. 2 Comparison of the residual errors of $I(t)$ (without vaccination) for equation (1).

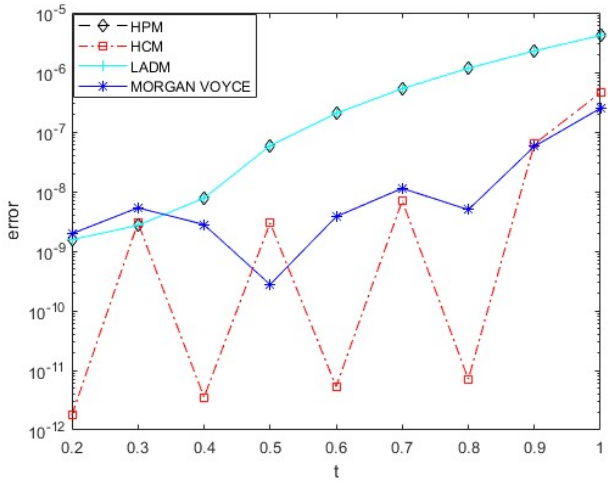

Fig. 3 Comparison of the residual errors of $R(t)$ (without vaccination) for equation (1).

Table 1 Comparisons of the values of $S(t)$ for HPM, HCM, LADM and M-VCM for example 1.

t	$S(t)$ (HPM)	$S(t)$ (HCM for L=5)	$S(t)$ (LADM)	$S(t)$ M-VCM
0.2	19.39842557	19.39842556	19.39842557	19.3984255735
0.4	18.79461232	18.79461227	18.79461232	18.7946122770
0.6	18.18993727	18.18993677	18.18993727	18.1899367871
0.8	17.58578366	17.58578115	17.58578366	17.5857811555
1.0	16.98352883	16.98352001	16.98352883	16.9835200246

Table 2 Comparisons of the residual errors ER_S for HPM, HCM, LADM and M-VCM for example 1.

t	ER_S (HPM)	ER_S (HCM for L=5)	ER_S (LADM)	ER_S M-VCM
0.2	2.2416×10^{-8}	1.0080×10^{-9}	2.2417×10^{-8}	3.8045×10^{-9}
0.4	6.6425×10^{-7}	1.0201×10^{-9}	6.6427×10^{-7}	4.4120×10^{-9}
0.6	4.6388×10^{-6}	1.0363×10^{-9}	4.6389×10^{-6}	5.2320×10^{-9}
0.8	1.7832×10^{-5}	1.0566×10^{-9}	1.7833×10^{-5}	6.3120×10^{-9}
1.0	4.9171×10^{-5}	1.9918×10^{-7}	4.9172×10^{-5}	1.9256×10^{-7}

Table 3 Comparisons of the values of $I(t)$ for HPM, HCM, LADM and M-VCM for example 1.

t	$I(t)$ (HPM)	$I(t)$ (HCM for L=5)	$I(t)$ (LADM)	$I(t)$ M-VCM
0.2	15.54049369	15.54049370	15.54049369	15.5404936937
0.4	16.08106363	16.08106367	16.08106363	16.081063672
0.6	16.62033527	16.62033570	16.62033527	16.6203357015
0.8	17.15693346	17.15693567	17.15693345	17.156935671
1.0	17.68949465	17.68950236	17.68949464	17.6895023739

Table 4 Comparisons of the residual errors ER_I for HPM, HCM, LADM and M-VCM for example 1.

t	ER_I (HPM)	ER_I (HCM for L=5)	ER_I (LADM)	ER_I M-VCM
0.2	2.0373×10^{-8}	1.0080×10^{-9}	2.0375×10^{-8}	8.9114×10^{-9}
0.4	5.9888×10^{-7}	1.0201×10^{-9}	5.9892×10^{-7}	9.0230×10^{-9}
0.6	4.1424×10^{-6}	1.0363×10^{-9}	4.1426×10^{-6}	9.0493×10^{-9}
0.8	1.5741×10^{-5}	1.0566×10^{-9}	1.5741×10^{-5}	8.9952×10^{-9}
1.0	4.2787×10^{-5}	1.9918×10^{-7}	4.2789×10^{-5}	4.4706×10^{-7}

Table 5 Comparisons of the values of $R(t)$ for HPM, HCM, LADM and M-VCM for example 1.

t	$R(t)$ (HPM)	$R(t)$ (HCM for L=5)	$R(t)$ (LADM)	$R(t)$ Morgan-Voyce
0.2	10.06108073	10.06108073	10.06108073	10.0610807329
0.4	10.12432405	10.12432405	10.12432405	10.1243240513
0.6	10.18972750	10.18972751	10.18972750	10.1897275117
0.8	10.25728304	10.25728317	10.25728304	10.2572831742
1.0	10.32697698	10.32697760	10.32697698	10.3269776025

Table 6 Comparisons of the residual errors ER_R for HPM, HCM, LADM and M-VCM for example 1.

t	ER_R (HPM)	ER_R (HCM for L=5)	ER_R (LADM)	ER_R M-VCM
0.2	1.5573×10^{-9}	1.7802×10^{-12}	1.5573×10^{-9}	1.9663×10^{-9}
0.4	7.7657×10^{-9}	3.5556×10^{-12}	7.7656×10^{-9}	2.7950×10^{-9}
0.6	2.0477×10^{-7}	5.3304×10^{-12}	2.0477×10^{-7}	3.7913×10^{-9}
0.8	1.1701×10^{-6}	7.1090×10^{-12}	1.1701×10^{-6}	4.9600×10^{-9}
1.0	4.1334×10^{-6}	4.5486×10^{-7}	4.1334×10^{-6}	2.4930×10^{-7}

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Example 2. *Consider equation (1) for interval* $0 \leq t \leq 1$. *The parameter values are selected such that* $N_S = 0.95$, $N_I = 0.05$, $N_R = 0$, $\beta = 0.2$, $\gamma = 0.1$, $\nu = 0.005, 0.01, 0.02, 0.03, 0.06$.

We selected these parameters from study [14]. The residual errors for $S(t)$, $I(t)$, $R(t)$ *are shown in Tables 7-11 and presented in Figures 4-8 for different values of* ν. *For small* ν *values, the errors of* $S(t)$ *and* $R(t)$ *are smaller, while the error of* $I(t)$ *does not change much. The smallest errors of* $I(t)$ *are obtained for the value* $\nu = 0.02$. *The smallest errors of* $S(t)$ *and* $R(t)$ *are obtained for the values* $\nu = 0.01$ *and* 0.03. *Thus, we can say that the best approximation is obtained for* $\nu = 0.02$.

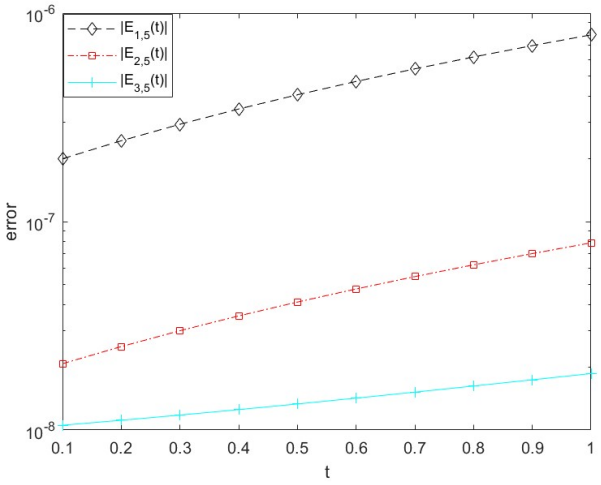

Fig. 4 The residual errors for equation (1), $\nu = 0.005$ via M-VCM.

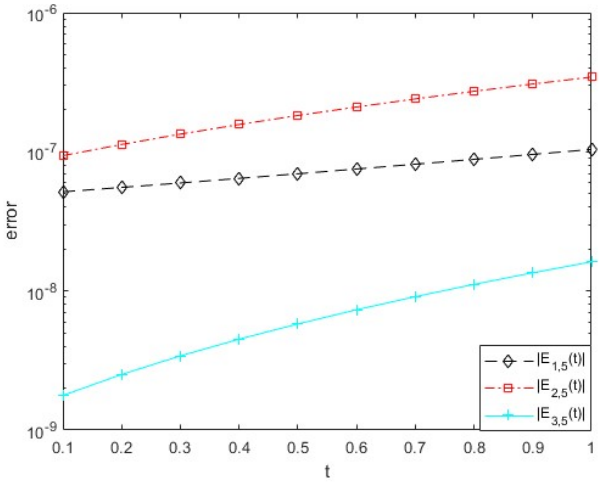

Fig. 5 The residual errors for equation (1), $\nu = 0.01$ via M-VCM.

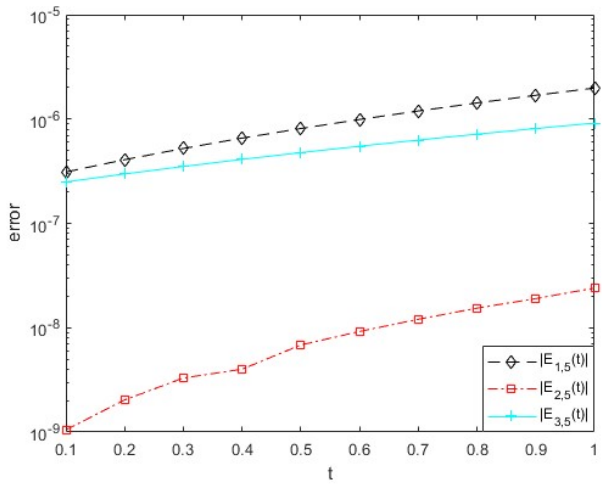

Fig. 6 The residual errors for equation (1), $\nu = 0.02$ via M-VCM.

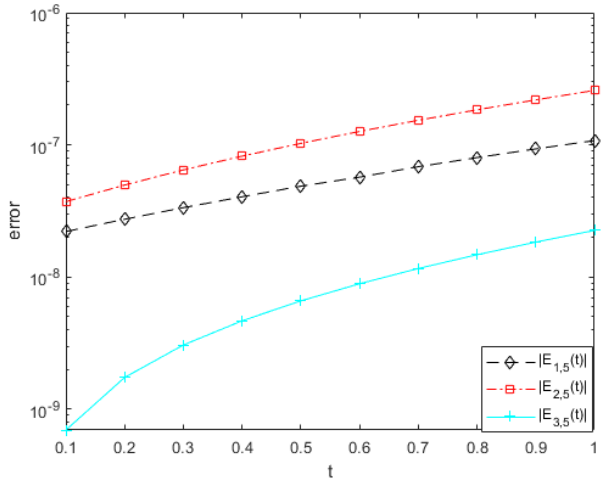

Fig. 7 The residual errors for equation (1), $\nu = 0.03$ via M-VCM.

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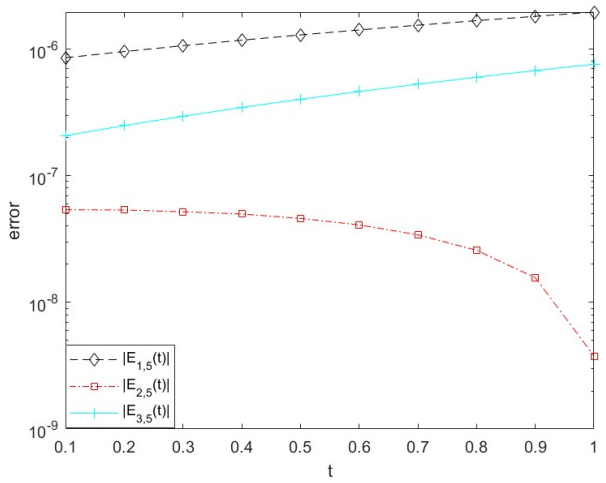

Fig. 8 The residual errors for equation (1), $\nu = 0.06$ via M-VCM.

Table 7 The residual errors for equation (1), $\nu = 0.005$ via M-VCM.

t	ER_S	ER_I	ER_R
0.1	2.0017×10^{-7}	2.0805×10^{-8}	1.0544×10^{-8}
0.2	2.4419×10^{-7}	2.5143×10^{-8}	1.1137×10^{-8}
0.3	2.9299×10^{-7}	2.9958×10^{-8}	1.1797×10^{-8}
0.4	3.4678×10^{-7}	3.5273×10^{-8}	1.2529×10^{-8}
0.5	4.0577×10^{-7}	4.1114×10^{-8}	1.3337×10^{-8}
0.6	4.7020×10^{-7}	4.7505×10^{-8}	1.4226×10^{-8}
0.7	5.4027×10^{-7}	5.4471×10^{-8}	1.5200×10^{-8}
0.8	6.1622×10^{-7}	6.2036×10^{-8}	1.6262×10^{-8}
0.9	6.9824×10^{-7}	7.0221×10^{-8}	1.7416×10^{-8}
1	7.8657×10^{-7}	7.9046×10^{-8}	1.8661×10^{-8}

Table 8 The residual errors for equation (1), $\nu = 0.01$ via M-VCM.

t	ER_S	ER_I	ER_R
0.1	5.1591×10^{-8}	9.3905×10^{-8}	1.7736×10^{-9}
0.2	5.5453×10^{-8}	1.1294×10^{-7}	2.5089×10^{-9}
0.3	5.9708×10^{-8}	1.3399×10^{-7}	3.4104×10^{-9}
0.4	6.4395×10^{-8}	1.5712×10^{-7}	4.4980×10^{-9}
0.5	6.9554×10^{-8}	1.8243×10^{-7}	5.7929×10^{-9}
0.6	7.5227×10^{-8}	2.1000×10^{-7}	7.3172×10^{-9}
0.7	8.1458×10^{-8}	2.3993×10^{-7}	9.0941×10^{-9}
0.8	8.8295×10^{-8}	2.7229×10^{-7}	1.1148×10^{-8}
0.9	9.5794×10^{-8}	3.0717×10^{-7}	1.3503×10^{-8}
1	1.0402×10^{-7}	3.4464×10^{-7}	1.6184×10^{-8}

Table 9 The residual errors for equation (1), $\nu = 0.02$ via M-VCM.

t	ER_S	ER_I	ER_R
0.1	3.1084×10^{-7}	1.0500×10^{-9}	2.4970×10^{-7}
0.2	4.0778×10^{-7}	2.0300×10^{-9}	2.9760×10^{-7}
0.3	5.2230×10^{-7}	3.2950×10^{-9}	3.5110×10^{-7}
0.4	6.5640×10^{-7}	4.0000×10^{-9}	4.1066×10^{-7}
0.5	8.1100×10^{-7}	6.8300×10^{-9}	4.7650×10^{-7}
0.6	9.9065×10^{-7}	9.2000×10^{-9}	5.4925×10^{-7}
0.7	1.1948×10^{-7}	1.2043×10^{-8}	6.2904×10^{-7}
0.8	1.4266×10^{-7}	1.5405×10^{-8}	7.1630×10^{-7}
0.9	1.6880×10^{-6}	1.9000×10^{-8}	8.1140×10^{-7}
1	1.9819×10^{-6}	2.4010×10^{-8}	9.1494×10^{-7}

Table 10 The residual errors for equation (1), $\nu = 0.03$ via M-VCM.

t	ER_S	ER_I	ER_R
0.1	2.2100×10^{-8}	3.7200×10^{-8}	6.9600×10^{-10}
0.2	2.7380×10^{-8}	4.9700×10^{-8}	1.7400×10^{-9}
0.3	3.3490×10^{-8}	6.4701×10^{-8}	3.0619×10^{-9}
0.4	4.0500×10^{-8}	8.2200×10^{-8}	4.6670×10^{-9}
0.5	4.8700×10^{-8}	1.0270×10^{-7}	6.6010×10^{-9}
0.6	5.7000×10^{-8}	1.2640×10^{-7}	8.9000×10^{-9}
0.7	6.8390×10^{-8}	1.5347×10^{-7}	1.1605×10^{-8}
0.8	8.0117×10^{-8}	1.8420×10^{-7}	1.4750×10^{-8}
0.9	9.3220×10^{-8}	2.1914×10^{-7}	1.8380×10^{-8}
1	1.0780×10^{-7}	2.5836×10^{-7}	2.2531×10^{-8}

Table 11 The residual errors for equation (1), $\nu = 0.06$ via M-VCM.

t	ER_S	ER_I	ER_R
0.1	8.5647×10^{-7}	5.4000×10^{-8}	2.0762×10^{-7}
0.2	9.5890×10^{-7}	5.3700×10^{-8}	2.4940×10^{-7}
0.3	1.0660×10^{-6}	5.2000×10^{-8}	2.9568×10^{-7}
0.4	1.1792×10^{-6}	4.9930×10^{-8}	3.4662×10^{-7}
0.5	1.2972×10^{-6}	4.6100×10^{-8}	4.0240×10^{-7}
0.6	1.4202×10^{-6}	4.0964×10^{-8}	4.6333×10^{-7}
0.7	1.5484×10^{-6}	3.4200×10^{-8}	5.2955×10^{-7}
0.8	1.6819×10^{-6}	2.5800×10^{-8}	6.0120×10^{-7}
0.9	1.8205×10^{-6}	1.5672×10^{-8}	6.7870×10^{-7}
1	1.9641×10^{-6}	3.7710×10^{-9}	7.6213×10^{-7}

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5 Conclusions

In this paper, we used the M-VCM to find the approximate solution of the governing model. Several applications were presented to show the efficiency and accuracy of the proposed method. To display the accuracy of the approximate solutions, these solutions are once again substituted into equation (1) using MATLAB software. Thus, this work offers a different measure for the reliability of the approximate solutions. The obtained approximate solutions and errors have been compared by the HPM [18] and the LADM [20]. These comparisons have shown that the presented method to find approximate solutions to the SIR model from epidemic models is more efficient and valuable. Tables 2, 4, and 6 illustrated that the numerical solutions of HPM [18] and LADM [20] are nearly the same. As can be seen from the figures and tables, minimal errors were obtained in this study, indicating that the results are very close to the exact solutions. In particular, in the second example, we can accurately determine the vaccination rate needed to control the epidemic. In conclusion, all these findings show that our method is efficient and feasible for solving the nonlinear ODE systems. The most significant advantage of the proposed method is that all calculations can be easily and quickly applied with the help of MATLAB code. In addition, the fact that the SIR model is a real-life problem increases the importance of our study. One of the most significant advantages of the method is changing the system equation (1) into an easily solved nonlinear algebraic equation system. As a further study, this method can be applied to other epidemic models for longer time periods.

6 Declarations

6.1 Conflict of interest:

The authors declare no conflict of interest.

6.2 Funding:

Not applicable.

6.3 Author's contribution:

Ö.İ.-Methodology, Supervision, Resources, and Methodology, Validation, Conceptualization and Formal analysis; G.Ş.-Data Curation, Writing-Original Draft, Investigation and Visualization. All authors read and approved the final submitted version of this manuscript.

6.4 Acknowledgement:

Not applicable.

6.5 Data availability statement:

All data that support the findings of this study are included within the article.

6.6 Using of AI tools:

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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