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Original Study

Dynamic nature of analytical soliton solutions of the (1+1)-dimensional Mikhailov-Novikov-Wang equation using the unified approach

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Abstract

In this work, the dynamical study of the (1+1)-dimensional Mikhailov-Novikov-Wang (MNW) equation via the unified method is investigated. This technique is used to obtain the soliton solutions, including the trigonometric function solution, the periodic function solution, the exponential function solution, the elliptic function solution, and other soliton-form solutions. All the obtained results in this work utilizing an effective unified method help gain a better understanding of the physical meaning and behaviour of the equation, thus sheding light on the significance of investigating diverse nonlinear wave phenomena in physics and ocean engineering. These derived results are entirely new and never repeated in previous works done by the other authors. For the interest of visual presentation and physical illustrations, we plot the graphical demonstrations of some of the specified solutions in 3-dimensional, contour, and 2-dimensional plots by using *Mathematica* software. Consequently, we observe that the acquired solutions of the MNW equations are anti-bell-shape, kink wave solution, solitary wave, periodic solution, multisoliton, and different types of soliton solutions.

Keywords: Analytical soliton solutions, unified method, solitons, Mikhailov-Novikov-Wang equation, dynamic nature, mathematical methods.
AMS 2020 codes: 35C07; 35Dxx; 34Axx.

1 Introduction

These nonlinear evolution equations have attracted a large section of mathematicians and physicists due to the fact that they have many applications in human life. These equations are typically used to explain a variety of complex natural phenomena. Nonlinear evolution equations have been noticed in plasma physics, optical fibers, fluid mechanics, and other applications. A great number of mathematicians have been trying to attain the exact solitary wave solutions of these complex phenomena in various areas of engineering and science including chemistry, plasma physics, nonlinear optics, biology, fluid dynamics, etc. Until now, various researchers have used many efficient and powerful methods to construct exact analytical solutions for these partial differential equations such as the Hirota method [1], the F-expansion method [2], the tanh function method [3], the modified extended Tanh expansion method [4, 5], the generalized Kudryashov method [6], the Darboux transformation method [7,8], the sine-cosine method [9], the Bäcklund transformation method [10,11],

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the Lie symmetry method [12–17], the Homogeneous balance method [18], the generalized exponential rational function method [19, 20], the generalized Riccati equation mapping (GREM) method [21, 22], the unified method [23, 24], and many more [25–27]. The unified method is the most useful and efficient technique for obtaining exact analytical solutions. In current periods, many researchers have utilized the unified technique and constructed different types of soliton solutions.

Hence, analyzing MNW equation can provide a good knowledge of different nonlinear scientific phenomena in physics [28, 29]. (1+1)-dimensional MNW equation has been studied by some researchers recently. Raza et al. employed the $\exp(-\phi(\xi))$-expansion method, singular manifold method, and generalized projective Riccati equations method on the equation to acquired new soliton solutions [30]. Ray and Singh obtained the kink-type multisoliton solutions of the MNW equation by using simplified Hirota's method [31]. Akbulut et al. used the generalized Kudryashov method, exponential rational function method, and modified extended tanh-function method and obtained exact solutions [32]. With the help of the (G'/G)-expansion method, Bekir et al. obtained new exact soliton solutions of the MNW equation [33], whereas in this paper, we constructed new analytical soliton solutions with the aid of an effective and powerful unified method. Studying this equation can give a useful understanding of numerous impressive nonlinear scientific phenomena in oceanography and physics.

In this paper, we aim to find the exact analytical solution of the MNW equation using a unified method. We consider MNW equation [34–37], which can be expressed as follows,

$$u_{tt} - u_{xxxt} - 8u_x u_{xt} - 4u_t u_{xx} + 2u_x u_{xxxx} + 24u_x^2 u_{xx} + 4u_{xx} u_{xxx} = 0, \tag{1}$$

where $u = u(x,t)$ represents the wave amplitude. This equation derived by utilizing the perturbative symmetry method to the classification of integrable equations. Furthermore, MNW equation belongs to a hierarchy where the Boussinesq equation is a member of this class.

The used method is efficient, straightforward, and very powerful in order to investigate nonlinear partial differential equations (NPDEs) as they give a broad range of solitary wave solutions. Some families of exact analytical solutions of the MNW equation are studied via the help of *Wolfram Mathematica* for soft symbolic computation. By using this integral method, we derived analytical soliton solutions in the different form of solitary wave solutions, trigonometric solutions, hyperbolic solutions, soliton solutions, rational solutions, and many more kinds of solutions. All the obtained soliton solutions including few arbitrary constants and unknown coefficient parameters from the unified method have immense wide applications in mathematical science engineering. These obtained solutions are more beneficial and important in the area of fluid dynamics, optical physics, mathematical physics, plasma physics, and various fields of sciences. The newly exact analytical outcomes are illustrated in the diverse forms of anti-bell-shape solitons, kink waves, and periodic solitons. These obtained results are entirely new and never proposed in previous works done by other authors. Additionally, the wide range of dynamics, characteristics, and suitable arbitrary constant parameters of these new exact analytical results are depicted using 3-dimensional plots, contour plots, and 2-dimensional plots representation, with the appropriate selection of the employed rational functions, hyperbolic functions, trigonometric functions, and other arbitrary parameters. This kind of study shows positive results in the areas of development and advanced research.

We have organized this paper in the following sections: In section 2, we use a wave transformation to transform the MNW equation into an ordinary differential equation. In section 3, a detailed description of the proposed techniques is presented. Moreover, this section is devoted to the applications of the proposed methodology to (1+1)-dimensional Mikhailov-Novikov-Wang equation. In section 4, the results and discussion of the obtained solutions of the governing model are presented. Section 5 contains conclusions and important remarks.

2 Methodology of the unified method

Suppose that a general PDE is as follows,

$$\Phi(u_x,\ u_t,\ u_{xt},\ u_{xx}, u_{xxx}, \cdots) = 0. \tag{2}$$

Using the wave transformation

$$u(x,t) = U(\chi),\ \ \chi = \zeta\, x - \delta\, t, \tag{3}$$

equation (2) converts into an ODE

$$\Theta(U',\ U'',\ U''\cdots) = 0, \tag{4}$$

in which ζ and δ real/complex constant parameters. The solution of equation (4) can furnish as follows

$$U(X) = G_0 + \sum_{i=1}^{N} G_i\, \xi(X)^i + \sum_{i=1}^{N} \frac{H_i}{\xi(X)^i}, \tag{5}$$

where G_0, G_i, H_i $(1 \leq i \leq N)$ are the unknown arbitrary parameters and explicit invariant solution $\xi(X)$ satisfies the following equation

$$\xi'(X) = \xi^2(X) + M. \tag{6}$$

In the three cases, equation (6) confirms nine types of exact solutions as follows.

Case 1: Hyperbolic function solutions (when M is negative)

$$\begin{cases} (i) \quad \xi(X) & = \frac{-\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(X+\phi))}{P\sinh(2\sqrt{-M}(X+\phi)) + Q}, \\ (ii) \quad \xi(X) & = \frac{\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(X+\phi))}{P\sinh(2\sqrt{-M}(X+\phi)) + Q}, \\ (iii) \quad \xi(X) & = -\sqrt{-M} + \frac{2P\sqrt{-M}}{P+\cosh(2\sqrt{-M}(X+\phi)) - \sinh(2\sqrt{-M}(X+\phi))}, \\ (iv) \quad \xi(X) & = \sqrt{-M} + \frac{2P\sqrt{-M}}{P+\cosh(2\sqrt{-M}(X+\phi)) - \sinh(2\sqrt{-M}(X+\phi))}. \end{cases} \tag{7}$$

Case 2: Trigonometric function solutions (when M is positive)

$$\begin{cases} (v) \quad \xi(X) & = \frac{-\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(X+\phi))}{P\sin(2\sqrt{M}(X+\phi)) + Q}, \\ (vi) \quad \xi(X) & = \frac{\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(X+\phi))}{P\sin(2\sqrt{M}(X+\phi)) + Q}, \\ (vii) \quad \xi(X) & = -i\sqrt{M} + \frac{2iP\sqrt{M}}{P+\cos(2\sqrt{M}(X+\phi)) - i\sin(2\sqrt{M}(X+\phi))}, \\ (viii) \quad \xi(X) & = i\sqrt{M} + \frac{2iP\sqrt{M}}{P+\cos(2\sqrt{M}(X+\phi)) - i\sin(2\sqrt{M}(X+\phi))}. \end{cases} \tag{8}$$

Case 3: Rational function solutions (when $M = 0$)

$$(ix) \quad \xi(X) = -\frac{1}{X+\phi}, \tag{9}$$

when $P \neq 0$, ϕ and Q are arbitrary parameters.

3 Exact analytical solution of the MNW equation

The main objective of this section is to construct the exact analytical solutions of the MNW equation. Incorporating (3) and (1) yields

$$U''(\delta^2+12\zeta^2U'(\delta+2\zeta^2U')+4\zeta^5U''')+\zeta^3(\delta+2\zeta^2U')U''''=0. \tag{10}$$

Integrating equation (10) with respect to χ and taking the integration constant zero, we obtained

$$6\zeta^2\delta(U')^2+8\zeta^4(U')^3+\zeta^5(U'')^2+\zeta^3\delta U'''+U'(\delta^2+2\zeta^5U''')=0. \tag{11}$$

With the help of the principle of balancing for $U'\,U'''$ and U'^3, we contain $N=1$. Inserting $N=1$ in equation (5), we have

$$U(X)=G_0+G_1\xi(X)+\frac{H_1}{\xi(X)}. \tag{12}$$

Combining equation (6), (12) and (11), we get the following three solution sets

$$G_0=G_0,\ G_1=-2\zeta,\ H_1=0,\ \delta=4\zeta^3M, \tag{13}$$
$$G_0=G_0,\ G_1=0,\ H_1=2\zeta M,\ \delta=4\zeta^3M, \tag{14}$$
$$G_0=G_0,\ G_1=-2\zeta,\ H_1=2\zeta M,\ \delta=16\zeta^3M. \tag{15}$$

Using equation (3), equation (7)-(9), and equation (13), we obtain the subsequent exact analytical solutions of the MNW equation (1)

$$u_{11}(x,t)=G_0-2\,\zeta\left(\frac{-\sqrt{-(P^2+Q^2)M}-P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-4\zeta^3M\,t+\phi))}{P\sinh(2\sqrt{-M}(\zeta\,x-4\zeta^3M\,t+\phi))+Q}\right), \tag{16}$$

$$u_{12}(x,t)=G_0-2\zeta\left(\frac{\sqrt{-(P^2+Q^2)M}-P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-4\zeta^3M\,t+\phi))}{P\sinh(2\sqrt{-M}(\zeta\,x-4\zeta^3M\,t+\phi))+Q}\right). \tag{17}$$

We can see various simulations of the solutions of the equation (17) in Figure 1.

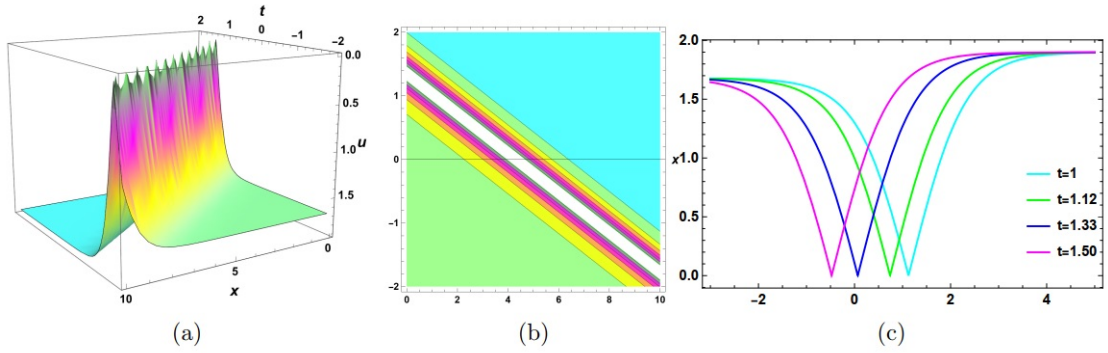


Fig. 1 Anti-bell-shape profile observed via 3D for the solutions of the equation (17) in this figure with the $G_0=0.11,\ \zeta=2,\ P=0.021,\ Q=397,\ \phi=3,\ M=-0.2$ and wave propagation for different values of $t=1,\ 1.12,\ 1.33,\ 1.50$ with the $-3\le x\le 5$ shows through 2D plot.

$$u_{13}(x,t) = \frac{4\,\zeta\,P\sqrt{-M}}{\sinh(2\sqrt{-M}(\zeta\,x-4\zeta^3 M\,t+\phi))-P-\cosh(2\sqrt{-M}(\zeta\,x-4\zeta^3 M\,t+\phi))}$$
$$G_0+2\,\zeta\,\sqrt{-M},, \tag{18}$$

$$u_{14}(x,t) = \frac{4\,\zeta\,P\sqrt{-M}}{\sinh(2\sqrt{-M}(\zeta\,x-4\zeta^3 M\,t+\phi))-P-\cosh(2\sqrt{-M}(\zeta\,x-4\zeta^3 M\,t+\phi))}$$
$$+G_0-2\,\zeta\,\sqrt{-M}, \tag{19}$$

Various simulations of the solutions of the equation (19) may be observed by Figure 2.

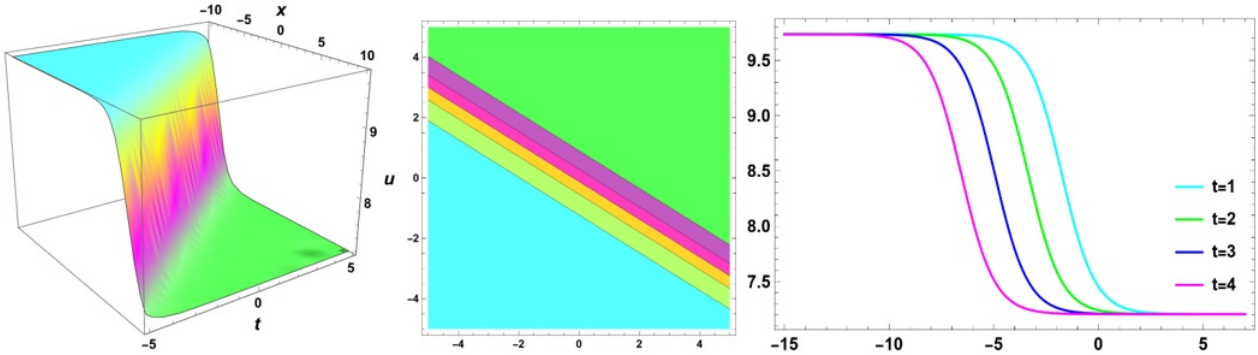


Fig. 2 Kink wave profile observed via 3D for the solutions of the equation (19) in this figures with the $G_0 = 11,\ \zeta = 2,\ P = 1,\ Q = 7,\ \phi = 0.3,\ M = -0.1$ and wave propagation for different values of $t = 1,\ 2,\ 3,\ 4$ with the $-15 \le x \le 7$ shows through 2D Plot.

$$u_{15}(x,t) = G_0+2\,\zeta\left(\frac{\sqrt{(P^2+Q^2)M}+P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))}{P\sin(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))+Q}\right), \tag{20}$$

$$u_{16}(x,t) = G_0-2\,\zeta\left(\frac{\sqrt{(P^2+Q^2)M}-P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))}{P\sin(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))+Q}\right),, \tag{21}$$

$$u_{17}(x,t) = -\frac{4\,i\,\zeta\,P\sqrt{M}}{P+\cos(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))-i\sinh(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))}$$
$$G_0+2\,i\,\zeta\,\sqrt{M}, \tag{22}$$

$$u_{18}(x,t) = -\frac{4\,i\,\zeta\,P\sqrt{M}}{P+\cos(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))-i\sin(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))}$$
$$+G_0-2\,i\,\zeta\,\sqrt{M}, \tag{23}$$

$$u_{19}(x,t) = G_0+\frac{2\,\zeta}{\zeta\,x-4\zeta^3 M\,t+\phi}. \tag{24}$$

Several views of the solutions of the equation (21) may be observed by Figure 3.

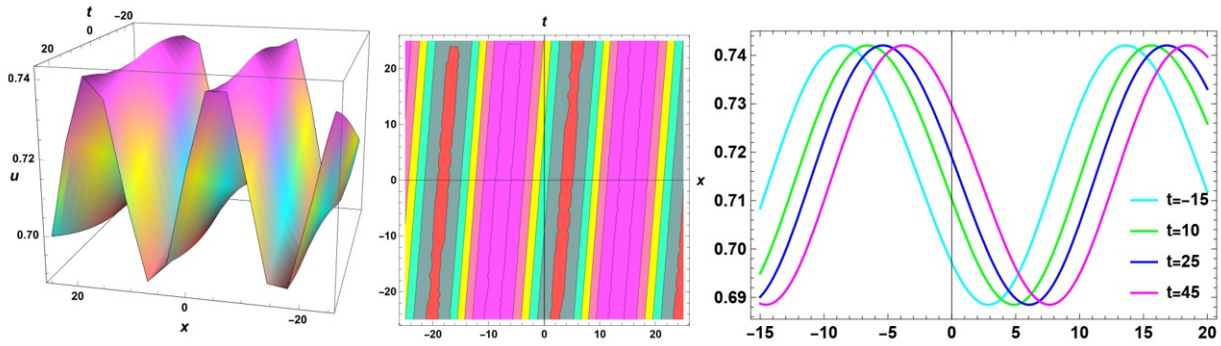


Fig. 3 Periodic wave profile observed via 3D for the solutions of the equation (21) in this figures with the $G_0 = 1,\ \zeta = 0.2,\ P = 0.2,\ Q = 3,\ \phi = 2,\ M = 0.5$ and wave propagation for different values of $t = -15,\ 10,\ 25,\ 45$ with the $-15 \leq x \leq 20$ shows through 2D Plot.

Using equation (3), equation (7)-(9), and equation (14), we obtain the subsequent exact analytical solutions of the MNW equation (1)

$$u_{21}(x,t) = G_0 + 2\,\zeta\,M\left(\frac{P\sinh(2\sqrt{-M}(\zeta\,x - 4\zeta^3 M\,t + \phi)) + Q}{-\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x - 4\zeta^3 M\,t + \phi))}\right), \tag{25}$$

$$u_{22}(x,t) = G_0 + 2\,\zeta\,M\left(\frac{P\sinh(2\sqrt{-M}(\zeta\,x - 4\zeta^3 M\,t + \phi)) + Q}{\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x - 4\zeta^3 M\,t + \phi))}\right), \tag{26}$$

$$u_{23}(x,t) = G_0 + 2\,\zeta\,M\left(\sqrt{-M} - \frac{2P\sqrt{-M}}{P + \cosh(2\sqrt{-M}\Xi) - \sinh(2\sqrt{-M}\Xi)}\right)^{-1}, \tag{27}$$

$$u_{24}(x,t) = G_0 + 2\,\zeta\,M\left(\sqrt{-M} + \frac{2P\sqrt{-M}}{P + \cosh(2\sqrt{-M}\Xi) - \sinh(2\sqrt{-M}\Xi)}\right)^{-1}. \tag{28}$$

We may plot the several simulations of the solutions of the equation (26) in Figure 4.

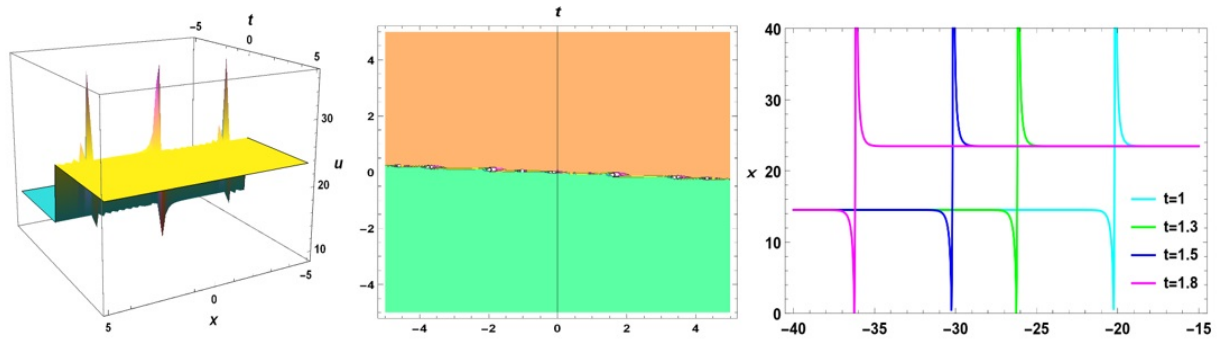


Fig. 4 Interaction between kink wave and soliton wave profile observed via 3D Plot for the solutions of the equation (26) in this figure with the $G_0 = 0.11,\ \zeta = 2,\ P = 0.021,\ Q = 397,\ \phi = 3,\ M = -0.2$ and wave propagation for different values of $t = 1,\ 1.3,\ 1.5,\ 1.8$ with the $-5 \leq x \leq 5$ shows through 2D graph.

$$u_{25}(x,t) = G_0 + 2\,\zeta\,M\left(\frac{-\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x - 4\zeta^3 M\,t+\phi))}{P\sin(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))+Q}\right), \tag{29}$$

$$u_{26}(x,t) = G_0 + 2\,\zeta\,M\left(\frac{P\sin(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))+Q}{\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x-4\zeta^3 M\,t+\phi))}\right), \tag{30}$$

$$u_{27}(x,t) = G_0 - 2\,\zeta\,M\left(i\sqrt{M} - \frac{2iP\sqrt{M}}{P+\cos(2\sqrt{M}\Xi) - i\sin(2\sqrt{M}\Xi)}\right)^{-1}, \tag{31}$$

$$u_{28}(x,t) = G_0 + 2\zeta\,M\left(i\sqrt{M} + \frac{2iP\sqrt{M}}{P+\cos(2\sqrt{M}\Xi) - i\sin(2\sqrt{M}\Xi)}\right)^{-1}, \tag{32}$$

$$u_{29}(x,t) = G_0 - 2\,\zeta\,M(\zeta\,x - 4\zeta^3 M\,t+\phi), \tag{33}$$

where $\Xi = (\zeta\,x - 4\zeta^3 M\,t+\phi)$. Figure 5 presents several simulations of the solutions of the equation (29).

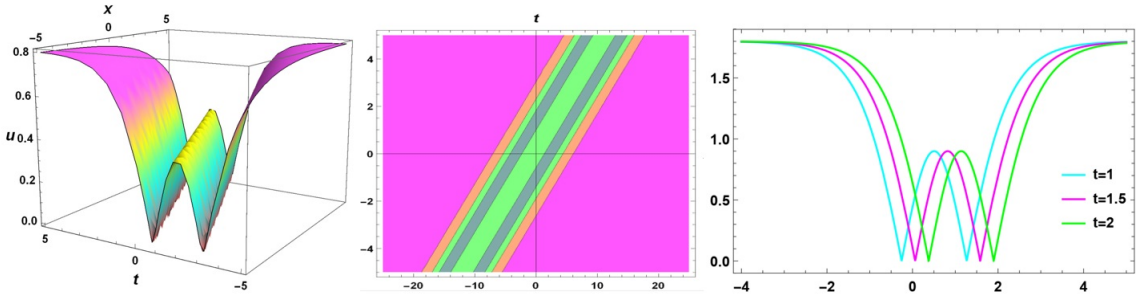


Fig. 5 W-shape wave profile observed via 3D for the solutions of the equation (29) in this figure with the $G_0 = 0.17,\ \zeta = 1.2,\ P = 0.61,\ Q = 1.7,\ \phi = 3,\ M = 1$ and wave propagation for different values of $t = 1,\ 1.5,\ 2$ with the $-3 \le x \le 5$ shows through 2D plot.

Using equation (3), equation (7)-(9), and equation (15), we obtain the subsequent exact analytical solutions of the MNW equation (1)

$$\begin{aligned} u_{31}(x,t) = G_0 &- 2\,\zeta\left(\frac{-\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))}{P\sinh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))+Q}\right) \\ &+ 2\,\zeta\,M\left(\frac{P\sinh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))+Q}{-\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))}\right), \end{aligned} \tag{34}$$

$$\begin{aligned} u_{32}(x,t) = G_0 &- 2\,\zeta\left(\frac{-\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))}{P\sinh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))+Q}\right) \\ &+ 2\,\zeta\,M\left(\frac{P\sinh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))+Q}{\sqrt{-(P^2+Q^2)M} - P\sqrt{-M}\cosh(2\sqrt{-M}(\zeta\,x-16\zeta^3 M\,t+\phi))}\right). \end{aligned} \tag{35}$$

Figure 6 introduces several simulations of the solutions of the equation (34).

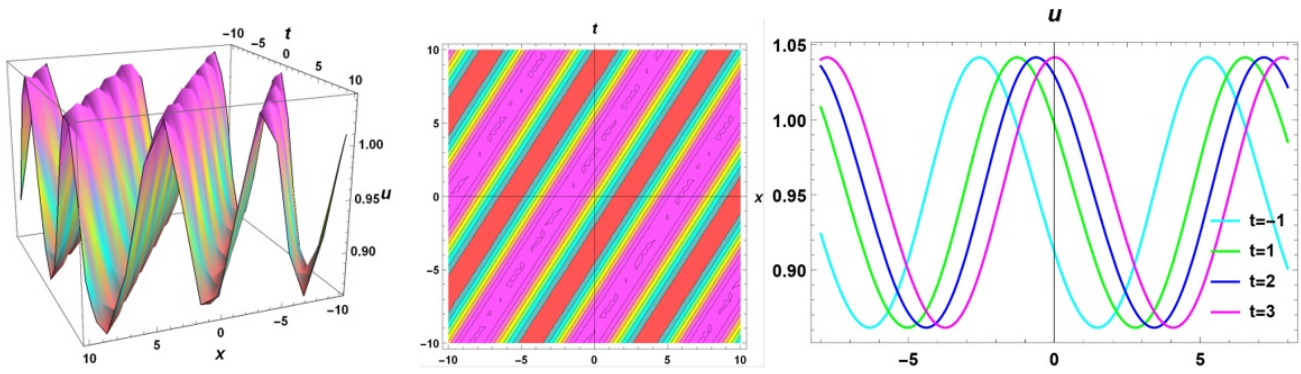


Fig. 6 Solitary wave profile observed via 3D for the solutions of the equation (34) in this figure with the $G_0 = 0.5,\ \zeta = 0.17,\ P = 3,\ Q = 6,\ \phi = 23,\ M = -1.4$ and wave propagation for different values of $t = -1,\ 1,\ 2,\ 3$ with the $-10 \le x \le 10$ shows through 2D plot.

$$u_{33}(x,t) = \frac{4\,\zeta\,P\sqrt{-M}}{\sinh(2\sqrt{-M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) - P - \cosh(2\sqrt{-M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}$$
$$G_0 + 2\,\zeta\,\sqrt{-M} + 2\,\zeta\,M\left(\sqrt{-M} - \frac{2P\sqrt{-M}}{P + \cosh(2\sqrt{-M}\Xi_1) - \sinh(2\sqrt{-M}\Xi_1)}\right)^{-1}, \tag{36}$$

$$u_{34}(x,t) = \frac{4\,\zeta\,P\sqrt{-M}}{\sinh(2\sqrt{-M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) - P - \cosh(2\sqrt{-M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}$$
$$+ G_0 - 2\,\zeta\,\sqrt{-M} + 2\,\zeta\,M\left(\sqrt{-M} + \frac{2P\sqrt{-M}}{P + \cosh(2\sqrt{-M}\Xi_1) - \sinh(2\sqrt{-M}\Xi_1)}\right)^{-1}, \tag{37}$$

$$u_{35}(x,t) = G_0 + 2\,\zeta\left(\frac{\sqrt{(P^2+Q^2)M} + P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}{P\sin(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) + Q}\right)$$
$$+ 2\,\zeta\,M\left(\frac{-\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}{P\sin(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) + Q}\right), \tag{38}$$

$$u_{36}(x,t) = G_0 - 2\,\zeta\left(\frac{\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}{P\sin(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) + Q}\right)$$
$$+ 2\,\zeta\,M\left(\frac{P\sin(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) + Q}{\sqrt{(P^2+Q^2)M} - P\sqrt{M}\cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}\right), \tag{39}$$

$$u_{37}(x,t) = -\frac{4\,i\,\zeta\,P\sqrt{M}}{P + \cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) - i\sinh(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}$$
$$G_0 + 2\,i\,\zeta\,\sqrt{M} - 2\,\zeta\,M\left(i\sqrt{M} - \frac{2iP\sqrt{M}}{P + \cos(2\sqrt{M}\Xi_1) - i\sinh(2\sqrt{M}\Xi_1)}\right)^{-1}, \tag{40}$$

$$u_{38}(x,t) = -\frac{4\,i\,\zeta\,P\sqrt{M}}{P + \cos(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi)) - i\sin(2\sqrt{M}(\zeta\,x - 16\zeta^3 M\,t + \phi))}$$
$$+ G_0 - 2\,i\,\zeta\,\sqrt{M} + 2\zeta\,M\left(i\sqrt{M} + \frac{2iP\sqrt{M}}{P + \cos(2\sqrt{M}\Xi_1) - i\sin(2\sqrt{M}\Xi_1)}\right)^{-1}, \tag{41}$$

$$u_{39}(x,t) = G_0 + \frac{2\,\zeta}{\zeta\,x - 16\zeta^3 M\,t + \phi} - 2\,\zeta\,M(\zeta\,x - 16\zeta^3 M\,t + \phi), \tag{42}$$

where $\Xi_1 = (\zeta\ x - 16\zeta^3 M\ t + \phi)$. Via Figure 7, we observe the several simulations of the solutions of the equation (38).

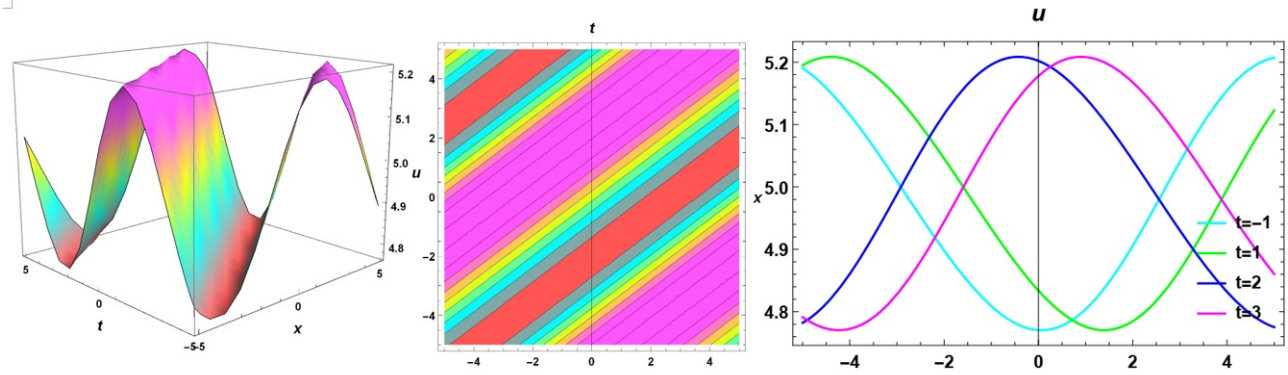


Fig. 7 Periodic wave profile observed via 3D for the solutions of the equation (38) in this figure with the $G_0 = 5,\ \zeta = 0.351,\ P = 0.4,\ Q = 3,\ \phi = 13,\ M = 0.672$ and wave propagation for different values of $t = -1,\ 1,\ 2,\ 3$ with the $-5 \le x \le 5$ shows through 2D plot.

4 Result and discussion

Examining the analytic soliton solutions employing the powerful integral unified technique is new and different from the earlier results acquired by employing diverse approaches. We received the eclectic forms of analytical soliton wave solutions such as hyperbolic function, traveling wave, rational function, and trigonometric function solutions for the Mikhailov-Novikov-Wang equation. Consequently, our obtained solutions have been verified by using the symbolic computation *Mathematica* software after placing them back into the MNW equation. To explain the physical phenomena of these solutions via 3D, contour, and 2D plots by using the suitable values of unknown coefficients and arbitrary constant parameters, the observations are given in the following manner: Figures 1 and 2 show the anti-bell shape wave and kink shape wave profiles for solutions (17) and (19). Figures 3, 4, and 5 illustrate the evolutionary profile of solitary waves for solutions (21), (26), and (29). Figures 6 and 7 depict the interaction between soliton and kink wave and W-shape wave soliton profiles for solutions (34), and (38). The dynamical behavior of the analytical soliton solutions is presented by setting the suitable values of unknown coefficients and free parameter constants.

5 Conclusion

In summary, we successfully implemented the unified method to obtain a broad range of new exact analytical solutions to the considered MNW equation. We acquired various kinds of exact solutions, such as rational, trigonometric, exponential, hyperbolic, and combinations of their solutions with arbitrary constant parameters. The applied method is computationally efficient, powerful, and more reliable than other analytical techniques. The studied results are completely different from those presented in literature. We employed graphical illustrations to discuss the behavior and nature of the solutions achieved for better understand. The physical signifcance of the specified analytical solitary wave outcomes has been exhibited through the 3D, contour, and 2D shapes. Consequently, anti-bell-shape, kink wave, solitary wave, and periodic wave profiles were observed for the solu-

tions (17), (19), (21), (26), (29), (34), and (38). Solitary wave structures of solitons construct a comprehensive view to indicate the dealing of various biological and physical systems. The growth of prosperous analytical soliton results will provide to immense research in various areas for different dynamical systems. The outcomes obtained by the method utilized in this paper can be immensely beneficial in plasma physics, mathematical physics, nonlinear waves, fluid dynamics, and nonlinear sciences. In future investigation, our work lays the foundation for several promising areas of future research. One possible direction is the extension of our methodology to tackle even more intricate higher-order nonlinear evolution equations that arise in fields such as plasma physics, mathematical physics, and nonlinear sciences.

6 Declarations

6.1 Conflict of interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

6.2 Authors' contributions:

A.K.-Methodology, Writing-Review and Editing and Supervision; S.K.-Resources, Writing-Original Draft and Methodology, Validation, Conceptualization and Formal analysis, Investigation and Visualization. All authors read and approved the final submitted version of this manuscript.

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6.5 Data availability statement:

All data that support the findings of this study are included within the article.

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