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Original Study

Optimizing industrial growth through alternative forest biomass resources: A mathematical model using DDE

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Abstract

This paper focuses on the classification of forest biomass into two categories: premature and mature forest biomass. The third variable considered is industrialization. The growth of the wood-based industry is believed to be closely tied to the population of mature forest biomass. Any scarcity of mature population could have a negative impact on industrialization, thus mature forest biomass is provided as an alternative for industrial growth. Industrialization growth is assumed to be based on a modified Leslie-Gower equation. The positivity and boundedness of the system are calculated using the comparison theorem. Stability analysis is done about nonzero equilibrium points with the help of the Routh-Hurwitz theorem. When there is no delay in the system, the system is stable. At $\tau < 1.8$, the system shows asymptotic stability, but at $\tau \geq 1.8$, system shows Hopf-bifurcation and periods oscillations occur. Furthermore, sensitivity analysis is examined about different parameters of the systems. MATLAB is used to draw the numerical simulation.

Keywords: Premature forest biomass, mature forest biomass, industrialization, stability, delay differential equation, sensitivity analysis, Matlab.
AMS 2020 codes: 65P30; 65P40.

1 Introduction

Environmental protection is inadequate, allowing industrialization and population growth to have a substantial influence on the alarming and ongoing degradation of natural forests. Each nation has forest reserves, which are natural resources that are essential for maintaining a variety of ecosystems and living things. The saturation point of woods for humanity is influenced by human awareness of their use. Random development has a negative impact on forestry assets. It is generally recognized that the degradation of forest management materials, owing to rapid human urbanization and industrialization is a severe challenge that the world faces, as it affects the sustainability of the environment studied by Nugrahani et al. [1]. Repetto and Holmes [2] examined the effect of growth on environmental destruction in emerging regions. Many scientists have examined the impact of degradation on forest assets [3–5]. Koya and teru [6] worked on the causes of the purposeful clearing of forest land and conservation. Kumar and Dipesh [7–9] examined the allelopathic effect in competing plant populations using delay and also examined the competition between wood based industries and synthetic based

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industries [10]. Hallam et al. [11–13] studied the critical significance of harmful absorption by living organisms. Panja [14] observed that if we increase the area of forest biomass, the rate of climate change will slow down. Peng et al. [15] worked on a nonlinear dynamical model for growth interaction between root biomass and above-ground biomass. Leslie [16] created the Leslie Gower model, and the model gives the idea that a drop in predatory species has a mutual link with the availability of per capita predominant feed. Chen and Chen [17], and Zhang et al. [18] examined that predators move to another community; however, their development would be restricted because its favoured meal prey is not abundant. Ruan [19] investigated many types of stabilities, such as conditional solidity, oscillations in the predator-prey model, discontinuous time-lags, and absolute stability. Hassard et al. [20] studied the different properties of hopf-bifurcation. Saltelli et al. [21] and Wu et al. [22] worked on techniques that consider numerous levels of variation, including model evaluation and parameterization. Akinyemi et al. [23] studied the nonlinear partial differential equation using q-HATM. Gao et al. [24] studied some unique characteristics of the new coronavirus infection under the impact of the fractional operator. Akinyemi et al. [25] examined the temporal progression of perturbations in a medium with weak stability or instability using Schrödinger. Ilhan et al. [26] studied the dynamics of carbon dioxide gas in the atmosphere using q-HATM. A lot of work has been done on forest biomass, and industrialization using mathematical models [1–3]. However, the excessive cutting of forest wood leads to the deacceleration of industrialization. So, this research paper provides a new perspective on the growth of industry by showing its dependence on pre-mature forest biomass. However, premature forest biomass is not immediately available. As it takes time for a tree to grow to its fullest capacity this delay period can't be studied with the help of ordinary differential equations. This complexity of forest biomass-industrialization is thus analyzed in this research paper with the help of a proposed mathematical model consisting of a system of non-linear delay differential equations.

The goal of this research is to examine the effect of the delay in the availability of premature forest biomass as an alternative to industrialization. It has been carried out by considering the delayed logistic growth equation of pre-mature forest biomass. The industry growth which actually is dependent on mature forest biomass is shown by taking a modified Leslie-Gower model.

The remaining sections of this work are organized as follows. Section 2 deals with the formulation of a mathematical model. Section 3 deals with boundedness, and the positivity of the model. In section 4, we examine the equilibrium points and the stability of the model using a different lemma dealing with the nature of the root of the transcendental equation with the help of the theory of equation. The analytical finding is supported by numerical simulation in section 5. In section 6 sensitivity analysis has been performed with the help of different parameters. The result and discussion are included in section 7. Findings and observations of this manuscript are concluded in section 8.

2 Mathematical model

We have proposed a mathematical model, variables of the dynamical model are premature forest biomass F_p, mature forest biomass F_m and industrialization forest biomass I_n. The mathematical formulation of the forest biomass complexities [27] is described by a given set of nonlinear differential equations:

$$\frac{dF_p}{dt} = aF_p\left(1 - \frac{F_p(t-\tau)}{k}\right) - \gamma_1 F_p + \gamma_2 F_p, \tag{1}$$

$$\frac{dF_m}{dt} = \gamma_1 F_p - q_1 H F_m - n_1 F_m, \tag{2}$$

$$\frac{dI_n}{dt} = \left(m_1 - \frac{m_2 I_n}{m_3 + F_m}\right) I_n - n_2 I_n, \tag{3}$$

where $F_P \geq 0, F_m \geq 0, I_n \geq 0$ and $F_P(t-\tau)$=Constant for $t \in [0, \tau]$. The parameters of the dynamical model are: a is the natural growth rate of premature forest biomass, k is the carrying capacity, γ_1 is the transformation rate from premature to mature, γ_2 is the rate of plantation of a new tree, q_1 is the cutting down rate of mature forest

biomass industrialization, H is the harvesting effect, n_1 is the natural decay rate for mature forest biomass, n_2 is the decrease rate of industrialization in the absence of forest biomass, m_1 is the maximum growth rate of industrialization, m_2 is the reduce rate of industrialization and $\frac{1}{m_3}$ is the rate of protection of forestry biomass provided by the government, τ is the delay parameter.

3 Analysis of the model

In this part of the paper, we study the boundedness and positivity of the model considered in this work.

3.1 Boundedness of the model

The bounded solution of dynamical system (1)-(3) is: From equations (1) and (2)

$$F = F_p + F_m,$$

and

$$\frac{dF}{dt} = \frac{dF_P}{dt} + \frac{dF_m}{dt}.$$

Let $\varphi = \min(\gamma_1, \gamma_2, q_1, H,\ n_1)$ and $F_P(t-\tau) \approx F_P$

$$\frac{dF}{dt} \leq a - \varphi.$$

As $t \to \infty$, $F \leq \frac{a}{\varphi}$, using comparision theorem. So, $0 \leq F \leq \frac{a}{\varphi} \Rightarrow 0 \leq F_P + F_m \leq \frac{a}{\varphi}$.
From equation (3), it may be obtained as below

$$\frac{dI_n}{dt} = \left(m_1 - \frac{m_2 I_n}{m_3 + F_m}\right) I_n - n_2 I_n.$$

Let $\chi = \min(m_1, m_2, m_3,\ n_2)$

$$\frac{dI_n}{dt} \leq \chi \frac{a}{\varphi} - \chi I_n,$$

when $t \to \infty$, $\frac{dI_n}{dt} \to 0$, using comparision theorem

$$0 \leq \chi \frac{a}{\varphi} - \chi I_n \Rightarrow \chi I_n \leq \chi \frac{a}{\varphi},$$

$$I_n \leq \frac{a}{\varphi}.$$

Hence, the dynamical system equations (1)-(3) lies in three-dimensional region.
$D = \left[(F_P, F_m, I_n)\, \varepsilon R^3_+ : 0 \leq F_P + F_m \leq \frac{a}{\varphi},\ 0 \leq I_n \leq \frac{a}{\varphi}\right]$ as $t \to \infty$, $\forall$ positive value
$\{(F_P(0) > 0, F_m(0) > 0, I_n(0) > 0), F_P(t-\tau) \approx F_P,\ Constant\ \forall\ t\varepsilon[-\tau, 0]\}\, \varepsilon D \subset R^3_+$,
where $\varphi = \min(\gamma_1, \gamma_2, q_1, H,\ n_1)$ and $\chi = \min(m_1, m_2, m_3,\ n_2)$.

3.2 Positivity of the model

Positive results indicate that the system is adaptive. All the solutions given by equations (1)-(3) are positive where initial constraints are

$$\{(F_P > 0, F_m > 0, I_n > 0), F_P(t-\tau) \approx F_P]\}.$$

The $F_P(t) > 0, F_m(t) > 0, I_n(t)$ of the dynamical model remains positive $\forall\, t > 0$. From equation (2)

$$\frac{dF_m}{dt} \geq -(q_1 H F_m + n_1 F_m),$$

$$\frac{dF_m}{dt} \geq -(q_1 H + n_1) F_m,$$

$$\frac{dF_m}{F_m} \geq -(q_1 H + n_1) dt,$$

then, we obtain the following result

$$F_m \geq e^{-(q_1 H + n_1)t}.$$

Similarly, we calculate the values of F_p and I_n.

4 Equilibrium points of the model

We calculate the equilibriums point of dynamical system equations (1)-(3)

$$E_1\,(0,0,0)\,,\; E_2\,(F_P, F_m, 0)\,,\; E_3\,(0,0,I_n)\,,\; E_4(F_P^*, F_m^*, I_n^*).$$

We studied the dynamical model about non-zero equilibrium points $E_4(F_P^*, F_m^*, I_n^*)$:
From equation (1) and $F_P\,(t-\tau) \approx F_P$

$$\frac{dF_P^*}{dt} = 0 \Longrightarrow a F_P^* \left(1 - \frac{F_P^*}{k}\right) - \gamma_1 F_P^* + \gamma_2 F_P^*$$

$$F_P^* \left[a\left(1 - \frac{F_P^*}{k}\right) - \gamma_1 + \gamma_2\right] = 0$$

$$F_P^* \neq 0,\; a\left(1 - \frac{F_P^*}{k}\right) - \gamma_1 + \gamma_2 = 0$$

$$F_P^* = \frac{(\gamma_1 - \gamma_2 - a)k}{a}.$$

The rest of the terms are calculated in a similar manner being $F_m^* = \frac{\gamma_1 F_P^*}{q_1 H + n_1}$ and $I_n^* = \frac{(m_1 - n_2)(m_3 - F_m^*)}{m_2}$.

4.1 Stability of the model

It is here investigated how the dynamical model's behaviour changes at the equilibrium points $E^*(F_P^*,\, F_m^*,\, I_n^*)$ according to equations (1)-(3). The following is the exponentially typical equation for $E_4\,(F_P^*, F_m^*, I_n^*)$:

$$\frac{dF_P^*}{dt} = a F_P^* \left(1 - \frac{F_P^*(t-\tau)}{k}\right) - \gamma_1 F_P^* + \gamma_2 F_P^*, \tag{4}$$

$$\frac{dF_m^*}{dt} = \gamma_1 F_P^* - q_1 H F_m^* - n_1 F_m^*, \tag{5}$$

$$\frac{dI_n^*}{dt} = \left(m_1 - \frac{m_2 I_n^*}{m_3 + F_m}\right) I_n^* - n_2 I_n^*. \tag{6}$$

The characteristic equation of the dynamical system equation (4)-(6) is:

$$\lambda^3 + A_1\lambda^2 + B_1\lambda + C_1 + e^{-\lambda\tau}\left(A_2\lambda^2 + B_2\lambda + C_2\right) = 0, \tag{7}$$

where the following parameters are given

$$A_1 = n_2 - m_1 + q_1H + a + \gamma_1 + \gamma_2 + 2\frac{I_n^* m_2}{F_m^* + m_3},$$

$$B_1 = q_1Hm_1 - q_1Hm_2 + 2q_1H\frac{I_n^* m_2}{F_m^* + m_3} + m_1a - n_2a - 2\frac{I_n^* m_2}{F_m^* + m_3} - q_1Ha - m_1\gamma_1 + n_2\gamma_1 + 2\gamma_1\frac{I_n^* m_2}{F_m^* + m_3} + q_1H\gamma_1 + m_1\gamma_2 - n_2\gamma_2 - q_1H\gamma_2 - 2\gamma_2\frac{I_n^* m_2}{F_m^* + m_3},$$

$$C_1 = q_1m_1aH - q_1n_2aH - 2aq_1H\frac{I_n^* m_2}{F_m^* + m_3} - q_1\gamma_1m_1H + q_1\gamma_1n_2H + 2q_1\gamma_1H\frac{I_n^* m_2}{F_m^* + m_3} + q_1m_1\gamma_2H - q_1n_2\gamma_2H + 2q_1\gamma_2H\frac{I_n^* m_2}{F_m^* + m_3},$$

$$A_2 = \frac{a}{k}F_P^*,\ B_2 = \left(-m_1 + n_2 + 2\frac{I_n^* m_2}{F_m^* + m_3} + q_1H\right)\frac{a}{k}F_P^*,$$

$$C_2 = \left(q_1Hm_1 + q_1Hn_2 + 2q_1H\frac{I_n^* m_2}{F_m^* + m_3}\right)\frac{a}{k}F_P^*.$$

Suppose that $\lambda = i\phi$ is the solution of equation (7), if

$$(i\phi)^3 + A_1(i\phi)^2 + B_1(i\phi) + C_1 + e^{-\lambda\tau}\left(A_2(i\phi)^2 + B_2(i\phi) + C_2\right) = 0. \tag{8}$$

Separate real and imaginary part of equation (7)

$$C_1 - A_1\phi^2 + \left(C_2 - A_2\phi^2\right)\cos\phi\tau + B_2\phi\sin\phi\tau = 0, \tag{9}$$

$$B_1\phi - \phi^3 + B_2\phi\cos\phi\tau - \left(C_2 - A_2\phi^2\right)\sin\phi\tau = 0. \tag{10}$$

Squaring and adding the equations (9)-(10), we get

$$\phi^6 + \left(A_1^2 - A_2^2 - 2B_1\right)\phi^4 + \left(B_1^2 - B_2^2 - 2A_1C_1\right)\phi^2 + \left(C_1^2 - C_2^2\right) = 0. \tag{11}$$

Let $x = \left(A_1^2 - A_2^2 - 2B_1\right)$, $y = \left(B_1^2 - B_2^2 - 2A_1C_1\right)$, $z = \left(C_1^2 - C_2^2\right)$.
Let $\phi^2 = l$, then equation (11) becomes

$$l^3 + xl^2 + yl + z = 0. \tag{12}$$

Lemma 1. *If $z < 0$, equation (12) has at least one real positive root.*

Proof. Let $j(l) = l^3 + xl^2 + yl + z$

$$j(0) = z < 0,\ \lim_{l\to\infty} j(l) = \infty$$

So $\exists\ \phi_0 \in (0,\infty)$, s.t. $j(l_0) = 0$.

Lemma 2. *If $z \geq 0$, then mandatory for equation (12) principal square root is $D = x^2 - 3y \geq 0$.*

Proof. Let $j(l) = l^3 + xl^2 + yl + z$

$$\Rightarrow j'(l) = 3l^2 + 2xl + y \tag{13}$$

$$j'(l) = 0 \Rightarrow 3l^2 + 2xl + y = 0.$$

Equation (13) has two roots as given

$$l_{1,2} = \frac{-2x \pm \sqrt{4x^2 - 12y}}{6}, \Rightarrow l_{1,2} = \frac{-x \pm \sqrt{D}}{3}. \tag{14}$$

Equation (13) has no positive root if $D < 0$. So, $j(l)$ is monotone increase function ϕ. We observed if $j(0) = z \geq 0$, which represent that equation (12) have no principal root. If $D \geq 0$, then $l_1 = \frac{-x+\sqrt{D}}{3}$ have local minima of $j(\phi)$.

Lemma 3. *When $z \geq 0$, equation (12) has principal root if $l_1 > 0$ & $j(l_1) \leq 0$.*

Proof. Let $l_1 \leq 0$ or $l_1 > 0$ & $j(l_1) > 0$. If $l_1 \leq 0$ & $q(l)$ is increasing for $l \geq l_1$ & $j(l) = z \geq 0$, which shows $j(l)$ have no principal square zero. If $l_1 > 0$ & $j(l_1) > 0$. Therefore $l_2 = \frac{-x-\sqrt{D}}{3}$ is the relative maximum. Which show $j(l_1) \leq j(l_2)$. So $j(l) = z \geq 0$, as $j(l)$ have no principal roots.
Suppose, that equation (12) has positive roots. WLOG, assume that there are three positive roots, l_1, l_2 *and* l_3. The roots of equation (11) are

$$\omega_1 = \sqrt{l_1}\ ,\ \omega_2 = \sqrt{l_2}\ ,\ \omega_3 = \sqrt{l_3}\ .$$

From equation (10) $\sin\phi\tau = \frac{B_2\phi - \phi^3}{d}$ Which gives

$$\tau = \frac{1}{\phi}\left[sin^{-1}\left(\frac{B_2\phi - \phi^3}{d}\right) + 2(i-1)\pi\right]\ ; i = 1,2,3,\cdots$$

Let $\tau_r{}^{(i)} = \frac{1}{\omega_r}\left[sin^{-1}\left(\frac{B_2\phi_r - \phi_r{}^3}{d}\right) + 2(i-1)\pi\right]\ ; r = 1,2,3\ ; i = 0,1,2,\cdots$
At that time $\mp\phi_r$ is the imagined root of equation (11) where $\tau = \tau_r^i,\ r = 1,2,3\ ; i = 0,1,2,3\cdots,\ \lim_{j\to\infty} \tau_r^i = \infty,\ r = 1,2,3,4.$
Thus, define

$$\tau_0 = \tau_{r_0}{}^{(i_0)} = \min_{1\leq r\leq 3,\ i\geq 1}\left[\tau_r{}^{(i)}\right]\ ,\ \ \phi_0 = \phi_{r_0}. \tag{15}$$

Assume that

$$\lambda(\tau) = \psi(\tau) + i\phi(\tau) \tag{16}$$

equation (7) have roots and fulfilling the condition:

$$\psi(\tau_0) = 0\ \&\ \phi(\tau_0) = \phi_0.$$

Assume that $j'(l_0) \neq 0$, to ensure that $\mp\phi_0$ are simple completely imagined roots of equation (7), with $\tau = \tau_0$ and $\lambda(\tau)$ fulfilling sufficient and necessary criterion.

Lemma 4. *Let $l_0 = \phi_0^2$. If $\tau = \tau_0$. Then sign $[\psi'(\tau_0)] = sign[j'(l_0)]$*

Proof. Putt $\lambda(\tau)$ in equation (7) and differentiation with regard to τ, we get

$$\frac{d\lambda}{d\tau}\left[3\lambda^2+2A_1\lambda+B_1+\left(\left(A_2\lambda^2+B_2\lambda+C_2\right)(-\tau)+(2A_2\lambda+B_2)\right)e^{-\lambda\tau}\right]=\lambda\left(A_2\lambda^2+B_2\lambda+C_2\right)e^{-\lambda\tau}.$$

Then, we reach the following result

$$\left(\frac{d\lambda}{d\tau}\right)^{-1}=\frac{\left(3\lambda^2+2A_1\lambda+B_1\right)e^{\lambda\tau}}{\lambda\left(A_2\lambda^2+B_2\lambda+C_2\right)}+\frac{(2A_2\lambda+B_2)}{\lambda\left(A_1\lambda^2+B_2\lambda+C_2\right)}-\frac{\tau}{\lambda}.$$

From equations (9) and (10):

$$\begin{aligned}\upsilon'(\tau_0)&=Re\left[\frac{\left(3\lambda^2+2A_1\lambda+B_1\right)e^{\lambda\tau}}{\lambda\left(A_2\lambda^2+B_2\lambda+C_2\right)}\right]+Re\left[\frac{(2A_2\lambda+B_2)}{\lambda\left(A_1\lambda^2+B_2\lambda+C_2\right)}\right]\\&=\frac{1}{g}\left[3{\phi_0}^6+2x{\phi_0}^4+y{\phi_0}^2\right].\end{aligned}$$

Where $g=\left[\left(C_2-A_1\phi^2\right)^2+\left(B_2\phi\right)^2\right]$ and shows $g>0$ and $\phi_0>0$.

$$\Rightarrow sign\left[\psi'(\tau_0)\right]=sign[j'(l_0)].$$

5 Numerical application

The dynamical system is visually represented using the pertinent parametric parameters:

$$a=1.5,\ k=10,\ \gamma_1=0.5,\ \gamma_2=0.03,\ q_1=1,$$

$$H=2.5,\ n_1=0.1,\ n_2=0.1,\ m_1=0.4,\ m_1=2.2,\ m_3=4$$

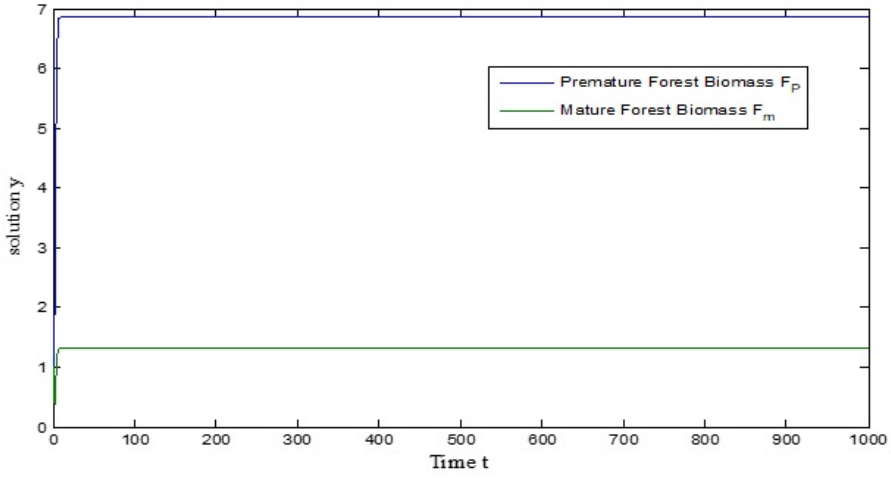


Fig. 1 Forest biomass in the absence of Industrialization Forest biomass $(F_p^*, F_m^*, 0)$.

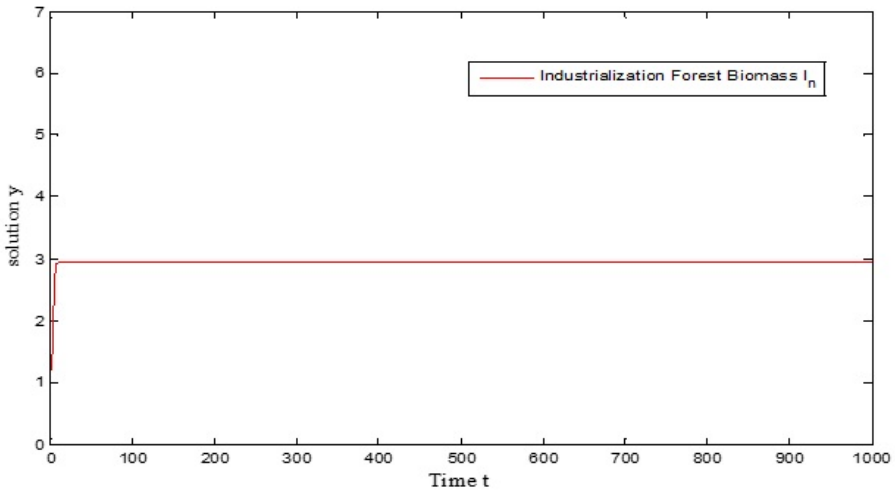


Fig. 2 Forest biomass, with neither premature forest biomass nor mature forest biomass, only industrialization $(0,0,I_n^*)$.

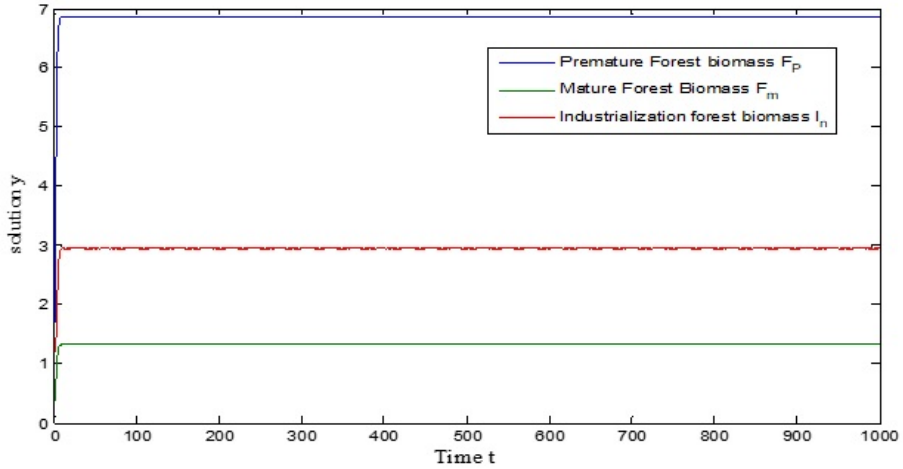


Fig. 3 The equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ is stable in the absence of delay, i.e., $\tau = 0$.

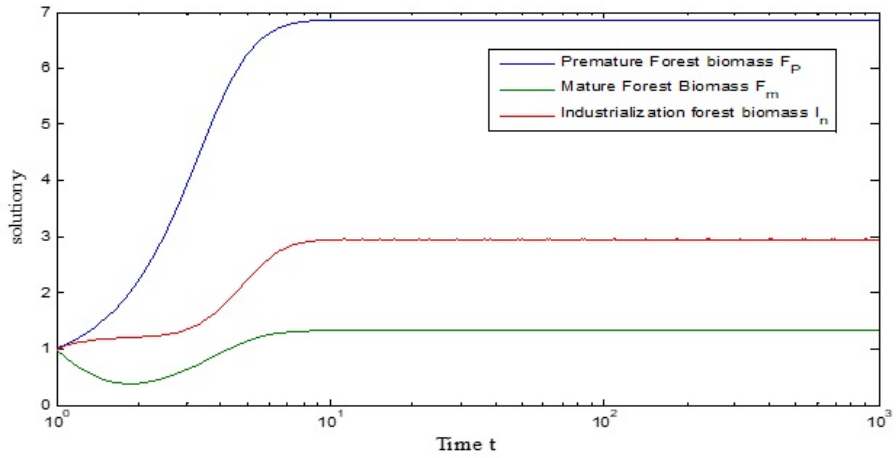


Fig. 4 Log phase view of the equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ in the absence of delay, i.e., $\tau = 0$.

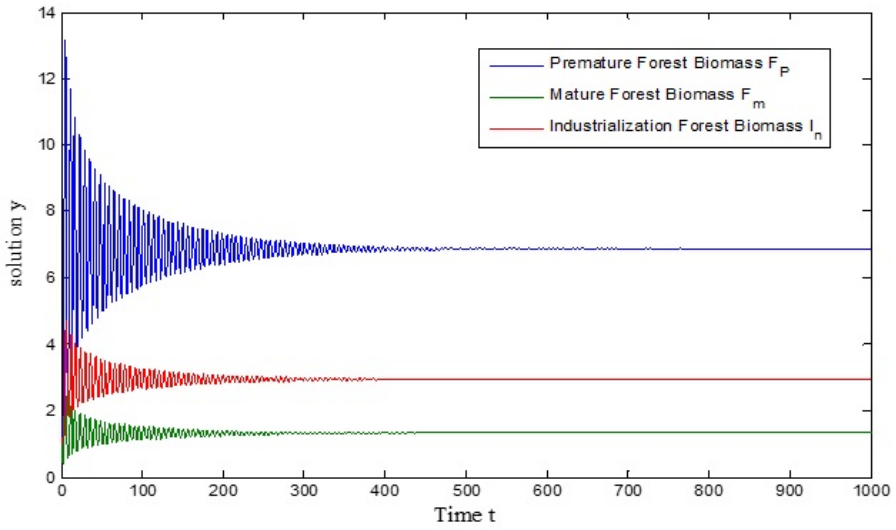


Fig. 5 The equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ shows asymptotic stability at $\tau < 1.8$.

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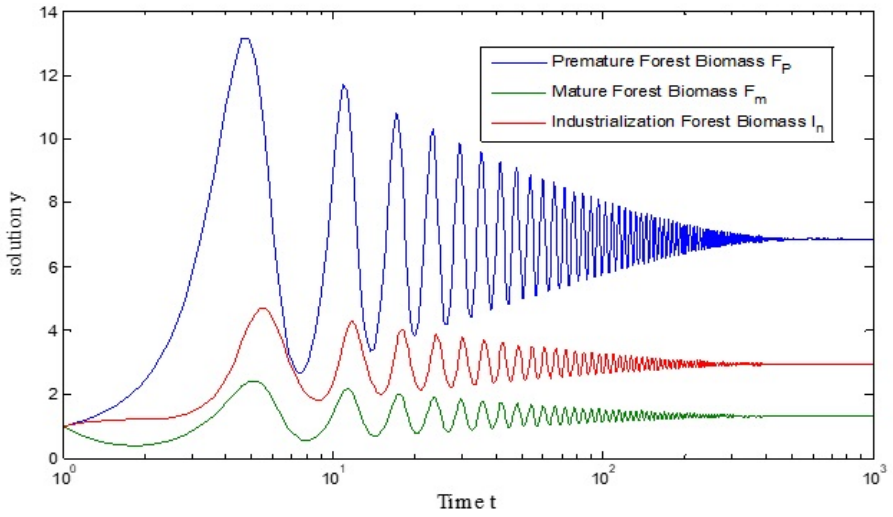


Fig. 6 Log phase view of the equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ at $\tau < 1.8$.

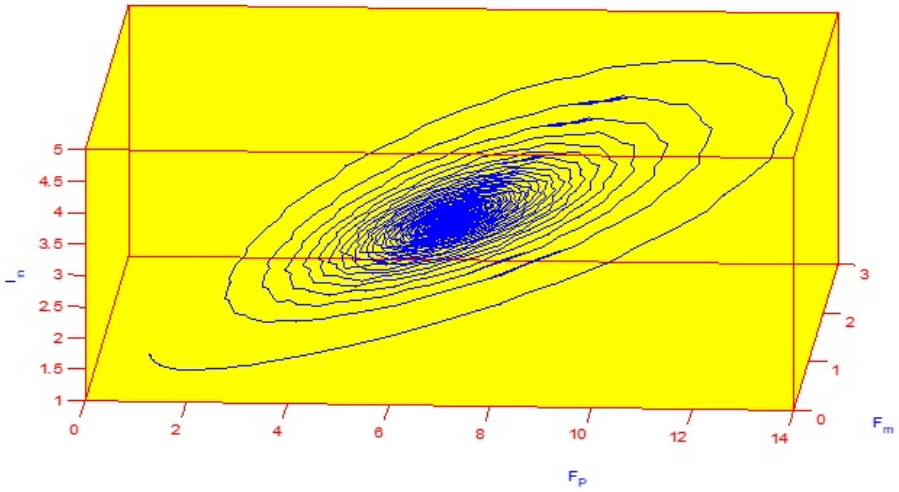


Fig. 7 Phase diagram of the equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ at $\tau < 1.8$.

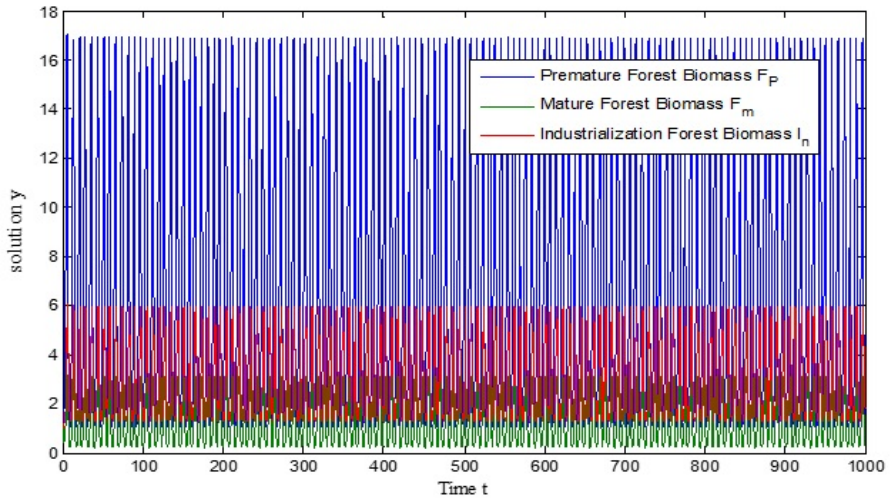


Fig. 8 Hopf-Bifurcation of the equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ at $\tau \geq 1.8$.

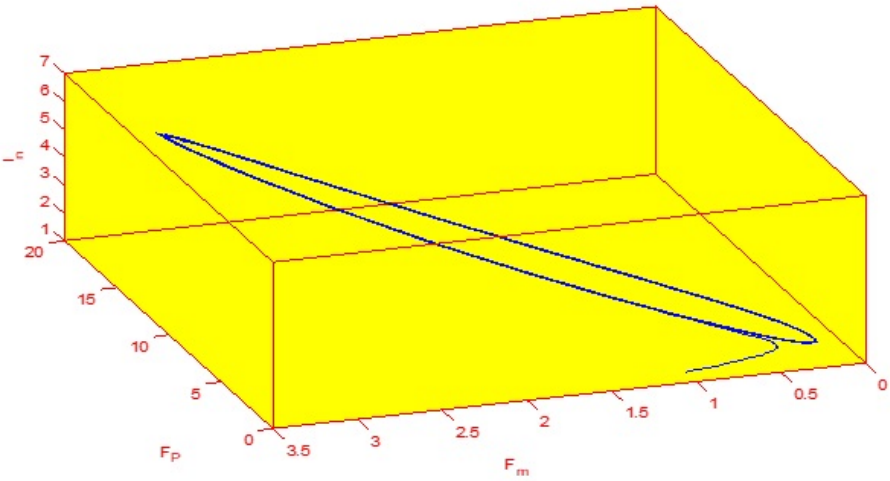


Fig. 9 Phase diagram of the equilibrium $E_4(F_p^*, F_m^*, I_n^*)$ at $\tau \geq 1.8$.

6 Sensitivity analysis of dynamical model

In order to calculate the coefficient of generalised sensitivity, Rihan [28] and Thomaseth and Cobelli [29] use the "direct Method". The direct technique works by assuming that all parameters are fixed, then calculating the sensitivity coefficient $(\gamma_2, a,\ k,\ q_1,\ H,\ n_1,\ n_2,\ m_1,\ m_2, m_3)$ by using the sensitivity equation and the initial solution of the equations (1)-(3). The partial derivatives of the solution with regard to every criterion in this instance i.e., we take the γ_1 parameter, then the partial derivatives of the solution $(F_P, F_m, I_n\ )$ with regard to γ_1 provide the criteria of the sensitivity equations (17), (18) and (19) given as

$$\frac{dS_1}{dt} = (\gamma_2 - \gamma_1 + a)S_1 - \left(\frac{a}{k}\left[(F_P S_1(t-\tau) + S_1 F_P(t-\tau))\right]\right), \tag{17}$$

$$\frac{dS_2}{dt} = \gamma_1 S_1 - (q_1 H - n_1) S_2, \tag{18}$$

$$\frac{dS_3}{dt} = m_1 S_3 - 2S_3 \frac{m_2 I_n}{m_3 + F_m} - n_2 S_3, \tag{19}$$

where $S_1 = \frac{\partial F_P}{\partial \gamma_1}$, $S_2 = \frac{\partial F_m}{\partial \gamma_1}$, $S_3 = \frac{\partial I_n}{\partial \gamma_1}$. The sensitivity analysis equations (17)-(19) is studied with original equations of system (1)-(3) to approximate the sensitivity of the variable (F_P, F_m, I_n) to the parameter γ_1.

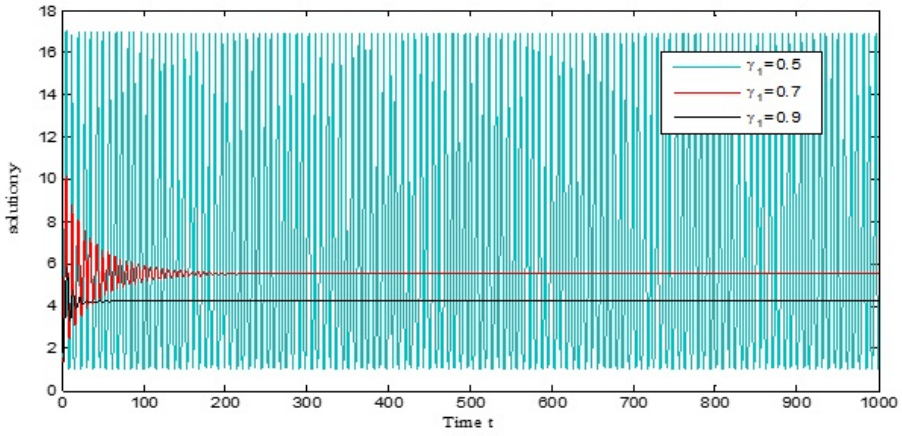


Fig. 10 Sensitivity Analysis of F_p with regard to γ_1 parameter.

Similarly, we calculate the sensitivity analysis of F_m, I_n with regard to different parameters of the dynamical system. In Figure 10, when we change the values of γ_1 parameter, the dynamical system shows the variations in the stability.

7 Results and discussion

Figure 1 represents the forest biomass of premature and mature in the absence of industrialization. Figure 2 represents the industrialization in the absence of premature and mature forest bimass. Figure 3 shows the stability of the dynamic system $E_4(F_P^*, F_m^*, I_n^*)$ in the absence of delay parameter. Figure 4 represents the log scale view of the stability of dynamic system. Figure 5 shows the dynamic system $E_4(F_P^*, F_m^*, I_n^*)$ loses its stability and move for asymptotical stability. Figures (6,7) represent the log scale view and phase diagram of dynamic system $E_4(F_P^*, F_m^*, I_n^*)$, when the value of delay parameter is less than the threshold value. Figure 8 shows the hopf-bifurcation and limit cyclics of the dynamic system $E_4(F_P^*, F_m^*, I_n^*)$ when delay crosses the threshold value. Figure 9 represent the phase diagram of the dynamical system $E_4(F_P^*, F_m^*, I_n^*)$. Figure 10 shows the sensitivity analysis of the dynamic system, how the various parameter source shows the uncertinuity in mathematical the overall contribution in uncertainty model.

8 Conclusion

In this paper, the industry growth dependent on mature forest biomass is shown by taking into account the modified Leslie-Gower model. The premature forest biomass is provided as an alternative for industrialization in case of shortage of mature forest biomass. It has been done by considering the delayed logistic growth equation of premature forest biomass. When the premature biomass gets delayed, the entire dynamics representing this forest biomass based on industrialization switches its behaviour from stable to unstable and showing limit cycles.

When there is no delay in the system, the dynamical system is stable ($\tau = 0$). When the value of the delay parameter is below the threshold value ($\tau < 1.8$), the system loses stability and moves for the asymptotic stability. When it crosses the threshold value ($\tau \geq 1.8$), the system loses its asymptoic stability and hopf-bifurcation occur. Furthermore, we examine the sensitivity analysis of the dynamical system using "Direct Method". The model indicates that in the lack of mature forest biomass, we may preserve sustainability through regulated growth of industrial activity. It has been noticed that industrialisation has harmed the ecosystem. Growing industry and urbanisation have resulted in the conversion of forest areas to cities and, as a consequence, the degradation of forest areas.

9 Declarations

9.1 Conflict of interest:

According to the authors of this paper, there are no conflicts of interest to report regarding the article that is being presented.

9.2 Authors' contributions:

Dipesh.-Writing original manuscript, conceptualization, formal analysis, investigation, methodology, and software. P.K.-Software and formal analysis. C.C.-Review and editing. The paper has been submitted with the knowledge and consent of all authors.

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9.5 Data availability statement:

All data that support the findings of this study are included within the article.

References

[1] Ramdhani V., Jaharuddin, Nugrahani E.H., Dynamical system of modelling the depletion of forestry resources due to crowding by industrialization, Applied Mathematical Sciences, 9(82), 4067-4079, 2015.
[2] Repetto R., Holmes T., The role of population in resource depletion in developing countries, Population and Development Review, 9(4), 609-632, 1983.
[3] Dubey B., Narayanan A.S., Modelling effects of industrialization, population and pollution on a renewable resource, Nonlinear Analysis: Real World Applications, 11(4), 2833-2848, 2010.
[4] Dubey B., Upadhyay R.K., Hussain J., Effects of industrialization and pollution on resource biomass: a mathematical model, Ecological Modelling, 167(1-2), 83-95, 2003.
[5] Shukla J.B., Dubey B., Modelling the depletion and conservation of forestry resources: effects of population and pollution, Journal of Mathematical Biology, 36, 71-94, 1997.
[6] Teru A.H., Koya P.R., Mathematical modelling of deforestation of forested area due to lack of awareness of human population and its conservation, Mathematical Modelling and Applications, 5(2), 94-104, 2020.
[7] Kumar P., Dipesh, Effect of time delay on dynamic of plant competition under allelopathy, Mathematical Methods in the Applied Sciences, 45(16), 9308-9321, 2022.
[8] Kumar P., Dipesh, Effect of time-lag on two mutually competing plant populations under allelochemicals, Journal of Physics: Conference Series, 2267(1), 012019, 2022.
[9] Dipesh, Kumar P., Investigating the impact of toxicity on plant growth dynamics through the zero of a fifth-degree exponential polynomial: A mathematical model using DDE, Chaos Solitons and Fractals, 171(113457), 2023.

[10] Dipesh, Kumar P., Delay differential equation model of forest biomass and competition between wood-based industries and synthetic-based industries, Mathematical Methods in the Applied Sciences, 46(9), 10602-10616, 2023.
[11] Hallam T.G., Clark C.E., Jordan G.S., Effects of toxicants on populations: A qualitative approach II. first order kinetics, Journal of Mathematical Biology, 18, 25-37, 1983.
[12] Hallam T.G., Clark C.E., Lassiter R.R., Effects of toxicants on populations: A qualitative approach I. Equilibrium environmental exposure, Ecological Modelling, 18(3-4), 291-304, 1983.
[13] Hallam T.G., De Luna J.T., Effects of toxicants on populations: A qualitative: approach III. Environmental and food chain pathways, Journal of Theoretical Biology, 109(3), 411-429, 1984.
[14] Panja P., Is the forest biomass a key regulator of global warming?: A mathematical modelling study, Geology Ecology and Landscapes, 6(1), 66-74, 2022.
[15] Zhou X., Yang M., Liu Z., Li P., Xie B., Peng C., Dynamic allometric scaling of tree biomass and size, Nature Plants, 7(1), 42-49, 2021.
[16] Leslie P.H., Some further notes on the use of matrices in population mathematics, Biometrika, 35(3/4), 213-245, 1948.
[17] Chen L., Chen F., Global stability of a Leslie-Gower predator-prey model with feedback controls, Applied Mathematics Letters, 22(9), 1330-1334, 2009.
[18] Zhang N., Chen F., Su Q., Wu T., Dynamic behaviors of a harvesting Leslie-Gower predator-prey model, Discrete Dynamics in Nature and Society, 2011(473949), 1-15, 2011.
[19] Ruan S., Absolute stability, conditional stability and bifurcation in Kolmogorov-type predator-prey systems with discrete delays, Quarterly of Applied Mathematics, 59(1), 159-173, 2001.
[20] Hassard B.D., Kazarinoff N.D., Wan Y.H., Theory and applications of Hopf bifurcation, Cambridge University Press, 1-311, 1981.
[21] Saltelli A., Tarantola S., Campolongo F., Ratto M., Sensitivity analysis in practice: a guide to assessing scientific models, Wiley, USA, 1-232, 2004.
[22] Wu F.C., Tsang Y.P., Second-order monte carlo uncertainty/variability analysis using correlated model parameters: application to salmonid embryo survival risk assessment, Ecological Modelling, 177(3-4), 393-414, 2004.
[23] Akinyemi L., Akpan U., Veeresha P., Rezazadeh H., Inc M., Computational techniques to study the dynamics of generalized unstable nonlinear Schrödinger equation, Journal of Ocean Engineering and Science, DOI:10.1016/j.joes.2022.02.011, 2022.
[24] Ilhan E., Veeresha P., Baskonus H.M., Fractional approach for a mathematical model of atmospheric dynamics of CO2 gas with an efficient method, Chaos Solitons and Fractals, 152(111347), 1-16, 2021.
[25] Akinyemi L., Veeresha P., Ajibola S.O., Numerical simulation for coupled nonlinear Schrödinger-Korteweg-de Vries and Maccari systems of equations, Modern Physics Letters B, 35(20), 2150339, 2021.
[26] Gao W., Veeresha P., Cattani C., Baishya C., Baskonus H.M., Modified predictor-corrector method for the numerical solution of a fractional-order SIR model with 2019-nCoV, Fractal and Fractional, 6(2), 92, 2022.
[27] Chaudhary M., Dhar J., Misra O.P., A mathematical model for the conservation of forestry biomass with an alternative resource for industrialization: a modified Leslie Gower interaction, Modeling Earth Systems and Environment, 1(43), 1-10, 2015.
[28] Rihan F.A., Sensitivity analysis for dynamic systems with time-lags, Journal of Computational and Applied Mathematics, 151(2), 445-462, 2003.
[29] Thomaseth K., Cobelli C., Generalized sensitivity functions in physiological system identification, Annals of Biomedical Engineering, 27, 607-616, 1999.