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Original Study

On the stochastic observation for the nonlinear system of the emigration and migration effects via artificial neural networks

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Abstract

The aim of this work is to solve a mathematical model based on the migration and emigration effects. The designed mathematical model shows one of the forms of prey-predator. The migration factor represents a step function for both normal and individuals that is restrictions or movement of people. The numerical solutions of the designed model are presented using the stochastic computational schemes based on the artificial neural networks (ANNs) together with the Levenberg-Marquardt back propagation (LMB), i.e., ANNs-LMB for solving the model based on the migration and emigration effects. Three different cases have been performed to solve the model based on the migration and emigration effects with the ANNs-LMB solver in terms of authentication, training, sample statistics and testing. The selection of the data is chosen as 80%, 10%, 10% for training, testing and authentication, respectively. The numerical results obtained through the ANNs-LMB of the model based on the migration and emigration effects will be compared with the Runge-Kutta method. The results of the model based on the migration and emigration effects using the ANNs-LMB are provided to reduce the mean square error (MSE). For the capability and efficiency of the proposed ANNs-LMB, the numerical results are provided using correlation, error histograms, regression and MSE.

Keywords: A mathematical model, migration and emigration effects, artificial neural network, numerical results, Runge-Kutta scheme.
AMS 2020 codes: 34A34; 65L06; 33F05.

1 Introduction

In recent times peoples have not only faced world wars but also many other challenges like, climate change, water deficits, lack of food, mental health problems violence, lack of education, unemployment, government corruption, maritime safety and pollution and many more, but despite all of these global challenges mass migrations, deportations and internal displacement of people is one of the most serious issues and threats for the world [1–3]. People migrate from one place to another or from one country to another with intentions of permanent or temporary settlement. Migrations on a very large scale have caused numerous impacts, like economical, security, business, environmental, residential, educational problems and many others. In Europe more than 50 million citizens were deprived of their houses due to the compulsory migration of people from different countries and recently very large numbers of people migrated from Afghanistan to many other countries of the world. Recently many studies have been carried out to analyse the impact of migrations and to check the connection

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between them. The main goal of applied mathematicians is to understand the physical situation, develop the mathematical model and analyze it mathematically. Nowadays much attention has been paid to developing and studying different mathematical models in every field of life as mentioned in [4–8]. Different mathematical approaches have been used to study the derived models and to analyze them.
In this study, novel nonlinear mathematical model given in terms of coupled system of ordinary differential equations has been studied via the ANNs-LMB approach. This model is designed on the basis of prey-predator form, where normal people are preys $u(\tau)$ in the population and predators $v(\tau)$ in the population. The mathematical model representing the prey and predator, along with general initial conditions is given as

$$\begin{aligned} \frac{du(\tau)}{d\tau} &= \beta u(\tau) - \alpha u(\tau)v(\tau) + C(u), \;\; u(0) = i_1, \\ \frac{dv(\tau)}{d\tau} &= \delta u(\tau)v(\tau) - \gamma v(\tau) + D(v), \;\; v(0) = i_2. \end{aligned} \tag{1}$$

The description of each variable used in the mathematical model of (1) is described in Table 1.

Table 1 Parameter's detail of each parameter of equation (1).

Parameter	Descriptions
$u(\tau)$	Normal people population
$v(\tau)$	Predator' population
$C(u)$	Emigration/Migration in the normal population
$D(v)$	Emigration/Migration in the population of attack
α	Attack rate
β	Individuals birth rate
δ	New recruited attack rate
γ	Captured/killed rate
i_1/i_2	Initial conditions

The aim of this study is to present the numerical performances of the nonlinear model using the stochastic computational schemes based on the ANNs together with the LMB, i.e., ANNs-LMB for solving the model based on the migration and emigration effects. Three different cases are performed to solve the equation (1) based on the migration and emigration effects with the ANNs-LMB solver in terms of authentication, training, sample statistics and testing. The selection of the data is chosen as 80%, 10%, 10% for training, testing and authentication, respectively. The stochastic procedures along with other properties of mathematical models have been explored to solve various applications [9–24]. The general novel features of the ANNs-LMB for solving the plastic waste management in the ocean system are provided as follows:

* A novel ANNs-LMB procedure is explored to solve the model based on the migration and emigration effects.
* The comparison of the obtained performances using the ANNs-LMB with the reference outcomes is presented for solving the model based on the migration and emigration effects.
* The overlapping of the outcomes is presented, which shows the correctness of the designed stochastic ANNs-LMB scheme for solving the nonlinear model.
* The solutions of two cases $b < c$ and $b > c$ of the kidney dysfunction model have been presented using the stochastic process.
* The performances of the absolute error (AE) proved in good ranges for solving the nonlinear model using the stochastic schemes.

The remaining parts of the work are described as follows: The solutions of the nonlinear model using the ANNs-LMB are given in section 2. The ANNs-LMB with necessary explanation is described in section 3. The

section 4 presents the general properties and novelties of this paper.

2 Methodology: ANNs-LMB

The designed ANNs-LMB procedures are applied for solving the model based on the migration and emigration effects.

* Essential procedures are applied using the ANNs-LMB.

* Execution procedures based on the ANNs-LMB solvers are presented for solving the nonlinear model.

The suitable optimization ANNs-LMB procedures are derived in Figure 1 and Figure 2 by using the multilayer procedures. The single neuron structure is derived in Figure 3. The ANNs-LMB is provided with the 'nftool' that is the building process in 'Matlab' solver using the designated data as 80%, 10% and 10% for training, testing and validation, respectively.

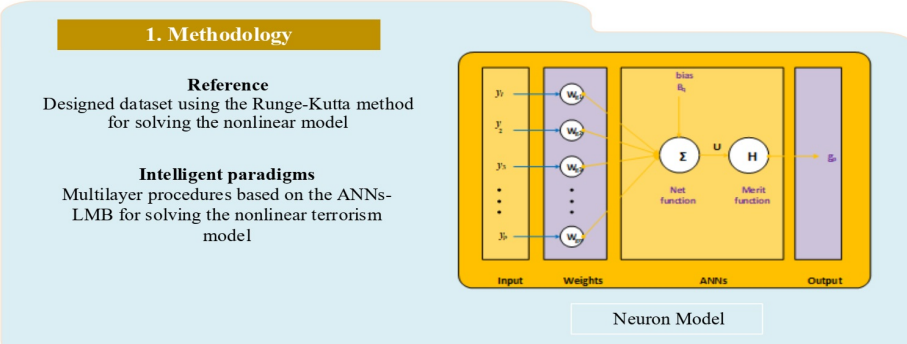


Fig. 1 Proposed ANNs-LMB for solving the nonlinear model in methodology.

3 Numerical results

The current section shows the numerical results of three different cases based on the nonlinear model using the proposed ANNs-LMB. The mathematical representations of each case are given by following cases;

Case 1 Consider the attack people rate is studied for $\alpha = 0.1$, whereas the other suitable measures are $\delta = 0.09, \beta = 0.4, \gamma = 2, D(y) = D = 4, C(x) = C = 4, u(0) = 100$ and $v(0) = 8$.

$$\begin{aligned} \frac{du(\tau)}{d\tau} &= 0.4u(\tau) - 0.1u(\tau)v(\tau) + 4, \quad u(0) = 100, \\ \frac{dv(\tau)}{d\tau} &= 0.09u(\tau)v(\tau) - 2v(\tau) + 4, \quad v(0) = 8, \end{aligned} \tag{2}$$

Case 2 Consider the attack people rate is studied for $\alpha = 0.5$, whereas the other suitable measures are $\delta = 0.09, \beta = 0.4, \gamma = 2, D(y) = D = 4, C(x) = C = 4, u(0) = 100$ and $v(0) = 8$.

$$\begin{aligned} \frac{du(\tau)}{d\tau} &= 0.4u(\tau) - 0.5u(\tau)v(\tau) + 4, \quad u(0) = 100, \\ \frac{dv(\tau)}{d\tau} &= 0.09u(\tau)v(\tau) - 2v(\tau) + 4, \quad v(0) = 8, \end{aligned} \tag{3}$$

Case 3 Consider the attack people rate is studied for $\alpha = 0.9$, whereas the other suitable measures are $\delta = 0.09, \beta = 0.4, \gamma = 2, D(y) = D = 4, C(x) = C = 4, u(0) = 100$ and $v(0) = 8$.

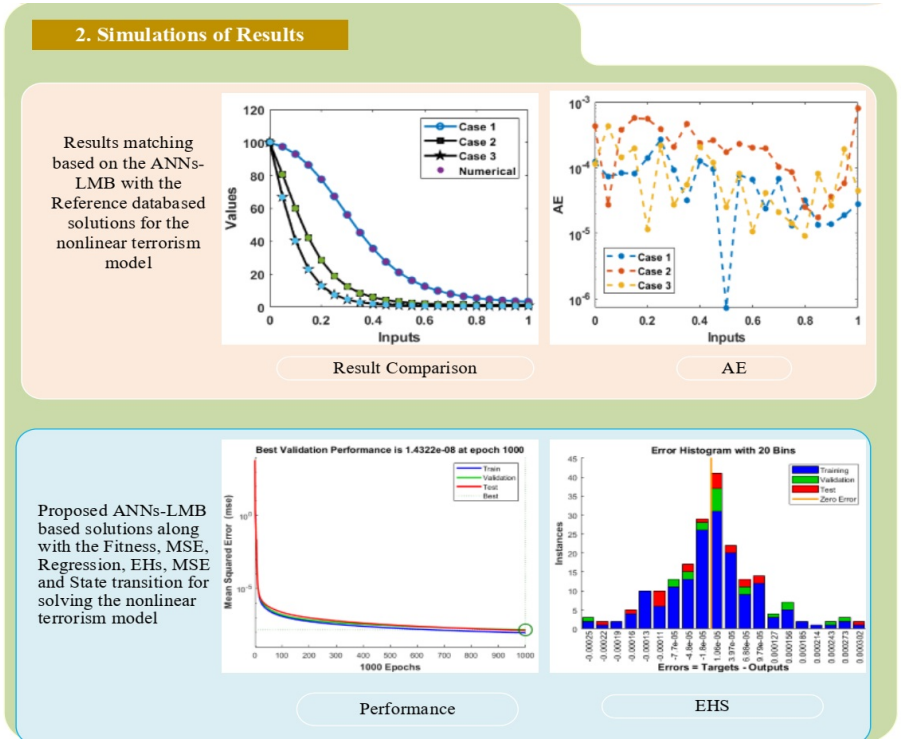


Fig. 2 Simulations of the proposed ANNs-LMB for solving the nonlinear model.

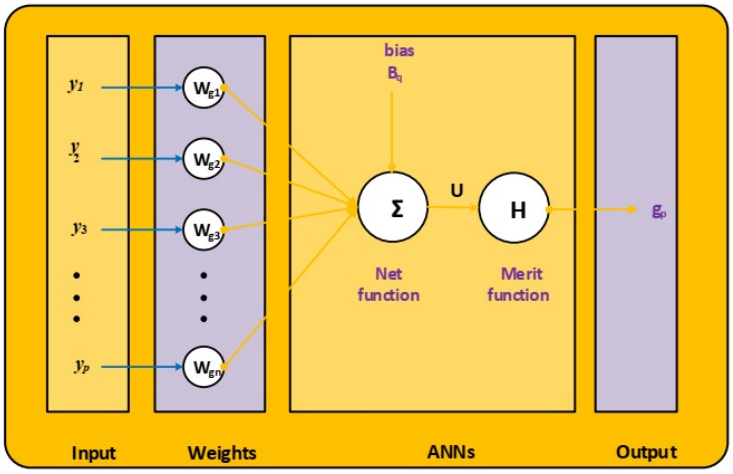


Fig. 3 Proposed structure based on the single neuron.

$$\begin{aligned} \frac{du(\tau)}{d\tau} &= 0.4u(\tau) - 0.9u(\tau)v(\tau) + 4, \quad u(0) = 100, \\ \frac{dv(\tau)}{d\tau} &= 0.09u(\tau)v(\tau) - 2v(\tau) + 4, \quad v(0) = 8, \end{aligned} \tag{4}$$

The numerical results are provided using the ANNs-LMB for solving the model based on the migration and emigration effects with [0, 1] inputs along by taking 0.01 step size. The accomplished performances of the ANNs-LMB solvers for solving the model based on the migration and emigration effects are illustrated in Figure 4.

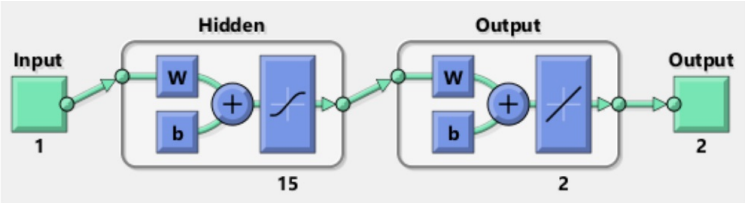


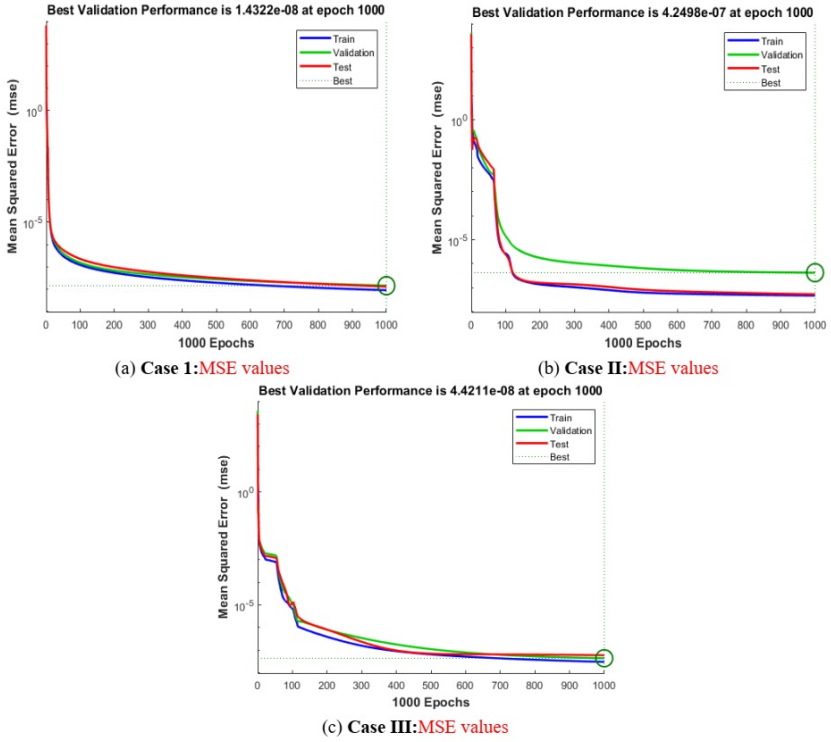
Fig. 4 Designed ANNs-LMB for solving the model based on the migration and emigration effects.

The numerical results of the nonlinear mathematical model based on migration and emigration effects via ANNs-LMB are presented in Figures (4-10). The coupled system of nonlinear ordinary differential equations given by equation (1) has been studied for three different cases for the parameter $\alpha = 0.1, \alpha = 0.5$ and $\alpha = 0.9$ where α is attack rate. Figure 5 depicts the numerical values of MSE (MSE) for the three different cases given in Figure 4 using proposed ANNs-LMB approach to solve the nonlinear mathematical model. It can easily be observed from Figure 5 that best validation performance is $4.2498e^{-07}$ and is obtained at epoch 1000. The training, testing, validation and best performance carried out based on MSE for all three cases are simulated in Figure 5. The state transition behavior of the nonlinear mathematical model by using ANNs-LMB approach is depicted in Figure 5 for all three cases. The gradient values for proposed ANNs-LMB approach to solve the nonlinear coupled system representing as model are obtained as $1.4322e^{-08}$, $4.2498e^{-07}$ and $4.4211e^{-08}$ at the Epoch of 1000 for first, second and third case respectively and are plotted in Figure 5. It can be also seen in Figure 5 that the parametric value of Mu at Epoch 1000 is $1e^{-06}$. The graphs drawn in Figure 5 indicate the precision, accuracy and convergence of proposed ANNs-LMB approach for solving the nonlinear mathematical model. The fitting curve plots for all three cases are drawn in Figure 6. The comparison via proposed ANNs-LMB approach to solve the nonlinear mathematical model is carried out in terms of training, validation, and test for each case and are drawn in Figure 6. Furthermore, the error plots for maximum values are also drawn for each case as depicted in Figure 6. The EHs plots for the proposed ANNs-LMB approach to solve nonlinear mathematical model are drawn in Figure 7 for each. Finally the plots for regression to solve the nonlinear mathematical model drawn for each case is given in Figure 7.

Table 2 Statistical performances of the nonlinear system.

Case	Train (MSE)	Test (MSE)	Validation (MSE)	Gradient	Performance	Epoch	Mu	Time
1	9.02E-09	1.43E-08	1.31E-08	4.18E-06	9.02E-08	1000	1.00E-06	04 Sec
2	4.72E-08	4.24E-07	5.31E-08	9.99E-08	1.00E-05	1000	1.00E-06	05 Sec
3	3.05E-08	6.42E-08	6.00E-08	9.92E-08	7.96E-06	1000	1.00E-05	04 Sec

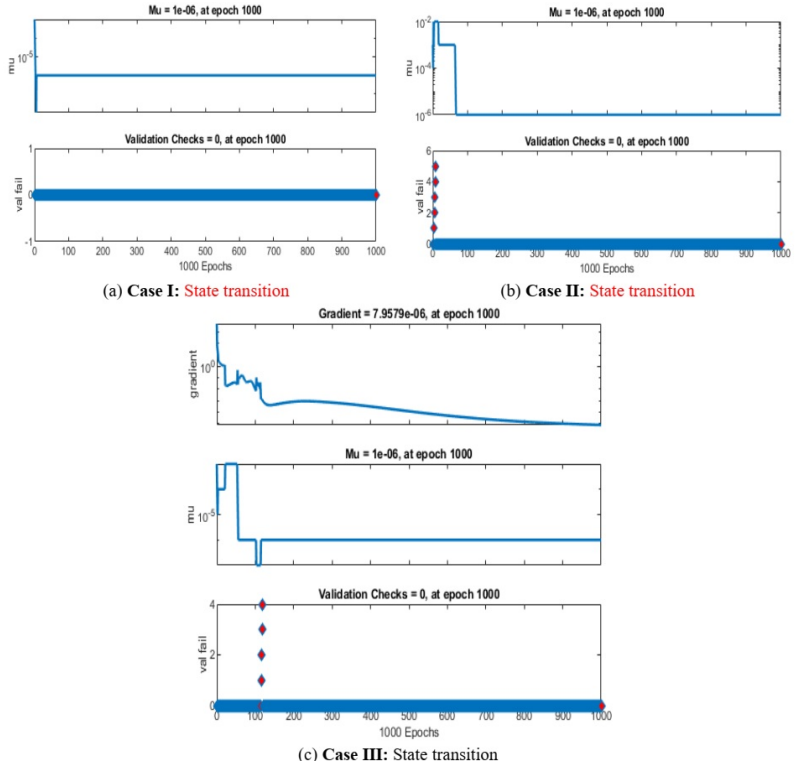
The Figures 10 and 11 depict the comparison performance based on the proposed ANNs-LMB to solve the nonlinear coupled system of ordinary differential equations for different cases. The numerical results for the parameters $T(y)$ and $F(y)$ for the different cases are given in Figure 10 for given nonlinear model. The exactness of proposed ANNs-LMB approach for solving the nonlinear model can be observed from the graphs given in Figure 10. The Figure 11 plots are drawn for the absolute error (AE) based on the proposed ANNs-LMB approach for studying the nonlinear coupled system of ordinary differential equations. Figure 11 represents the parametric values based on $T(y)$ and $F(y)$ for each case to solve novel mathematical model. It can easily be noticed from Figure 11 (a) that the absolute error for the parameter $T(y)$ lies between 10^{-4} to 10^{-6} i.e., $10^{-4} < T(y) < 10^{-6}$ for the case I, however for the I and III it lies between 10^{-3} to 10^{-5} i.e. $10^{-3} < T(y) < 10^{-5}$. Similarly, from Figure 11 (b) it can be seen that the absolute error for the parameter $F(y)$ lies between 10-4 to 10-6 i.e., $10^{-4} < F(y) < 10^{-6}$ for case I, however for the I and II it lies between 10^{-2} to 10^{-6} i.e., $10^{-2} < F(y) < 10^{-6}$. The correctness of proposed ANNs-LMB approach to solve the nonlinear mathematical model is clearly witnessed in the graphs drawn for absolute error given in Figure 11.



(a) **Case 1:**MSE values (b) **Case II:**MSE values

(c) **Case III:**MSE values

Fig. 5 Performance through MSE results using the proposed ANNs-LMB to solve the nonlinear dynamics of the mathematical model.



(a) **Case I:** State transition (b) **Case II:** State transition

(c) **Case III:** State transition

Fig. 6 State transition based on the ANNs-LMB for the nonlinear mathematical system.

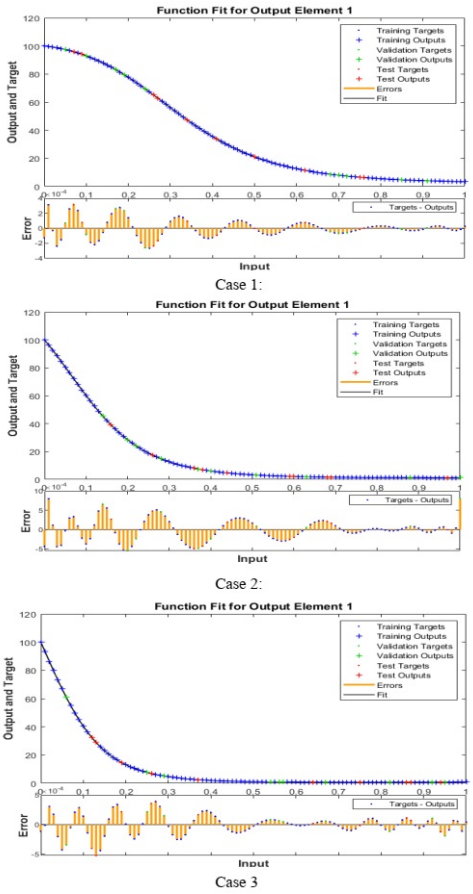


Fig. 7 Result comparison based ANNs-LMB to solve the nonlinear dynamics of the mathematical model.

4 Conclusions

In this study, the nonlinear mathematical model given in terms of coupled system of ordinary differential equations has been studied subject to migration and emigrations effects. The numerical solutions of the nonlinear model are obtained by using the ANNs-LMB. The model (along with table 2 parameters) has been studied for three different cases based on migration and emigration effects in terms of authentication, training, sample statistics and testing. For this purpose the 80%, 10%, 10% data is chosen for training, testing and authentication respectively. Furthermore, in order to validate the results obtained for designed nonlinear model via ANNs-LMB are also compared with the well-known Runge-Kutta method. The efficiency of the proposed ANNs-LMB approach for solving the nonlinear model is investigated by using correlation, error histograms, regression and MSE. It has been observed that the performance of AE appears in good ranges for solving the nonlinear model by employing stochastic schemes.

5 Declarations

5.1 Conflict of interest:

According to the authors of this paper, there are no conflicts of interest to report regarding the article that is being presented.

5.2 Funding:

No funding was received to assist with the preparation of this manuscript.

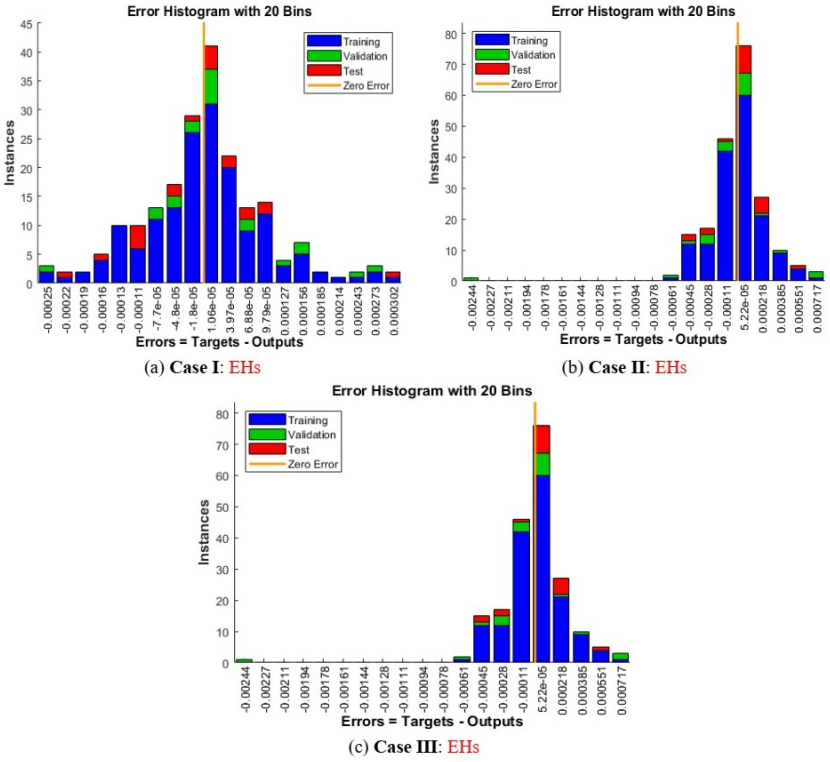


Fig. 8 EHs based on the ANNs-LMB for the nonlinear mathematical system.

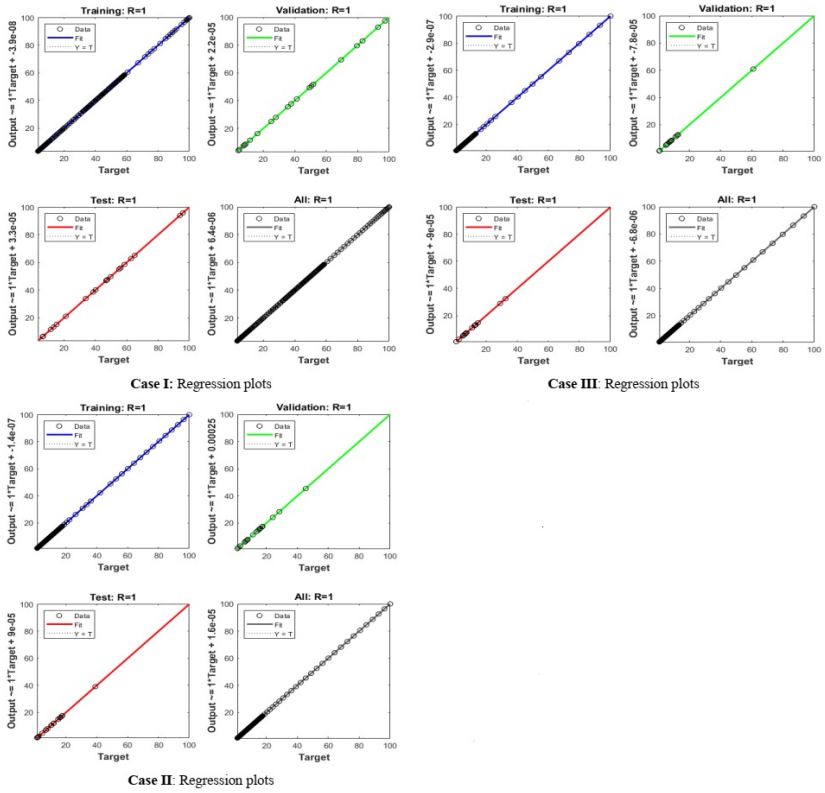
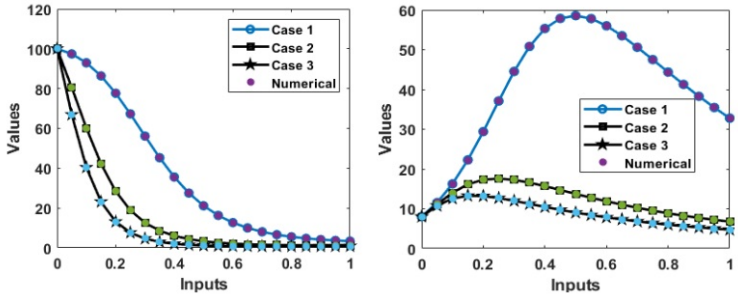
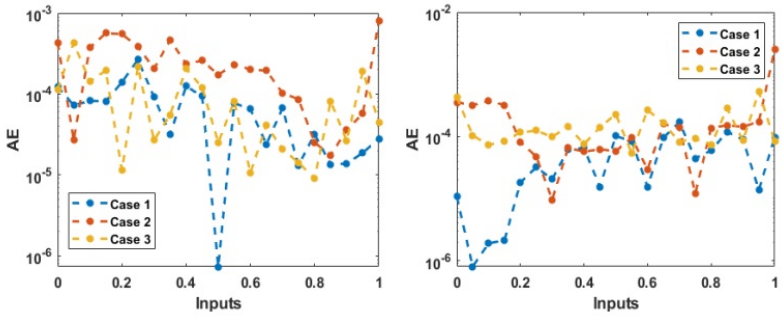


Fig. 9 Regression plots for the nonlinear mathematical model.



(a) **Case 1**: Results of the parameters $T(y)$ (b) **Case II**: Results of the parameters $F(y)$

Fig. 10 Comparison performance based on the proposed ANNs-LMB to solve the nonlinear dynamics of mathematical mode.l



(a) **Case I**: AE values for the parameters $T(y)$ (b) **Case II**: AE values for the parameters $F(y)$

Fig. 11 AE values based on the proposed ANNs-LMB to solve the nonlinear dynamics of mathematical model.

5.3 Authors Contribution:

J.L.G.G.- Conceptualization, Methodology, Software, Writing-Review Editing. All authors read and approved the final submitted version of this manuscript.

5.4 Acknowledgement:

The authors deeply appreciate the anonymous reviewers for their helpful and constructive suggestions, which can help further improve this paper.

5.5 Data availability statement:

All data that support the findings of this study are included within the article.

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