sciendo International Journal of Mathematics and Computer in Engineering 1(2) (2023) 149–170



International Journal of Mathematics and Computer in Engineering

https://sciendo.com/journal/IJMCE

Original Study

The analysis of exact solitons solutions in monomode optical fibers to the generalized nonlinear Schrödinger system by compatible techniques

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Communicated by Haci Mehmet Baskonus; Received: 06.06.2023; Accepted: 16.07.2023; Online: 20.07.2023

Abstract

In this research work, we employ the unified method, the extended sinh-Gordon equation expansion method (ShGEEM), and the extended rational sine-cosine/sinh-cosh method to derive the novel optical solitons solutions of the (2+1)-dimensional nonlinear dynamical conformable fractional generalized Schrödinger system in monomode optical fibers. We extract the optical soliton solutions in diverse forms like, dark, bright, combinations of dark-bright, periodic, and singular solutions, that are presented by trigonometric functions, and hyperbolic functions. The employed procedures are useful in order to clarify nonlinear partial differential equations (NLPDEs) and secure new exact solutions in addition to previously recovered ones. The accuracy of these answers has been verified for all extracted results using the Mathematica. The 3D surface plots, 2D line plots, and associated contour graphs are used to analyze the obtained solutions to visualize and support the theoretical conclusions using appropriate parameter values. The findings of this research demonstrate the efficacy of the approaches exploited in enhancing nonlinear dynamical behavior.

Keywords: Optical soliton solutions, nonlinear dynamical system, conformable derivative, novel approaches, integrability.
AMS 2020 codes: 35C07; 35Dxx; 45Kxx; 65Mxx; 45K05.

1 Introduction

Since the 18th century, scientists have been striving to simplify complex physical processes by modeling them using NLPDEs [1–10]. Studying optical solitons has implications for many areas of research and development. Optical soliton communication is particularly interesting in optical fiber communication. Data is transmitted using optical fiber, which uses light pulses that are directed along a long glass or plastic fiber. Optical fiber transmission is frequently done via metal cables to ensure the security of the delivered signs. Furthermore, optical fibers don't aid in the spreading of electromagnetic noise. In addition to the optical fiber, fibers are built to help propagate light while taking into account the necessary transmission distance and required force. Longer distances require single-mode fiber, while shorter ones employ multimode fiber. Compared to metal wires, these

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sciendo
ISSN 2956-7068
doi:10.2478/ijmce-2023-0012
sciendo

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filaments need better protection for their outer covering. Refractive index, materials, modes of light transmission, or some combination of these factors can be used to distinguish between various types of optical fibers [11–14]. In a similar way, optical soliton has the potential of significantly reducing the negative effects of nonlinearity on system performance, especially in high-speed communication networks. Many applications, such as fiber lasers and nonlinear optics, make use of optical solitons. Optical soliton communication also offers low error coding and a low relay cost in addition to its long relay distance. The width of the optical soliton has been expanded to the femtosecond scale, and modern communication systems are trending toward high speed, vast capacity, and extensive range. This means that femtosecond soliton communication is both necessary and suitable for issue resolution. Compressing optical solitons to increase the data rate in a high-speed optical communication system may lead to issues with their interactions [15–28].

With the definition of a soliton by Zabusky and Kruskal in 1965, the study of soliton dynamics has advanced rapidly. Recent years have seen a plethora of research on the solitary wave and exact solutions to differential equations. NLPDEs are used to describe a variety of phenomena in several applied sciences, including optics, plasma physics, and biomedical science. As a result, various analytical techniques have been developed in the prior study [29–38] to obtain the exact solutions of NLPDEs. With the help of analytical methods, the NLPDEs will be transformed into ordinary differential equations (ODEs) using the proper transforms. The parameters and coefficients used in transformations play a crucial role in the proposed model's physical applications and describe the behavior of the model's analytical solution. A number of physical conditions, such as water waves and nonlinear sciences, may be predicted using the nonlinear Schrödinger equation (NLSE) [39]. The NLSE is a popular model for describing the propagation of bright and dark soliton pulses via an optical fiber, and it is able to capture both benign and malignant dispersion ranges. Nonlinear optics is a great place to see the variety of optical solitons in action. Temporal optical solitons, spatial optical solitons, and spatiotemporal optical solitons all result from the compromise between the broadening of optical pulses due to dispersion (or diffraction) and the compression of optical pulses due to nonlinear processes.

The key idea of this work is to retrieve its optical soliton solutions by utilizing three novel schemes. The (2 + 1)-dimensional conformable fractional generalized nonlinear Schrödinger (CFGNLS) system [40, 41] is considered

$$\begin{aligned} iD_t^{\alpha}\Psi + \beta_1\Psi_{xy} + \beta_2\Psi\Phi = 0,\\ \beta_3\Phi_x + \beta_4(|\Psi|^2)_y = 0, \end{aligned} \tag{1}$$

where $0 < \alpha \leq 1$ represents the fractional order and $\beta_i, i = 1 \cdots 4$ are the arbitrary constants. More information can be found in refrence [42]. The traditional nonlinear models of integer order are generalized by nonlinear fractional models (NFMs) of non-integer order. Because current fractional-order models are more adaptable and versatile than the conventional integer order models, many physical processes have been represented using the fractional calculus.

We are inspired to identify soliton solutions of the considered system using three well-known methodologies: namely the unified method [43], the extended sinh-Gordon equation expansion method (ShGEEM) [44] and the extended rational sine-cosine/sinh-cosh method [45] to fill the gap left by prior findings. Soliton solutions for the CFGNLS system are important in soliton theory, physical sciences, and optical fibers. Extracting exact soliton solutions for the CFGNLS system is therefore critical for mathematics and physicists. To the best of our knowledge, such an abundance of solutions has never been documented in previous research. The current study's purpose is to figure out how to apply these strategies to obtain soliton solutions to the investigated model. It is also noted that the proposed methods are simple to use and provide a wide range of analytical solutions, which could lead to new types of soliton solutions with direct applications in optical fibers.

The paper is summarized as follows: In section 2, the conformable derivative and its properties have been discussed. Section 3 contains the extraction of solutions. The obtained solutions are graphically displayed in section 4. The conclusions are given in section 5.

2 Conformable derivative and its properties

The basics of conformable fractional derivatives are presented here. Also, it helps us comprehend the dynamics of physical processes. The diverse uses of conformable fractional derivatives show that there is a high demand for more accurate mathematical procedures when dealing with the real-world objects and processes. Fractional derivatives have the potential to successfully formulate a number of phenomena across several scientific disciplines. This is due to the fact that logical modeling of physical phenomena depends on both past and present time histories. Fractional models have been discovered to be more accurate and e effective than older classical models, and non-integer derivative operators have been found to be essential for the representation of physical processes. Classical models are unable to reveal the impact of memory due to their time consuming character, which is most apparent in non local and non conservative physical systems. When we compared the fractional derivative to the integer derivative, the fractional derivative has global correlations and can well describe the dependency process of function development. Fractional derivative model overcomes the theoretical and experimental inconsistencies of the classic integer derivative model, uses fewer parameters to get the desired effect, and provides more accurate mathematical and physical models.

Definition 1. Let $g:(0,\infty)\longrightarrow\mathbb{R}$ be a function. The conformable derivative of g of order α. For details see refrence [46].

$$\mathscr{T}_\alpha g(t)=\lim_{x\to 0}\frac{g(t+xt^{1-\alpha})-g(t)}{x},\forall\, t>0 \text{ and } 0<\alpha\leq 1.$$

Moreover, If g is α-differentiable in some interval $(0,\zeta)$ where $\zeta>0$, and $\lim_{t\to 0} g_t^\alpha(t)$ exists. We define $g_t^\alpha\,(0)=\lim_{t\to 0^+} g_t^\alpha(t)$. Some properties are given below.

Lemma 1. Let $\alpha\in(0,1]$ and g_1 and g_2 are α-conformable differentiable for all $t>0$. Then :
(i) $\mathscr{T}_\alpha(a_1g_1+a_2g_2)=a_1\mathscr{T}_\alpha(g_1)+a_2\mathscr{T}_\alpha(g_2),\qquad \forall\ a_1,a_2\in\mathbb{R}.$
(ii) $\mathscr{T}_\alpha(t^r)=rt^{r-\alpha},\quad \forall\ r\in\mathbb{R}.$
(iii) $\mathscr{T}_\alpha(\lambda)=0,\quad$ where λ is a constant function.
(iv) $\mathscr{T}_\alpha(g_1g_2)=g_1\mathscr{T}_\alpha(g_2)+g_2\mathscr{T}_\alpha(g_1).$
(v) $\mathscr{T}_\alpha(\frac{g_1}{g_2})=\frac{g_2\mathscr{T}_\alpha(g_1)-g_1\mathscr{T}_\alpha(g_2)}{g_2^2}$, provided $g_2\neq 0$.
(vi) If g_1 is a differentiable function then $\mathscr{T}_\alpha(g_1)(t)=t^{1-\alpha}\frac{\partial g_1}{\partial t}$.
For more information see references [47, 48].

3 Extraction of solutions

We solve Eq.(1), with the following wave transformation $\Psi(x,y,t)=R(\xi)e^{i\eta}$ and $\Phi(x,y,t)=S(\xi)$, $\xi=g_1x+h_1y+\frac{c_1t^\alpha}{\alpha}\quad \eta=g_2x+h_2y+\frac{c_2t^\alpha}{\alpha}$. The soliton's speed, wave number, and frequency are represented by g_i,h_i, and c_i, $i=1,2$. By inserting wave transformation into Eq.(1), we obtain the imaginary and real terms,

$$(c_1+\beta_1(g_1h_2+g_2h_1))R'(\xi)=0, \tag{2}$$

$$-c_2R-\beta_1g_2h_2R+\beta_1g_1h_1R''(\xi)+\beta_2R(\xi)S(\xi)=0, \tag{3}$$

and:

$$\beta_3g_1S'(\xi)+\beta_4h_1(R(\xi)^2)'=0. \tag{4}$$

Integrating Eq.(4). Considering the constant of integration to be zero, it becomes

$$S(\xi) = -\frac{h_1\beta_4}{g_1\beta_3}R(\xi)^2. \tag{5}$$

On solving Eq.(2), it gives

$$c_1 = -\beta_1(g_1h_2 + g_2h_1). \tag{6}$$

Plugging Eq.(5) into Eq.(3), yields

$$-(c_2 + \beta_1 g_2 h_2)R(\xi) + \beta_1 g_1 h_1 R''(\xi) - \frac{h_1\beta_2\beta_4}{g_1\beta_3}R(\xi)^3 = 0. \tag{7}$$

3.1 Solutions via the unified method

Using the concept of homogeneous balance principle in Eq. (7) yields, $n = 1$. So, the solution of Eq.(7) is given:

$$R(\xi) = a_0 + \sum_{j=1}^{n}\left(a_j k^j + b_j k^{-j}\right),$$

$$R(\xi) = a_0 + a_1 k + b_1 k^{-1}, \tag{8}$$

where the unknown in Eq.(8) are constants that must be calculated. On manipulating the Eqs. (8) and (7) we get

$$\textbf{Set}-\textbf{1}: \quad a_0 = 0,\ a_1 = \frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{\beta_2}\sqrt{\beta_4}},\ b_1 = 0,\ c_2 = \beta_1\left(2g_1h_1l - g_2h_2\right).$$

$$\textbf{Set}-\textbf{2}: \quad a_0 = 0,\ a_1 = 0,\ b_1 = -\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}},\ c_2 = \beta_1\left(2g_1h_1l - g_2h_2\right).$$

$$\textbf{Set}-\textbf{3}: \quad a_0 = 0,\ a_1 = -\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{\beta_2}\sqrt{\beta_4}},\ b_1 = \frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}},\ c_2 = \beta_1\left(8g_1h_1l - g_2h_2\right).$$

For **Set-1**,
Case-1 • **Hyperbolic solutions**:
If $l < 0$,

$$\Psi_{1,1}(x,y,t) = \frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)} - A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right) + B\right)} \times$$
$$\exp\left(i\left(\frac{\beta_1 t^\alpha\left(2g_1h_1l - g_2h_2\right)}{\alpha} + g_2x + h_2y\right)\right). \tag{9}$$

$$\Phi_{1,1}(x,y,t) = -\frac{2\beta_1 g_1 h_1\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)} - A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right)\right)^2}{\beta_2\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right) + B\right)^2}. \tag{10}$$

$$\Psi_{1,2}(x,y,t) = -\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)} - A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha} + g_1x + g + h_1y\right)\right) + B\right)}$$
$$\times\exp\left(i\left(\frac{\beta_1 t^\alpha\left(2g_1h_1l - g_2h_2\right)}{\alpha} + g_2x + h_2y\right)\right). \tag{11}$$

$$\Phi_{1,2}(x,y,t)=-\frac{2\beta_1 g_1 h_1\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}{\beta_2\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}. \quad (12)$$

$$\Psi_{1,3}(x,y,t)=\frac{g_1\sqrt{-l}\left(1-\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}\times$$
$$\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (13)$$

$$\Phi_{1,3}(x,y,t)=\frac{2\beta_1g_1h_1l\left(1-\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}{\beta_2}. \quad (14)$$

$$\Psi_{1,4}(x,y,t)=\frac{g_1\sqrt{-l}\left(\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}-1\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}\times$$
$$\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (15)$$

$$\Phi_{1,4}(x,y,t)=\frac{2\beta_1g_1h_1l\left(\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}-1\right)^2}{\beta_2}. \quad (16)$$

Case-2 • Trigonometric solution:
If $l>0$,

$$\Psi_{1,5}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\times$$
$$\exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (17)$$

$$\Phi_{1,5}(x,y,t)=-\frac{2\beta_1g_1h_1\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}{\beta_2\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}. \quad (18)$$

$$\Psi_{1,6}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\left(A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)-\sqrt{l\left(A^2-B^2\right)}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\times$$
$$\exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (19)$$

$$\Phi_{1,6}(x,y,t) = -\frac{2\beta_1 g_1 h_1 \left(A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)-\sqrt{l\left(A^2-B^2\right)}\right)^2}{\beta_2\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}. \tag{20}$$

$$\Psi_{1,7}(x,y,t) = \frac{ig_1\sqrt{l}\left(1-\frac{2A}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}\times$$
$$\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\exp\left(i\left(\frac{\beta_1 t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right). \tag{21}$$

$$\Phi_{1,7}(x,y,t) = \frac{2\beta_1 g_1 h_1 l\left(1-\frac{2A}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}{\beta_2}. \tag{22}$$

$$\Psi_{1,8}(x,y,t) = -\frac{ig_1\sqrt{l}\left(1-\frac{2A}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}\times$$
$$\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}\exp\left(i\left(\frac{\beta_1 t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right). \tag{23}$$

$$\Phi_{1,8}(x,y,t) = \frac{2\beta_1 g_1 h_1 l\left(1-\frac{2A}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}{\beta_2}. \tag{24}$$

For Set-2,
Case-1 • Hyperbolic solutions:
If $l<0$,

$$\Psi_{2,1}(x,y,t) = -\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1 l\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\times$$
$$\exp\left(i\left(\frac{\beta_1 t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right). \tag{25}$$

$$\Phi_{2,1}(x,y,t) = -\frac{2\beta_1 g_1 h_1 l^2\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}{\beta_2\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}. \tag{26}$$

$$\Psi_{2,2}(x,y,t) = \frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1 l\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\times$$
$$\exp\left(i\left(\frac{\beta_1 t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right). \tag{27}$$

$$\Phi_{2,2}(x,y,t)=-\frac{2\beta_1 g_1 h_1 l^2\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}{\beta_2\left(\sqrt{l\left(-\left(A^2+B^2\right)\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}. \tag{28}$$

$$\Psi_{2,3}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\exp\left(i\left(\frac{\beta_1t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(1-\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}. \tag{29}$$

$$\Phi_{2,3}(x,y,t)=\frac{2\beta_1g_1h_1l}{\beta_2\left(1-\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}. \tag{30}$$

$$\Psi_{2,4}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\exp\left(i\left(\frac{\beta_1t^\alpha(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}-1\right)}. \tag{31}$$

$$\Phi_{2,4}(x,y,t)=\frac{2\beta_1g_1h_1l}{\beta_2\left(\frac{2A}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}-1\right)^2}. \tag{32}$$

Case-2 • **Trigonometric solution**:
If $l>0$,

$$\Psi_{2,5}(x,y,t)=-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\times \exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \tag{33}$$

$$\Phi_{2,5}(x,y,t)=-\frac{2\beta_1g_1h_1l^2\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}{\beta_2\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}. \tag{34}$$

$$\Psi_{2,6}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\times \exp\left(i\left(\frac{\beta_1t^\alpha\left(2g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \tag{35}$$

$$\Phi_{2,6}(x,y,t)=-\frac{2\beta_1g_1h_1l^2\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)^2}{\beta_2\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)^2}. \tag{36}$$

$$\Psi_{2,7}(x,y,t)=\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\exp\left(i\left(\frac{\beta_1 t^{\alpha}(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(1-\frac{2A}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}. \quad (37)$$

$$\Phi_{2,7}(x,y,t)=\frac{2\beta_1g_1h_1l}{\beta_2\left(1-\frac{2A}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}. \quad (38)$$

$$\Psi_{2,8}(x,y,t)=-\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\exp\left(i\left(\frac{\beta_1 t^{\alpha}(2g_1h_1l-g_2h_2)}{\alpha}+g_2x+h_2y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(1-\frac{2A}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}. \quad (39)$$

$$\Phi_{2,8}(x,y,t)=\frac{2\beta_1g_1h_1l}{\beta_2\left(1-\frac{2A}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}\right)^2}. \quad (40)$$

For Set-3,

Case-1 • Hyperbolic solutions:

If $l<0$,

$$\Psi_{3,1}(x,y,t)=\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\right.$$
$$\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(-A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}+$$
$$\left.\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\right]\times$$
$$\exp\left(i\left(\frac{\beta_1t^{\alpha}\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (41)$$

$$\Phi_{3,1}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\right.$$
$$\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(-A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}+$$
$$\left.\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\right]^2. \quad (42)$$

$$\Psi_{3,2}(x,y,t)=\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(-A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\Bigg]\times\exp\left(i\left(\frac{\beta_1t^\alpha\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (43)$$

$$\Phi_{3,2}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(-A^2-B^2\right)}-A\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(-A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\Bigg]^2. \quad (44)$$

$$\Psi_{3,3}(x,y,t)=\exp\left(i\left(\frac{\beta_1t^\alpha\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right)\times\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{-l}-\frac{2A\sqrt{-l}}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}+\frac{2\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\Bigg]. \quad (45)$$

$$\Phi_{3,3}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{-l}-\frac{2A\sqrt{-l}}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}+\frac{2\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\Bigg]^2. \quad (46)$$

$$\Psi_{3,4}(x,y,t)=\exp\left(i\left(\frac{\beta_1 t^{\alpha}\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right)\times\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\right.$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{2A\sqrt{-l}}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}-\sqrt{-l}\right)}-$$
$$\left.\frac{2\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\right]. \quad (47)$$

$$\Phi_{3,4}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\right.$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{2A\sqrt{-l}}{A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}-\sqrt{-l}\right)}-$$
$$\left.\frac{2\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{-l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-\sinh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cosh\left(2\sqrt{-l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\right]^2. \quad (48)$$

Case-2 • Trigonometric solution:
If $l>0$,

$$\Psi_{3,5}(x,y,t)=\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\right.$$
$$\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}+$$
$$\left.\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\right]\times$$
$$\exp\left(i\left(\frac{\beta_1t^{\alpha}\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (49)$$

$$\Phi_{3,5}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\left[\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\right.$$
$$\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}+$$
$$\left.\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\right]^2. \quad (50)$$

$$\Psi_{3,6}(x,y,t)=\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\Bigg]\times\exp\left(i\left(\frac{\beta_1t^\alpha\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right). \quad (51)$$

$$\Phi_{3,6}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\Bigg[-\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}Bg_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1l\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{l\left(A^2-B^2\right)}-A\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l\left(A^2-B^2\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}-\frac{\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+B\right)}\Bigg]^2. \quad (52)$$

$$\Psi_{3,7}(x,y,t)=\exp\left(i\left(\frac{\beta_1t^\alpha\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right)\times\Bigg[-\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(i\sqrt{l}-\frac{2iA\sqrt{l}}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}+\frac{2i\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\Bigg]. \quad (53)$$

$$\Phi_{3,7}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\Bigg[-\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(i\sqrt{l}-\frac{2iA\sqrt{l}}{A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)}\right)}+\frac{2i\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A-i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\Bigg]^2. \quad (54)$$

$$\Psi_{3,8}(x,y,t)=\exp\left(i\left(\frac{\beta_1 t^{\alpha}\left(8g_1h_1l-g_2h_2\right)}{\alpha}+g_2x+h_2y\right)\right)\times\left[\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\right.$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{2iA\sqrt{l}}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}-i\sqrt{l}\right)}-$$
$$\left.\frac{2i\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\right].\quad(55)$$

$$\Phi_{3,8}(x,y,t)=-\frac{h_1\beta_4}{\beta_3g_1}\left[\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}}+\right.$$
$$\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1l}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{2iA\sqrt{l}}{A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)}-i\sqrt{l}\right)}-$$
$$\left.\frac{2i\sqrt{2}A\sqrt{\beta_1}\sqrt{\beta_3}g_1\sqrt{l}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(A+i\sin\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)+\cos\left(2\sqrt{l}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)\right)\right)}\right]^2.\quad(56)$$

Case-3 • Plane wave solution:
If $l=0$,

$$\Psi_{3,9}(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1e^{i\left(-\frac{\beta_1g_2h_2t^{\alpha}}{\alpha}+g_2x+h_2y\right)}}{\sqrt{\beta_2}\sqrt{\beta_4}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)}.\quad(57)$$

$$\Phi_{3,9}(x,y,t)=-\frac{2\beta_1g_1h_1}{\beta_2\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+g+h_1y\right)^2}.\quad(58)$$

3.2 Solutions via the extended ShGEEM

Let the solution Eq.(7) can be expressed as

$$R(\varpi)=b_1\sinh(\varpi)+a_1\cosh(\varpi)+a_0,\quad(59)$$
$$R(\xi)=\pm b_1\,i\,\mathrm{sech}(\xi)\pm a_1\tanh(\xi)+a_0,\quad(60)$$
$$R(\xi)=\pm b_1\,\mathrm{csch}(\xi)\pm a_1\coth(\xi)+a_0.\quad(61)$$

On manipulating the Eqs. (59) with its derivatives and (7) we get
Set-1:

$$a_0=0,\ a_1=0,\ b_1=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{\beta_2}\sqrt{\beta_4}},\ c_2=\beta_1\left(g_1h_1-g_2h_2\right).\quad(62)$$

• The bright and singular optical solitons are obtained

$$\Psi_1(x,y,t)=\frac{i\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\exp\left(i\left(\frac{\beta_1(g_1h_1-g_2h_2)t^{\alpha}}{\alpha}+g_2x+h_2y\right)\right)\ \mathrm{sech}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^{\alpha}}{\alpha}+g_1x+h_1y\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}.\quad(63)$$

$$\Phi_1(x,y,t)=\frac{2\beta_1 g_1 h_1 \operatorname{sech}^2\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\beta_2}. \tag{64}$$

$$\Psi_2(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1\exp\left(i\left(\frac{\beta_1(g_1h_1-g_2h_2)t^\alpha}{\alpha}+g_2x+h_2y\right)\right)\operatorname{csch}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}. \tag{65}$$

$$\Phi_2(x,y,t)=-\frac{2\beta_1 g_1 h_1 \operatorname{csch}^2\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\beta_2}. \tag{66}$$

Set-2:

$$a_0=0,\ a_1=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\ b_1=-\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\ c_2=-\frac{1}{2}\beta_1\left(g_1h_1+2g_2h_2\right). \tag{67}$$

• The combined singular and bright-dark optical solitons are attained

$$\Psi_3(x,y,t)=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\coth\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}-\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\operatorname{csch}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\times\exp\left(i\left(-\frac{\beta_1\left(g_1h_1+2g_2h_2\right)t^\alpha}{2\alpha}+g_2x+h_2y\right)\right). \tag{68}$$

$$\Phi_3(x,y,t)=-\frac{\beta_4h_1\left(\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\coth\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}-\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\operatorname{csch}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\right)^2}{\beta_3g_1}. \tag{69}$$

$$\Psi_4(x,y,t)=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\tanh\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}-\frac{i\sqrt{\beta_1}\sqrt{\beta_3}g_1\operatorname{sech}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\times\exp\left(i\left(-\frac{\beta_1\left(g_1h_1+2g_2h_2\right)t^\alpha}{2\alpha}+g_2x+h_2y\right)\right). \tag{70}$$

$$\Phi_4(x,y,t)=-\frac{\beta_4h_1\left(\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\tanh\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}-\frac{i\sqrt{\beta_1}\sqrt{\beta_3}g_1\operatorname{sech}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\right)^2}{\beta_3g_1}. \tag{71}$$

Set-3:

$$a_0=0,\ a_1=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1}{\sqrt{\beta_2}\sqrt{\beta_4}},\ b_1=0,\ h_1=-\frac{c_2+\beta_1g_2h_2}{2\beta_1g_1}. \tag{72}$$

• The dark and singular solitons are extracted

$$\Psi_5(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1e^{i\left(\frac{c_2t^\alpha}{\alpha}+g_2x+h_2y\right)}\tanh\left(\frac{\beta_1t^\alpha\left(\frac{g_2(c_2+\beta_1g_2h_2)}{2\beta_1g_1}-g_1h_2\right)}{\alpha}-\frac{y(c_2+\beta_1g_2h_2)}{2\beta_1g_1}+g_1x\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}. \tag{73}$$

$$\Phi_5(x,y,t)=\frac{(c_2+\beta_1 g_2 h_2)\tanh^2\left(\frac{\beta_1 t^\alpha\left(\frac{g_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}-g_1 h_2\right)}{\alpha}-\frac{y(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}+g_1 x\right)}{\beta_2}. \tag{74}$$

$$\Psi_6(x,y,t)=\frac{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_3}g_1 e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\coth\left(\frac{\beta_1 t^\alpha\left(\frac{g_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}-g_1 h_2\right)}{\alpha}-\frac{y(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}+g_1 x\right)}{\sqrt{\beta_2}\sqrt{\beta_4}}. \tag{75}$$

$$\Phi_6(x,y,t)=\frac{(c_2+\beta_1 g_2 h_2)\coth^2\left(\frac{\beta_1 t^\alpha\left(\frac{g_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}-g_1 h_2\right)}{\alpha}-\frac{y(c_2+\beta_1 g_2 h_2)}{2\beta_1 g_1}+g_1 x\right)}{\beta_2}. \tag{76}$$

3.3 Solutions via the extended rational sine-cosine method

Assume that the solution Eq.(7) can be expressed in the form

$$R(\xi)=\frac{\alpha_0\sin(\mu\xi)}{\alpha_2+\alpha_1\cos(\mu\xi)}, \tag{77}$$

where the unknown in Eq.(77) are constants that must be calculated. On manipulating the Eqs.(77) and (7) we get

$$\textbf{Set}-\textbf{1}:\quad \alpha_0=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu\alpha_1}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\ \alpha_2=-\alpha_1,\ c_2=\frac{1}{2}\beta_1\left(g_1h_1\mu^2-2g_2h_2\right),\ \alpha_1=\alpha_1.$$

$$\textbf{Set}-\textbf{2}:\quad \alpha_0=\frac{\sqrt{\beta_3}\sqrt{g_1}\alpha_1\sqrt{c_2+\beta_1 g_2 h_2}}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{h_1}},\ \alpha_2=0,\ \mu=\frac{\sqrt{c_2+\beta_1 g_2 h_2}}{\sqrt{2}\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}},\ \alpha_1=\alpha_1.$$

Consequently, the trigonometric solutions of Eq.(1) for the sets 1, 2 can be expressed as:

$$\Psi_1(x,y,t)=-\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu\cot\left(\frac{1}{2}\mu\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}\times\exp\left(i\left(\frac{\beta_1 t^\alpha\left(g_1h_1\mu^2-2g_2h_2\right)}{2\alpha}+g_2x+h_2y\right)\right). \tag{78}$$

$$\Phi_1(x,y,t)=-\frac{\beta_1 g_1 h_1\mu^2\cot^2\left(\frac{1}{2}\mu\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)\right)}{2\beta_2}. \tag{79}$$

$$\Psi_2(x,y,t)=\frac{\sqrt{\beta_3}\sqrt{g_1}\sqrt{c_2+\beta_1 g_2 h_2}e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2x+h_2y\right)}\tan\left(\frac{\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{h_1}}. \tag{80}$$

$$\Phi_2(x,y,t)=-\frac{(c_2+\beta_1 g_2 h_2)\tan^2\left(\frac{\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2h_1-g_1h_2)t^\alpha}{\alpha}+g_1x+h_1y\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)}{\beta_2}. \tag{81}$$

OR

Let the solution of Eq.(7) can be expressed in the following form

$$R(\xi)=\frac{\alpha_0\cos(\mu\xi)}{\alpha_2+\alpha_1\sin(\mu\xi)}, \tag{82}$$

where the unknown in Eq.(82) are constants that must be calculated. On manipulating the Eqs. (82) and (7) we get

$$\textbf{Set}-\textbf{3}: \quad \alpha_0=\frac{\sqrt{\beta_3}\sqrt{g_1}\alpha_1\sqrt{c_2+\beta_1 g_2 h_2}}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{h_1}},\ \alpha_2=-\alpha_1,\ \mu=-\frac{\sqrt{2}\sqrt{c_2+\beta_1 g_2 h_2}}{\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}},\ \alpha_1=\alpha_1.$$

$$\textbf{Set}-\textbf{4}: \quad \alpha_0=\frac{\sqrt{\beta_3}\alpha_1\left(c_2+\beta_1 g_2 h_2\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu},\ \alpha_2=0,\ g_1=\frac{c_2+\beta_1 g_2 h_2}{2\beta_1 h_1\mu^2},\ \alpha_1=\alpha_1.$$

Consequently, the periodic solutions of Eq.(1) for the sets 3, 4 can be expressed as:

$$\Psi_3(x,y,t)=-\frac{\sqrt{\beta_3}\sqrt{g_1}\sqrt{c_2+\beta_1 g_2 h_2}e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\cos\left(\frac{\sqrt{2}\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2 h_1-g_1 h_2)t^\alpha}{\alpha}+g_1 x+h_1 y\right)}{\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)}{\sqrt{\beta_2}\sqrt{\beta_4}\sqrt{h_1}\left(\sin\left(\frac{\sqrt{2}\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2 h_1-g_1 h_2)t^\alpha}{\alpha}+g_1 x+h_1 y\right)}{\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)+1\right)}. \tag{83}$$

$$\Phi_3(x,y,t)=-\frac{\left(c_2+\beta_1 g_2 h_2\right)\cos^2\left(\frac{\sqrt{2}\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2 h_1-g_1 h_2)t^\alpha}{\alpha}+g_1 x+h_1 y\right)}{\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)}{\beta_2\left(\sin\left(\frac{\sqrt{2}\sqrt{c_2+\beta_1 g_2 h_2}\left(\frac{\beta_1(-g_2 h_1-g_1 h_2)t^\alpha}{\alpha}+g_1 x+h_1 y\right)}{\sqrt{\beta_1}\sqrt{g_1}\sqrt{h_1}}\right)+1\right)^2}. \tag{84}$$

$$\Psi_4(x,y,t)=\frac{\sqrt{\beta_3}\left(c_2+\beta_1 g_2 h_2\right)e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\cot\left(\mu\left(\frac{\beta_1 t^\alpha\left(-\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1\mu^2}-g_2 h_1\right)}{\alpha}+\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1\mu^2}+h_1 y\right)\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu}. \tag{85}$$

$$\Phi_4(x,y,t)=-\frac{\left(c_2+\beta_1 g_2 h_2\right)\cot^2\left(\mu\left(\frac{\beta_1 t^\alpha\left(-\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1\mu^2}-g_2 h_1\right)}{\alpha}+\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1\mu^2}+h_1 y\right)\right)}{\beta_2}. \tag{86}$$

3.4 Solutions via the extended rational sinh-cosh method

Assume that the solution of Eq.(7) can be expressed as

$$R(\xi)=\frac{\alpha_0\sinh(\mu\xi)}{\alpha_2+\alpha_1\cosh(\mu\xi)}, \tag{87}$$

where the unknown in Eq.(87) are constants that must be determined. On manipulating the Eqs. (87) and (7) we get

$$\textbf{Set}-\textbf{5}: \quad \alpha_0=-\frac{\sqrt{\beta_3}\alpha_1\left(c_2+\beta_1 g_2 h_2\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu},\ \alpha_2=0,\ g_1=-\frac{c_2+\beta_1 g_2 h_2}{2\beta_1 h_1\mu^2},\ \alpha_1=\alpha_1.$$

$$\textbf{Set}-\textbf{6}: \quad \alpha_0=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu\alpha_1}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\ \alpha_2=-\alpha_1,\ h_1=-\frac{2\left(c_2+\beta_1 g_2 h_2\right)}{\beta_1 g_1\mu^2},\ \alpha_1=\alpha_1.$$

Consequently, the dark and singular optical soliton solutions of Eq.(1) for the sets 5,6 can be expressed as:

$$\Psi_5(x,y,t) = -\frac{\sqrt{\beta_3}\,(c_2+\beta_1 g_2 h_2)\,e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\tanh\left(\mu\left(\frac{\beta_1 t^\alpha\left(\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}-g_2 h_1\right)}{\alpha}-\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}+h_1 y\right)\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu}. \quad (88)$$

$$\Phi_5(x,y,t) = \frac{(c_2+\beta_1 g_2 h_2)\tanh^2\left(\mu\left(\frac{\beta_1 t^\alpha\left(\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}-g_2 h_1\right)}{\alpha}-\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}+h_1 y\right)\right)}{\beta_2}. \quad (89)$$

$$\Psi_6(x,y,t) = \frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\coth\left(\frac{1}{2}\mu\left(\frac{\beta_1 t^\alpha\left(\frac{2g_2(c_2+\beta_1 g_2 h_2)}{\beta_1 g_1 \mu^2}-g_1 h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1 g_2 h_2)}{\beta_1 g_1 \mu^2}+g_1 x\right)\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}}. \quad (90)$$

$$\Phi_6(x,y,t) = \frac{(c_2+\beta_1 g_2 h_2)\coth^2\left(\frac{1}{2}\mu\left(\frac{\beta_1 t^\alpha\left(\frac{2g_2(c_2+\beta_1 g_2 h_2)}{\beta_1 g_1 \mu^2}-g_1 h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1 g_2 h_2)}{\beta_1 g_1 \mu^2}+g_1 x\right)\right)}{\beta_2}. \quad (91)$$

OR

Let the Eq.(7) has the following form of solution

$$R(\xi) = \frac{\alpha_0\cosh(\mu\xi)}{\alpha_2+\alpha_1\sinh(\mu\xi)}, \quad (92)$$

where the unknown in Eq.(92) are constants that must be found. On manipulating the Eqs. (92) and (7) we get

$$\textbf{Set}-\textbf{7}: \quad \alpha_0 = \frac{\sqrt{\beta_3}\alpha_1\,(c_2+\beta_1 g_2 h_2)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu},\ \alpha_2 = 0,\ g_1 = -\frac{c_2+\beta_1 g_2 h_2}{2\beta_1 h_1 \mu^2},\ \alpha_1 = \alpha_1.$$

$$\textbf{Set}-\textbf{8}: \quad \alpha_0 = \frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu\alpha_1}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}},\ \alpha_2 = -i\alpha_1,\ h_1 = -\frac{2\,(c_2+\beta_1 g_2 h_2)}{\beta_1 g_1 \mu^2},\ \alpha_1 = \alpha_1.$$

The optical solitons for Eq.(1) for the sets 7, 8 can be expressed as:

$$\Psi_7(x,y,t) = \frac{\sqrt{\beta_3}\,(c_2+\beta_1 g_2 h_2)\,e^{i\left(\frac{c_2 t^\alpha}{\alpha}+g_2 x+h_2 y\right)}\coth\left(\mu\left(\frac{\beta_1 t^\alpha\left(\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}-g_2 h_1\right)}{\alpha}-\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}+h_1 y\right)\right)}{\sqrt{2}\sqrt{\beta_1}\sqrt{\beta_2}\sqrt{\beta_4}h_1\mu}. \quad (93)$$

$$\Phi_7(x,y,t) = \frac{(c_2+\beta_1 g_2 h_2)\coth^2\left(\mu\left(\frac{\beta_1 t^\alpha\left(\frac{h_2(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}-g_2 h_1\right)}{\alpha}-\frac{x(c_2+\beta_1 g_2 h_2)}{2\beta_1 h_1 \mu^2}+h_1 y\right)\right)}{\beta_2}. \quad (94)$$

$$\Psi_8(x,y,t)=\frac{\sqrt{\beta_1}\sqrt{\beta_3}g_1\mu e^{i\left(\frac{c_2t^\alpha}{\alpha}+g_2x+h_2y\right)}\cosh\left(\mu\left(\frac{\beta_1t^\alpha\left(\frac{2g_2(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}-g_1h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}+g_1x\right)\right)}{\sqrt{2}\sqrt{\beta_2}\sqrt{\beta_4}\left(\sinh\left(\mu\left(\frac{\beta_1t^\alpha\left(\frac{2g_2(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}-g_1h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}+g_1x\right)\right)-i\right)}. \quad (95)$$

$$\Phi_8(x,y,t)=\frac{(c_2+\beta_1g_2h_2)\cosh^2\left(\mu\left(\frac{\beta_1t^\alpha\left(\frac{2g_2(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}-g_1h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}+g_1x\right)\right)}{\beta_2\left(\sinh\left(\mu\left(\frac{\beta_1t^\alpha\left(\frac{2g_2(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}-g_1h_2\right)}{\alpha}-\frac{2y(c_2+\beta_1g_2h_2)}{\beta_1g_1\mu^2}+g_1x\right)\right)-i\right)^2}. \quad (96)$$

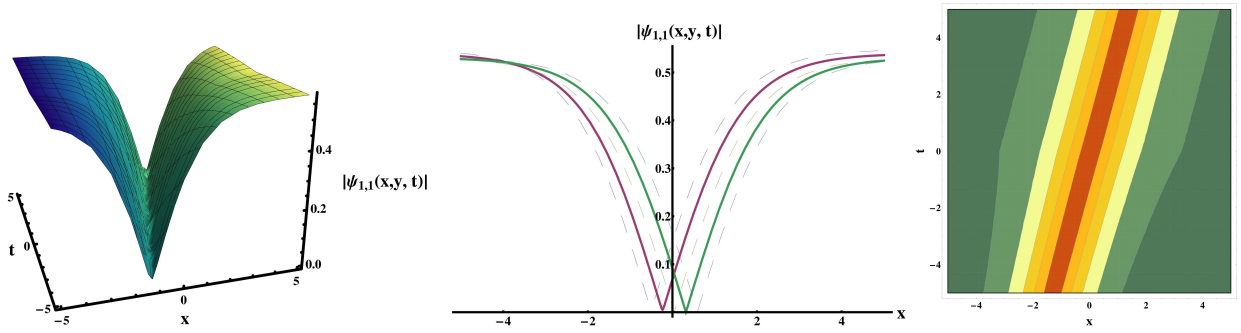


Fig. 1 Diagrammatic representations to dark soliton solution of the equation (9) for arbitrary values of parameters.

4 Discussion

In this section, a comparison of the obtained results is made with the previously reported findings, which advocates the novelty of this work. In the available literature, Akinyemi et al. [42] discussed the above system of equations and secured diverse optical soliton solutions by engaging three methods. The originality of our results is further proved by the fact that when compared to the results acquired by other approaches [49–52], only a minority of the obtained solutions are similar to those previously realized. The analyzed some solutions presented here appear novel and have not been previously reported. Although these techniques are straightforward and easy to implement, they introduce many novel sorts of solutions to nonlinear mathematical models, and thus have very robust performance. The pictorial view of the dark, bright, periodic, and singular optical soliton solutions for the equations (9, 73), (63), (33, 83), and (88) can be seen in Figures (1-6), respectively. The findings show that the computational approaches taken are successful, simple, and applicable even to the complicated phenomena and this work will also be beneficial to a large number of engineering and physical model specialists, in our opinion. The obtained solutions can be effective for physical phenomena related to signal processing through sound waves, water wave mechanics, and traffic flow models, especially in monomode fiber optics. Our findings shed light on nonlinear phenomena in dispersive waves, which have many practical applications in engineering and applied physics, and are hence of fundamental importance. We also anticipate that the answers we have retrieved will have a profound impact on the future of optical fiber communication technologies and provide valuable insight into the physics of higher-dimensional nonlinear waves that arise in a variety of physical context settings.

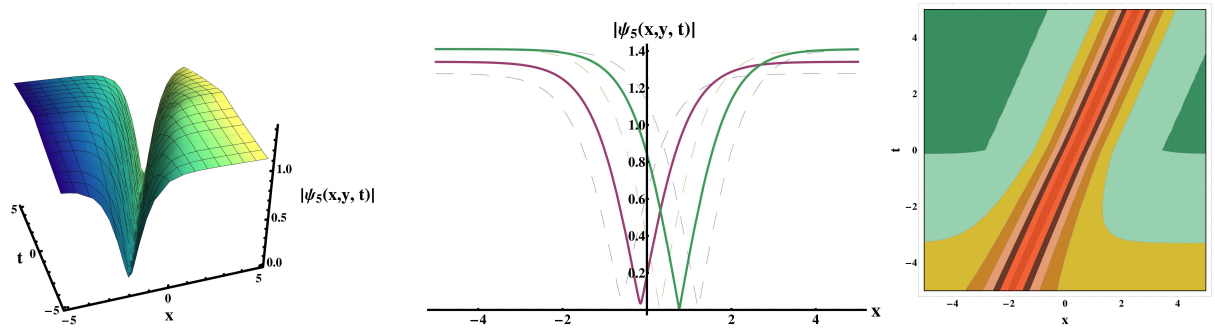


Fig. 4 Diagrammatic representations to dark soliton solution of the equation (73), for arbitrary values of parameters.

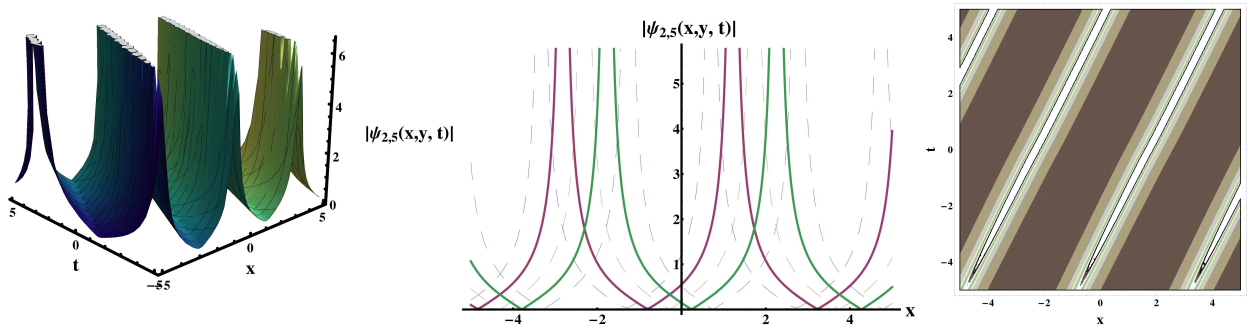


Fig. 2 Diagrammatic representations to periodic solution of the equation (33), for arbitrary values of parameters.

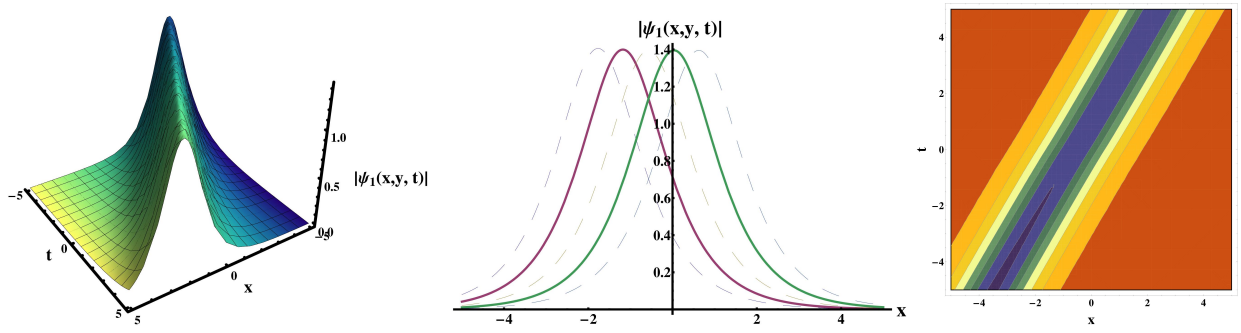


Fig. 3 Diagrammatic representations to bright soliton solution of the equation (63), for arbitrary values of parameters.

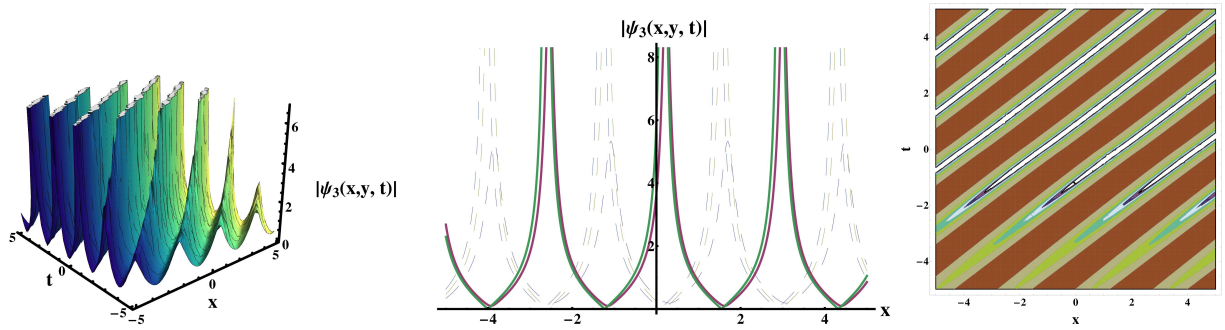


Fig. 5 Diagrammatic representations singular periodic solution of the equation (83), for arbitrary values of parameters.

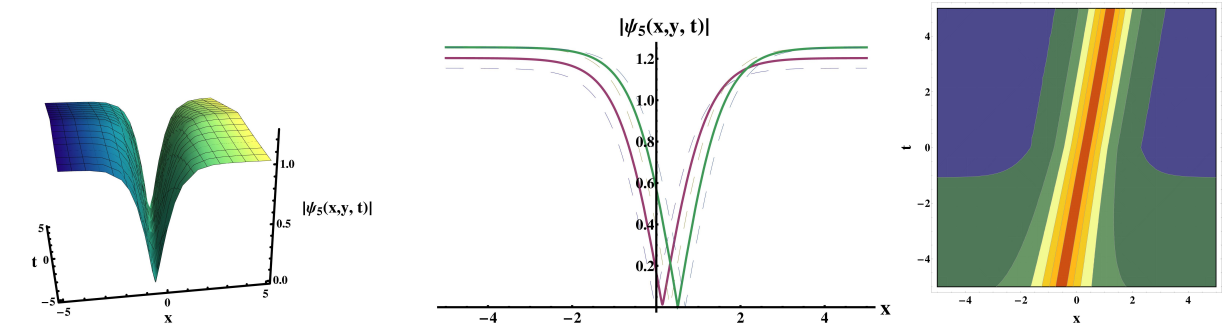


Fig. 6 Diagrammatic representations to dark soliton solution of the equation (88), for arbitrary values of parameters

5 Concluding remarks

In this article, the CFGNLS system has been successfully investigated. Three analytical techniques are presented to retrieve an effective and further generic unreachable soliton solution. Diverse exact wave structures for the proposed system have been retrieved, particularly dark, bright, singular, combo bright-dark, and singular periodic waves. These results have some other physical meanings; for example, the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, the hyperbolic secant arises in the profile of a laminar jet and hyperbolic cotangent arises in the Langevin function for magnetic polarization [53]. Moreover, dark optical soliton describes the solitary waves with lower intensity than the background, and the singular optical soliton is a solitary wave with discontinuous derivatives; examples of such solitary waves include compactions, which have finite (compact) support, and peakons, whose peaks have a discontinuous first derivative. The obtained solutions can be effective for physical phenomena related to signal processing through sound waves, water wave mechanics, and traffic flow models, especially in monomode fiber optics. To verify all the constructed solutions by putting them back into the considered system, a computer package Mathematica is used. Evidence from existing models in nonlinear science and engineering demonstrates that the proposed approaches are robust, simple, and effective for obtaining accurate wave solutions to new nonlinear fractional differential equations. We also believe that this research is timely and will be very valuable to a wide range of professionals who are working on the similar mathematical models. The achievements of this study are imperative in the field of nonlinear fiber optics since these soliton solutions will be necessary to execute in the telecommunications industry. The acquired results are fresh, correct and not reported earlier in the literature.

6 Declarations

6.1 Competing interests:

The authors declare that there is no conflict of interest regarding the publication of this paper.

6.2 Authors' contributions:

M.B.-Methodology, Writing-Review and Editing, Supervision; H.H.-Resources, Writing-Original Draft, Methodology; A.W.-Validation, Conceptualization, Formal analysis; M.F.-Data Curation, Investigation, Visualization. The paper has been submitted with the knowledge and consent of all authors.

6.3 Funding:

Not applicable.

6.4 Acknowledgement:

Thank you so much to Editor-in-Chief Prof. Dr. Haci Mehmet Baskonus for his guidelines and opinions throughout this process.

6.5 Data availability statement:

All data that support the findings of this study are included within the article.

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