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Original Study

Levenberg-Marquardt backpropagation neural network procedures for the consumption of hard water-based kidney function

Zulqurnain Sabir[1,2†], Muhammad Umar[1]

[1]Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan

[2]Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon

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Abstract

Water resources in Nusa Tenggara Timur have great concentrations based on magnesium and calcium ions thus being referred to as "hard water". Prolonged hard water consumption has become a reason of kidney disfunction that can cause additional illnesses, like cerebrovascular pathologies and diabetes. Hence, it is crucial to comprehend how drinking hard water affects renal functions. The current study shows the kidney dysfunction model based on hard water consumption by applying the stochastic procedures of the Levenberg-Marquardt backpropagation neural networks (LMBNNs). The kidney dysfunction model of hard water consumption depends upon human components and water. Human dynamics is further divided into susceptible, infected and recovered, while water components are categorized into calcium and magnesium concentration. The log-sigmoid transfer function along with 20 hidden neurons is used to present the solutions of the model. Three cases of the model have been numerically stimulated and the correctness of the stochastic technique is perceived by using the comparison of proposed and reference Adam databased solutions along with the negligible absolute error. Training, validation and testing performances have been applied to reduce the values of the mean square error. Moreover, the statistical performances using the transition of state, error histograms and regression/correlation have been validated to authenticate the reliability of the scheme.

Keywords: Kidney, nonlinear model, Levenberg-Marquardt, transfer function, neural networks.
AMS 2020 codes: 92B20; 62M45; 62Mxx.

1 Introduction

Life cannot exist without water in fact it is crucial to peoples health to consume enough water. Peoples health would be impacted by hard water having higher concentrations of substances, like magnesium and calcium. Consuming hard water for a long time can cause kidney dysfunction thus causing various illnesses, including cerebrovascular pathologies and diabetes [1]. In recent years this kind of problems have been faced above all by the developing countries [2]. Most of the residents in Nusa Tenggara Timur (NTT) have ingested water from sources that are utilized for daily purposes and have a greater percentage of magnesium and calcium. This is possibly one of the causes of the higher number of renal disease sufferers in NTT that is among the four

†Corresponding author.
Email address: zulqurnain_maths@hu.edu.pk

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provinces with the highest prevalence of kidney disease [3]. It is of paramount importance to comprehend how drinking hard water affects peoples health. To achieve the desirable outputs and decisions in the healthcare system with evidence, a thorough evaluation process is required. However, when statistics are accessible, such evaluations and outcomes can be attained. Before analyzing different sorts of information, these questions are to be addressed. There are various questions to be inquired. Finding better ways to reduce the incidence of illnesses or limiting the transmission of infections is the end goal of evaluations of public health issues [4]. The use of mathematical modeling to comprehend complicated systems is an effective strategy. Notwithstanding a variety of mathematical systems that have been created to study the dynamics of Wolbachia on dengue dynamics [5], susceptible/infected/recovered model [6], epidemiological model [7], waterborne spread and control of diseases [8], spread of malaria disease system [9], Rabies epidemic model [10], and epidemiology [11] some of mathematical systems have been designed to comprehend the kidney related illnesses [12–15], while some other models have been constructed to examine the impact of hard water consumption on the kidney. The current investigations show the numerical simulations of kidney dysfunction mathematical model on hard water consumption by applying the stochastic frameworks using the Levenberg-Marquardt backpropagation neural networks (LMBNNs). Recently, these procedures have been used in the eye surgery system [16], prevention category in HIV model [17] and Emden-Fowler model used in the theory of electromagnetics [18]. However, the stochastic LMBNNs has not been used to solve the kidney dysfunction mathematical model on hard water consumption before. The mathematical form of the kidney dysfunction mathematical model on hard water consumption depends upon the human components as well as water. Human components are classified into three dynamics, whereas water components are divided into calcium and magnesium concentration, mathematically given as [19]:

$$\begin{aligned}
\frac{dS}{dt} &= A - \beta\lambda(W)S(t) - \mu S(t),\\
\frac{dI}{dt} &= \beta\lambda(W)S(t) - \gamma I(t) - \mu I(t),\\
\frac{dZ}{dt} &= \gamma I(t) - \mu R(t),\\
\frac{dR}{dt} &= bW(t)(1 - \frac{w(t)}{K}) - cW(t).
\end{aligned} \tag{1}$$

In the above model, the human population is supposed to be constant with no deceased. Population of humans with susceptible birth at rate A. When people normally drink hard water for a time, they become afflicted with kidney dysfunction $\beta\lambda(W)$. γ represents the rate of recovered compartments. b is the compound concentration (calcium and magnesium) water enhance rate with limited increment rate K [20]. The process of treating water causes a reduction at rate c in the number of compounds (calcium and magnesium). β shows the ratio of ingesting magnesium and calcium through the water, $\lambda(W)$ presents the consumptions probability, where $0 < \lambda < 1$. Moreover, the individual's probability of keeping the kidney dysfunction shows the magnesium/calcium concentration in the water that is governed as $\lambda(W) = \frac{W(t)}{K+W(t)}$.

Few new features are presented as:

* The solutions of the c based on hard water consumption are presented successfully by applying the stochastic LMBNNs.

* The stability of the kidney dysfunction model based on the hard water consumption is also presented in this study.

* The log-sigmoid transfer function together with 20 numbers of hidden neurons is used to solve the model.

* The solutions of two cases $b < c$ and $b > c$ of the kidney dysfunction model have been presented using the stochastic process.

* The exactness of the procedure is performed through the proposed and reference solutions along with the absolute error (AE) values.

* The statistical measures based on the transition of state (TS), error histograms (EHs) and regression/correlation have been authenticated for the reliability of the stochastic procedure.

The remaining sections of this work are shown as: The stability of the kidney dysfunction model is given in section 2. Mathematical model is provided in section 3. The solutions are shown in section 4 and the conclusions are reported in section 5.

2 Stability of the kidney dysfunction model

The current section shows the stability of the kidney dysfunction model. It is noted that the endemic state of steady is realistic if $b>c$, *i.e.,* non-negative. In the system (1), the increasing infected individuals are based on the water category. It is also performed that the stability of kidney dysfunction-free steady state E_1 for $b < c$, else unstable. Moreover, the widespread equilibrium becomes stable E_1 for $b > c$. In this regard, the following theorem is presented as:

Theorem 1. *E_1 shows the locally stable behavior for $b < c$, while E_2 becomes stable for $b > c$.*

Proof. For local stability of E_1, a Jacobian matrix based on the model (1) is presented along with the typical form and Routh-Hurwitz standards to check its constancy. The Jacobian matrix is given as:

$$\mathrm{J}=\begin{bmatrix} -\mu & 0 & 0 & -\frac{bA}{K\mu} \\ 0 & -\gamma-\mu & 0 & \frac{bA}{K\mu} \\ 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where J is the form of characteristic polynomial.

$$\lambda^3(\lambda+p_3)+\lambda(p_2\lambda+p_1)+p_0=0, \tag{2}$$

where $p_{=}-\mu^2(\mu+\gamma)(b-c), p_1=\mu^3+3(b-c)\mu^2+(\mu^2-c(b-c)\mu)\gamma, p_2=-(3\mu+\gamma)(b+c)+\mu(3\mu+2\gamma)$ and $p_3=3\mu+\gamma+c-b$.

The steady state E_1 takes the form of local stable based on the Routh-Hurwitz standard, when $p_0>0, p_1>0, p_3>0$ and $p_1p_2p_3-p_1^2-p_0p_3^2>0$.

If $b < c$, then $p_0, p_1, p_3 > 0$. Moreover, $p_1p_2p_3-p_1^2-p_0p_3^2=2\mu(\mu-b-c)^2(2\mu+\gamma)^2(\mu+c-b+\gamma)$.

If $b < c$, then this condition is performed $p_1p_2p_3-p_1^2-p_0p_3^2>0$ and E_1 is locally stable. The stability is also determined based on the eigenvalues, which are $\mu,\mu,\gamma\mu$ and $b\ \ c$. The negative eigenvalues are performed for $b < c$ with locally stable E_1. The same process is also used to represent the local stability of E_2. The similar procedural steps perform that $p_0>0, p_1>0, p_3>0$ and $p_1p_2p_3-p_1^2-p_0p_3^2>0$. If $b > c$, then $p_0, p_1, p_3 > 0$ and $p_1p_2p_3-p_1^2-p_0p_3^2=2\mu(\mu-b-c)^2(2\mu+\gamma)^2(\mu+c-b+\gamma)$. Moreover, the eigen values are negative for $b > c$ and E_2 perform locally stable. The kidney dysfunction-free stable state is expected to be reached before infection class increases, which is shown as $\frac{dI(t)}{dt}$ that is

$\beta\lambda(W)S(t)-(\gamma+\mu)I(t)>0$ or $\dfrac{\beta\lambda(W)S(t)}{K+W(t)}>(\mu+\gamma)I(t)$.

3 Methodology

The kidney dysfunction model methodology based LMBNNs is shown in this section by using the essential implementation of stochastic procedure along with some numerical steps. Figure 1 provides the step-by-step workflow representations of the numerical operator for the kidney dysfunction model. In the first phase, the problem's outline is discussed, and then the model's mathematical expression that depends upon the nonlinear equations is provided. The third stage presents the LMBNNs procedural processes, and the final phase describes the result computations and evaluations.

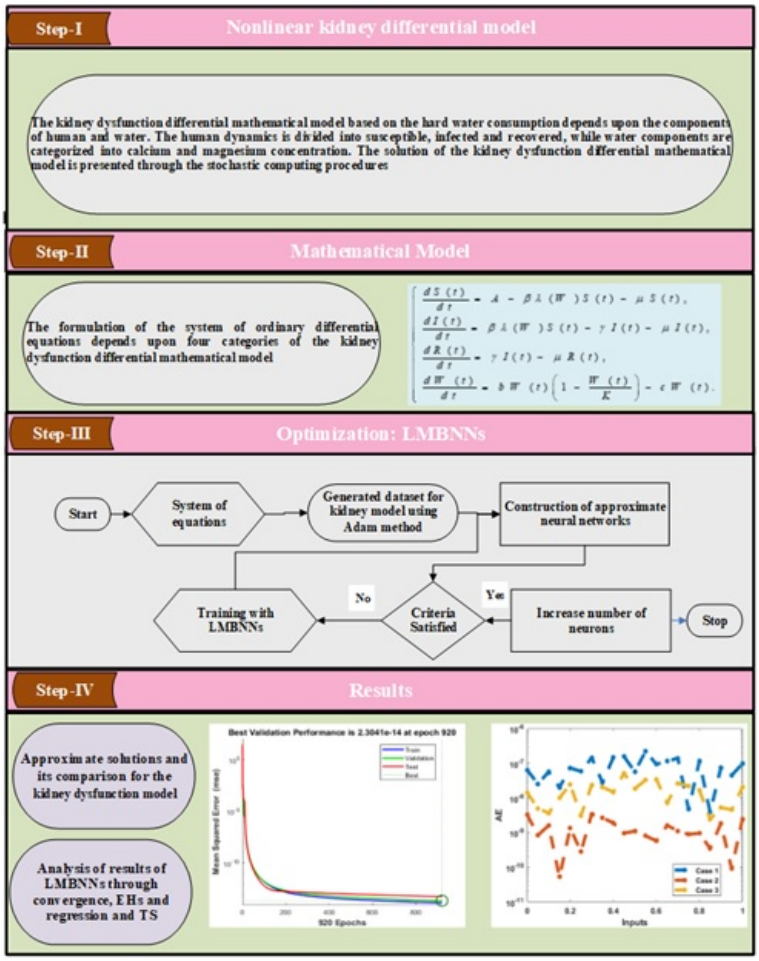
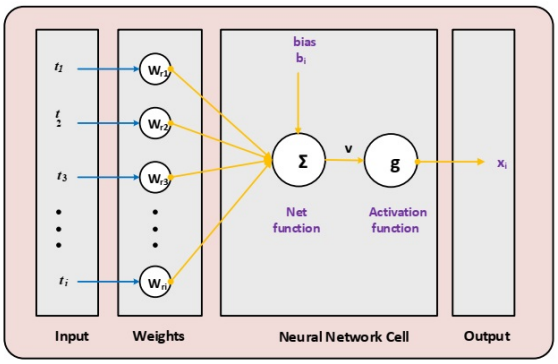


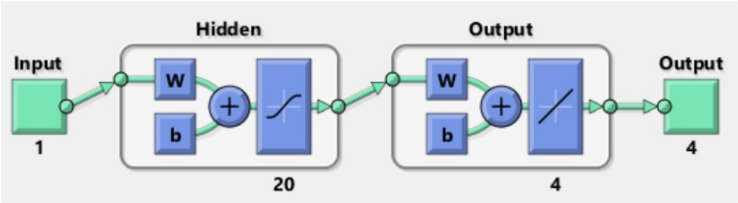
Fig. 1 Workflow of the proposed LMBNNs structure to solve the kidney disease model.

The dataset for several iterations of the kidney dysfunction model using the standard results is shown in Figure 2(a) using the "NDSolve" command of Mathematica. The function procedure (NDSolve) is functional to the default setting of the parameters to execute and terminate the tolerances. The comparison evaluations also make use of the data generated for the kidney dysfunction model. The stochastic LMBNNs is applied to create the general perception of the model for a single neuron which is shown in Figure 2(b). Additionally, a more advanced form of the multi-layer neural network for the kidney dysfunction model is presented to simulate the form of solution dynamics. The LMBNNs is presented through the command 'nftool' in the 'Matlab' using the suitable hidden neurons including learning, testing and substantiation data. The executing practice and parameter setting based on the designed approach to solve the model is given in Table 1. Premature convergence in terms of adjustments or alterations in a poor performance. Hence, after coordinating extensive study and experiment,

the input parameters will be integrated with great care.



(a) A single neuron structure



(b) Layer structure based on the single, hidden layers with 20 neurons and output layers

Fig. 2 Single neuron and different layers structure for the kidney dysfunction model.

Table 1 Executing practice and parameter setting

Parameter	Settings
Neurons	20
Testing	0.11
Maximum mu	0.E+9
Maximum learning of iteration	1100
Validation	0.7
Increasing factor of mu	13
Decreeing mu factor	0.3
Dataset	Adam technique
Training	0.72
Minimum gradient	0.E-7
Hidden/Input/output layers	Single
Fitness (MSE)	0
Adam method and stoppage criteria	Default
Adaptive parameter of mu	0.0006
Sample selection	Randoom

4 Numerical simulations

This part of this work represents the solutions of the kidney dysfunction model for three cases, given as: The first case is based on $b < c$ by taking $\mu = \frac{1}{65}, \gamma = \frac{1}{45}, \beta = \frac{1}{100}, K = 60, b = \frac{1}{10}, c = \frac{2}{5}, A = 2, S_0 = 999, I_0 = 1, W_0 = 20$ and $R_0 = 0$ in equation (1) as:

$$\begin{aligned}
\frac{dS(t)}{dt} &= 2 - \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{65}S(t),\\
\frac{dI(t)}{dt} &= \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{45}I(t) - \frac{1}{65}I(t),\\
\frac{dR(t)}{dt} &= \frac{1}{45}I(t) - \frac{1}{65}R(t),\\
\frac{dW(t)}{dt} &= 0.1\left(1 - \frac{W(t)}{60}\right)W(t) - 0.4W(t),
\end{aligned} \tag{3}$$

The second case is based on $b > c$ by taking $\mu = \frac{1}{65}, \gamma = \frac{1}{45}, \beta = \frac{1}{100}, K = 60, b = \frac{1}{5}, c = \frac{1}{10}, A = 2, S_0 = 999, I_0 = 1, W_0 = 20$ and $R_0 = 0$ in equation (1) as:

$$\begin{aligned}
\frac{dS(t)}{dt} &= 2 - \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{65}S(t),\\
\frac{dI(t)}{dt} &= \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{45}I(t) - \frac{1}{65}I(t),\\
\frac{dR(t)}{dt} &= \frac{1}{45}I(t) - \frac{1}{65}R(t),\\
\frac{dW(t)}{dt} &= \frac{1}{5}\left(1 - \frac{W(t)}{60}\right)W(t) - \frac{1}{10}W(t),
\end{aligned} \tag{4}$$

The third case is based on $b > c$ by using $\mu = \frac{1}{65}, \gamma = \frac{1}{45}, \beta = \frac{1}{100}, K = 60, b = \frac{1}{4}, c = \frac{1}{10}, A = 2, S_0 = 999, I_0 = 1, W_0 = 20$ and $R_0 = 0$ in equation (1) as:

$$\begin{aligned}
\frac{dS(t)}{dt} &= 2 - \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{65}S(t),\\
\frac{dI(t)}{dt} &= \frac{1}{100}\left(\frac{W(t)}{60+W(t)}\right)S(t) - \frac{1}{45}I(t) - \frac{1}{65}I(t),\\
\frac{dR(t)}{dt} &= \frac{1}{45}I(t) - \frac{1}{65}R(t),\\
\frac{dW(t)}{dt} &= \frac{1}{4}\left(1 - \frac{W(t)}{60}\right)W(t) - \frac{1}{10}W(t),
\end{aligned} \tag{5}$$

The calculated outputs that have been achieved through the designed procedure for solving the kidney dysfunction mathematical model with input interval 0 and 1 using twenty neurons and data selection based on the training, testing and authentication. The obtained performances using the LMBNNs for the kidney dysfunction mathematical model together with best validation performances and TS are given in Figure 3. These best validation measures are calculated as $1.29193\text{x}10^{-11}$, $2.30408\text{x}10^{-14}$ and $1.17106\text{x}10^{-11}$ at epochs 1000, 920 and 973. The gradient values are also performed in Figure 3, which are reported as $1.2676\text{x}10^{-7}$, $9.9924\text{x}10^{-8}$ and $9.9906\text{x}10^{-8}$. This convergence performance is validated through these illustrations. The fitting curve graphs have been plotted in Figure 4 for the numerical solutions of the kidney dysfunction model, which represent the correctness of the procedure. The EHs measures using the training, verification and testing are also presented in Figure 4, which are $-1.5\text{x}10^{-7}$, $-1.80\text{x}10^{-11}$ and $-2.2\text{x}10^{-7}$ for the kidney dysfunction model. The regression (Reg) plots are given in Figure 5 that are calculated as 1 for each variation of the kidney dysfunction model.

This signifies the precision of the proposed technique for the kidney dysfunction model. The MSE values for different performances are given in Table 2 for the kidney dysfunction model.

Table 2 Data representation through LMBNNs for the kidney dysfunction model.

Case	Test (MSE)	Train (MSE)	Validation(MSE)	Gradient	Mu	Epoch	Performance	Time
1	2.23E-12	2.57E-12	1.29E-11	1.27E-07	1.00E-09	1000	2.57E-12	02 Sec
2	5.81E-14	1.45E-14	2.30E-14	9.99E-08	1.00E-11	920	1.45E-14	03 Sec
3	3.44E-12	2.33E-12	1.17E-11	9.99E-08	1.00E-09	973	2.33E-12	03 Sec

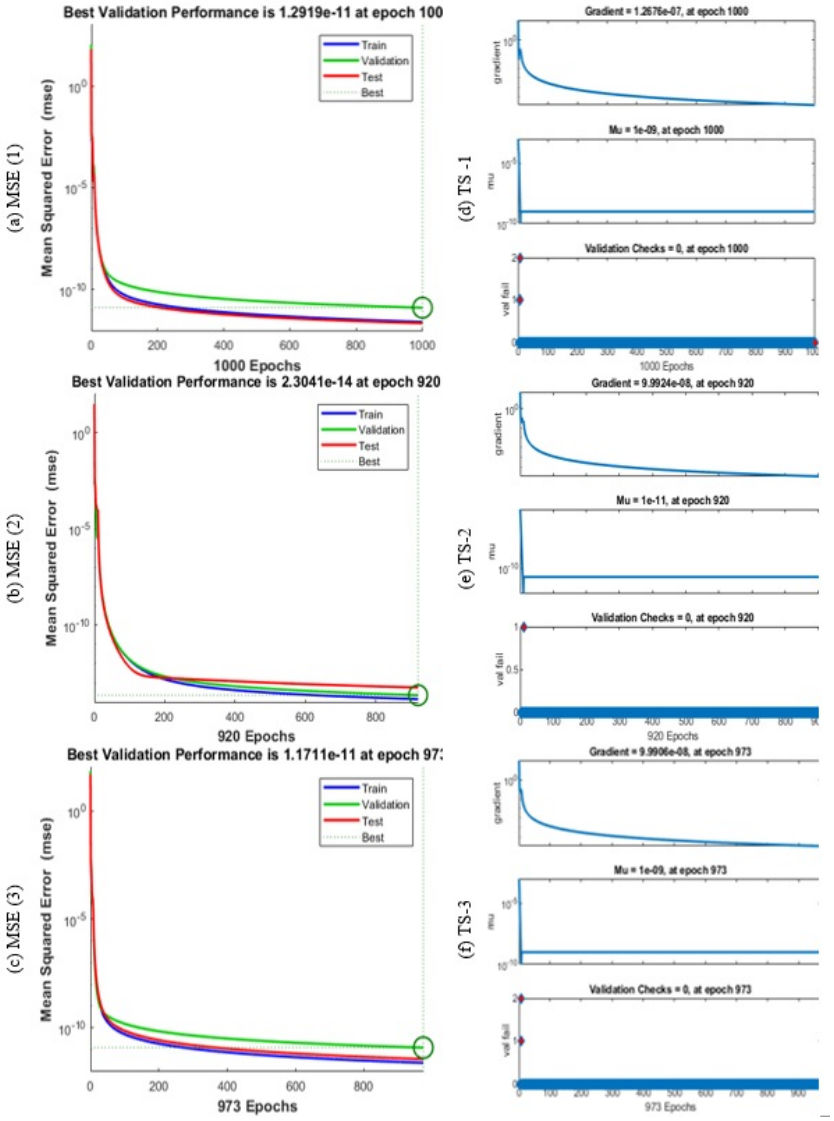


Fig. 3 MSE and TS for the kidney dysfunction mathematical model.

Figures 6 and 7 show the comparison plots and AE for each dynamic of the kidney dysfunction mathematical

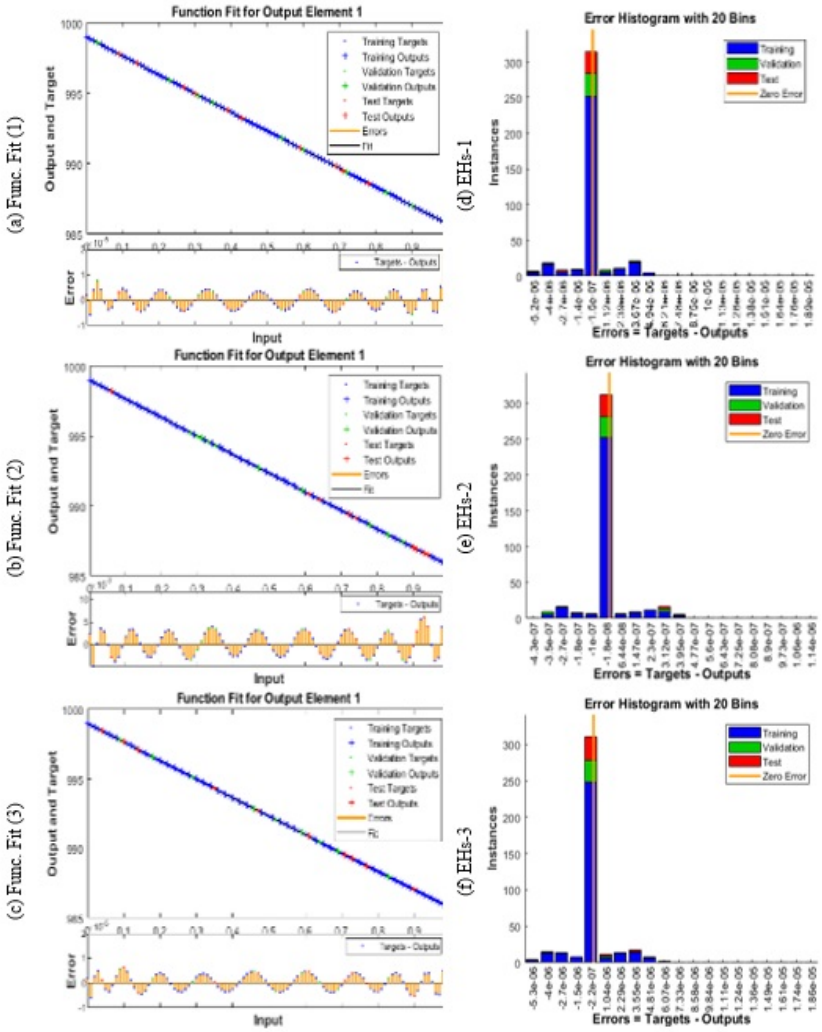


Fig. 4 The values of MSE and EHs for the kidney dysfunction mathematical model.

model by applying the designed stochastic procedure. The comparison performances have been presented in Figure 6, which shows the overlapping of the results. These matching solutions enhance the accuracy procedure for kidney dysfunction mathematical model. Figure 7 shows the AE illustrations through the LMBNNs for the kidney dysfunction model, which is divided into the components of human and water. The AE for susceptible is reported as 10^{-05} to 10^{-08}, 10^{-06} to 10^{-09} and 10^{-05} to 10^{-07} for case 1 to 3. The AE for infected are presented as 10^{-07} to 10^{-09}, 10^{-09} to 10^{-11} and 10^{-08} to 10^{-09} for case 1 to 3. The AE performances for the recovered dynamics are presented as 10^{-07} to 10^{-09} for the 1^{st} and 3^{rd} case and AE values for the 2^{nd} case are noticed as 10^{-08} to 10^{-11}. Moreover, these values for the water components are reported as 10^{-06} to 10^{-08}, 10^{-09} to 10^{-10} and 10^{-07} to 10^{-10} for case 1 to 3. These small values of AE show the reliability of the technique for the kidney dysfunction mathematical model.

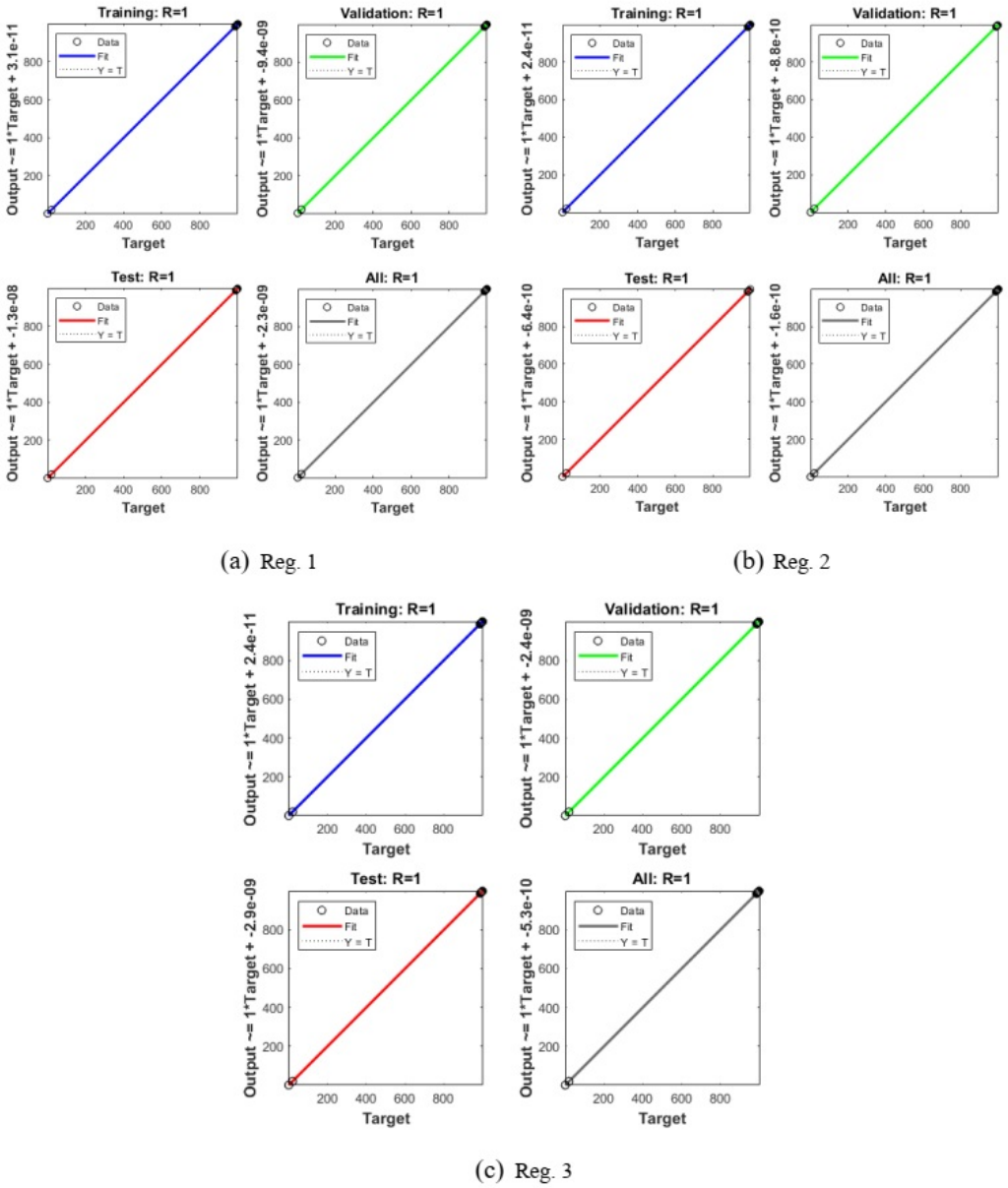


Fig. 5 Reg. values for the kidney dysfunction mathematical model.

5 Conclusions

There are various water resources that have a greater concentration based on magnesium and calcium ions called "hard water". Hence, it is now necessary to understand how drinking hard water damages the kidneys. The current work presents the numerical results of the model based on the kidney dysfunction model by applying the stochastic Levenberg-Marquardt backpropagation neural networks. The kidney dysfunction model is categorized into the components of human and water. The dynamics of human is further classified into three classes, while water component contains calcium and magnesium concentration. Twenty neurons along with log-sigmoid transfer function have been used to solve the model. The stability of the kidney dysfunction model based on hard water consumption has also been presented. The correctness is observed through the comparisons of the results along with the negligible AE, which is calculated as 10^{-06} to 10^{-07}. Validation, training, and testing values have been used to reduce the mean square error. Furthermore, the statistical operators have been given

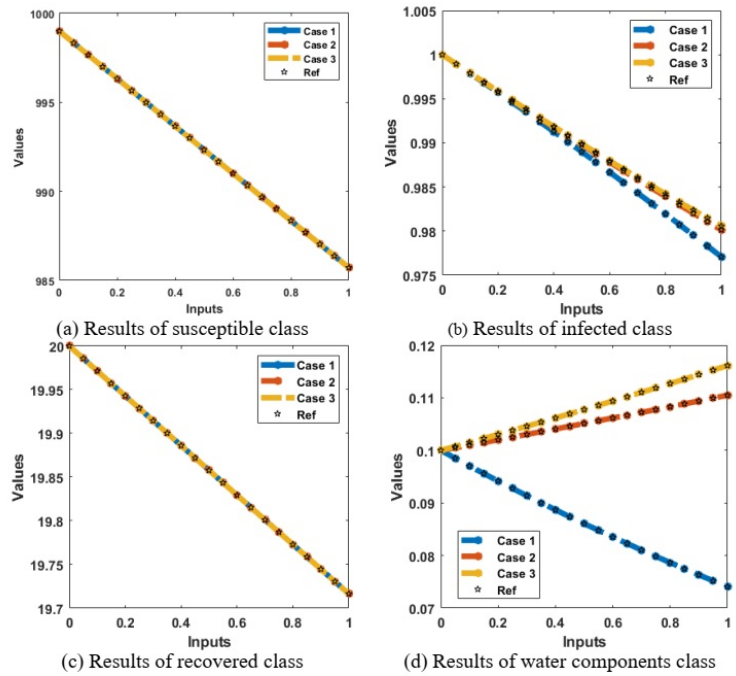


Fig. 6 Results performances for the kidney dysfunction mathematical model.

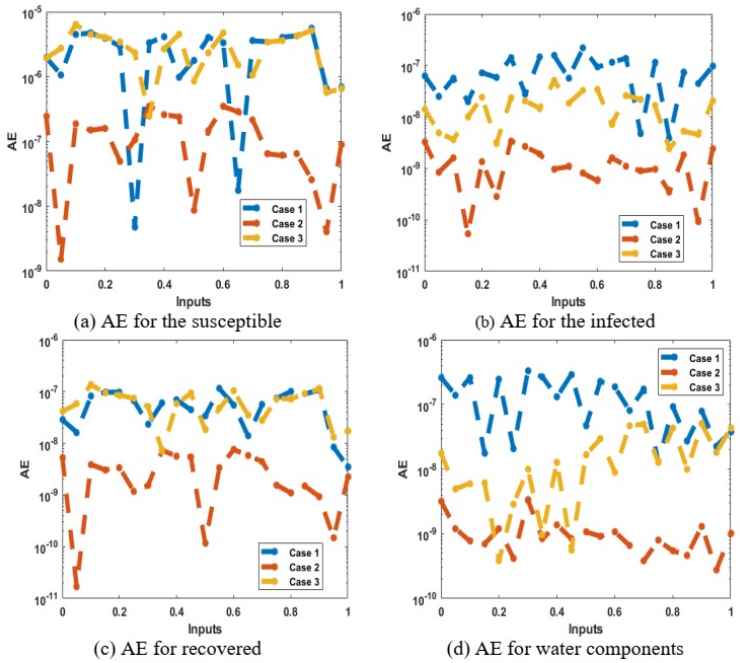


Fig. 7 AE performances for the kidney dysfunction mathematical model.

to authenticate the consistency of the method. In future, the designed scheme may be used for the solutions of fractional order systems and coupled/dynamical equations.

6 Declarations

6.1 Conflict of interest:

According to the authors of this paper, there are no conflicts of interest to report regarding the article that is being presented.

6.2 Funding:

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6.3 Authors Contribution:

Z.S.- Conceptualization, Methodology, Software, Writing-Review Editing. M.U.-Formal Analysis, Validation, Writing-Original Draft. All authors read and approved the final submitted version of this manuscript.

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6.5 Data availability statement:

All data that support the findings of this study are included within the article (and any supplementary files).

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