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Original Study

Fractional SIZR model of Zombie infection

Hossein Jafari[1,2], Pranay Goswami[3], Ravi Shankar Dubey[4]†, Shivani Sharma[5], Arun Chaudhary[6]

[1]Department of Mathematics, University of Mazandaran, Babolsar, Iran

[2]Department of Mathematical Sciences, University of South Africa, UNISA0003, South Africa

[3]School of Liberal Studies, Dr B R Ambedkar University Delhi, Delhi, India

[4]Department of Mathematics, Amity School of Applied Science, Amity University Rajasthan, Jaipur, India

[5]Department of Mathematics, SGND Khalsa College, University of Delhi, Delhi, India

[6]Department of Mathematics, Rajdhani College, University of Delhi, Delhi, India

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Abstract

This research paper investigates the SIZR model related to Zombie infection outbreaks according to a time-dependent infection rate. The proposed model is extended to the fractional order using different fractional derivative operators. The solution of the proposed model by numerical schemes, is briefed. Graphical representations provide us with a better understanding of this mathematical model. Lastly, as observed in movies, we conclude that Zombie infections can generate the destruction and collapse of human development and it is of paramount importance to deal with Zombies as early as possible in order to avoid putting our civilization under threatening and destructive circumstances.

Keywords: SIZR model, fractional derivative, numerical solution.
AMS 2020 codes: 92B05; 92C60; 26A33.

1 Introduction

Fictional characters have always intrigued and fascinated people. From movies to TV shows to video games, these fictional characters have somehow become a part of us. Authors invent these characters suggesting the possibility that someday they might actually come to life. One of the most popular and exploited among fictional characters is ‘Zombie’. The word ‘Zombie’ originated from the West African word ‘nZambi’ or ‘Zumbi’. It was introduced into the English dictionary as Zombie in the year 1819. A Zombie is a walking corpse that feeds on human life. Zombies can be perceived as the victims of some serious infectious disease outbreak. Their brains are almost dead and their sole purpose is to run after other people’s life, by biting and infecting them as well. Zombies gained popularity from movies like ‘Resident Evil’, ‘Zombieland’ etc as well as TV shows like

†Corresponding author.
Email address: ravimath13@gmail.com

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'The Walking Dead' that also helped increase their popularity. In movies, TV shows and video games, Zombies are depicted as dead corpses grunting and limping around the city, searching for human bodies to relish their flesh and convert them to one of their kind. They are deemed walking dead. Although having the body of a dead person, they are still alive and their only task is that of infecting other humans. There is no possibility of treating them in any way. The only way to stop their proliferation is by killing them. Zombies are an exaggerated description of an infectious disease. This disease is so severe that once infected, there is no turning back [1]. The movie 'World War Z', shows how a Zombie outbreak takes place and spreads around the whole globe. The protagonist embarks on an adventure to discover a cure against the outbreak while tackling boisterous Zombies and eventually succeeds in his mission. Apart from a fictional tale or story, Zombies can be seen as a viral disease that can become a challenge to the human race. Their distribution can be considered as the source of any other viral illness spreading.

Mathematical models can make decisions, save lives, help in policy and much more. They are beneficial for understanding the conditions needed to sustain life and provide a way to study and predict spreading behavior. The concept of calculus plays an important role in formulating these mathematical models. In this paper, we will investigate the SIZR mathematical model of the occurrence of Zombie attacks by examining the outbreak of these attacks. A SIZR model determines the number of people infected over time by transmission-infected diseases in a closed population. These models are obtained in such a way that they contain equations associated with the occurrence of $S(t)$ susceptible individuals, $I(t)$-infected individuals and the number of people who have been transformed into Zombies and those who have been removed $R(t)$.

This research paper is organized as follows: section 2 gives background information about the SIZR model related to Zombie outbreak. Section 3 presents various important fractional operators. In sections 4 and 5 we have analysed the SIZR model in the sense of Caputo and C-F derivatives of fractional order, respectively. In section 6, we discuss about the physical interpretation and section 7 contains the conclusion and future directions.

2 SIZR model for outbreak of Zombie with time dependent infection rate

In the **SIZR** model for Zombie infection, we have considered the four different classes of individuals as **S:** Susceptible group of individuals, **I:** Symptomatically infected group of individuals, **Z:** Individuals who have become infected, **R:** Removed group of individuals.
We assume that when Zombies attack the susceptible individuals, they leave wounds containing Zombies slobber in it. Then the slobbering fluid gets mixed with the blood of susceptible individuals thus infecting them. After the Zombies attack the susceptible class of individuals first move to the class of infected individuals. The infected individuals can either die by natural death or otherwise become Zombies. Zombies move to the removed group by having their brains destroyed or by removing their heads. This removed group comprehends humans who have died either by natural death or due to a Zombie attack. Individuals in the removed class can also restore to life and become a Zombie again. The following assumptions are also made

- Zombies have thirst for human flesh only and can transmit infection only to human beings.
- The new Zombies can only arise through (a) the susceptible individuals who came in contact with Zombies, (b) those who have restored to life from the removed class of individuals.

Furthermore, a time dependence on the parameter β the transmission rate [2], is introduced in this model, so the effect of transmission will be given as $\beta_0 e^{-\varsigma t}$. In this paper the choice of negative exponential function is

justified by a step by step implementation of rules and by rise in consciousness and awareness in individuals. Here, β_0 is the infection rate at the beginning of the Zombie infection and ς determines the change in the time of infection [2] time infection. Keeping in mind the aforementioned assumptions the SIZR model is formulated as [3]

$$\begin{aligned}
\frac{dS}{dt} &= N - \beta_0 e^{-\varsigma t} S(t) Z(t) - \omega S(t), \ S(0) = S_0, \\
\frac{dI}{dt} &= \beta_0 e^{-\varsigma t} S(t) Z(t) - \chi I(t) - \omega I(t), \ \ I(0) = I_0, \\
\frac{dZ}{dt} &= \chi I(t) + \delta R(t) - \rho S(t) Z(t), \ \ Z(0) = Z_0, \\
\frac{dR}{dt} &= \omega S(t) + \omega I(t) + \rho S(t) Z(t) - \delta R(t), \ \ R(0) = R_0.
\end{aligned} \tag{1}$$

Where $N, \omega, \delta, \rho, \chi$ represents the birth rate, rate of death, the rate at which deceased individuals resurrect and become Zombies, the rate by which Zombies move to the removed class by annihilating their brains and demolishing their heads, the rate by which infected individuals become Zombies respectively. The SIZR model is demonstrated in Figure 1.

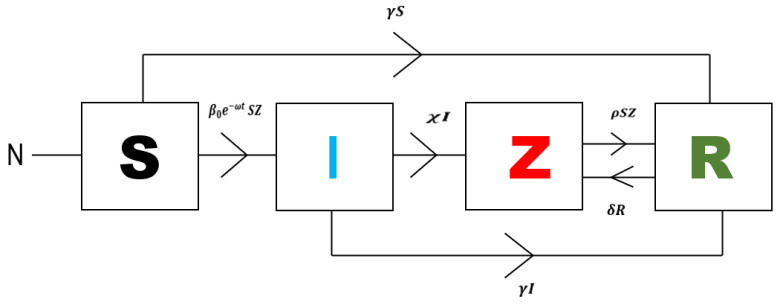


Fig. 1 The SIZR model for Zombie attack

Theorem 1. *[4] Assuming that all the parameters which are defined in the above model are positive, the solution of model $S(t), I(t), Z(t), R(t)$ with non-negative initial values are positive $\forall t > 0$.*

Proof. Define Δ=$Sup\{t > 0 : S_0, I_0, Z_0, R_0 \geq 0\}$. Consider the following equation

$$\frac{dS}{dt} = N - \beta_0 e^{-\varsigma t} S(t) Z(t) - \omega S(t),$$

$$\frac{dS}{dt} \geq -\omega S(t). \tag{2}$$

On solving, we get $S(t) = A_1 e^{-\omega t}$, where A_1 is the constant of integration.
Using the fact that $S(0) = S_0 > 0$, we get

$$S(t) = S_0 e^{-\omega t} \geq 0. \tag{3}$$

Next Consider,

$$\frac{dI}{dt} = \beta_0 e^{-\varsigma t} S(t) Z(t) - \chi I(t) - \omega I(t),$$

$$\frac{dI}{dt} \geq -(\chi + \omega) I(t). \tag{4}$$

On solving, we get $I(t) = A_2 e^{(\chi+\omega)t}$, where A_2 is constant of integration.
Using the fact that $I(0) = I_0 > 0$, we get

$$I(t) = I_0 e^{-(\chi+\omega)t} \geq 0. \tag{5}$$

In a similar way, we get

$$Z(t) = Z_0 e^{-\rho t S(t)} \geq 0. \tag{6}$$

$$R(t) = R_0 e^{-(\delta)t} \geq 0. \tag{7}$$

3 Fractional SIZR model of Zombie Infection

We will now extend the SIZR model of Zombie infection to the fractional mathematical Zombie model infection using different fractional derivative operators. Furthermore, to ensure that the right- and left-hand sides of the consequent fractional model possess the same dimension $(time)^{-\eta}$, all the parameters having the dimension $(time)^{-1}$ are replaced by the power of ν, while the other parameters remain unchanged. The concept of fractional derivatives [5–8] is instrumental in providing a better understanding of real-world problems which exhibit non-local behaviors. Different fractional derivative operators have their significance. Riemann-Liouville proposed the definition with singular kernel [5] and this was the first most accepted definition in the field of fractional calculus after that new development was introduced in the field of calculus of fractional order, which involves the definition of the derivative of fractional order with the non-singular kernel, one of which was presented by Caputo [5]. This definition is based on the concept of power law, so it does not work for the problems that exhibit the fading memory process. The next improvement in this area was made by Caputo and Fabrizio [9] by suggesting the definition that can deal with the process that exhibits a fading memory process with the exponential decay accompanied by Delta-Dirac characteristic. Lastly, the definition of fractional derivatives and integrals that can easily handle the process by exhibiting a passage from fading memory to power law have been introduced in [10] and the applications are given in [11–34]. We will now discuss the detailed analysis of the proposed fractional mathematical SIZR model in the sense of Caputo and C-F fractional operators.

4 SIZR model of Zombie infection with Caputo operator

In this section we define SIZR model of Zombie infection based on the well known Caputo operator as following

$$\begin{aligned}
{}^{\mathscr{C}}\zeta_t^{\eta} S(t) &= N^{\eta} - \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \omega^{\eta} S(t),\ S(0) = S_0,\\
{}^{\mathscr{C}}\zeta_t^{\eta} I(t) &= \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \chi^{\eta} I(t) - \omega^{\eta} I(t),\ I(0) = I_0,\\
{}^{\mathscr{C}}\zeta_t^{\eta} Z(t) &= \chi^{\eta} I(t) + \delta^{\eta} R(t) - \rho^{\eta} S(t) Z(t),\quad Z(0) = Z_0,\\
{}^{\mathscr{C}}\zeta_t^{\eta} R(t) &= \omega^{\eta} S(t) + \omega^{\eta} I(t) + \rho^{\eta} S(t) Z(t) - \delta^{\eta} R(t),\ R(0) = R_0,
\end{aligned} \tag{8}$$

here, ${}^{\mathscr{C}}\zeta_t^{\eta}$ denotes the Caputo derivative of fractional order η.

Definition 1. [5] Let f on $\mathbb{R}$ be an integrable function, $0 < \eta < 1$ the fractional Caputo derivative with order η is

$${}^{\mathscr{C}}\zeta_t^{\eta} f(t) = \frac{1}{\omega(1-\eta)} \int_0^t \frac{1}{(t-\tau)^{\eta}} \frac{d}{d\tau} f(\tau) d\tau. \tag{9}$$

Where ${}^{\mathscr{C}}\zeta_t^{\eta}$ represents Caputo derivative of order η.

Theorem 2. *[30] Assuming that positive constants L and $\bar{L}$ exist such that the following holds*

- *Lipschitz Condition:* $\forall v_1, v_2 \in \mathbb{R}$ *and* $\forall t \in [t_0, T]$

$$|g(t, v_1(t)) - g(t, v_2(t))| \leq L|v_1 - v_2|. \tag{10}$$

- *Linear growth condition:* $\forall (v,t) \in \mathbb{R} \times [[t_0, T]]$

$$|g(t,v)|^2 \leq \bar{L}(1 + |v|^2). \tag{11}$$

Then the Cauchy problem with Caputo derivative allows a unique solution.

4.1 Numerical scheme for the SIZR model in frame of Caputo Derivative

Consider the SIZR model for Zombie infection in frame of Caputo fractional derivative operator

$$\begin{aligned}
{}^{\mathscr{C}}\zeta_t^{\eta} S(t) &= N^{\eta} - \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \omega^{\eta} S(t), \ S(0) = S_0, \\
{}^{\mathscr{C}}\zeta_t^{\eta} I(t) &= \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \chi^{\eta} I(t) - \omega^{\eta} I(t), \ I(0) = I_0, \\
{}^{\mathscr{C}}\zeta_t^{\eta} Z(t) &= \chi^{\eta} I(t) + \delta^{\eta} R(t) - \rho^{\eta} S(t) Z(t), \ Z(0) = Z_0, \\
{}^{\mathscr{C}}\zeta_t^{\eta} R(t) &= \omega^{\eta} S(t) + \omega^{\eta} I(t) + \rho^{\eta} S(t) Z(t) - \delta^{\eta} R(t), \ R(0) = R_0.
\end{aligned} \tag{12}$$

Here, ${}^{\mathscr{C}}\zeta_t^{\eta}$ denotes the fractional Caputo derivative of order η.
To find the numerical solution [31], consider

$${}_0^C\zeta_t^{\eta} \psi(t) = k(t, \psi(t)), \ t \geq 0, \quad \psi(0) = \psi_0. \tag{13}$$

Using the fundamental theorem, we rewrite the above equation as

$$\psi(t) = \psi(0) + \frac{1}{\omega(\eta)} \int_0^t (t - \tau)^{\eta - 1} k(\psi, \tau) d\tau, \tag{14}$$

at $t = t_{p+1}$, we have

$$\psi_{p+1} = \psi(t_{p+1}) = \psi(0) + \frac{1}{\omega(\eta)} \int_0^{t_{p+1}} (t_{p+1} - \tau)^{\eta - 1} k(\psi, \tau) d\tau, \tag{15}$$

at $t = t_p$, we have

$$\psi_p = \psi(t_p) = \psi(0) + \frac{1}{\omega(\eta)} \int_0^{t_p} (t_p - \tau)^{\eta - 1} k(\psi, \tau) d\tau, \tag{16}$$

From the above two equations, we get

$$\psi(t_{p+1}) - \psi(t_p) = \frac{1}{\omega(\eta)} \left[\int_0^{t_{p+1}} (t_{p+1} - \tau)^{\eta - 1} k(\psi, \tau) \, d\tau - \int_0^{t_p} (t_p - \tau)^{\eta - 1} k(\psi, \tau) d\tau \right]. \tag{17}$$

Now, applying the handled scheme with Lagrange polynomial interpolation, we have the following numerical iteration formula

$$\begin{aligned}
S_{p+1} = \frac{\eta h^{\eta}}{\omega(\eta + 2)} \sum_{m=0}^{r} f_1(t_m, S_m) \left[(p - m + 1)^{\eta} (p - m + 2 + 2\eta) - (p - m)^{\eta} (p - m + 2 + 2\eta) \right] \\
- \frac{h^{\eta}}{\omega(\eta + 2)} \sum_{m=0}^{r} f_1(t_{m-1}, S_{m-1}) \left[(p - m + 1)^{\eta + 1} - (p - m)^{\eta} (p - m + 1 + \eta) \right],
\end{aligned} \tag{18}$$

where

$$f_1(t,S) = N^{\eta} - \beta_0^{\eta}\, e^{-\varsigma t}\, S(t)\, Z(t) - \omega^{\eta}\, S(t). \tag{19}$$

$$I_{p+1} = \frac{\eta h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_2(t_m, I_m)\left[(p-m+1)^{\eta}(p-m+2+2\eta) - (p-m)^{\eta}(p-m+2+2\eta)\right] - \frac{h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_2(t_{m-1}, I_{m-1})\left[(p-m+1)^{\eta+1} - (p-m)^{\eta}(p-m+1+\eta)\right], \tag{20}$$

where

$$f_2(t,I) = \beta_0^{\eta}\, e^{-\varsigma t}\, S(t)\, Z(t) - \chi^{\eta}\, I(t) - \omega^{\eta}\, I(t). \tag{21}$$

$$Z_{p+1} = \frac{\eta h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_3(t_m, Z_m)\left[(p-m+1)^{\eta}(p-m+2+2\eta) - (p-m)^{\eta}(p-m+2+2\eta)\right] - \frac{h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_3(t_{m-1}, Z_{m-1})\left[(p-m+1)^{\eta+1} - (p-m)^{\eta}(p-m+1+\eta)\right] \tag{22}$$

and also

$$f_3(t,Z) = \chi^{\eta}\, I(t) + \delta^{\eta}\, R(t) - \rho^{\eta}\, S(t)\, Z(t). \tag{23}$$

$$R_{p+1} = \frac{\eta h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_4(t_m, R_m)\left[(p-m+1)^{\eta}(p-m+2+2\eta) - (p-m)^{\eta}(p-m+2+2\eta)\right] - \frac{h^{\eta}}{\omega(\eta+2)} \sum_{m=0}^{r} f_4(t_{m-1}, R_{m-1})\left[(p-m+1)^{\eta+1} - (p-m)^{\eta}(p-m+1+\eta)\right], \tag{24}$$

and

$$f_4(t,R) = \omega^{\eta}\, S(t) + \omega^{\eta}\, I(t) + \rho^{\eta}\, S(t)\, Z(t) - \delta^{\eta}\, R(t). \tag{25}$$

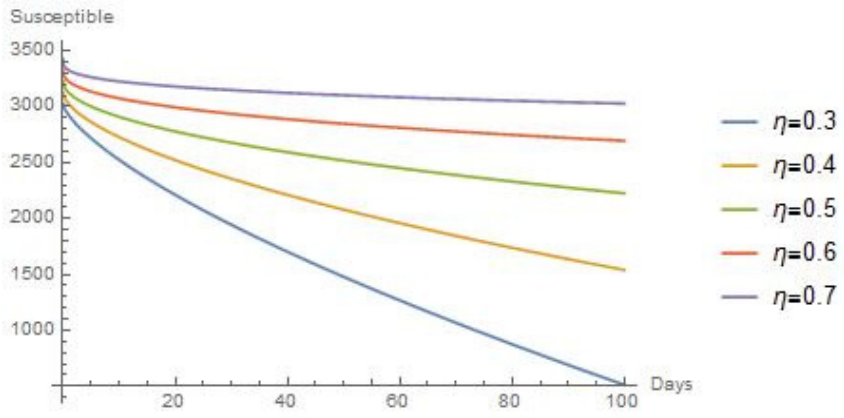


Fig. 2 2D simulation of S(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (18).

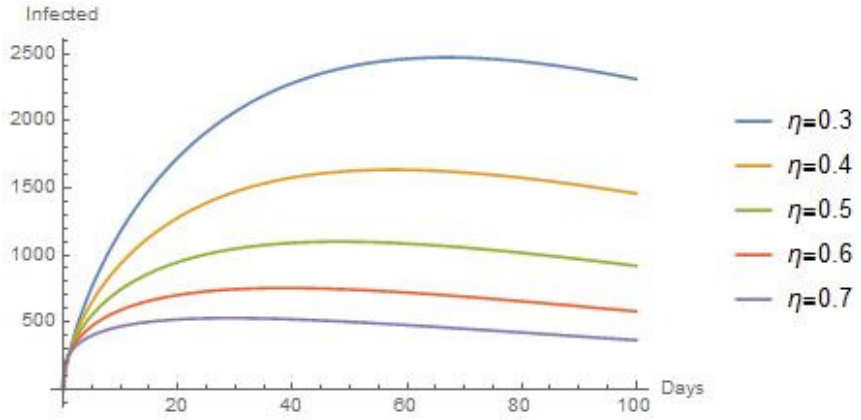


Fig. 3 2D simulation of I(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (20).

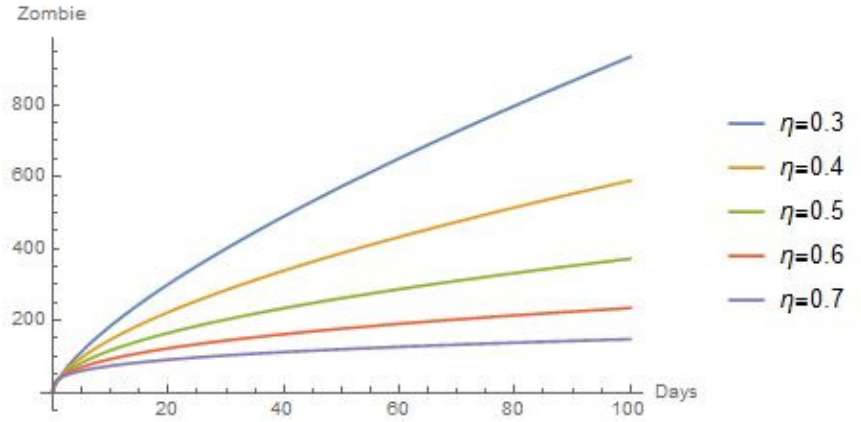


Fig. 4 2D simulation of Z(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (22).

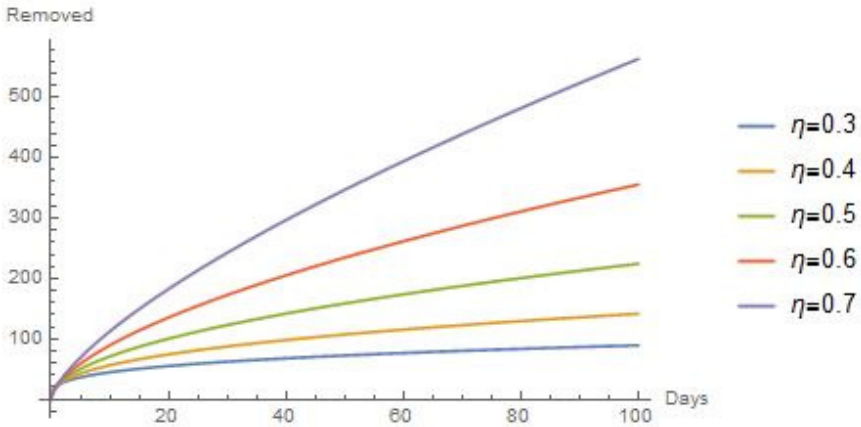


Fig. 5 2D simulation of R(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (24).

5 SIZR Model of Zombie infection With Caputo-Fabrizio

We have

$$\begin{aligned}
{}^{\mathscr{CF}}\zeta_t^{\eta} S(t) &= N^{\eta} - \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \omega^{\eta} S(t),\ S(0) = S_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} I(t) &= \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \chi^{\eta} I(t) - \omega^{\eta} I(t),\ I(0) = I_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} Z(t) &= \chi^{\eta} I(t) + \delta^{\eta} R(t) - \rho^{\eta} S(t) Z(t),\ Z(0) = Z_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} R(t) &= \omega^{\eta} S(t) + \omega^{\eta} I(t) + \rho^{\eta} S(t) Z(t) - \delta^{\eta} R(t),\ R(0) = R_0,
\end{aligned} \tag{26}$$

here, ${}^{\mathscr{CF}}\zeta_t^{\eta}$ denotes the Caputo Fabrizio fractional derivative of order η.

Definition 2. [9] Let f on $\mathbb{R}$ be an integrable function, $t > 0,\ \ 0 < \eta < 1$, the Caputo-Fabrizio fractional derivative of order η is defined as

$${}_0^{CF}\zeta_t^{\eta}(f(t)) = \frac{N(\eta)}{1-\eta}\int_0^t exp\left(\frac{-\eta(t-\tau)}{1-\eta}\right) f'(\tau) d\tau. \tag{27}$$

Where ${}_0^{CF}\zeta_t^{\eta}$ represents the fractional Caputo-Fabrizio derivative of order η, $N(\eta)$ is a normalization function such that $N(0) = N(1) = 1$.

Definition 3. [9] Let f on $\mathbb{R}$ be an integrable function, $t > 0,\ \ 0 < \eta < 1$, the Caputo-Fabrizio time fractional integral of order η is given as

$$\mathscr{I}^{\mathscr{CF}}{}_t^{\beta_0}(f(t)) = \frac{2(1-\eta)}{(2-\eta)N(\eta)} f(t) + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t f(\tau)\, d\tau. \tag{28}$$

Where $N(\eta)$ is the normalization function such that $N(0) = N(1) = 1$.

5.1 Numerical scheme for the SIZR model in frame of Caputo-Fabrizio operator

Consider the SIZR model for Zombie infection in frame of fractional Caputo-Fabrizio derivative

$$\begin{aligned}
{}^{\mathscr{CF}}\zeta_t^{\eta} S(t) &= N^{\eta} - \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \omega^{\eta} S(t),\ S(0) = S_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} I(t) &= \beta_0^{\eta} e^{-\varsigma t} S(t) Z(t) - \chi^{\eta} I(t) - \omega^{\eta} I(t),\ I(0) = I_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} Z(t) &= \chi^{\eta} I(t) + \delta^{\eta} R(t) - \rho^{\eta} S(t) Z(t),\ Z(0) = Z_0,\\
{}^{\mathscr{CF}}\zeta_t^{\eta} R(t) &= \omega^{\eta} S(t) + \omega^{\eta} I(t) + \rho^{\eta} S(t) Z(t) - \delta^{\eta} R(t),\ R(0) = R_0,
\end{aligned} \tag{29}$$

here, ${}^{\mathscr{CF}}\zeta_t^{\eta}$ denotes the fractional Caputo Fabrizio derivative of order η.

To illustrate the method [35], consider

$${}_0^{CF}\zeta_t^{\eta}\phi(t) = k(t, \phi(t)),\ \ t \geq 0,\ \ \ \phi(0) = \phi_0. \tag{30}$$

Using the fundamental theorem, we rewrite the above equation as

$$\psi(t) - \psi(0) = \frac{1-\eta}{G(\eta)} k(t, \psi(t)) + \frac{\eta}{G(\eta)}\int_0^t k(\tau, \psi(\tau)) d\tau. \tag{31}$$

At $t = t_{p+1}$, for p= 0,1,2 $\cdots$, equation (31) is given as

$$\psi(t_{p+1}) - \psi(0) = \frac{1-\eta}{G(\eta)} k(t_p, \psi(t_p)) + \frac{\eta}{G(\eta)}\int_0^{t_{p+1}} k(\tau, \psi(\tau)) d\tau. \tag{32}$$

At $t = t_p$, for p= 0,1,2 $\cdots$, equation (31) is given as

$$\psi(t_p) - \psi(0) = \frac{1-\eta}{G(\eta)} g(t_{p-1}, \psi(t_{p-1})) + \frac{\eta}{G(\eta)} \int_0^{t_p} k(\tau, \psi(\tau)) d\tau. \tag{33}$$

From the above two equations, we get

$$\psi(t_{p+1}) - \psi(t_p) = \frac{1-\eta}{G(\eta)} [k(t_p, \psi(t_p)) - k(t_{p-1}, \psi(t_{p-1}))] + \frac{\eta}{G(\eta)} \int_{t_p}^{t_{p+1}} k(\tau, \psi(\tau)) d\tau. \tag{34}$$

Considering k(τ, $\psi(\tau)$) through Lagrange polynomial interpolation,

$$k(\tau, \psi(\tau)) = \frac{\tau - t_{m-1}}{t_m - t_{m-1}} g(t_m, \psi_{t_m}) + \frac{\tau - t_m}{t_{m-1} - t_m} k(t_{m-1}, \psi_{t_{m-1}}), \tag{35}$$

where $\psi(t_m)$ is a function at time t_m and $\psi(t_{m-1})$ is a function at time t_{m-1}. Substituting the value of $k(\tau, \psi(\tau))$ in equation (34), we get

$$\psi_{p+1} - \psi_p = \frac{1-\eta}{G(\eta)} [k(t_p, \psi(t_p)) - k(t_{p-1}, \psi(t_{p-1}))] + \frac{\eta}{G(\eta)} \int_{t_p}^{t_{p+1}} \left(\frac{k(t_p, \psi_p)}{h} (\tau - t_{p-1}) - \frac{k(t_{p-1}, \psi_{p-1})}{h} (\tau - t_p) \right) d\tau. \tag{36}$$

Substituting $h = t_m - t_{m-1}$ and after solving, we have

$$\psi_{p+1} = \psi_0 + \left(\frac{1-\eta}{G(\eta)} + \frac{3h}{2\,G(\eta)} \right) k(t_p, \psi(t_p)) - \left(\frac{1-\eta}{G(\eta)} + \frac{\eta\, h}{2\,G(\eta)} \right) k(t_{p-1}, \psi(t_{p-1})). \tag{37}$$

Using this concept [35], the numerical scheme for the fractional model of Zombie attack in the sense of fractional Caputo-Fabrizio derivative is given as

$$S_{p+1} = S_0 + \left(\frac{1-\eta}{G(\eta)} + \frac{3h}{2\,G(\eta)} \right) f_1(t_p, \psi(t_p)) - \left(\frac{1-\eta}{G(\eta)} + \frac{\eta\, h}{2\,G(\eta)} \right) f_1(t_{p-1}, \psi(t_{p-1})), \tag{38}$$

where

$$f_1(t, S) = N^\eta - \beta_0^\eta\, e^{-\varsigma t}\, S(t)\, Z(t) - \omega^\eta\, S(t).$$

Moreover, I is

$$I_{p+1} = I_0 + \left(\frac{1-\eta}{G(\eta)} + \frac{3h}{2\,G(\eta)} \right) f_2(t_p, \psi(t_p)) - \left(\frac{1-\eta}{G(\eta)} + \frac{\eta\, h}{2\,G(\eta)} \right) f_2(t_{p-1}, \psi(t_{p-1})), \tag{39}$$

where

$$f_2(t, I) = \beta_0^\eta\, e^{-\varsigma t}\, S(t)\, Z(t) - \chi^\eta\, I(t) - \omega^\eta\, I(t).$$

For Z, it is given

$$Z_{p+1} = Z_0 + \left(\frac{1-\eta}{G(\eta)} + \frac{3h}{2\,G(\eta)} \right) f_3(t_p, \psi(t_p)) - \left(\frac{1-\eta}{G(\eta)} + \frac{\eta\, h}{2\,G(\eta)} \right) f_3(t_{p-1}, \psi(t_{p-1})), \tag{40}$$

being

$$f_3(t, Z) = \chi^\eta\, I(t) + \delta^\eta\, R(t) - \rho^\eta\, S(t)\, Z(t).$$

For R, it is written as

$$R_{p+1} = R_0 + \left(\frac{1-\eta}{G(\eta)} + \frac{3h}{2\,G(\eta)}\right) f_3(t_p, \psi(t_p)) - \left(\frac{1-\eta}{G(\eta)} + \frac{\eta\, h}{2\,G(\eta)}\right) f_3(t_{p-1}, \psi(t_{p-1})), \tag{41}$$

which is

$$f_4(t,R) = \omega^{\eta} S(t) + \omega^{\eta} I(t) + \rho^{\eta} S(t) Z(t) - \delta R(t).$$

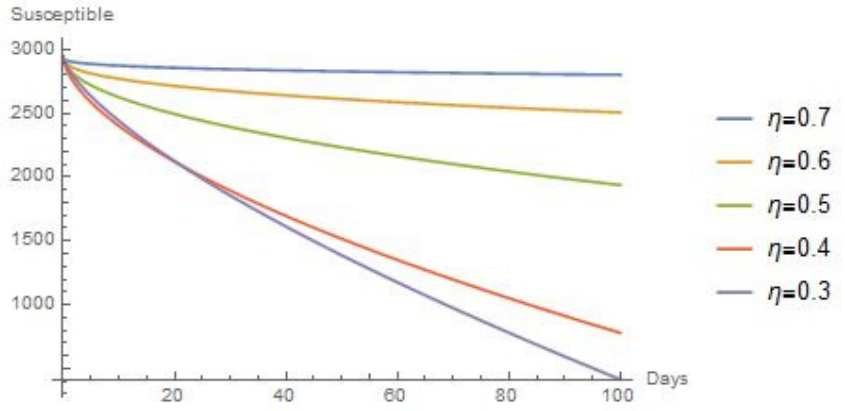


Fig. 6 2D simulation of S(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (38).

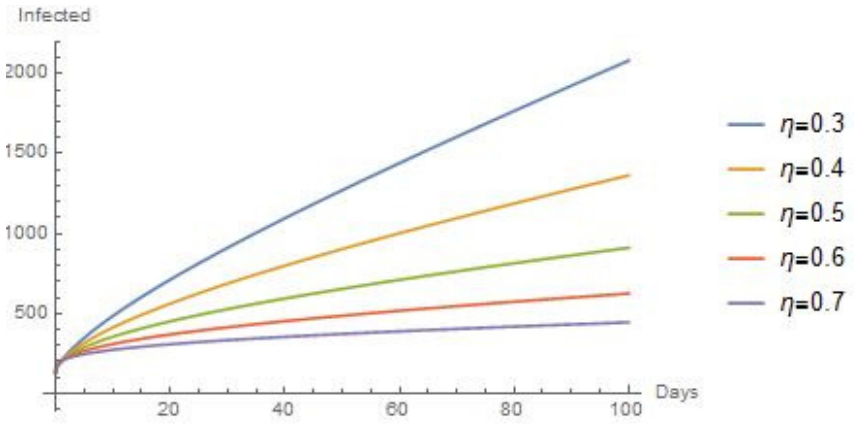


Fig. 7 2D simulation of I(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (39).

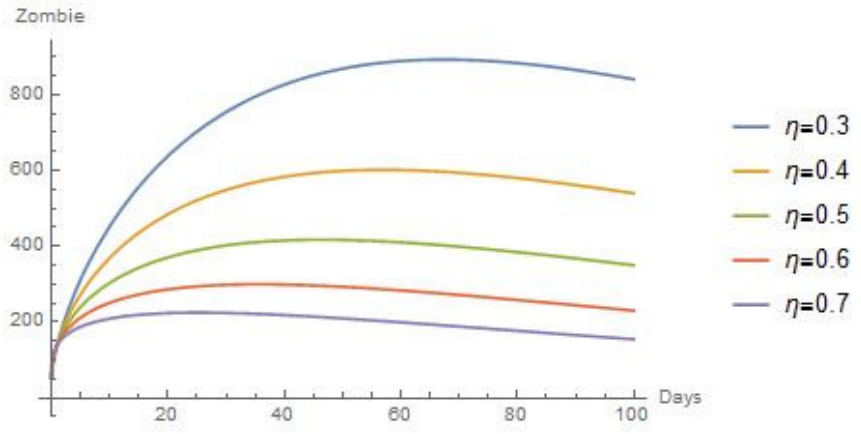


Fig. 8 2D simulation of Z(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (40).

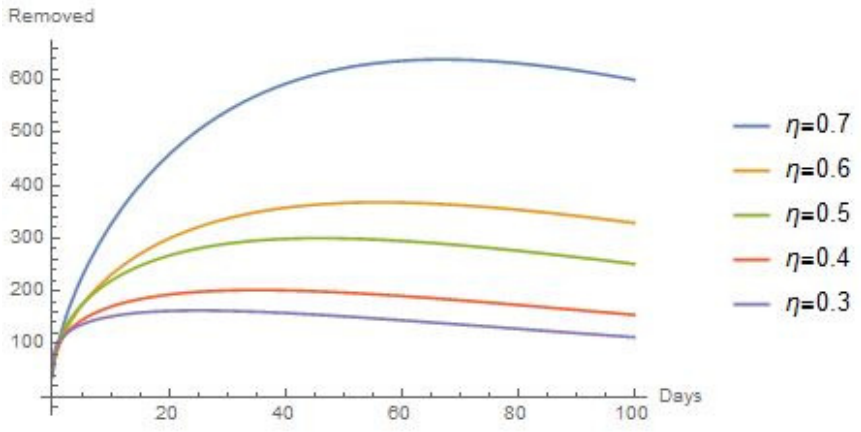


Fig. 9 2D simulation of R(t) for $\varsigma = 0.1, \omega = 0.0001, \chi = 0.005, \delta = 0.0001, \rho = 0.005$ in equation (41).

6 Physical interpretation

Figures (2) and (6) show that on decreasing the order of fractional derivative the susceptible individuals highly decrease. Figures (3) and (7) show that on increasing the order of fractional derivative (close to 1) the infected individuals decrease well but the infection cannot be completely eradicated since there is no available vaccine and this is also because the proposed model shows that the removed group of Zombies can also restore to life. Figures (4) and (8) show that on increasing the order of fractional derivative (close to 1) the Zombies decrease but the Zombie group cannot be completely eradicated since there is no available vaccine and this is also because the proposed model shows that the removed group of Zombies can also restore to life. Figures (5) and (9) present that on increasing the order of fractional derivative (close to 1) the removed individuals keep on increasing, since the only way to get out of the infection is to destroy the Zombies.

To understand the physical interpretation, we have seen that for $\varsigma = 0.01$ the infection is very high and for $\varsigma = 0.1$, the infection rate sharply declines. This shows the effect of time dependency on the infection rate

β_0. Furthermore, we concluded that the Zombie's infection of humans is highly disastrous. Although strong and rigid quarantine can help extirpate the infection. A cure with proper vaccination can also be a way of eliminating the infection. In the future, we can also consider the SIZR model for Zombie infection with parameters involving quarantine and vaccination. Lastly, as observed in movies, we conclude that Zombie infection can open to the destruction and collapse of human development and it is of paramount importance to deal with Zombies as early as possible, otherwise civilization will be exposed to threatening and destructive circumstances.

7 Conclusion and future directions

In this work, we have considered the SIZR model applied to the Zombie infection with a time-dependent infection rate. The SIZR model is then extended to the fractional order using Caputo and Caputo-Fabrizio fractional derivative operators. We briefed the numerical solutions of fractional SIZR model in the sense of each fractional derivative operator. Graphical representations provide us with a better understanding of the fractional SIZR model. This SIZR model is much different from the other infectious disease SIR and SEIR models as in this SIZR model, the removed Zombies can again return to life. This is an unexpected plot which is not practical in real life but with this SIZR model we have tried to illustrate the significance of mathematical modeling in different circumstances and challenges in the field of medical science. Figures (2), (3), (4) and (5) represent the behaviour of the susceptible, infected, Zombie and removed individuals for different fractional ordered derivative η in the sense of Caputo fractional derivative operator. Figures (6), (7), (8) and (9) represent the behavior of the susceptible, infected, Zombie and removed individuals for different fractional ordered derivative η in the sense of the Caputo-Fabrizio fractional derivative operator. Moreover, we have determined that the harm inflicted upon humans by Zombies is exceedingly calamitous. While implementing a robust and stringent isolation protocol can aid in eradicating the contagion, a remedy through appropriate immunization also holds promise for obliterating the infection. Additionally, in forthcoming scenarios, we can contemplate employing the SIZR model to analyze the parameters associated with quarantine and vaccination for Zombie afflictions. Lastly, drawing from cinematic depictions, we assert that the contagion of Zombies can lead to the annihilation and downfall of human progress, necessitating urgent action against the undead. Failure to do so will ensnare civilization in ceaseless turmoil and pernicious circumstances.

8 Declarations

8.1 Conflict of interest:

According to researchers, there are no conflicts of interest to report regarding the article that is being presented.

8.2 Funding:

Authors have no founding source.

8.3 Authors Contribution:

H.J.-Conceptualization, Methodology, Software. P.G.-Formal Analysis. S.S.-Validation. R.S.D.-Writing-Original Draft. A.C.-Writing-Review Editing. All authors read and approved the final submitted version of this

manuscript.

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8.5 Data availability statement:

All data that support the findings of this study are included within the article (and any supplementary files).

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