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On related fixed points in two d-complete topological spaces [1]

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Abstract

In this paper, using the method of Hicks, we prove some new related fixed point theorems for pair of mappings on two d-complete topological spaces under the suitable conditions. We also derive some related fixed point theorems from our main results. Finally, by using w-continuity, we establish a related fixed point theorem in d-complete Hausdorff topological spaces.

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1 Introduction

In 1981, Fisher proved a fixed point theorem for compositions of two mappings on two complete metric spaces and also gave the relation of between the fixed points for these two mappings.

Let (V, d) and (Y, ϱ) be complete metric spaces. Fisher proved in [7] that if P is a mapping of V into Y and R is a mapping of Y into V satisfying the inequalities

$$\varrho(Pv, PRy) \leq r \max\{d(v, Ry), \varrho(y, Pv), \varrho(y, PRy)\}$$
$$d(Ry, RPv) \leq r \max\{\varrho(y, Pv), d(v, Ry), d(v, RPv)\}$$

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for all v in V and y in Y, where $0 \leq r < 1$, then RP has a unique fixed point z in V and PR has a unique fixed point w in Y. Further, $Pz = w$ and $Rw = z$.

Fisher's result is the first related fixed point theorem for a pair of mappings on two metric spaces. Since then several authors have proved various extensions and generalizations of this theorem in many different directions. For example [1, 3, 8, 9, 10, 20, 23, 24, 28, 29] and others.

In 1992 Hicks [12] proved several generalizations of Banach's fixed point theorem in a large class of non-metric space so-called d-complete topological space. Since then in [2, 4, 6, 11, 13, 14, 15, 17, 21, 22, 25, 26, 27] have proved several fixed point theorems in this space.

In this paper, using the method of Hicks [12], we prove some related fixed point theorems in two d-complete topological spaces under the suitable conditions. We also derive some related fixed point theorems from our main results. Finally, by using w-continuity, we establish a related fixed point theorem in d-complete Hausdorff topological spaces.

We now recall the definition of d-complete topological space.

Definition 1 (*See [12]*) *Let (V, τ) be a topological space and $d : V \times V \to [0, \infty)$ such that $d(v, y) = 0$ if and only if $v = y$. Then V is said to be d-complete if $\sum_{k=1}^{\infty} d(v_k, v_{k+1}) < \infty$ implies that the sequence $\{v_k\}$ is convergent in (V, τ).*

For details on d-complete topological spaces, we refer to Iseki [16] and Kasahara [18, 19].

Complete metric spaces and complete quasi-metric spaces are examples of d-complete topological spaces. The d-complete semi-metric spaces form an important class of examples of d-complete topological spaces (see [12]).

Example 1 ([*17*]) *Let $V = \mathbb{R}$, ϱ be a metric on V and let τ be a metric topology on V induced by ϱ. Define $d : V \times V \to [0, \infty)$ by $d(v, y) = |e^{\mathrm{sgn}(v-y)} - 1|$, where* sgn *is a function defined by* $\mathrm{sgn}(v) = \frac{v}{|v|}$ *if $v \neq 0$ and* $\mathrm{sgn}(0) = 0$. *Since d does not satisfy the symmetry property, d is neither a metric nor a semi-metric on V. Also (V, τ) is a d-complete topological space.*

2 Main Results

Let V and Y be nonempty sets, let P be a mapping of V into Y and let R be a mapping of Y into V.

Suppose v_0 be an arbitrary point in V. Define sequences $\{v_n\}$ in V and $\{y_n\}$ in Y by

$$v_n = (RP)^n v_0, \quad y_n = P(RP)^{n-1} v_0$$

for $n = 1, 2, ...$ In this paper, we use these sequences in our theorems.

Define the following sets

$$\Re_V(v_0) = \{(RP)^n v_0 : n = 1, 2, ...\}, \quad \Re_Y(v_0) = \{P(RP)^{n-1} v_0 : n = 1, 2, ...\}.$$

Definition 2 (*See*[5]) *Let* (V,τ) *and* (Y,τ') *be topological spaces. A real-valued function* $\Gamma : V \times Y \to [0,\infty)$ *is said to be weak lower semi-continuous* (*w.l.s.c.*) *at* $(v,y) \in V \times Y$ *iff* $\{v_n\}$ *and* $\{y_n\}$ *are sequences in* V *and* Y *respectively and*

$$\lim_{n\to\infty} v_n = v, \lim_{n\to\infty} y_n = y \quad \textit{implies} \quad \Gamma(v,y) \leq \lim_{n\to\infty} \sup \Gamma(v_n, y_n).$$

We now prove the following related fixed point theorem in two d and ϱ-complete topological spaces.

Theorem 1 *Let* (V,τ) *be a d-complete topological space and* (Y,τ') *be a* ϱ*-complete topological spaces, let* P *be a mapping of* V *into* Y *and let* R *be a mapping of* Y *into* V*. Suppose there exists* $v_0 \in V$ *such that*

$$\max\{d(v,RPv), \varrho(y,PRy)\} \leq \gamma(v) - \gamma(RPx) + \varphi(y) - \varphi(PRy) \tag{1}$$

for all v *in* $\Re_V(v_0)$ *and* y *in* $\Re_Y(v_0)$*, where* $\gamma : V \to [0,\infty)$ *and* $\varphi : Y \to [0,\infty)$*. Then*

(*a*) $\lim_{n\to\infty} v_n = \lim_{n\to\infty} (RP)^n v_0 = z$ *and*
$\lim_{n\to\infty} y_n = \lim_{n\to\infty} P(RP)^{n-1} v_0 = w$ *exist.*

(*b*) $Rw = z$ *and* $Pz = w$ *if and only if* $\Gamma : V \times Y \longrightarrow [0,\infty)$, $\Gamma(v,y) = d(v,Ry)$ *and* $\Upsilon : V \times Y \longrightarrow [0,\infty)$, $\Upsilon(v,y) = \varrho(y,Pv)$ *are w.l.s.c. at* (z,w).

Further, if (*b*) *holds, then* $RPz = z$ *and* $PRw = w$.

Proof. (a) Using inequality (1) we have

$$\begin{aligned}
S_n &= \sum_{k=1}^{n} \max\{d(v_k, v_{k+1}), \varrho(y_k, y_{k+1})\} \\
&= \sum_{k=1}^{n} \max\{d(v_k, RPv_k), \varrho(y_k, PRy_k)\} \\
&\leq \sum_{k=1}^{n} [\gamma(v_k) - \gamma(RPv_k) + \varphi(y_k) - \varphi(PRy_k)] \\
&= \sum_{k=1}^{n} [\gamma(v_k) - \gamma(v_{k+1}) + \varphi(y_k) - \varphi(y_{k+1})] \\
&= \gamma(v_1) - \gamma(v_{n+1}) + \varphi(y_1) - \varphi(y_{n+1}) \leq \gamma(v_1) + \varphi(y_1).
\end{aligned}$$

Therefore $\{S_n\}$ is bounded above and also non-decreasing and so convergent. Thus, we have

$$\sum_{k=1}^{\infty} \max\{d(v_k, v_{k+1}), \varrho(y_k, y_{k+1})\} < \infty$$

and so

$$\sum_{k=1}^{\infty} d(v_k, v_{k+1}) < \infty \quad \text{and} \quad \sum_{k=1}^{\infty} \varrho(y_k, y_{k+1}) < \infty. \tag{2}$$

Since (V, τ) is d-complete and (Y, τ') is ϱ-complete, the sequence $\{v_n\}$ has a limit z in V and the sequence $\{y_n\}$ has a limit w in Y.

(b) Assume that $Rw = z$, $Pz = w$ and $\{v_n\}$ and $\{y_n\}$ are sequences in V and Y respectively with $v_n \to z$, $y_n \to w$. Then we have,

$$\Gamma(z, w) = d(z, Rw) = 0 \leq \limsup d(v_n, Ry_n) = \limsup \Gamma(v_n, y_n)$$

and

$$\Upsilon(z, w) = \varrho(w, Pz) = 0 \leq \limsup \varrho(y_n, Pv_n) = \limsup \Upsilon(v_n, y_n)$$

and so Γ and Υ are w.l.s.c. at (z, w).

Now let $v_n = (RP)^n v_0, \quad y_n = P(RP)^{n-1} v_0$ and Γ, Υ are w.l.s.c. at (z, w). It follows from (2) that $\lim_{n\to\infty} d(v_n, v_{n+1}) = 0$ and $\lim_{n\to\infty} \varrho(y_n, y_{n+1}) = 0$. Then from (a) we have

$$\begin{aligned} 0 \leq d(z, Rw) = \Gamma(z, w) &\leq \limsup \Gamma(v_n, y_n) = \limsup d(v_n, Ry_n) \\ &= \limsup d((RP)^n v_0, RP(RP)^{n-1} v_0) = 0 \end{aligned}$$

and

$$\begin{aligned} 0 \leq \varrho(w, Pz) = \Upsilon(z, w) &\leq \limsup \Upsilon(v_n, y_n) = \limsup \varrho(y_n, Pv_n) \\ &= \limsup \varrho(P(RP)^{n-1} v_0, P(RP)^n v_0) \\ &= \limsup \varrho(y_n, y_{n+1}) = 0. \end{aligned}$$

Thus $Rw = z$, $Pz = w$.

We now assume that (b) holds. Then we have

$$RPz = Rw = z, \; PRw = Pz = w.$$

Remark 1 *Since*

$$\max\{d(v, RPv), \varrho(y, PRy)\} \leq d(v, RPv) + \varrho(y, PRy),$$

Theorem 1 holds for the following inequality.

$$d(v, RPv) + \varrho(y, PRy) \leq \gamma(v) - \gamma(RPv) + \varphi(y) - \varphi(PRy). \tag{3}$$

If we put $(V, \tau) = (Y, \tau')$ and $d = \varrho$, then from Theorem 1 we have the following result.

Corollary 1 *Let (V,τ) be a d-complete topological space and let R, P be mappings of V into itself. Suppose there exists $v_0 \in V$ such that*

$$\max\{d(v,RPv), d(y,PRy)\} \leq \gamma(v) - \gamma(RPv) + \varphi(y) - \varphi(PRy)$$

for all v, y in $O_{RP}(v_0,\infty)$, where $\gamma : V \to [0,\infty)$, $\varphi : V \to [0,\infty)$ and

$$O_{RP}(v_0,\infty) = \{Pv_0, RPv_0, ..., P(RP)^{n-1}v_0, (RP)^n v_0, ...\}.$$

Then

(a) $\lim_{n\to\infty} v_n = \lim_{n\to\infty}(RP)^n v_0 = z$ *and*
$\lim_{n\to\infty} y_n = \lim_{n\to\infty} P(RP)^{n-1}v_0 = w$ *exist.*

(b) $Rw = z$ *and* $Pz = w$ *if and only if* $\Gamma : V \times V \longrightarrow [0,\infty)$, $\Gamma(v,y) = d(v,Ry)$ *and* $\Upsilon : V \times V \longrightarrow [0,\infty)$, $\Upsilon(v,y) = d(y,Pv)$ *are w.l.s.c. at* (z,w).

Further, if (b) holds, then $RPz = z$ and $PRw = w$. Also if $z = w$, z is a common fixed point of R and P.

Note that the similar result can also be obtained for inequality (3).

Now let $(V,\tau) = (Y,\tau')$, $d = \varrho$ and $R = P$. Then the set

$$\Re_V(v_0) = \{P^n v_0 : n = 1, 2, ...\}$$

is equal to $O_P(v_0,\infty)$, which is called the orbit of v_0.

If we put $(V,\tau) = (Y,\tau')$, $d = \varrho$, $R = I$ and $\gamma = \varphi$ where I is a identity map of V, then from inequality (3) we have the following Hicks's result [12].

Corollary 2 *Let (V,τ) be a d-complete topological space and let P be a self-mapping of V. Suppose there exists $v_0 \in V$ such that*

$$d(v,Px) \leq \gamma(v) - \gamma(Pv)$$

for all $v \in O_P(v_0,\infty)$, where $\gamma : V \to [0,\infty)$. Then,

(a) $\lim_{n\to\infty} v_n = \lim_{n\to\infty} P^n v_0 = z$ *exists.*

(b) $Pz = z$ *if and only if* $\Gamma : V \longrightarrow [0,\infty)$, $\Gamma(v) = d(v,Pv)$ *is w.l.s.c. at* z.

3 Some Results

In this section, we derive some related fixed point results in two metric spaces by considering the Theorem 1 in previous section.

Theorem 2 *Let (V,τ) be a d-complete topological space and (Y,τ') be a ϱ-complete topological spaces, let P be a mapping of V into Y and let R be a mapping of Y into V. Suppose there exists $v_0 \in V$ such that*

$$d(RPv,(RP)^2v)+\varrho(PRy,(PR)^2y)\leq h[d(v,RPv)+\varrho(y,PRy)] \tag{4}$$

for all v in $\Re_V(v_0)$ and y in $\Re_Y(v_0)$, where $0\leq h<1$. Then

(a) $\lim_{n\to\infty} v_n=\lim_{n\to\infty}(RP)^n v_0=z$ *and*
$\lim_{n\to\infty} y_n=\lim_{n\to\infty} P(RP)^{n-1}v_0=w$ *exist.*

(b) $Rw=z$ *and* $Pz=w$ *if and only if* $\Gamma: V\times Y\longrightarrow[0,\infty)$, $\Gamma(v,y)=d(v,Ry)$ *and* $\Upsilon: V\times Y\longrightarrow[0,\infty)$, $\Upsilon(v,y)=\varrho(y,Pv)$ *are w.l.s.c. at* (z,w).

Further, if (b) holds, then z is a fixed point of RP and w is a fixed point of PR.

Proof. Define, $\gamma(v)=\frac{1}{1-h}d(v,RPv)$, for all $v\in V$ and $\varphi(y)=\frac{1}{1-h}\varrho(y,PRy)$, for all $y\in Y$. Then from inequality (4) we have

$$\begin{aligned} &d(v,RPv)+\varrho(y,PRy)-h[d(v,RPv)+\varrho(y,PRy)]\\ &\qquad\leq d(v,RPv)+\varrho(y,PRy)-[d(RPv,(RP)^2v)+\varrho(PRy,(PR)^2y)]. \end{aligned}$$

Thus, we get

$$\begin{aligned} d(v,RPv)+\varrho(y,PRy)\leq{}& \frac{1}{1-h}[d(v,RPv)+\varrho(y,PRy)]\\ &-\frac{1}{1-h}[d(RPv,(RP)^2v)+\varrho(PRy,(PR)^2y)] \end{aligned}$$

and so

$$d(v,RPv)+\varrho(y,PRy)\leq\gamma(v)-\gamma(RPv)+\varphi(y)-\varphi(PRy).$$

The results follow immediately from inequality (3) and Theorem 1.

Corollary 3 *Let (V,τ) be a d-complete topological space and (Y,τ') be a ϱ-complete topological spaces, let P be a mapping of V into Y and let R be a mapping of Y into V. Suppose there exists $v_0 \in V$ such that*

$$\max\{d(RPv,(RP)^2v),\varrho(PRy,(PR)^2y)\}\leq(h/2)[d(v,RPv)+\varrho(y,PRy)]$$

for all v in $\Re_V(v_0)$ and y in $\Re_Y(v_0)$, where $0\leq h<1$. Then

(a) $\lim_{n\to\infty} v_n=\lim_{n\to\infty}(RP)^n v_0=z$ *and*
$\lim_{n\to\infty} y_n=\lim_{n\to\infty} P(RP)^{n-1}v_0=w$ *exist.*

(b) $Rw=z$ *and* $Pz=w$ *if and only if* $\Gamma: V\times Y\longrightarrow[0,\infty)$, $\Gamma(v,y)=d(v,Ry)$ *and* $\Upsilon: V\times Y\longrightarrow[0,\infty)$, $\Upsilon(v,y)=\varrho(y,Pv)$ *are w.l.s.c. at* (z,w).

Further, if (b) holds, then z is a fixed point of RP and w is a fixed point of PR.

Proof. We have

$$\begin{aligned} d(RPv,(RP)^2v)+\varrho(PRy,(PR)^2y) &\\ &\le 2\max\{d(RPv,(RP)^2v),\varrho(PRy,(PR)^2y)\}\\ &\le 2(h/2)[d(v,RPv)+\varrho(y,PRy)]\\ &= h[d(v,RPv)+\varrho(y,PRy)]. \end{aligned}$$

The results follow immediately from Theorem 2.

Theorem 3 *Let (V,τ) be a d-complete topological space and (Y,τ') be a ϱ-complete topological spaces, let P be a mapping of V into Y and let R be a mapping of Y into V. Suppose there exists $v_0\in V$ such that*

$$d(RPv,RPv')+\varrho(PRy,PRy')\le h[d(v,v')+\varrho(y,y')] \tag{5}$$

for all v,v' in $\Re_V(v_0)$ and y,y' in $\Re_Y(v_0)$, where $0\le h<1$. Then

(a) $\lim_{n\to\infty}v_n=\lim_{n\to\infty}(RP)^nv_0=z$ *and* $\lim_{n\to\infty}y_n=\lim_{n\to\infty}P(RP)^{n-1}v_0=w$ *exist.*

(b) *$Rw=z$ and $Pz=w$ if and only if $\Gamma:V\times Y\longrightarrow[0,\infty)$, $\Gamma(v,y)=d(v,Ry)$ and $\Upsilon:V\times Y\longrightarrow[0,\infty)$, $\Upsilon(v,y)=\varrho(y,Pv)$ are w.l.s.c. at (z,w).*

Further, if (b) holds, then z is a unique fixed point of RP and w is a unique fixed point of PR.

Proof. Let $v'=RPv$ and $y'=PRy$ in inequality (5). Then we obtain inequality (4). The results follow immediately from Theorem 2.

Now suppose that RP has a second fixed point z'. Then using inequality (5), for $y=y'$, we have

$$d(z,z')=d(RPz,RPz')+\varrho(PRy,PRy)\le h[d(z,z')+\varrho(y,y)]=hd(z,z'),$$

proving that $z=z'$, since $h<1$. Similarly we can prove that w is the unique fixed point of PR.

Corollary 4 *Let (V,τ) be a d-complete topological space and (Y,τ') be a ϱ-complete topological spaces, let P be a mapping of V into Y and let R be a mapping of Y into V. Suppose there exists $v_0\in V$ such that*

$$\max\{d(RPv,RPv'),\varrho(PRy,PRy')\}\le(h/2)[d(v,v')+\varrho(y,y')]$$

for all v,v' in $\Re_V(v_0)$ and y,y' in $\Re_Y(v_0)$, where $0\le h<1$. Then

(a) $\lim_{n\to\infty}v_n=\lim_{n\to\infty}(RP)^nv_0=z$ *and* $\lim_{n\to\infty}y_n=\lim_{n\to\infty}P(RP)^{n-1}v_0=w$ *exist.*

(b) *$Rw = z$ and $Pz = w$ if and only if $\Gamma : V \times Y \longrightarrow [0, \infty)$, $\Gamma(v, y) = d(v, Ry)$ and $\Upsilon : V \times Y \longrightarrow [0, \infty)$, $\Upsilon(v, y) = \varrho(y, Pv)$ are w.l.s.c. at (z, w).*

Further, if (b) holds, then z is a unique fixed point of RP and w is a unique fixed point of PR.

Proof. We have

$$\begin{aligned} d(RPv, RPv') + \varrho(PRy, PRy') &\leq 2\max\{d(RPv, RPv'), \varrho(PRy, PRy')\} \\ &\leq 2(h/2)[d(v, v') + \varrho(y, y')] = h[d(v, v') + \varrho(y, y')]. \end{aligned}$$

The results follow immediately from Theorem 3.

4 Further Result

In this section, by using the w-continuity, we prove a related fixed point theorem in two d and ϱ-complete Hausdorff topological spaces.

Before establishing our result, we need the following definition.

Definition 3 *Let (V, τ) and (Y, τ') be topological spaces. A function $P : V \to Y$ is w-continuous at v if $v_n \to v$ implies $Pv_n \to Pv$ as $n \to \infty$.*

Theorem 4 *Let (V, τ) be a d-complete Hausdorff topological space and (Y, τ') be a ϱ-complete Hausdorff topological spaces, let P be a mapping of V into Y and let R be a mapping of Y into V satisfying the inequalities*

$$d(RPv, (RP)^2 v) \leq \varphi(d(v, RPv)), \tag{6}$$

$$\varrho(PRy, (PR)^2 y) \leq \varphi(\varrho(y, PRy)) \tag{7}$$

for all v in V and y in Y, where $\varphi : [0, \infty) \to [0, \infty)$ is a nondecreasing function with $\varphi(0) = 0$.

Suppose that R and P are w-continuous. Then there exist z in V and w in Y such that $Rw = z$ and $Pz = w$ if and only if there exists $v_0 \in V$ with

$$\sum_{n=1}^{\infty} \varphi^n(d(v_0, RPv_0)) < \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \varphi^n(\varrho(Pv_0, PRPv_0)) < \infty.$$

Further, z is a fixed point of RP and w is a fixed point of PR.

Proof. If $Rw = z$ and $Pz = w$, then $d(z, RPz) = \varrho(Pz, PRPz) = 0$. Since $\varphi^n(0) = 0$,

$$\sum_{n=1}^{\infty} \varphi^n(d(z, RPz)) < \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \varphi^n(\varrho(Pz, PRPz)) < \infty$$

hold.

Conversely, suppose that there exists an v_0 in V with

$$\sum_{n=1}^{\infty} \varphi^n(d(v_0, RPv_0)) < \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \varphi^n(\varrho(Pv_0, PRPv_0)) < \infty$$

We now consider the sequences $\{v_n\}$ in V and $\{y_n\}$ in Y defined by

$$v_n = (RP)^n v_0, \quad y_n = P(RP)^{n-1} v_0$$

for $n = 1, 2, ...$

Using inequality (6) we have

$$\begin{aligned} d(v_n, v_{n+1}) &= d(RPv_{n-1}, (RP)^2 v_{n-1}) \\ &\leq \varphi(d(v_{n-1}, RPv_{n-1}) = \varphi(d(v_{n-1}, v_n)) \end{aligned}$$

and in general,

$$d(v_n, v_{n+1}) \leq \varphi^n(d(v_0, v_1)) = \varphi^n(d(v_0, RPv_0)),$$

since φ is nondecreasing. It follows from assumption that

$$\sum_{n=1}^{\infty} d(v_n, v_{n+1}) \leq \sum_{n=1}^{\infty} \varphi^n(d(v_0, RPv_0)) < \infty.$$

Similarly using inequality (7) we have

$$\begin{aligned} \varrho(y_n, y_{n+1}) &= \varrho(PRy_{n-1}, (PR)^2 y_{n-1}) \\ &\leq \varphi(\varrho(y_{n-1}, PRy_{n-1}) = \varphi(\varrho(y_{n-1}, y_n)) \end{aligned}$$

and

$$\varrho(y_{n+1}, y_{n+2}) \leq \varphi^n(\varrho(y_1, y_2)) = \varphi^n(\varrho(Pv_0, PRPv_0))$$

for $n = 1, 2, ...$ It follows from assumption that

$$\begin{aligned} \sum_{n=1}^{\infty} \varrho(y_n, y_{n+1}) &= \varrho(y_1, y_2) + \sum_{n=2}^{\infty} \varrho(y_n, y_{n+1}) \\ &\leq \varrho(y_1, y_2) + \sum_{n=1}^{\infty} \varphi^n(\varrho(Px_0, PRPv_0)) < \infty. \end{aligned}$$

Since (V, τ) is d-complete and (Y, τ') is ϱ-complete, $\lim_{n\to\infty} v_n = z$ in V and $\lim_{n\to\infty} y_n = w$ in Y exist. Since R and P are w-continuous and (V, τ) and (Y, τ') are Hausdorff we have

$$z = \lim_{n\to\infty} x_n = \lim_{n\to\infty} (RP)^n v_0 = R(\lim_{n\to\infty} P(RP)^{n-1} v_0) = R(\lim_{n\to\infty} y_n) = Rw$$

and

$$w = \lim_{n\to\infty} y_n = \lim_{n\to\infty} P(RP)^{n-1}v_0$$
$$= P(\lim_{n\to\infty} (RP)^{n-1}v_0) = P(\lim_{n\to\infty} v_{n-1}) = Pz.$$

Thus

$$RPz = Rw = z,\ PRw = Pz = w$$

and so z is a fixed point of RP and w is a fixed point of PR.

5 Conclusions

We, here, proved some new related fixed point theorems for pair of mappings in two d-complete topological spaces under the suitable conditions. Thus we obtained Hicks's result [12] which is a spacial case of our result. We also derived some related fixed point theorems from our main results. Finally, by using w-continuity, we established a fixed point theorem in d-complete Hausdorff topological spaces.

Note that as the mapings are satisfied a system of at least two contractive conditions in the results of Fisher and others, the mapings are satisfied only one contractive condition in our main results.

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