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Economically optimised target state of uneven-aged forest management for main forest types in Slovakia

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Abstract
The study presented a bioeconomic modelling approach for an uneven-aged mixed forest management planning. Regression models for transition (increment), and ingrowth (regeneration) used the National Forest Inventory (NFI) of Slovakia and regional inventory data. Mortality was based on salvage logging records. Models were specific for five tree species within three forest types (FT) (oak with hornbeam and beech, beech, mixed fir-beech-spruce). Net timber prices were calculated with regard to stem quality. Tree growth depended on crown characteristics. The regression models were adjusted to three main geobiotope (GBT) sites per FT. Forest growth was simulated with the density-dependent stand-level matrix transition model. Financial optimisation of harvest was sensitive to an interest rate. Long-time optimisation stabilised in a steady state equilibrium characterised by a stable diameter distribution. Target diameters were specific for site and tree species, and were highest for fir, a dense crown, a good stem quality, and a lower interest rate. Standing timber volume varied from 150 m^3 ha^{-1} (oak forests, 2% interest rate) to 400 m^3 ha^{-1} (beech and fir-beech-spruce forests, 0.5% interest rate). Harvested volume varied from 38 to 93 m^3 ha^{-1} per 10 years, stand basal area (ba) varied from 19 to 36 m^2 ha^{-1} depending on the site, timber price, and interest rate. The discussion pointed out that the relative low volume of the oak FT resulted from the light-demanding characteristics of oak. The mean of oak mosaic structures was lower compared to the high level of more storeys present in the single tree selection structures in beech and mixed fir-beech-spruce forests.

Key words: simultaneous non-linear optimisation; subplex optimisation algorithm; continuous cover forestry; close-to-nature forest management; single tree selection management

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1. Introduction

The Slovak Act on forests (326/2005 as amended 6/2022) aims at promoting close-to-nature forest management by financial support (§55, 1s), while it restricts clear-felling to light-demanding species; plantations, and forest reconstructions (§18, 2). Such a goal to extend close-to-nature forest management demands a more precise definition how an ideal forest should look like.

The most ambitious form of close-to-nature forest management is small-scaled especially when applied by single tree or group wise selection felling (Bauhus et al. 2013), also called plenter management (Schütz 2006) and associated with uneven-aged and partly mixed forest management as promoted by Pro Silva Slovakia (Saniga & Szanyi 1998; Saniga & Bruchánik 2009). Selection management is challenging due to a complex inventory and planning (Gurnaud 1878; Biolley 1887; Dvorak 2000; Favre & Oberson 2002). Nevertheless, forests managed according to these principles for a long time such as the example of Smolnícka osada (Kulla et al. 2017, 2018; Roessiger et al. 2018) are rare. Currently, there is only a little experience about such a management at a large scale as forests with a selection vertical structure cover only 0.9% of Slovak forest area (Šebeň 2017).

Current forest management planning in Slovakia is based on yield tables (Halaj et al. 1987) using data from even-aged and pure forests that are designed to remain such structures until a final clearcut. Such yield tables are inflexible to react to stand density (Halaj et al. 1987) except for rare cases of tables reflecting the reduction of stand increment due to low stocking (Assmann & Franz 1963) while the density suppresses the tree growth in uneven-aged forests (Buongiorno & Michie 1980; Peng

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2000). Stand productivity increased due to the climate change, e.g. by 5–77% for beech compared to yield tables (Pretzsch et al. 2014), suggesting that older yield tables currently recommend too low amount of thinning. Hence, the use of even-aged yield tables counteracts the trend towards uneven-aged forests.

Therewith, uneven-aged management planning requires specific growth and yield models comparable to yield tables for uneven-aged forests (Peng 2000). A challenge is the typically missing information about the age and the site quality index (SI) (height dependent on age) in uneven-aged forests (Peng 2000).

The density-dependent matrix transition model introduced by Buongiorno & Michie (1980) is an alternative planning tool based on diameter classes rather than age classes. The matrix model considers increment (transition probability), ingrowth frequency (regeneration), and mortality probability. Simulations may result in an equilibrium (Peng 2000) defined e.g. as "trees moving up from below equal outgrowth plus removal" (Schütz 2006). Further, as simulations can start from varying initial stand states, the matrix tool is designed to model an uneven-aged forest development.

Moreover, it is possible to couple the matrix model with an optimisation (Rämö & Tahvonen 2014; Tahvonen & Rämö 2016) of optimal harvest per diameter class, tree species, and time period; and therewith allowing a detailed planning. Further, subclasses can be implemented within a tree species, considering various stem qualities (adapting timber prices), and various crown properties (impact on increment) (Roessiger et al. 2016, 2018). This makes modelling ecologically and economically more realistic by diversifying harvest diameters per species.

An optimisation objective can maximise or minimise ecological indicators, ecosystem services, social, or economic aspects (Ficko et al. 2016, 2018), but predominantly financial indicators such as the net present value are in use. An adequate optimisation objective is better than a predefined stand management scheme as applied in yield tables as it allows more and new options and considers the above-mentioned increased productivity (Pretzsch et al. 2014).

The matrix model is driven by ecological regression functions dependent on actual measured (plot) inventory data. The required data base needs two repeated measurements at a single tree level, including the identification, state, and diameter of a tree. Alternatively, to shorten the time until inventory data is available, measurements on bore ring analyses can also be used (Kulla et al. 2017, 2018; Roessiger et al. 2018; Bošeľa et al. 2019).

Matrix models are only valid for a single forest stand of the specific region, or the strata represented by the inventory data (Ficko et al. 2016, 2018; Roessiger et al. 2016, 2018, 2019, 2022). That is a shortcoming compared to more general yield tables which can consider the site with a SI based on height (Halaj et al. 1987). Up to now only few cases of matrix models are sensitive to the SI: (1) The countrywide matrix model for Norway of Bollandsås et al. (2008) considering the SI individually for Scots pine (*Pinus sylvestris* L.), Norway spruce (*Picea abies* [L.] Karst.), and birch (*Betula pubescens* Ehrh., *B. pendula* Roth.). (2) The model considering uneven-sized diameter distributions by Pukkala et al. (2011, 2013) for two different site types in Finland. These models were used by Rämö & Tahvonen (2014) and by Tahvonen & Rämö (2016) for the optimisation. (3) The SOUTHPRO matrix model by Lin et al. (1998) uses the productivity rating related to the stand volume growth to consider the site for loblolly pine (*Pinus taeda* L.), soft and hard hardwood species in the southern United States.

Inventory plot data of the NFI of Slovakia (Šebeň 2017) are not related to SI but to GBT. The concept of aggregated GBT site units is based on the assumption that each site variant represents a potential forest biotope with a combined category of an altitudinal vegetation zone (Zlatník 1976), an edaphic-trophic gradient (nutrients), and an edaphic-hydric (water) gradient (Kulla & Bošeľa 2013, 2014). Kulla & Bošeľa (2014) indicated a typical range of site indices and a typical potential (natural) regeneration species composition derived from theoretical findings and data from reserves for each GBT. Therefore, the goal of this study is to overcome limiting of the model to a specific region and to develop a more general forest management planning tool that can be applied at a wider scale of FTs, tree species, and GBT forest sites in Slovakia to enhance the application of close-to-nature management. For this purpose, a matrix modelling approach based on Roessiger et al. (2016, 2018, 2019, 2022) was applied.

Research question: Optimal uneven-aged forest management is specific for geobiotope sites differing in tree species composition, diameter distribution, standing and harvested volume.

2. Material and methods

2.1. National Forest Inventory and salvage harvest records of Slovakia related to the concept of geobiotope

Data from NFI Slovakia (Šebeň 2017) covers two complete inventory cycles performed between the years 2005/2006 and 2015/2016. As NFI contained few data about forests with fir, the data was complemented with the regional inventory from the typical selection forest of Smolnícka osada (Kulla et al. 2018; Roessiger et al. 2018). From this data set, single plots were selected that were representative for specific FTs (Table 1).

From complete inventory data only multi-layered forests were selected, representing uneven-aged forest management with typical mixing proportions of tree species

Table 1. Modelled tree species (MTS), selection criteria, and number of selected inventory plots per forest type (FT), based on data from National Forest Inventory (NFI) and Smolnícka osada.

Forest type (FT)	OA Oak FT	BE Beech FT	FI Fir-(Fir-Beech-Spruce mixed) FT
Modelled tree species (MTS)	Oak OA *Quercus* (*robur* L., *petraea* [Mattuschka] Liebl., *cerris* L.), Beech BE *Fagus sylvatica* L., Hornbeam HB *Carpinus betulus* L. (missing HB data estimated from BE)	Beech BE *Fagus sylvatica* L.	Beech BE *Fagus sylvatica* L., Fir FI *Abies alba* Mill., Spruce SP *Picea abies* [L.] Karst.
Number of plots (NFI)	115	174	87
Number of plots (Smolník)		2	21
Criteria to select plots (NFI)	OA (*robur* L. *pedunculate* + *petraea* [Mattuschka] Liebl. sessile) > 20%; MTS > 80%	BE > 80%	Mixture of at least 2 MTS (BE > 5%; SP > 5%, FI > 1%); MTS > 80% (exception if FI > 0% then MTS > 60%)
Criteria to select plots (Smolník)			Mixture of at least 2 MTS (BE > 5%; SP > 5%, FI > 1%); MTS > 80%; SP < 10%; FI >30%

Table 2. Numbering and character of aggregated geobiotope (GBT) site units per forest type (FT).

Oak OA Forest Type			Beech BE Forest Type			Fir-Beech-Spruce FI Forest Type		
Character	Model	Label	Character	Model	Label	Character	Model	Label
Mean	FT	10	Mean	FT	30	Mean	FT	50
Base	GBT	23	Base	GBT	34	Base	GBT	54
Dry	GBT	22	Acidic	GBT	33	Acidic	GBT	53
Wet	GBT	14	Nutrient rich	GBT	35	Nutrient rich	GBT	55

(Table 1). Relevant selection criteria were the presence of the main tree species, a near natural tree species mixture typical for the FT, and a high degree of naturalness at the inventory plot.

Inventory tree data was used to determine actual increments of single trees and ingrowth frequencies per FT to calibrate the matrix model at various GBT site levels (Kulla & Bošeľa 2014) Table 2. Mortality data results from a rich dataset about salvage harvest records related to GBT (Information System Forest Management 2022).

2.2. Economically and ecologically realistic matrix modelling

To implement GBTs (Table 2), the density dependent matrix transition model by Kulla et al. (2018) and Roessiger et al. (2018, 2019) was applied to actual data and study requirements. It considered subclasses of timber quality and crown characteristics per tree species and 23 dbh (diameter at breast height) classes with means from 10 to 98 cm (class width of 4 cm). Simulation and harvest cycles were 10 year long, applied over a total span of 400 years.

Ingrowth frequency

$$Ing_t = \frac{\kappa}{\lambda} * \left(\frac{ba_{t-10}/c}{\lambda}\right)^{\kappa-1} * exp^{-\left(\frac{ba_{t-10}/c}{\lambda}\right)^{\kappa}} \quad [1]$$

Ingrowth [Ing_t, eq. 1] was modelled with the Weibull (2-parameter) probability density function individually per FT and species using the "nls" function (R package minpack.lm) to estimate the scale parameter Lambda λ, and the calibration parameter c to allow for larger values of ba at the x-axis (Table 3). The shape parameter Kappa κ was fixed at a value of 2. The number of ingrowth (Fig. 1) at plots from 0 to 20 m^2 was replaced with zero ingrowth (Roessiger et al. 2016). In the simulations, the ingrowth was related to the maximum ba during the 30-year period covering the span from the regeneration until reaching the 10 cm dbh class.

Site adaptation of ingrowth according to aggregated geobiotope site units

The number of ingrowth per FT estimated from inventory was adapted to GBT by multiplying it with a coefficient calculated from potential (natural) regeneration species composition per GBT presented by Kulla & Bošeľa (2014) divided by the weighted mean composition from the GBTs per FT.

Table 3. Statistics of the ingrowth model of the modelled species per forest type.

FT	MTS	Scale parameter λ			Calibrating parameter c			RSE	DGF
		Estim.	SD	sig	Estim.	SD	sig		
OA	BE	0.1230	0.0497	*	479.9	547.4		19.73	113
	OA	0.0263	0.0180		826.0	377.1	*	79.92	113
	HB	0.0846	0.0239	***	391.3	100.8	***	20.65	113
BE	BE	0.0302	0.0087	***	1096.0	244.5	***	62.35	174
FI	BE	0.0227	0.0083	**	1354.0	390.8	***	81.17	106
	FI	0.0497	0.0108	***	814.0	215.5	***	29.99	106
	SP	0.0627	0.0198	**	542.3	150.2	***	28.55	106

Significance level *** 0.001 ** 0.01 * 0.05 . 0.1

Estimate, standard deviation SD, significance, relative standard error RSE, degrees of freedom DGF, Modelled species MTS beech BE, oak OA, hornbeam HB, fir FI, and spruce SP per forest type FT.

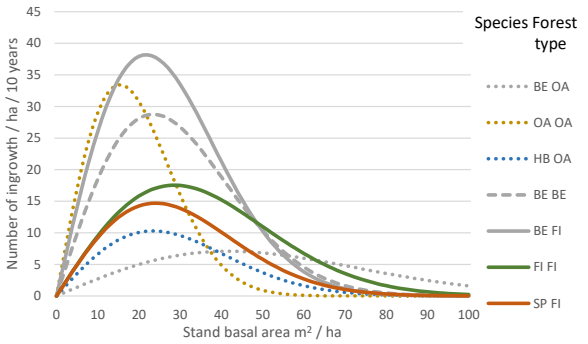

Fig. 1. Ingrowth frequency functions per tree species BE beech, FI fir, HB hornbeam, OA oak, SP spruce and per BE, FI, OA forest type FT.

As NFI data was available only from dbh class of ≥ 14 cm, while the modelling started at dbh of 10 cm we used an adaptation coefficient: The time duration for trees to transit from dbh-class 10 cm to dbh-class 14 cm was multiplied with mean mortality rate during this span. This adaptation resulted in increasing the number of specific ingrowth by 0.7% to 8% (Table 4). In GBT 14 there was no beech ingrowth.

Coefficients for crown class allocation adjusting the transition probability

Following the approach of Roessiger et al. (2018), the regression analyses of transition probabilities depended on crown characteristics. Two categories of crown types were implemented: 1 dense crown DC, and 0 sparse crown SC (Table 5).

Transition probability

Transition probabilities [T, eq. 2] specific for species (s) in a period (t) represent dbh increments per 10 years. They decrease with the ba, and they increase with the dbh and with the crown density (Table 6). They specify the allocation of trees into dbh classes in a decade using a multinomial logistic regression (R function multinom). The indicator k specifying the coefficient c is 1 in the case of trees "remaining in the same class", while in the case of "transition into the next class" it is 2, and for "transition into the next-but-one class" it is 3 (Table 6). k = 1 is the reference category, which means that the coefficients are 0.

$$T_{k,dbh,s,t} = \frac{exp(c_{0,k} + c_{1,k} * dbh_{t-10} + c_{2,k} * ba_{t-10})}{\sum_{k=1}^{3} exp(c_{0,k} + c_{1,k} * dbh_{t-10} + c_{2,k} * ba_{t-10})} \quad [2]$$

Increment adjusted to the site quality index of the geobiotope

Kulla & Bošeľa (2014) presented a range of site quality indices (absolute theoretical mean height; Halaj et al. 1987) SI per GBT. Based on the mean SI of GBTs (Table 8, left side), the mean increment per species I_{SI} in a specific GBT (Table 8, right side) was modelled dependent on the SI of the GBT [linear model, eq. 3] (Table 7).

$$I_{SI} = c_{Int} + c_{SI} * SI \quad [3]$$

In the first adjustment step, a normalised single tree increment I_{Norm} [eq. 4] was calculated from each single tree increment I_{Ind} by adjusting it from the SI of the GBT (where the tree was located) SI_{Ind} to the mean SI (of all

Table 4. Main ecological characteristics of ingrowth regression models per forest type (cases 10, 30, 50), and adjustment to geobiotopes (others).

FT	GBT	Maximum number of ingrowths per ha in 10 years at optimal stand basal area					Optimal stand basal area m² ha⁻¹ for maximum number of ingrowth				
		BE	OA	HB	FI	SP	BE	OA	HB	FI	SP
OA	10	7.1	33.4	10.3			42	15	24		
	14		40.1	11.4				15	24		
	22	1.3	47.7	12.6			42	15	24		
	23	2.5	43.7	12.6			42	15	24		
BE	30	28.8					23				
	33	33.5					23				
	34	31.0					23				
	35	33.5					23				
FI	50	38.2			17.5	14.7	22			29	24
	53	31.8			16.8	31.7	22			29	24
	54	36.9			24.9	15.9	22			29	24
	55	32.6			24.9	15.3	22			29	24

Modelled species MTS beech BE, oak OA, hornbeam HB, fir FI, and spruce SP per forest type FT and geobiotope GBT.

Table 5. Categorisation of trees into crown types based on visual crown density and crown proportion length to total stem height (Cpl). Proportions of crown types (in %) in the inventory data per forest type FT and species used to allocate the number of ingrowth into crown type categories.

Crown type	Category	Crown characteristic			Forest type FT, species						
		Sparse	Medium	Dense	OA-FT			BE-FT	FI-FT		
					BE	OA	HB	BE	BE	FI	SP
Dense DC	1	No	If cpl > 50%	Yes	61.1	59.9	69.5	60.7	51.5	59.0	68.6
Sparse SC	0	Yes	If cpl < 50%	No	38.9	40.1	30.5	39.3	48.5	41.0	31.4

Beech BE, oak OA, hornbeam HB, fir FI, and spruce SP, crown proportion length to total stem height Cpl.

Table 6. Multinomial logistic regression models for transition probability, only the models for average conditions of the forest types FT 10, 30, 50 are shown.

FT species			k	Intercept			dbh			Ba			Crown		
				Est	SD	Sig	Est	SD	Sig	Est	SD	Sig	Est	SD	Sig
OA	10	BE	2	0.334	0.643		0.066	0.015	***	–0.044	0.016	**	0.436	0.271	
			3	1.263	1.176		0.116	0.023	***	–0.171	0.035	***	0.289	0.518	
		OA	2	–0.457	0.237	.	0.075	0.007	***	–0.046	0.007	***	0.321	0.109	**
			3	–2.449	0.590	***	0.120	0.013	***	–0.108	0.018	***	0.220	0.260	
		HB	2	–0.222	0.602		0.100	0.023	***	–0.081	0.015	***	0.379	0.234	
			3	–6.960	1.069	***	0.166	0.071	*	–0.336	0.084	***	8.959	1.070	***
BE	30	BE	2	0.155	0.162		0.072	0.005	***	–0.049	0.005	***	0.175	0.080	*
			3	–0.472	0.225	*	0.108	0.006	***	–0.111	0.007	***	0.313	0.132	*
FI	50	BE	2	0.722	0.304	*	0.041	0.008	***	–0.040	0.007	***	0.629	0.155	***
			3	–0.692	0.428		0.080	0.010	***	–0.087	0.011	***	0.981	0.262	***
		FI	2	0.600	0.464		0.064	0.010	***	–0.051	0.011	***	1.022	0.216	***
			3	1.064	0.513	*	0.086	0.011	***	–0.102	0.013	***	1.386	0.246	***
		SP	2	–0.118	0.396		0.012	0.010		–0.018	0.010	.	1.510	0.216	***
			3	–1.151	0.626	.	0.029	0.013	*	–0.070	0.016	***	2.650	0.458	***

Significance level *** 0.001 ** 0.01 * 0.05 . 0.1,

Estimate Est, standard deviation SD, significance Sig, multinomial category of transition k, diameter at breast height dbh, Ba stand basal area, Crown (see Table 5), beech BE, oak OA, hornbeam HB, fir FI, spruce SP.

Table 7. Statistics of the linear regression models for increment dependent on the site quality index represented by the absolute theoretical mean height (Halaj et al. 1987) of a geobiotope GBT per forest type FT and modelled tree species MTS.

FT	MTS	c_{Int} Intercept			c_{SI} Site quality index			AIC	F	p	RSE	DGF	Mul.	Adj.
		Estim.	SD	Sig	Estim.	SD	Sig						R^2	R^2
OA	BE	–6.151	4.760		0.329	0.172	.	1,234	3.6	0.057	1.98	291	0.012	0.009
	OA	–0.012	0.453		0.110	0.019	***	6,310	34.5	0.000	1.57	1,683	0.020	0.020
	HB	–0.637	1.729		0.083	0.064		1,626	1.7	0.197	1.43	454	0.004	0.001
BE	BE	0.122	0.767		0.108	0.028	***	15,328	9.3	0.002	2.31	3,395	0.004	0.004
FI	BE	1.484	1.485		0.071	0.057		4,255	1.5	0.219	2.63	889	0.002	0.001
	FI	–6.346	3.993		0.394	0.139	**	3,787	14.5	0.000	3.13	737	0.011	0.010
	SP	–0.454	2.392		0.122	0.074	.	2,500	2.7	0.099	2.60	524	0.005	0.003

Significance level *** 0.001 ** 0.01 * 0.05 . 0.1,

Estimate Estim., standard deviation SD, significance sig, Akaike Information Criterion AIC, F- and, p-value, relative standard error RSE, degrees of freedom DGF, multiple and adjusted coefficient of determination R^2, beech BE, oak OA, hornbeam HB, fir FI, and spruce SP.

trees in the FT) SI_{Mean} using the coefficient for SI (Table 7). Then I_{Norm} was transferred to the adapted increment I_{Ada} [eq. 5] by a relative shift of the increment specific for the GBT I_{SI} in relation to the mean increment of all measured trees of the FT I_{Mean}. The reason to implement I_{Norm} was to limit an over-proportional increase of I_{Ada} for high I_{Ind} compared to low I_{Ind}. With the data adjusted on I_{Ada}, an individual multinomial logistic transition probability was estimated for each species and GBT, but the results are not presented due to the high amount of model coefficients.

$$I_{Norm} = I_{Ind} + c_{SI} * (SI_{Mean} - SI_{Ind}) \quad [4]$$

$$I_{Ada} = I_{Norm} * \frac{I_{SI}}{I_{Mean}} \quad [5]$$

Mortality probability (salvage harvest proportion)

Salvage harvest records at a forest stand level were evaluated from the database on forest management plans and

Table 8. Main indicators of site quality index and resulting mean increment [eq. 3, Table 7] per forest type (cases 10, 30, 50), and adjustment to geobiotopes.

FT	GBT	SI site quality index [m]					I_{SI} Increment regression				
		BE	OA	HB	FI	SP	BE	OA	HB	FI	SP
OA	10	27.6	24.1	27.0			2.94	2.64	1.59		
	14	28.0	25.0	28.0			3.06	2.74	1.68		
	22	26.0	22.0	26.0			2.41	2.41	1.51		
	23	27.0	25.5	27.0			2.73	2.79	1.59		
BE	30	27.1					3.04				
	33	26.5					2.97				
	34	28.0					3.13				
	35	24.0					2.70				
FI	50	25.8			28.8	32.4	3.31			5.01	3.49
	53	24.0			27.5	31.0	3.18			4.48	3.33
	54	26.0			29.5	33.5	3.32			5.27	3.63
	55	23.0			29.0	30.0	3.11			5.07	3.21

SI site quality index, (Absolute theoretical mean height, Halaj et al. 1987), Modelled species MTS beech BE, oak OA, hornbeam HB, fir FI, spruce SP, forest type FT, geobiotope GBT.

forest management records during years 2012 to 2021 (Information System Forest Management 2022). Specific cases were excluded from original data: Unplausible data; mean stand dbh larger than 44 cm (data for dbh above 44 cm showed an unplausible decrease in mortality); data from single layered stands (as uneven-sized forests were modelled); protected sites (as only normally managed and special purpose forests should be considered).

As periods of salvage harvest varied between 1 and 10 years, mortality probability M was converted from a period of x years M_x to 10 years M_{10} (eq. 6) using a factor that limits the maximum to 1:

$$M_{10} = 1 - (1 - M_x)^{\frac{10}{x}} \quad [6]$$

A binary logistic regression M (eq. 7) dependent on the mean stand dbh in the period t 10 years ago was derived with glm function (R Core Team 2019) separately for each FT, GBT, and species (Table 9).

$$M_{dbh,\,s,\,t} = 1 / \left(1 + exp\left(-(c_0 + c_1 * dbh_{t-10})\right)\right) \quad [7]$$

Stand area in ha multiplied with species volume proportion was used as a weighting factor. For the matrix simulations, insignificant mortality models were replaced by those of comparable sites: GBT 14 was replaced by FT 10 (all species) and GBT 55 was replaced by FT 50 (only fir).

Volume, quality, price, and cost data

Timber volume calculation (Petráš & Pajtík 1991) per species and per FT was based on the tree height and dbh. Further, volume per FT was adjusted to GBT by the relevant SI (Table 8). The timber size and quality structure of harvested trees were derived from single tree assortment tables (Petráš & Nociar 1991).

The mean timber revenues in Slovakia were based on the timber quality and the size classes according to the price lists of the state forest enterprise from 01/2017–12/2021 with a specific price scenario 07–12/2021 for conifers (Fig. 2) (Forests of the Slovak Republic 2022). The oak price was modified due to the lower cerris oak price according to the mean cerris proportion at all oaks. Cerris oak prices per size class were calculated as mean prices of oak quality IIIB and oak quality V.

For each species separate prices for two groups of stem quality were determined:

- Good quality GT: A quality and B quality without damage.
- Bad quality BT: B quality with damage and C quality.

In the case of mortality, a price reduction coefficient was estimated as the mean of IIIA and IIIB quality divided by IIIC quality, also during the period 2017–2021. The reduction of the price due to the damage by mortality was considered at a level of 76.9% for beech, 36.1% for oak, 87.9% for hornbeam, and 68.8% for both spruce and fir of the undamaged timber price.

Table 9. Statistics of the binary logistic regression for mortality probability dependent on diameter dbh, individual for forest type FT and geobiotope GBT for modelled tree species MTS.

MTS	FT / GBT	Intercept c_0			Mean stand diameter c_1			AIC	NullDev	ResDev	DGF
		Estim.	SD	Sign	Estim.	SD	Sign				
BE	10	–5.358	0.413	***	0.0530	0.0114	***	1,291	1,092	1,067	5,947
	14	–2.571	2.698		–0.0762	0.0992		5	5	4	142
	22	–5.452	0.980	***	0.0546	0.0268	*	258	211	206	1,117
	23	–5.163	0.501	***	0.0490	0.0138	***	987	829	814	3,839
	30	–5.867	0.085	***	0.0717	0.0023	***	36,314	30,040	28,852	53,259
	33	–5.492	0.281	***	0.0648	0.0079	***	3,078	2,544	2,465	5,304
	34	–5.896	0.093	***	0.0721	0.0025	***	31,315	25,892	24,868	43,556
	35	–6.314	0.349	***	0.0856	0.0099	***	1,919	1598	1,507	4,394
	50	–5.429	0.148	***	0.0526	0.0042	***	8,724	7,984	7,804	30,222
	53	–6.057	0.615	***	0.0665	0.0176	***	620	637	620	3,449
	54	–5.335	0.165	***	0.0494	0.0047	***	7,003	6,379	6,250	23,480
	55	–6.118	0.433	***	0.0807	0.0130	***	1,091	963	917	3,285
OA	10	–5.257	0.237	***	0.0390	0.0068	***	5,690	4,517	4,482	13,981
	14	–4.602	1.276	***	0.0098	0.0350		158	108	108	633
	22	–5.082	0.585	***	0.0345	0.0173	*	878	695	691	2,491
	23	–5.427	0.283	***	0.0437	0.0080	***	4,349	3,485	3,454	9,865
HB	10	–6.672	0.458	***	0.0804	0.0191	***	576	706	689	11,880
	14	–7.284	3.285	*	0.0519	0.1328		7	10	10	577
	22	–6.872	1.099	***	0.0992	0.0475	*	115	121	117	2,003
	23	–6.696	0.523	***	0.0821	0.0218	***	446	559	545	8,937
FI	50	–4.494	0.186	***	0.0384	0.0049	***	8,944	7,457	7,388	23,453
	53	–4.660	0.405	***	0.0499	0.0109	***	1,870	1,604	1,580	4,500
	54	–4.523	0.214	***	0.0383	0.0056	***	6,799	5,616	5,563	17,036
	55	–4.161	1.205	***	0.0111	0.0324		233	202	202	1,910
SP	50	–3.408	0.039	***	0.0540	0.0011	***	72,830	57,695	55,115	40,804
	53	–3.159	0.091	***	0.0412	0.0027	***	15,503	12,402	12,154	8,183
	54	–3.370	0.045	***	0.0545	0.0013	***	50,231	39,842	37,824	28,658
	55	–3.852	0.150	***	0.0671	0.0043	***	6,918	5,253	4,963	3,955

Significance level *** 0.001 ** 0.01 * 0.05 . 0.1, Estimate, standard deviation SD, significance, Akaike Information Criterion AIC, null and residual deviance, degrees of freedom DGF, beech BE, oak OA, hornbeam HB, fir FI, and spruce SP.



Fig. 2. Timber market prices for modelled tree species MTS beech BE (also for hornbeam), oak OA, fir FI, and spruce SP.

Harvest costs consider the work costs of 8.53 € per working hour (Ministry of Agriculture and Rural Development of the Slovak Republic 2019), 2.62 € per hour for compensations for actual conditions following the time consumption standards (Ministry of Forest and Water Management of the Slovak Republic 1992a, b), and depreciation costs of 2.15 € per m^3 for tractor skidding. The net cash flow NCF (net timber price) is timber revenues minus harvest costs.

2.3. Financial optimisation and evaluation

The objective function to be maximised is the adapted net present value [NPV, eq. 8] based on the capital holding value (Deegen et al. 2000), which considers an existing stand at the beginning and the end. The interest rate i was set at 0.5%, 1%, and 2% for each GBT. T is the total simulation time of 40 periods (400 years), and t is a single period of 10 years. The discounted NCF was considered for the remaining stand r (before the first period and at the end of T), harvest h (at the beginning of each period) and mortality m (at the end of each period), all per ha. Each NCF summarises the species crown, stem, and dbh classes. The financial evaluation during the equilibrium (year 250) was carried out with annuity [Ann, eq. 9], (Möhring et al. 2006).

$$NPV_T = -NCF_{r_0} + \sum_{t=1}^{T=40} \left(\frac{NCF_{h_t}}{(1+i)^{(t*10-10)}} + \frac{NCF_{m_t}}{(1+i)^{(t*10)}} \right) + \frac{NCF_{r_T}}{(1+i)^{(T*10)}} \quad [8]$$

$$Ann_{t=25} = \left(NCF_{h_{t=25}} + NCF_{m_{t=25}}\right) * \frac{(1+i)^{10}}{(1+i)^{10}-1} \quad [9]$$

The dynamic optimisation was carried out with the Subplex algorithm (Rowan 1990), which was implemented in the R environment (R Core Team 2019) by King (2015). The algorithm decided about the proportion of trees harvested per each species, crown, stem, and dbh class and in each optimisation period. To shorten the time effort of the optimisation, one optimised harvest proportion was used twice in two consecutive periods, resulting in a 20-year optimisation period (representing the management planning period while harvest periods remained 10-year long).

The Subplex algorithm required a technical constraint for harvesting only a proportion of existing trees in each dbh class between 0% and 100%. An "If" function entered a high negative value in the case of more trees were harvested than were present and in the case of a negative number of trees.

A sufficient proportion of hornbeam is relevant to ensure high quality of oak stems by limiting the growth of branches while hornbeam itself has only a low timber value. No statistical effect of hornbeam proportion on stem quality was found. Therefore, a constraint for a minimum of 15% volume proportion of hornbeam in the oak FT was used. To avoid strong fluctuations in the harvested volume in consecutive periods, a constraint allowed a maximum 20% difference in the harvested volume compared to the previous harvest.

3. Results

The optimisation results approached an equilibrium after 40 to 120 years. After 300 to 360 years, ba and volume increased as the optimisation did not consider the relevance for ingrowth any longer. The equilibrium phase resulted in stable continuous dbh distributions, specific for GBTs and interest rates. The year 250 was selected to represent the equilibrium state in Fig. 3 and 4.

Ba and standing volume decreased with the higher interest rate (Fig. 3A and B). However, the harvested volume was the highest for interest rate 1%, while for i = 0.5% it was slightly (2–5%) lower, and for i = 2% it was substantially lower by 6–17% (Fig. 3C), consistently for all GBTs. The lower harvested volume for the low i of 0.5% was caused by the higher stand density, which resulted from larger target diameters, supressing ingrowth and increment. The lower harvested volume for high i of 2% was caused by lower target diameters reducing the production time, and supporting the increment. For each FT, the highest volume was in the GBT defined as the basic one: 23 34 54, independent from the interest rate. The second best in OA FT was 14 with wet conditions, in BE FT 33 the acidic, and in FI FT 55 the nutrient rich. The order for all FT and GBT followed the order of the SI of the main tree species (Table 8) determining the increment and volume while the ingrowth frequency and the mortality probability had a lower impact. For the standing ba there were exceptions as 22 was the best in OA FT for i of 0.5% and 1%.

The lowest mortality timber volume occurred in OA FT (Fig. 3D), and the highest one in BE FT due to the relatively high beech target diameters. Spruce had the highest relative mortality timber volume but its proportion in FI FT was relatively low. The high spruce mortality forced lower target diameters. Mortality volume decreased with the interest rate due to lower target diameters. High oak timber quality led to the highest financial

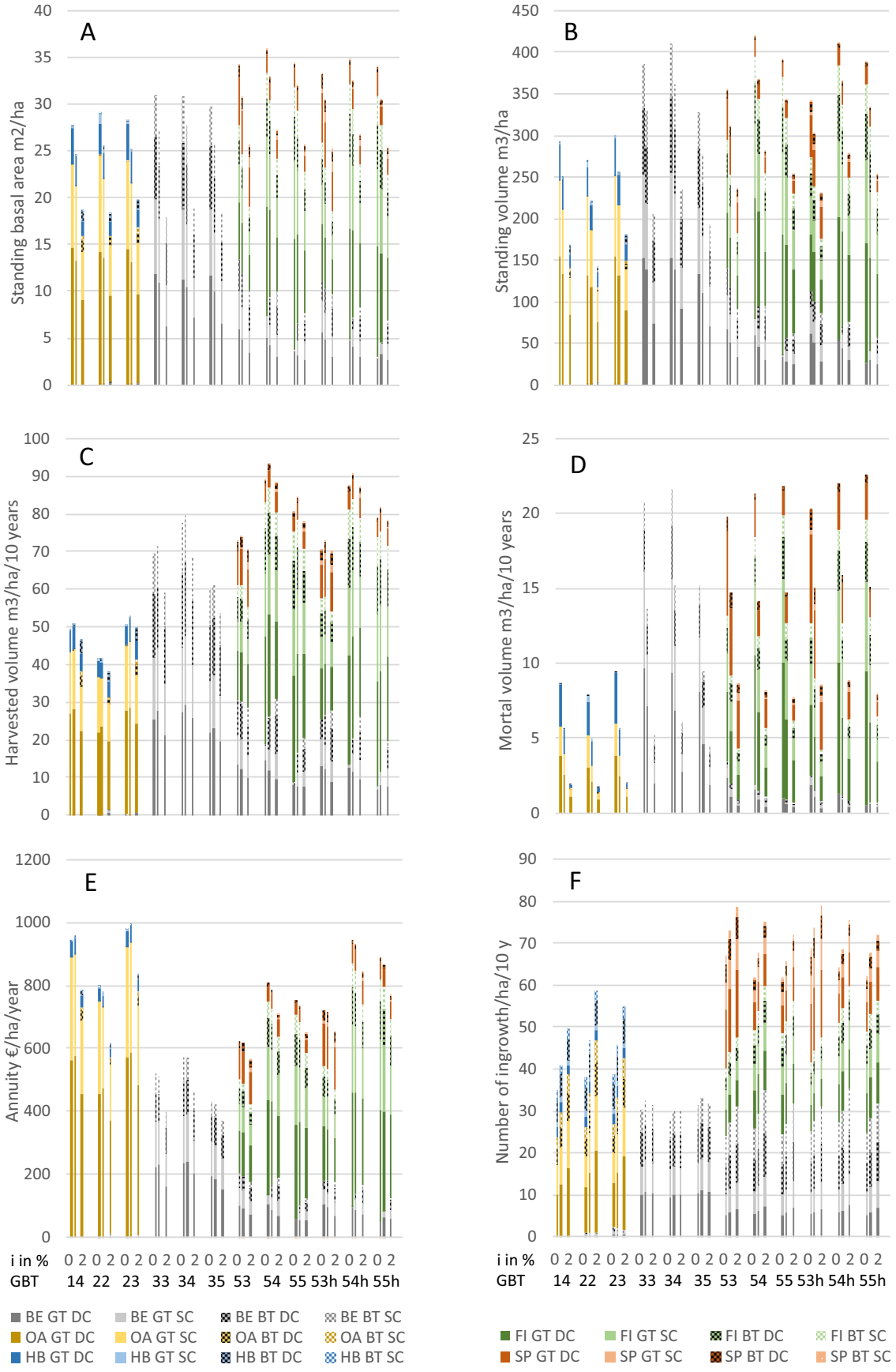

Fig. 3. Stand indicators during equilibrium for species BE beech, FI fir, HB hornbeam, OA oak, SP spruce, and subclasses GT good timber quality, BT bad timber quality, DC dense crown, SC sparse crown, per geobiotope GBT (h high price scenario), and interest rate i.

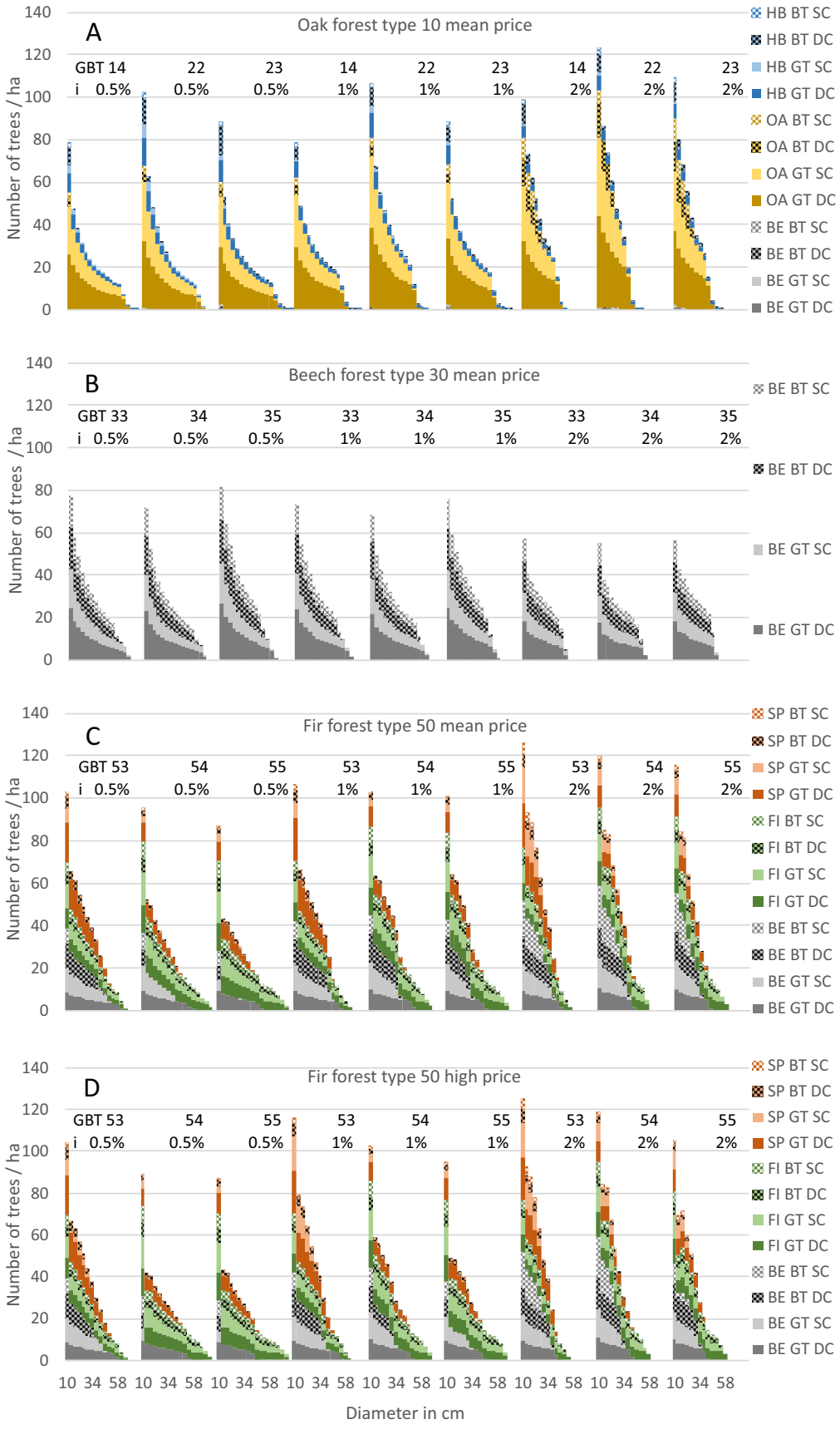

Fig. 4. Diameter distribution during equilibrium for species BE beech, FI fir, HB hornbeam, OA oak, SP spruce, and subclasses GT good timber quality, BT bad timber quality, DC dense crown, SC sparse crown, per geobiotope GBT, and interest rate i.

results (Fig. 3E), followed by the FI FT which benefited from higher prices for conifers compared to those of beech (Fig. 2).

Regarding dbh distributions (Fig. 4), a higher i resulted in a higher number of trees in smaller dbh classes as the lower ba supported the ingrowth (Fig. 3F). The lower ba also contributed to a faster growth and to lower target diameters for higher i, all leading to a more steeply decreasing dbh distribution. An exception was the BE FT (Fig. 4B), characterised by a lower number of trees in lower dbh classes compared to lower i because for i of 2% the standing ba already dropped below the ba optimal for the BE ingrowth (Table 4). It needs to be mentioned that the OA FT had a much lower ba than the BE FT but the optimal ba for the ingrowth of the light-demanding OA was also lower than for BE (Table 4).

BE ingrowth in BE FT was relatively low and only at a level comparable with beech ingrowth in FI FT. The number of beech trees declined strongly before they reached the diameter limit of ingrowth. The higher ingrowth potential in FI FT resulted in the highest standing timber volumes.

A mixed species structure was established in the FI FT (Fig. 4C). FI proportion was the highest in GBT 55 and high in GBT 54, while in GBT 53 FI and BE proportions were comparable. SP proportion was always the lowest. FI proportion decreased, and SP and partly BE proportion increased with the increasing i, representing that fir is the most shade-tolerant species under the higher ba under low i. On the contrary, in OA FT (Fig. 4A) nearly all beech trees with dbh of 10 cm were harvested, as they were outcompeted by the high price of OA. Only for i of 2%, a little BE proportion remained. A 15% proportion of HB was maintained only due to the constraint used within the optimisation. Dense crowns and high quality of stems resulted in higher target diameters compared to sparse crowns and low timber quality (Fig. 4), contributing to a diversified dbh structure additionally to the species mixture.

High prices of conifers in the second half of 2021 compared to the five-year mean in FI FT decreased the BE proportion by 20% (lower target diameter), and increased FI (by 3.6%) and SP (by 9.3%) proportions (greater target diameter). In the sum, high prices decreased the standing volume by 1.7% (Fig. 4B) to allow a higher increment and larger target diameters of conifers although the harvested volume decreased by 1.8% (Fig. 4C). The higher mortality of thicker conifers increased the mortality volume by 4% (Fig. 4D). The higher prices due to the larger target diameters of conifers financially overcompensated the effect of lower harvest. The annuity increased by 17% on average (Fig. 4E).

A high proportion of planned harvested broadleaved timber volume was pulpwood (BE FT 41%; OA 26%) followed by IIIB (BE FT 36%; OA 26%) (Fig. 5A; 5B), while from OA also a high veneer I proportion of 18% was obtained (Fig. 5A). The main assortment from conifers was saw timber SP 49% IIIA, FI 52% IIIB (Fig. 5D; 5E). Higher prices of conifers in the second half of 2021 compared to the five-year mean in FI FT outcompeted BE of bad timber quality improving the BE quality proportion (I 9.1%; II 14%; IIIA 2.8%) (Fig. 5C). The resulting larger target diameters of conifers increased also the proportion of I quality (SP 3.4%; FI 0.5%), but decreased II (SP –1.2%; FI –2.7%) and IIIA (SP –0.5%; FI –2.5%) quality proportions (Fig. 5D; 5E).

4. Discussion

4.1. Limitations of data and ecological modelling

The optimisation study demonstrated the adjustment of forest management planning by matrix modelling to individual site conditions. The interaction of growth and regeneration models of Kulla & Bošeľa (2014) as well as of mortality models (Information System Forest Management 2022) representing individual GBTs makes the matrix modelling more complex compared to yield tables that react to SI only with the increment. The relation of SI to GBT (Kulla & Bošeľa 2014) allows for modelling of transition / increment to further combinations of trees species with varying SI, also independent from GBT. Nevertheless, the transfer to GBT is limited by precise hypothetic assumptions (e.g. species compositions), circumstances, geobiocenological and climate conditions per GBT. Further, the model quality suffered from limited and missing data, requiring carefully interpretation of results: The individual SI of an inventory plot might differ from the mean SI per GBT (Kulla & Bošeľa 2014). This is problematic as plots of one GBT (site 3.4) dominated the BE dataset (represented 68%). Many plots from the BE FT were used for the calibration of the OA (29%) and FI FT (20%), overestimating the BE proportion. The increment increase with SI of BE at OA site and of FI was four and three times higher compared to the other increment models (Table 7). Only three increment calibration models were significant. Ingrowth data were available only for dbh of 14 cm and needed to be adapted to 10 cm. Ingrowth and transition models were partly insignificant. Nonetheless, the models were used. Ingrowth models were accepted as the general relations were realistic, mainly the order of species according to their demands on light conditions (Ficko et al. 2016, 2018; Roessiger et al. 2016, 2018, 2019, 2022). The insignificance was mainly caused by the low amount of data, mainly for fir.

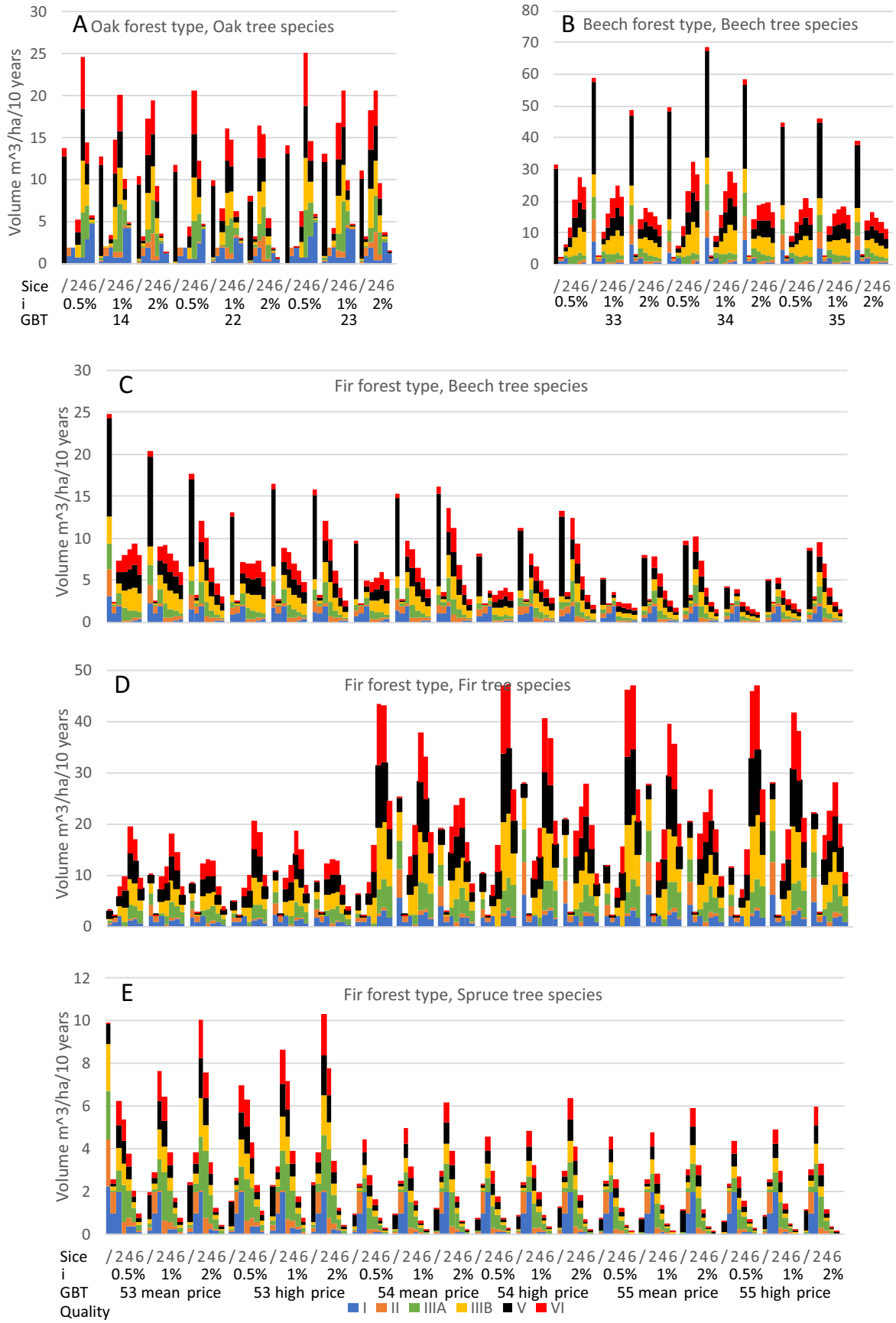

Fig. 5. Planned harvested timber volume assortment structure: Size: 1 < 20 cm; 6 ≥ 60 cm Quality: I cross-cut veneer; II rotary cut veneer; IIIA saw timber of higher quality; IIIB of lesser quality; V pulpwood; VI fuelwood; V and VI without size class "/" (Petráš et al. 2017).

4.2. Forest structure differs between Forest Types

The optimal ba for the ingrowth of oak was low compared to other tree species because oak is a light-demanding species. The high optimal ba of beech in the oak FT also indicated a lower effect of oak ba to cause shadow / competition compared to ba of other species. The light-demanding oak did not grow under the closed forest cover as species in the BE and FI FT. The mosaic structure composed of dense and sparse stand parts is typical for OA forests (Saniga & Bruchánik 2009), contrary to single or tree wise mixture of various size classes possible for uneven-aged forests of shade-tolerant species. The matrix model is designed to represent the second case (Buongiorno & Michie 1980).

That makes the comparison of the results for the FT difficult: While the BE and FI FT were represented by one optimal final state, the OA FT was represented by the mean state of dense and sparse mosaic structures (Saniga & Bruchánik 2009), leading to a relatively low ba and volume in the equilibrium. Distance-dependent single tree models (Peng 2000) with a realistic modelling of competition and regeneration dynamics in uneven-aged forests have the potential to describe the forest structure. But the dynamic optimisation of harvesting at single tree level gets more complex and is so far only possible with algorithms to determine harvests (Pukkala & Miina 1998; Härtl et al. 2010) or by a selection from previously simulated variants (Merganič et al. 2020, 2022). Hyytiäinen & Haight (2012) concluded that spatial growth models considering spatial irregularities and their consequences on regeneration dynamics and economics are still inadequately addressed. Vacek et al. (2019) found a negative influence of tree layer on spruce regeneration at 5 m distance from stem and on beech regeneration at 2 m distance. Therewith, dynamic and spatial economic optimisation of harvesting at single tree level and their effect on regeneration is still missing and needs to be developed.

4.3. Variations between regional management and ideal goals

Spruce FT was not evaluated as spruce forests in Slovakia are often managed using a system that is far away from a near-natural species composition and an uneven-aged diameter distribution. Roessiger et al. (2022) optimised the conversion of rapidly declining pure spruce forests affected by storm and bark beetle in the Beskydy region (Table 10). They simultaneously optimised the number of trees to be planted, and tested the sensitivity to initial states, afforestation costs, and sites. Species-specific models for GBT 53 in Beskydy were adjusted on growth conditions in GBTs 33, 34, and 54. The equilibria in all Beskydy GBTs are still 55% to 68% spruce dominated although GBTs 33 and 34 are originally beech GBTs. Due to BE, FI and SP in GBT 33 and 34, higher volumes were obtained compared to considering only BE in BE FT.

Further, the typical single-tree selection forest of Smolnícka osada (Stratum A, GBT 34, uneven-aged BE-FI forest with SP) achieved 50% higher volume than the GBT 34 (Fig. 3B). The neighbouring forest (Stratum B, GBT 33, uneven-aged SP-Pine (*Pinus sylvestris* L.) forest with FI and BE) has developed on former pastures, has a lower volume, and currently pine is outcompeted by shade-tolerant species (Kulla et al. 2018; Roessiger et al. 2022) (Table 10). The examples based on reality (representative plots of the region) differ from the GBT study (ideal selected plots) and reflect the necessity to support uneven-aged management by funding (Roessiger et al. 2022).

4.4. Impacts of variability in financial conditions on economics and equilibrium

Interest rates were used as a financial indicator of the character of the manager and his or her management preferences regarding time scheduling and risk (Fisher 1930). As these preferences are unknown and individual, relevant differences in optimisation results for three interest rates entail a risk to misinterpret the financial results. Further, Knoke et al. (2017) discuss the interest rate as decreasing with time.

Besides the choice of the interest rate, an inaccuracy of the financial results can be caused by varying harvesting and skidding costs, currency inflation, market interest rates (for financing and banking) that cannot be influenced by management. The scenarios about the actually most relevant fluctuations with high coniferous timber prices reflected the possible extent of inaccuracy by an increase of annuity by 15% to 19% (Fig. 3E).

5. Conclusions

SI, FT, and interest rates can cause substantial differences in diameter and species composition and volume level during a steady state equilibrium. Moreover, a comparison of optimisation studies with a similar modelling approach demonstrates that further regional differences, actual and historical management, species and diameter composition, and their dynamics can cause even stronger differences. Inventory data characterises these differences and therewith ecological modelling of ingrowth, transition, and mortality of the present tree species, each within the scale from light-demanding to shade-tolerant, and affecting species interactions in forest dynamics. Datasets representing even-aged forests do not contain these aspects and should not be used carelessly within

Table 10. Standing timber volume during equilibrium per geobiotope and species. Comparison of the results: National Forest Inventory of Slovakia extended by data from Smolnícka osada (Fig. 3B); with the optimisation studies of Roessiger et al. (2022) for the Beskydy region (interest rate 2%; 50% funding proportion of planting and silvicultural costs during 30 years); Kulla et al. (2018), Roessiger et al. (2018) for Smolnícka osada (interest rate 2%).

GBT	NFI + Smolnícka osada (Fig. 3B, for comparison)				Beskydy Roessiger et al. (2022)				Smolnícka osada Kulla et al. (2018), Roessiger et al. (2018)			
	BE	FI	SP	Sum	BE	FI	SP	Sum	BE	FI	SP	Sum
33	206			206	57	68	151	276	96	127	36	260
34	236			236	64	27	195	286	277	78	0	355
53	91	91	55	237	40	44	180	264				
54	95	159	27	281	43	48	183	275				

Beech BE, fir FI, spruce SP, geobiotope GBT, National Forest Inventory NFI.

a density-dependent matrix transition model designed for uneven-aged forests.

The actual species composition and the historical forest management often differ from the natural potential of a geobiotope derived from theoretical findings and data from reserves (Kulla & Bošeľa 2014). In such cases, the calibration of the natural potential in ecological models (especially ingrowth) is unrealistic. If close-to-nature management is still a goal, then transition paths (Roessiger et al. 2022) from real initial stand states might be more realistic and practical than the goal of an equilibrium.

Inaccuracy and fluctuations in financial and economic conditions (e.g. in timber prices) and limitations in ecological models and data lead to the discussion (Dvorak 2000; Roessiger et al. 2018) if an ideal equilibrium really exists (Schütz 2006). Mitscherlich (1952) pointed out that an equilibrium is a hypothetical concept and several optimal states for a forest exist. Schütz et al. (2012) recommended a frame of multiple optimal states to allow a wider potential for adaptation. Such a conclusion is in accordance with the continuous 200-year fluctuations of ba between 30 and 90 $m^2\ ha^{-1}$ found by Roessiger et al. (2018) resulting from the optimisation with interest rates lower than 1% rather than a fixed equilibrium. Fluctuations in ba are similar to varying stand development phases present in old-growth forests (Kulla et al. 2023) and caused by the requirement to establish low ba phases allowing ingrowth. A realistic ingrowth model needs to be adjusted to consider the requirements for space and light and the relevance of seed trees (Roessiger et al. 2016, 2018, 2022; Kulla et al. 2023). Forest management planning for uneven-aged, mixed, near-natural, continuous cover management remains challenging as it is rare and requires further research. Planning should be adapted to a forest, a site, or a region, and regularly updated with actual repeated inventory data, mortality / salvage records, and price and cost data to represent the actual dynamics.

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